Electron magnetohydrodynamic approximations and the magnetic drift wave

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It is shown that the approximations of electron magnetohydrodynamics (EMHD) are too restrictive to be realistic for at least plasmas of low-Z materials (for example, hydrogen and its isotopes). The simultaneously used assumptions of stationary ions and inertialess electrons are not self-consistent, especially within the framework of local theory. As a result the application of EMHD to describe the pure transverse magnetic drift wave suffers from weaknesses and such a mode does not seem to exist in magnetized plasmas.
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The endeavors for mathematical simplicity to describe physical phenomena are very interesting and important in general. However, the assumptions used in such delicate procedures become too restrictive or even self-contradictory in some cases. Such a model of stationary ions and inertialess electron fluids has widely been used to study magnetic field generation in laser-produced plasmas ([1–10] and references therein) and magnetic field evolution in magnetized plasmas [11–14]. In the presence of an external magnetic field this model is generally called electron magnetohydrodynamics (EMHD). During the past few years interest in this area has been continuously growing. The numerical simulation of the magnetic drift wave (MDW), which is believed to propagate in the magnetized inhomogeneous plasma within the limits of EMHD approximations, has been performed [14]. The coupling of this wave with another ion and hybrid modes has also been investigated [15]. A review article [16] presents a detailed discussion on EMHD along with several possible applications. Recently two-dimensional EMHD turbulence has been studied by numerical simulation [17]. These studies contain many useful discussions as well, apart from the discussion of EMHD and MDW.

It is important to note that the limit of inertialess electrons requires $\omega^2 \ll \omega_{pe}^2, \Omega_e^2$ and ions can be assumed to be stationary if $\omega_{pi}^2 \ll \omega^2$ [where $\omega_{pe}(\omega_{pi})$ are the electron and ion plasma oscillation frequencies, respectively, and Ω_e is the electron gyrofrequency]. The situation in magnetized plasmas in this case, even for the perpendicularly propagating perturbation $(\mathbf{k} \perp \mathbf{B}_0)$, is not like $\Omega_i \ll \omega \ll \Omega_e$ (where Ω_i is the ion gyrofrequency), which can provide more liberty in the choice of ω. Rather it turns out to be the same condition as in the case of unmagnetized plasmas [18–20], that is, $\omega_{pi} \ll \omega \ll \omega_{pe}$ or even stronger restrictions. Note that $\omega_{pi}/\Omega_i = c/v_A$ (where v_A is the Alfvén speed) and $\omega_{pe}/\Omega_e = c/v_A(m/M)^{1/2}$. In the nonrelativistic case $v_A/c \ll 1$ is always true. There can be three possibilities: $\omega_{pe}/\Omega_e \sim O(1)$, $\omega_{pe}/\Omega_e \ll 1$, and $1 \ll \omega_{pe}/\Omega_e$. The first possibility provides for the condition

$$\Omega_i \ll \omega_{pi} \ll \omega \ll \omega_{pe}, \Omega_e . \tag{1}$$

The second possibility implies

$$\Omega_i \ll \omega_{pi} \ll \omega \ll \omega_{pe} \ll \Omega_e . \tag{2}$$

In both these cases the situation is ultimately $\omega_{pi} \ll \omega \ll \omega_{pe}$, which means

$$\left(\frac{m}{M}\right)^{1/2} \ll \frac{\omega}{\omega_{pe}} \ll 1, \tag{3}$$

where $m(M)$ are the electron (ion) masses, respectively. This suggests that a smallness parameter, say ϵ, can be defined as $\omega_{pi}/\omega \sim O(\epsilon) \sim \omega/\omega_{pe}$, such that the inequalities (1)–(3) look like $\epsilon^2 \ll \epsilon \ll 1$, so that $\epsilon^2 \sim O(m/M)^{1/2}$ or $\epsilon \sim O(m/M)^{1/4}$.

The displacement current in Ampere's law is ignored, assuming $\omega/\omega_{pe} \ll \omega_{pe}/\Omega_e$. Note that in magnetized plasmas the condition $\omega \ll \omega_{pe}$ is not sufficient to ignore the displacement current. Then Ampere's law becomes

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j}. \tag{4}$$

Let us consider the inhomogeneous magnetized