Persistence of collective fluctuations in *N*-body metaequilibrium gravitating and plasma systems

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Starting from a metaequilibrium state (in the Vlasov limit), the time scale of the fluctuations exhibited by systems of one-dimensional charged particles is computed. This study is given both for plasma and gravitational systems. The use of the multiple water-bag model allows an analytical treatment for both collective and individual modes. These results are compared with those obtained by numerical simulations of *N*-body systems. Finally, it is numerically shown that collective effects are responsible for the long time scale of phase-space holes structures. [S1063-651X(98)10612-8]

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I. INTRODUCTION

The evolution of plasma or gravitational systems is usually divided into two parts. In a first step, collective effects drive the system toward a metaequilibrium state (that is, an equilibrium in the Vlasov limit); and in a second step, the collisional effects slowly thermalize the distribution function. This scenario has been used by many authors to explain the relaxation of objects like galaxies for which the evolution is driven by the sole collective effects—the universe being too young to be affected by collisional effects. After the model of violent relaxation proposed by Lynden-Bell [1], many researchers have studied this first step both from a theoretical and numerical point of view; see, for example, Gurzadyan [2], Miller [3], Reidl [4], Yamashiro [5], and more recently Tsuchiya [6].

The aim of this paper is to look at the evolution of the system after this first time for both plasma and gravitating systems. Then the distribution function is a metaequilibrium. It can be noticed that the first step of evolution involving only collective effects does not allow the system to reach exactly a metaequilibrium state. Eulerian simulations of gravitating systems [7] have shown the formation of holes in phase space, which remains unchanged for the entire simulation time and prevents the system from reaching completely such a metaequilibrium. Moreover, in the plasma case, these holes are responsible for the stopping of the Landau damping. It will be seen hereafter that these structures are strong enough to resist individual effects.

All distributions that depend only on the energy are solutions of the Vlasov equation. To compute the time scale and the frequency spectrum of the fluctuations, we need to solve the Vlasov equation linearized around this metaequilibrium. Here, a fundamental difference appears between plasma and gravitating systems. While this problem is easily solved by a double Fourier-Laplace transform in x and t (the Landau treatment) in the homogeneous plasma case, the gravitational problem implies a treatment of the inhomogeneous equilibria, which, strangely, has not been addressed very much. Then, in order to perform an analytical treatment (at least in a first step), the water-bag model is adopted hereafter: in such a model [8] the distribution function is constant by steps between two contours defined by a function $a_{\pm}(x,t)$, where the subscript $\pm$ refers, respectively, to upper $(+)$ and lower $(-)$ values of the contour.

Consequently, plasma and gravitational systems exhibit very different behavior and indeed analytical treatment. The comparison of the results is interesting but not obvious, since in the plasma homogeneous infinite case, we deal with a continuous spectrum of wave numbers (each with a resonance frequency), while, in the gravitational case, we have a discrete spectrum of eigenfrequencies.

The paper is organized as follows. After this introduction, Sec. II is devoted to the analytical treatment allowed by the multiple water-bag (MWB) model both for the plasma and the gravitational case. Section III gives numerical results for the plasma and Sec. IV for the gravitational case. In Sec. V the stability of structures initially dug out in the phase-space distribution of the gravitational case (holes) is numerically studied. Section VI gives our conclusions.

II. MULTIPLE WATER-BAG MODEL—LINEARIZED EQUATION AROUND EQUILIBRIUM

In order to obtain analytical results, we will limit this work to a one-dimensional system and a very simple distribution function. The simplest distribution function $f(x,v,t)$ one can think of is the one that has a single value A in a delimited area of the space phase. This model, called water bag, has been introduced in [8] by De Pack. He, and after him many authors, noticed that, with this simple model, analytical treatment can be performed [9]. An extension of this model is the multiple water bag, which presents several areas of constant value, each delimited by a ''bag.'' The MWB model can be obtained by the discretization of a continuous distribution function. Nevertheless it introduces discontinuities and the connection of the physical properties of the continuous distribution function and the discretized one deserves a careful treatment [10].

Here, to describe the metaequilibrium, we will take a

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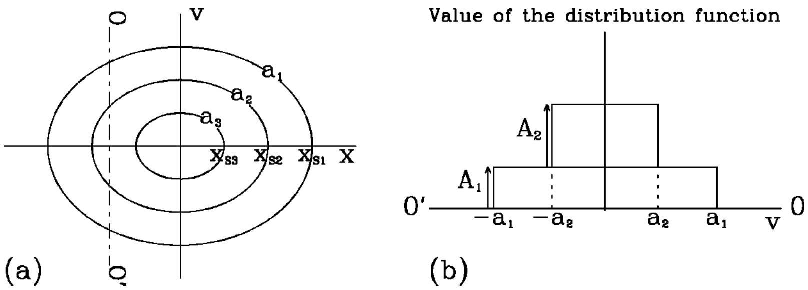

FIG. 1. (a) MWB model, (b) cutoff of (a) along the O′ O axis.

MWB, function of the energy alone (see Fig. 1). Then each bag is delimited by two contours, symmetric with respect to the x (space) axis, with, on the two contours of bag i, a given energy ε_i. The velocity of a particle moving on the border of bag number i is $\pm a_i(x)$ with

$$\varepsilon_i = \tfrac{1}{2} m a_i^2(x) + m\phi(x), \tag{1}$$

where $\phi(x)$ is the potential created by all the bags at point x.

Equation (1) depends on the square of the velocity a_i; this is the reason for the symmetry with respect to x. Let us call x_{si} the value for which the bag closes (because of the absence of neutralizing species, the bags always close in the one-dimensional gravitational case). For this value, we have $a_i(x_{si})=0$, that is,

$$\varepsilon_i = m\phi(x_{si}). \tag{2}$$

Now, let us perturb the MWB equilibrium. If we keep in mind the picture of the particle following the borders of the ith bag, its two velocities $V_{i_\pm}(x,t)$ obey the equations

$$\frac{\partial V_{i_\pm}}{\partial t} + V_{i_\pm}\frac{\partial V_{i_\pm}}{\partial x} = E, \tag{3}$$

where $V_{i_+} = a_i + v_{i_+}$ for the upper border in the phase-space plane, and $V_{i_-} = -a_i + v_{i_-}$ for the lower border.

The field E reads

$$E = E_0 + E_1, \tag{4}$$

where E_0 is the field created by the unperturbed MWB equilibrium and E_1 is the correction at first order for the perturbed one. In the plasma case $E_0=0$ and in the gravitational case it is given by the Poisson law

$$\frac{dE_0(x)}{dx} = -4\pi G \sum_i A_i 2a_i(x), \tag{5}$$

where the summation involves all the bags not yet closed at point x [see Fig. 1(b)].

The linearization of Eq. (3) gives for the two perturbed velocities of bag i

$$\frac{\partial v_{i+}}{\partial t} + \frac{\partial}{\partial x}(a_i v_{i+}) = E_1, \tag{6}$$

$$\frac{\partial v_{i-}}{\partial t} - \frac{\partial}{\partial x}(a_i v_{i-}) = E_1, \tag{7}$$

where $E_1(x,t)$ is the sum of the partial fields $E_{1i}(x,t)$ created by the particles of bag i:

$$E_1 = \sum_i E_{1i}, \tag{8}$$

and

$$\frac{\partial E_{1i}}{\partial x} = -4\pi G A_i (v_{i+} - v_{i-}). \tag{9}$$

In addition we will constrain the perturbed field E_{1i} to be equal to zero at $x=\pm x_{si}$. It means that the points $\pm x_{si}$ are held fixed. Obviously $E_{1i}=0$ for $|x|>x_{si}$.

Calculating the difference between Eqs. (6) and (7), and with the help of Eq. (9), we find, after integrating on x,

$$\frac{\partial E_{1i}}{\partial t} = -4\pi G A_i a_i (v_{i+} + v_{i-}). \tag{10}$$

Substituting $v_{i+} \pm v_{i-}$ by Eqs. (9) and (10) into the result of Eq. (6) plus (7), we obtain, after a Fourier transform on t, for each bag i,

$$a_i \frac{\partial}{\partial x} a_i \frac{\partial}{\partial x} E_{1i}(\omega,x) + \omega^2 E_{1i}(\omega,x) = -\omega_{J_i}^2 E_1(\omega,x), \tag{11}$$

where $\omega_{J_i}^2$ is the Jeans frequency associated with the bag i. $\omega_{J_i}^2$ reads

$$\omega_{J_i}^2(x) = 8\pi G A_i a_i(x) = 4\pi G n_i(x). \tag{12}$$

Equation (11) provides for each bag an equation connecting E_{1i} to E_1, while Eq. (8) will give the dispersion relation.

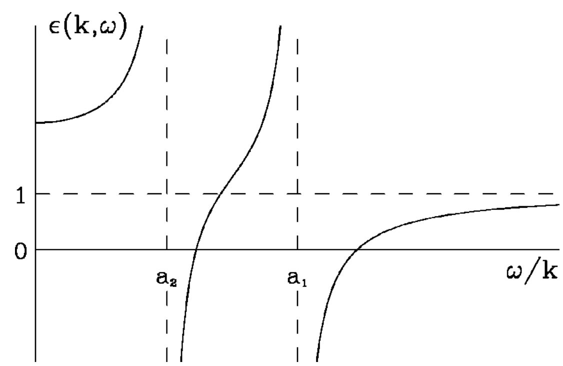

FIG. 2. Dielectric coefficient function of the frequency for a double water-bag distribution function.

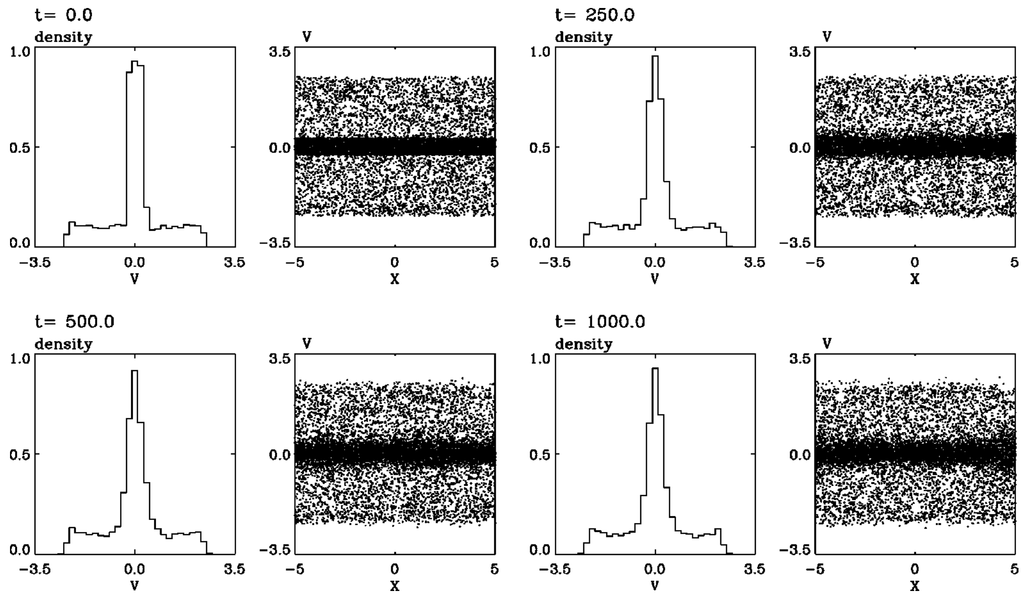

FIG. 3. Time evolution of a double water bag with $n\lambda_D=1000$ and $L=10\lambda_D$. The velocity distribution function and the representation in phase space are given.

For example, in the usual homogeneous plasma case, $a_i(x)=a_i$, and we can now take the Fourier transform on x of Eq. (11) to obtain

$$E_{1i}=\frac{\omega_{pi}^2}{\omega^2-k^2a_i^2}E_1. \tag{13}$$

The plasma frequency ω_{pi}^2 replaces the Jeans frequency, with, formally, $\omega_{pi}^2=-\omega_{Ji}^2$, because of the change of sign in the Poisson law. And, indeed, Eqs. (8) and (13) give the dispersion relation

$$\sum_i \frac{\omega_{pi}^2}{\omega^2-k^2a_i^2}=1, \tag{14}$$

which can be deduced from the general formulas. In the gravitational case, the eigenvalues ω of Eq. (11) are found with the constraint $E_{1i}(\pm x_{si})=0$.

It can be noticed that, if the velocity perturbation of the bags is taken independent of the time t, then $\omega=0$ is a solution of Eq. (11). This marginal mode corresponds to a translation at a constant velocity of the bags. The argument reads as follows. Deriving twice with respect to x (1), one obtains

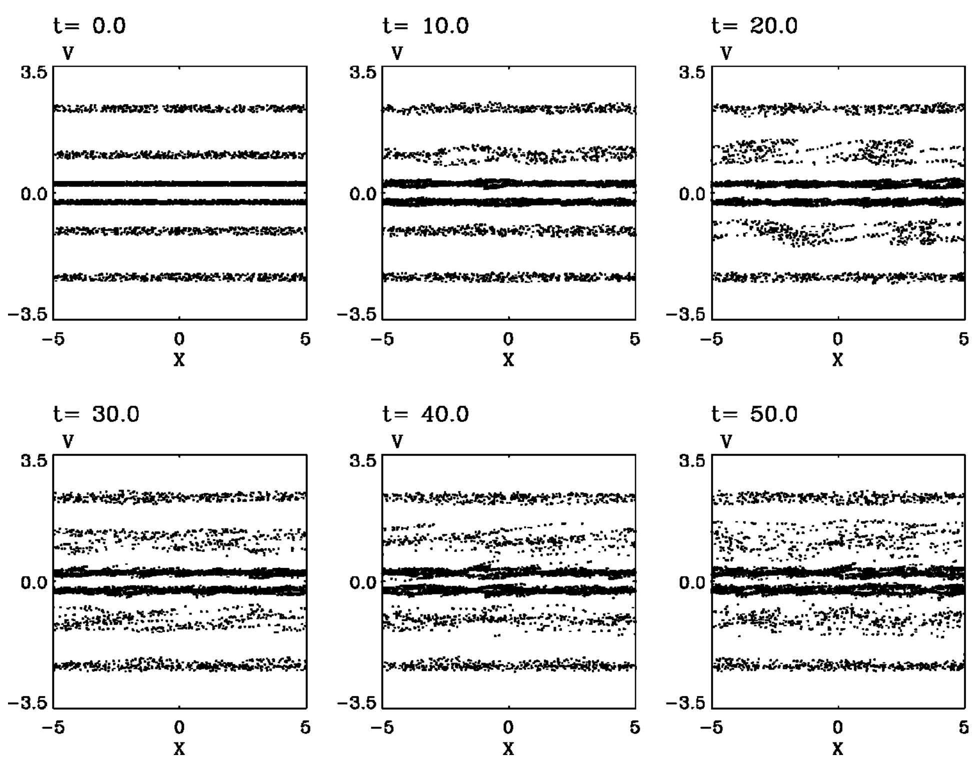

FIG. 4. Time evolution of labelized particles that belong to the double water bag with $n\lambda_D=2000$ and $L=10\lambda_D$ and initially localized into six ranges of velocities.

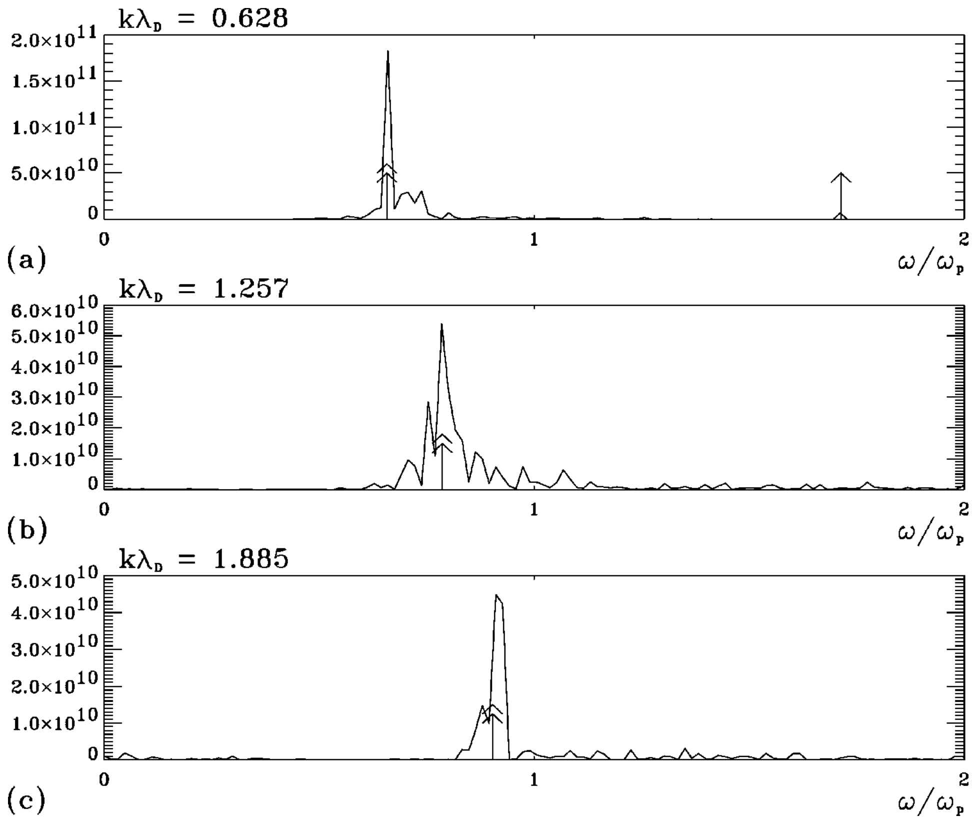

FIG. 5. Frequency spectrum of the $\rho_k(t)$, for (a) the fundamental wave number, (b) the second wave number, and (c) the third wave number, created by the particles of the double water bag with $n\lambda_D = 2000$ and $L = 10\lambda_D$. The samples are taken from $\omega_p t = 0$ to $\omega_p t = 400$, every 0.1. The single arrow gives the frequency excited by the Landau pole, while the doubled arrow gives the frequency excited by the second pole. For (b) and (c) the Landau pole is at frequencies higher than $2\omega_p^{-1}$.

$$\frac{d}{dx}\left(a_i \frac{d}{dx} a_i \right) + \frac{d^2\phi}{dx^2} = 0. \tag{15}$$

Supposing the entire system moves at velocity v, the field $E(x,t)$ reads, at $t = \Delta t$,

$$E(x,\Delta t) = E(x - v\Delta t, 0) = E(x,0) - \frac{dE_0(x)}{dx} v \Delta t \tag{16}$$

where the second term of the right-hand side is the perturbed field E_1 defined by Eq. (4). Now, taking Eqs. (5) and (8) into account Eq. (15) becomes

$$a_i \frac{d}{dx}\left(a_i \frac{d}{dx} E_{1i} \right) = -\omega_{j_i}^2 E_1, \tag{17}$$

which is indeed Eq. (11) with $\omega = 0$. Consequently, $\omega = 0$ corresponds to a translation of the entire system at constant velocity.

On the other hand, the period of rotation T of a particle of energy ε_A in the potential ϕ of the unperturbed multiple water bag is given by

$$T = 2\sqrt{2} \int_0^{x_A} \frac{dx}{\sqrt{\varepsilon_A / m - \phi(x)}}, \tag{18}$$

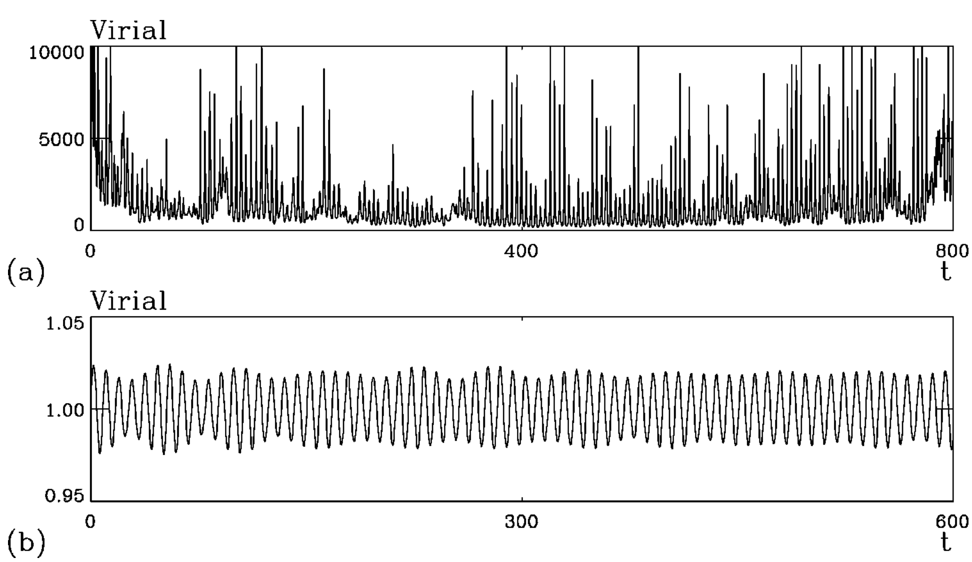

FIG. 6. Virial function of time for (a) the plasma and (b) the gravitational case.

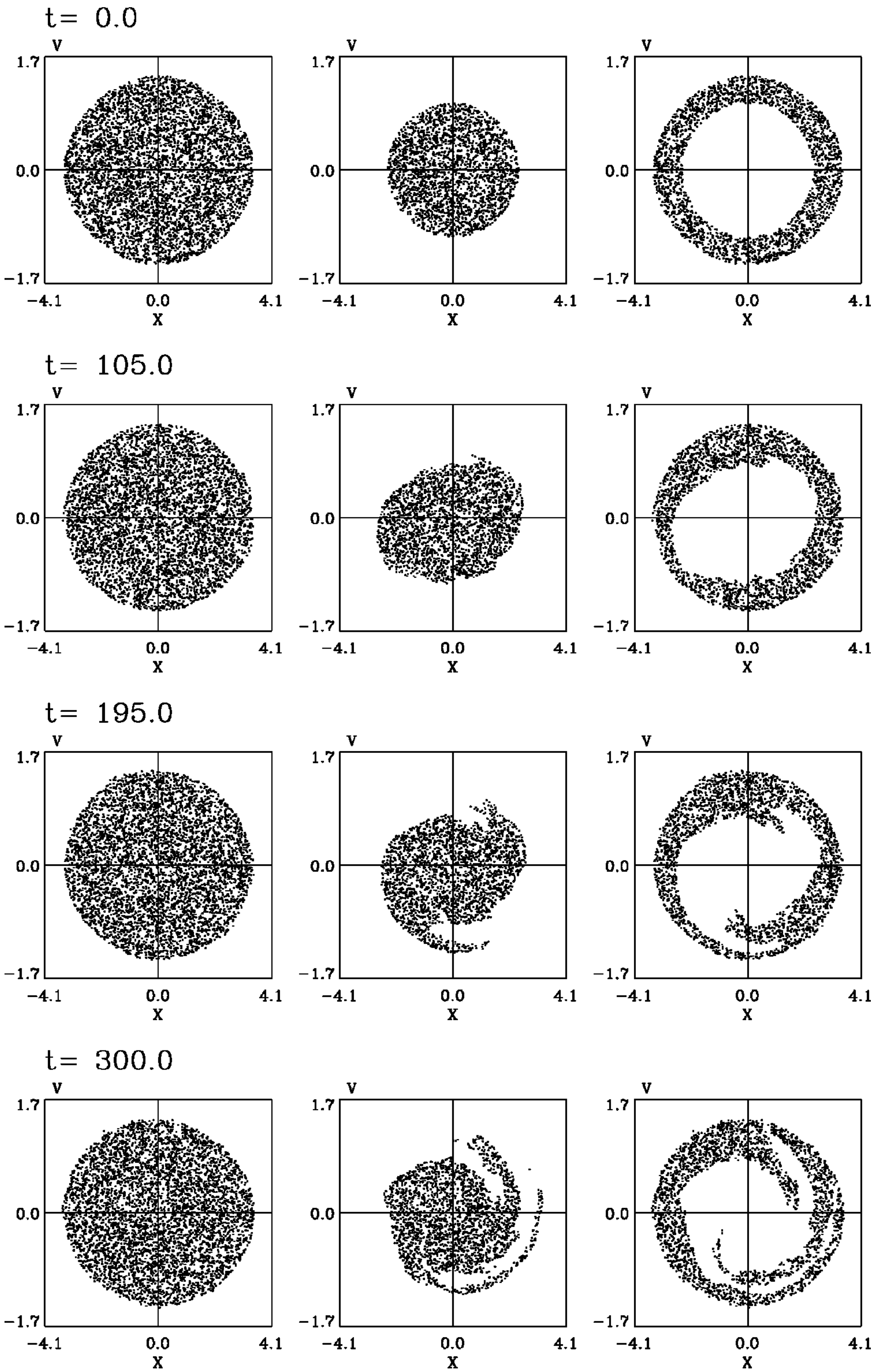

FIG. 7. Gravitational case: Metaequilibrium single water bag with $N=5000$ particles. Time evolution of, respectively, the overall distribution (first column), the $N/2$ particles of initial low energy (second column), and the $N/2$ particles of initial high energy (third column).

where x_A is the position of the particle when its velocity is equal to zero.

As already mentioned, the restriction to a one-dimensional (1D) system allows us to obtain the relatively simple system of equations as given by Eqs. (8) and (11). Another interesting point is, still for a 1D system, the existence of an exact code. As 1D particles are infinite plane sheets, each creates a field that is a constant. Consequently the total field is a piecewise constant and depends only on the relative positions of the particles. Each one experiences a uniform acceleration as long as it does not cross its neighbors, then it experiences a new field and a new accelerated motion. The program calculates the time at which crossings between two particles take place and keeps the position order relation between them. To have more precision, refer to [11]. The crucial property of this code is to be exact and have no error introduced, except the round-off errors due to the finite number of bits treated by the computer.

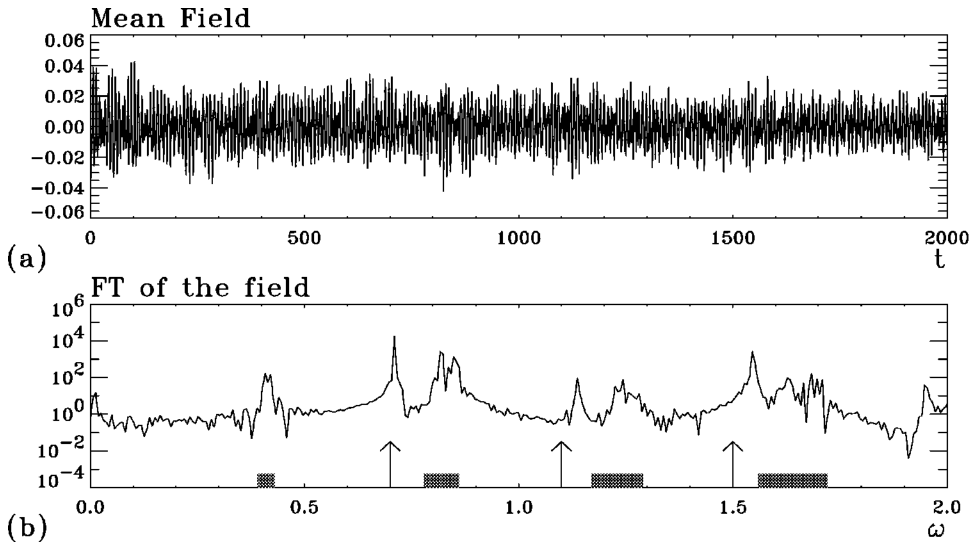

FIG. 8. (a) Time evolution and (b) time frequency spectrum of the averaged gravitational field (around the mean value) taken each 0.238 85 from $x=0$ to $x=2.3885$ for the water-bag case containing $N=10\,000$ particles. The field is sampled from $t=0$ to $t=2000$ each 0.05. In (b) the arrows indicate the collective modes, while the gray area gives the range occupied by the individual modes.

Consequently, this code will take into account all the effects, the individual as well as the collective ones. Moreover, it will give precisely the individual modes.

III. PLASMA CASE

In the plasma case, the space translation invariance (i.e., the homogeneous character of the metaequilibrium [12]) allows us to push the analytical treatment much further. The well-known classical first-order theory in the graininess parameter g [$g=(n\lambda_D)^{-1}$, where n is the density and λ_D is the Debye length] of the system uses both a Laplace transform in time and a Fourier transform in space to calculate the fluctuating field. The treatment is achieved supposing the independence of each wave number k, in agreement with the property of the Vlasov equation linearized around an infinite homogeneous metaequilibrium. The result shows a collective behavior around the resonance frequency for $k\lambda_D \ll 1$. These resonances are given by the dispersion relation, which reads for the Maxwellian distribution function (in the limit of small k)

$$\omega_k^2=\omega_p^2+3k^2V_T^2, \tag{19}$$

where ω_p is the plasma frequency and V_T the thermal velocity.

In the simple one bag case the dispersion relation [which has been recovered by the previous calculation, see Eq. (14)] reads

$$\omega_k^2=\omega_p^2+k^2a^2, \tag{20}$$

where $\pm a$ are the velocities delimiting the border of the bag. This relation shows that collective fluctuations will not happen in this case because the excited waves correspond to a phase velocity $(a^2+\omega_p^2/k^2)^{1/2}$ larger than a and no particles have a velocity larger then a. This important difference between a water bag and a Maxwellian distribution concerning the level of excitation of small k spectrum of the charge density has been studied and checked by numerical simulation in [13]. In order to exhibit some collective fluctuations we must consider at least two bags. In this case the dispersion relation is given by Eq. (14) for $i=1,2$. Figure 2, which gives the ϵ function of ω/k, indicates that the collective contribution is given by particles with velocities in the range $[a_2,a_1]$, that is, particles that belong to the outer bag (in phase space). The role and the importance of this second

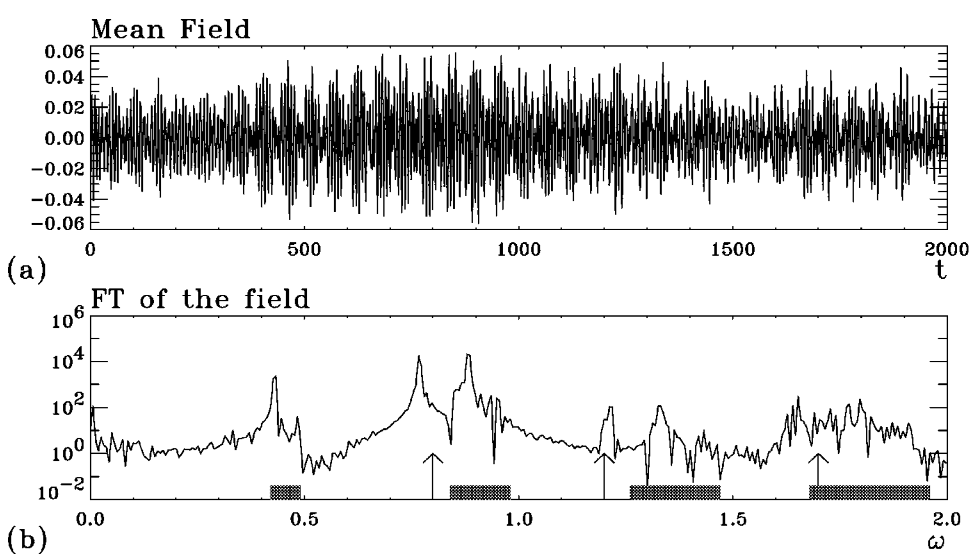

FIG. 9. (a) Time evolution and (b) time frequency spectra of the averaged gravitational field (around the mean value) taken each 0.2586 from $x=0$ to $x=2.586$ for the double water-bag case containing $N=10\,000$ particles. The field is sampled from $t=0$ to $t=2000$ each 0.05. In (b) the arrows indicate the collective modes, while the gray area gives the range of the individual modes.

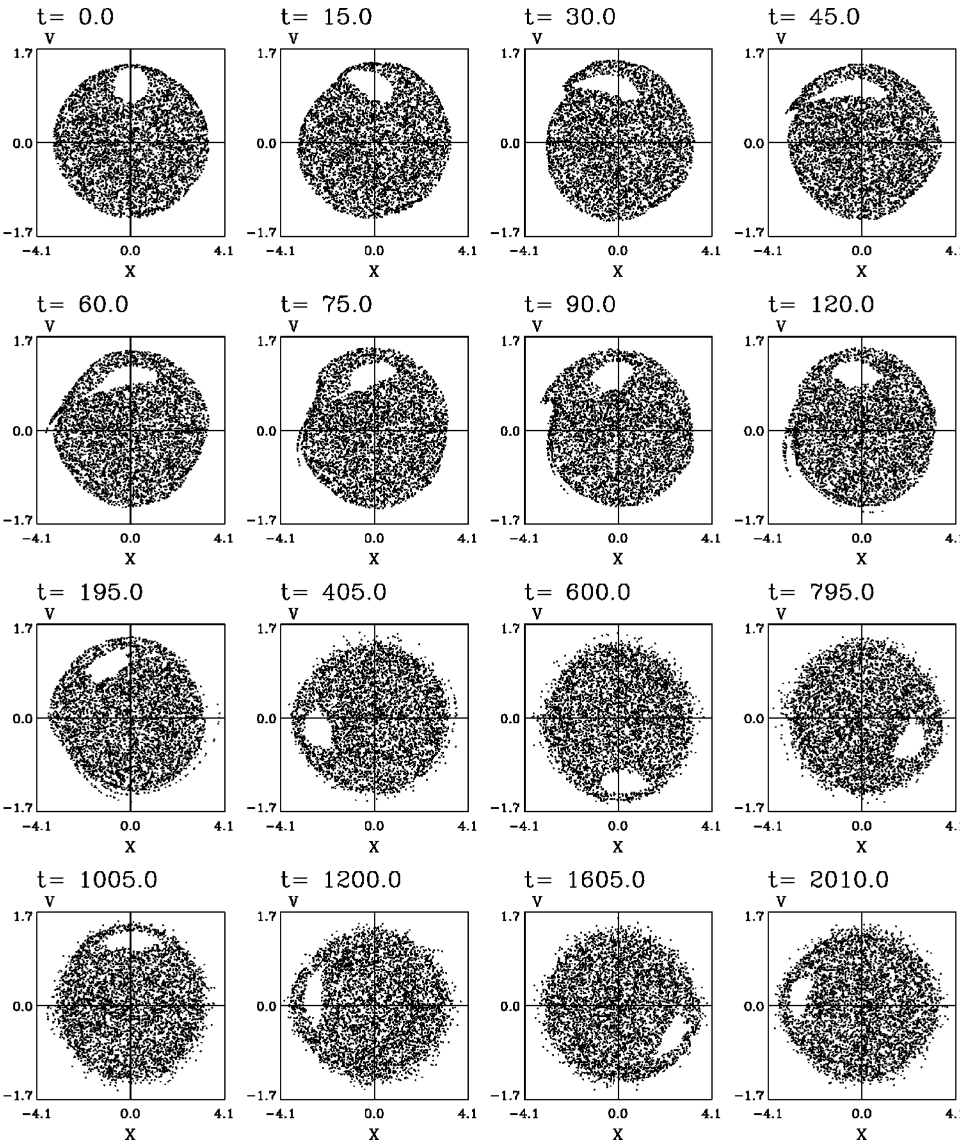

FIG. 10. Time evolution in phase space of a water bag containing $N=5000$ particles, in which a hole is created *ab initio*, for almost 130 rotations of the system.

pole are clearly exhibited by the double water bag, but similar results can be obtained with two electron plasmas at very different temperatures or with a mixture of electrons, positive and negative ions.

The level of excitation of a wave is proportional to $1-v_G/v_\Phi$, where v_G is the group velocity and v_Φ is the phase velocity [14]. This expression computed for the second pole (inside the water bag) shows that this level function of k goes to a maximum. The length of the system will be chosen in order to have a fundamental wave number not too far from this maximum. Fortunately, this will give rather small systems, which deemphasizes the role of the Landau pole (which as a matter of fact is not excited, but also not damped by the multiple water bag), allowing a better study of the second pole. Moreover, the computational effort will not be too heavy and more attention can be paid to the graininess parameter value.

Figure 3 shows the snapshots of the evolution of the distribution function taken at $\omega_p t=0$, 250, 500, 1000 for a system with length L equal to $10\lambda_D$ and containing 1000 particles (ions and electrons) by Debye length; this corresponds to a grain. It must be noticed that Balescu [15] has proved that no global evolution due to graininess can take place at time $\omega_p t$ proportional to $n\lambda_D$ in the one-dimensional case, and that the overall distribution will thermalize at time $\omega_p t$ proportional to $(n\lambda_D)^2$ [16]. On the other hand, test particles can relax in time $(n\lambda_D)\omega_p^{-1}$ (see [17]); these results have been numerically confirmed in the case of the water bag. In the double water-bag case, an undamped pole exists, with a phase velocity located in the outer bag. Consequently, in the regular fluctuations theory, an infinite level of excitation is present at this frequency. Of course, neglected phenomena (second order in graininess factor, for example) will bring a finite limit, but we should observe a quick destruction of a beam of particles located initially at this velocity. Turning to numerical simulation, we first observe that the global distribution function indeed does not change during the time of order $n\lambda_D \sim 1000$ (see Fig. 3). On the other hand, we show in Fig. 4 the evolution of six beams of particles that belong to the distribution but that are labeled in order to follow their motions; in order to have better insight, the number of particles is now of $n\lambda_D=2000$. Two of them represent the particles at the border of the inner bag, two the particles at the border of the outer bag, and two are formed with these particles that excite the fluctuations supported by the poles. It is

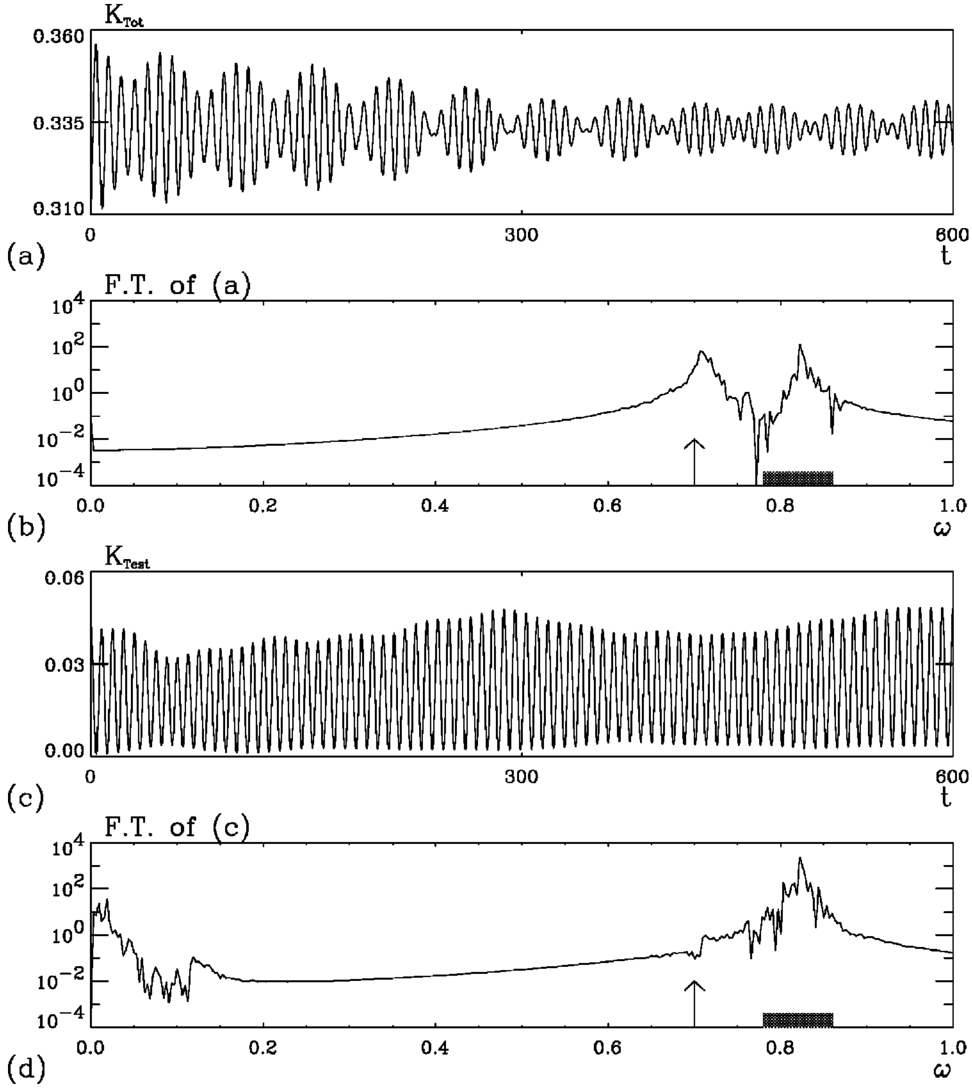

FIG. 11. (a) Total kinetic energy K_{Tot} function of time for the system represented, (b) frequency spectrum of K_{Tot}, (c) kinetic energy K_{Test} function of time for the test particles filling initially the hole, and (d) frequency spectrum of K_{Test}. In (b) and (d) the arrow indicates the collective modes, while the gray area gives the range of the individual modes calculated for the complete equilibrium water bag (with no holes).

clear from Fig. 4 that these two beams are quickly (around $\omega_p t = 40$) affected by these collective fluctuations. Let us point out that we are at the limit of validity of the usual first order in g theory since the regular collisional term (first order in g) exhibits singularities for the two beams with velocities equal to the phase velocity of the second pole (shown in Fig. 4). Indeed in time now much shorter than $n\lambda_D\omega_p^{-1}$ (20–40ω_p^{-1} while $n\lambda_D = 2000$) we see a strong destruction of these two beams while the others are much less affected.

The density Fourier transform on both space and time variables given in Figs. 5(a), 5(b), and 5(c) shows that the collective fluctuations are mostly supported by the largest wavelength allowed by the periodic system (that is, $k = 2\pi/L$). The other modes obtained with larger wavelengths are indeed present but less excited [see, for example, Figs. 5(b) and 5(c), which give the cases $k = 4\pi/L$ and $k = 6\pi/L$, respectively]. Figure 5(a) shows that, in fact, the Landau pole is excited at a very low level. It is initially excited and remains for the duration of the simulation because it exhibits no dumping [13].

The rapid destruction of a perturbed equilibrium system in the Vlasov limit has also been observed in the case of a Lorentzian velocity distribution function for which the energy diverges on small k because of a large number of particles in the tail. This indicates the nonphysical character of both distributions that have too slow a decrease in v of the velocity distribution and that have no particles of high velocity at all, but presents a sharp cutoff.

Nevertheless, in the latter case, it is an excellent model that allows analytical treatment, as can be seen in Sec. II and is numerically confirmed in the next section. Finally, the double water-bag model is a good approximation of a two electronic population plasma having two different temperatures—the Landau pole of the low-temperature population exciting the particles that belong to the high-temperature population.

IV. GRAVITATIONAL CASE

The change of sign in the interaction gives very different dynamical properties between plasma and gravitational systems; for example, the virial presents very large and periodic oscillations in the gravitational case while, in the plasma, it looks like a noise [see Figs. 6(a) and 6(b)]. Actually, the absence of neutralizing species is the point that prevents us from adopting a similar analytical treatment for both systems. Moreover, the neutralizing background is needed to treat the Jeans instability, which requires an infinite medium [18].

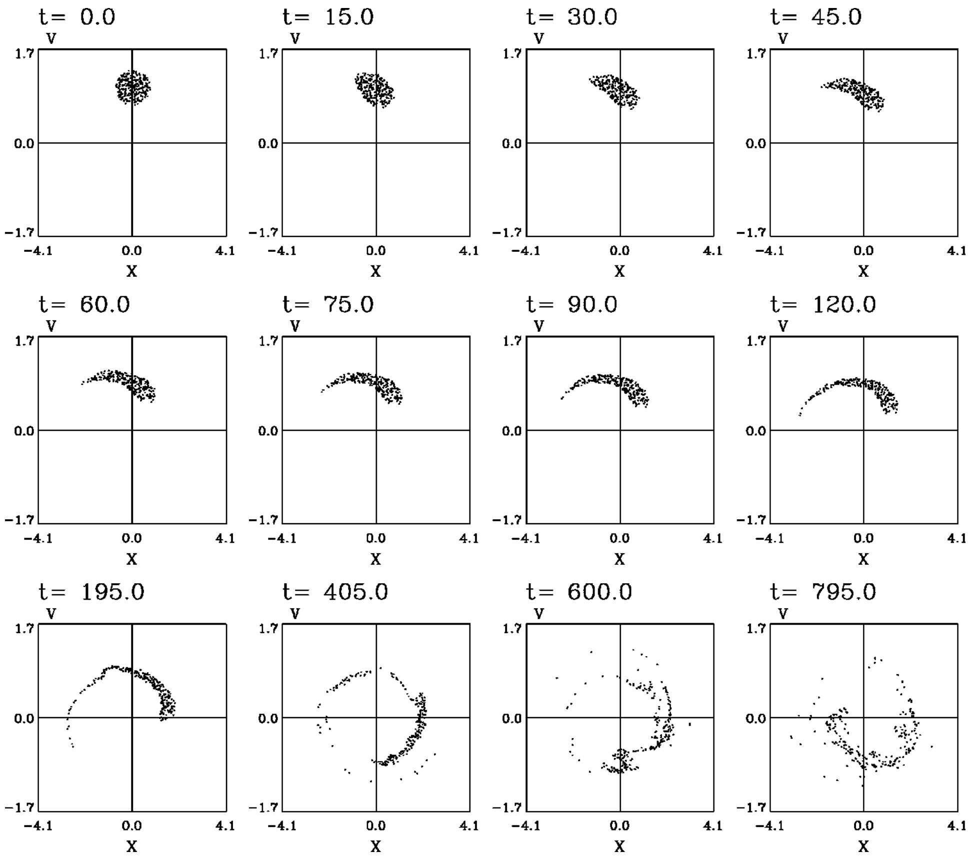

FIG. 12. Time evolution in phase space of a population of particles that belong to the water bag represented in Fig. 7 and initially localized in the same area as the hole of Fig. 10.

The lack of spatial homogeneity prevents the independence between the different wave numbers and imposes a proper mode analysis. As already mentioned, the calculus of such modes is easily done when we restrict consideration to the one-dimensional multiple water-bag distribution function. Nevertheless, even in that case, the determination of the modes become more and more difficult as the number of bags N increases since N coupled equations must be solved.

Systems (11),(8) have been numerically solved for the four first modes both for the single water bag and a double water bag. In that latter case we have two independent parameters A_2/A_1 and $\varepsilon_2/\varepsilon_1$ where A_i is the height of bag i and ε_i is the energy of the border of bag i (the bags are numbered from outside to inside). A shoot method, coupled with a Runge-Kutta scheme of order 4 [19], gives for the first collective modes, which are alternatively even and odd, respectively; $\omega=0$, $\omega=0.7$, $\omega=1.1$, $\omega=1.5$ for the single water bag and $\omega=0$, $\omega=0.8$, $\omega=1.2$, $\omega=1.7$ for the double water bag with $A_2=A_1$ and $\varepsilon_2=\varepsilon_1/2$. For values of ω large enough, the Jeans frequency ω_J does not play any role and the solutions are those of a string with fixed end points.

As already mentioned, the first mode $\omega=0$ is a marginal mode corresponding to an overall translation of the bags. In our numerical experiments, the initial conditions are chosen such that the total impulsion is zero and, consequently, this mode will not be excited.

On the other hand, the period of rotation of the particles in the field of the unperturbed metaequilibrium gives the individual particular modes. The oscillation frequencies vary from $\omega=0.43$ at the center to $\omega=0.39$ at the border for the single water bag and from $\omega=0.49$ to $\omega=0.42$ in the double water-bag case.

The numerically simulated system contains N particles of equal mass m uniformly distributed inside the phase-space contour of energy ε_i with the phase-space density A_i. The normalization is such that the total mass mN equals 1 and ε_1, and the maximum energy equals 1. With this normalization and taking $4\pi G=1$, the square of the Jeans frequency ω_{J0}^2 defined at $x=0$ is equal to 3/16 for the single water bag and $3(\sqrt{2}+1)/8(2\sqrt{2}+1)$ for the double water bag with $A_2/A_1=1$ and $\varepsilon_2=\varepsilon_1/2$.

Figure 7 shows the snapshots of the evolution of a single water bag for, respectively, the overall distribution, the population of initially high-energy particles, and the population of initially low-energy particles. The first column shows that no global change happens and the water-bag character of the distribution is conserved for the duration of the simulation. On the other hand, the two last columns show that the two populations mix but keep a strong cohesion. This process is very far from the idea of a smooth diffusion. Indeed, the Fourier transform (FT) of the field given in Fig. 8 exhibits the collective and individual modes (including the harmonics) theoretically determined before and the biggest mode is the first collective mode at $\omega=0.7$. In order to have better statistics and to have both even and odd contributions, the field is collected for 10 positions equally spaced out from 0 to x_{s1} and the averaged value taken around the mean value is given in Fig. 8.

The same kind of diagnostic can be obtained with the double water bag and Fig. 9 gives the time Fourier transform of the field collected under the same conditions as for the single water bag. Also, in that case, collective and individual modes, including the harmonics of the individual excitations

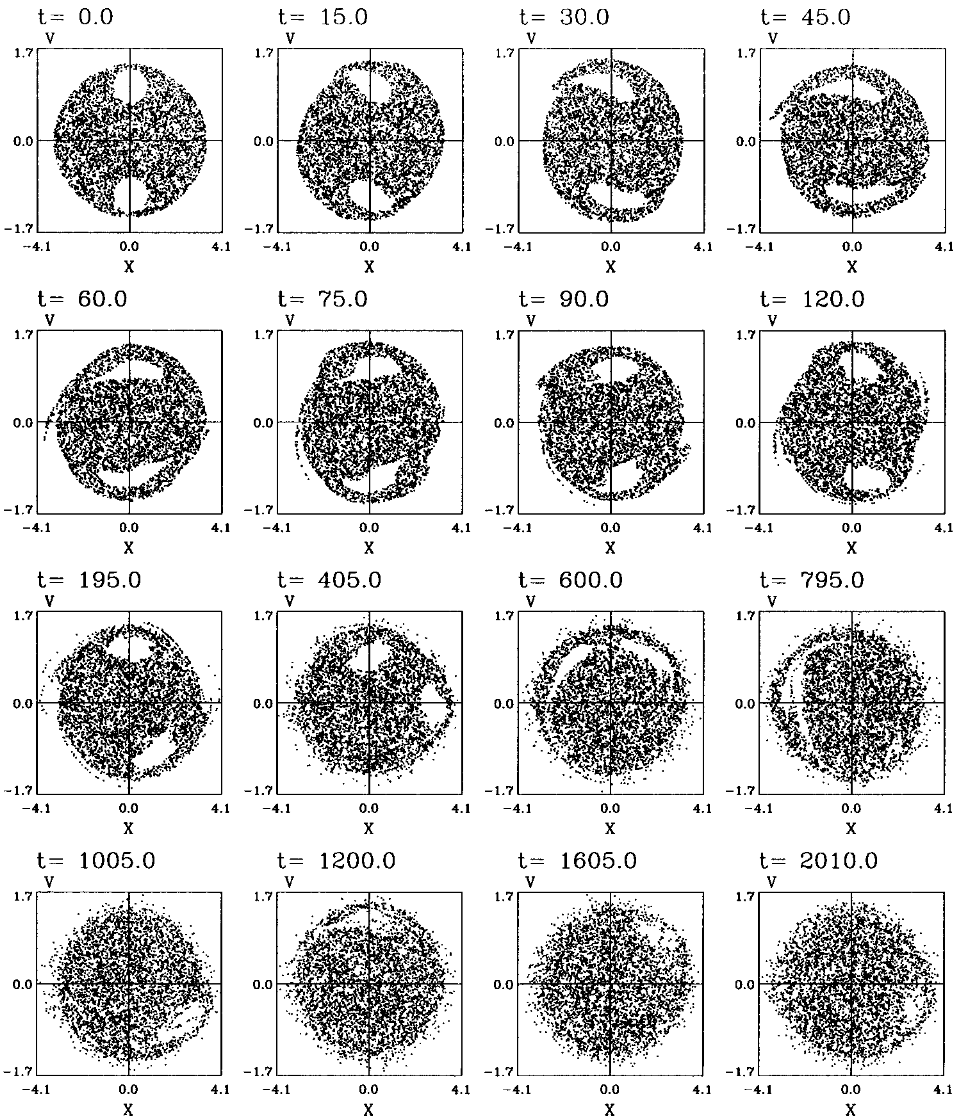

FIG. 13. Time evolution in phase space of a water bag containing $N=5000$ particles, in which two symmetric holes are created *ab initio*, for almost 130 rotations of the system.

are present and dominated by the first collective mode at $\omega \sim 0.8$.

For these two distribution functions, the single and double water bag, the numerical results indicate that collective modes are highly excited by the grainy character of the distribution function describing the systems.

V. STABILITY OF HOLES IN PHASE SPACE

Numerical simulations of systems outside equilibrium show that they do not relax toward an equilibrium: they develop arms that carry the excess kinetic energy and go around an empty zone of phase space. This process, which seems to be systematic, creates holes in phase space that remain for the duration of the simulation and prevent the system from reaching a complete metaequilibrium. Nevertheless, taking a time average allows one to obtain a distribution of the alone energy [7]. Moreover, numerical simulations reveal that the number of holes and their positions are closely related to the initial shape of the distribution function.

In order to study the behavior of this structure, a hole is created *ab initio* in the water-bag equilibrium [20]. Figure 10 shows the evolution of the hole for nearly 130 rotations of the system. The hole is still present at the end of the simulation and it resists the differential rotation of the particles that are localized at its border. In order to have a better insight into the behavior of this hole, it is initially filled with ''test particles'' that experience the field of the other but do not contribute to the field. Figures 11(a), 11(b), 11(c), and 11(d) give, respectively, the kinetic energy of the particles of the systems, its Fourier transform, the kinetic energy of the test particles and its Fourier transform. The excited modes, Fig. 11(d), are the individual modes, indicating that the holes rotate at the same velocity as the particles. Moreover, these holes triggered the collective mode, as can be seen in Fig. 11(b).

To show the long time scale of the hole, we go back to the complete water-bag equilibrium and follow the particles which are initially localized in the area of the previous hole. These particles are just ''labeled'' and have the same physical properties as the others. Figure 12 shows that the differential rotation between particles of low and high energy stretches the area occupied by these labeled particles. Nevertheless, the presence of another effect can be detected because the stretching is not complete.

Finally, we study numerically the behavior of symmetric structures. Starting from a symmetric distribution function, the Eulerian simulations (for which the Vlasov equation is directly integrated) show the formation of an even number of symmetric holes (two in the simulation given by Mineau [7]). In our case, the particle description of the system breaks the Liouville invariant and the holes may even disappear. The stability of two symmetric holes initially dug in the single water bag is numerically studied. Figure 13 shows the evolution of test particles initially localized in these holes. It reveals that the symmetry rapidly breaks: one of the holes goes to the center while the other goes near the border of the bag. Then, as already mentioned, their periods of rotation, given by the particles of its border, change and rapidly merge to form a single hole.

VI. CONCLUSION

This paper gives both analytical and numerical approaches of the determination of collective modes in the gravitational and plasma systems. These approaches are possible because of the restriction to 1D systems and to the multiple water-bag model. It must be pointed out that the theory of plasma assumes a neutral medium that is, most of the time, a uniform motionless background. The infinite gravitational system also needs a neutralizing background, but this system is unstable under Jeans instability and clusters into subsystems, the dimension of which is of the order of the Jeans length [21]. Here, the model studied is not infinite and does not need such an unphysical background.

N-body numerical simulations confirm pretty well the theoretical results and show that the gravitational system presents very strong collective behavior that in a certain sense is stronger than in the plasma case.

These collective effects triggered by the grainy nature of our system explain the very strange behavior of a labeled population, a fact already mentioned in Luwel and Severne [20]. Moreover, numerical simulations show that holes are structures that certainly play an important role in one-dimensional systems, a fact already noticed in the plasma case. With an initial hole, the water bag almost keeps its shape for a time large compared to the time necessary to destroy the same area filled with particles.

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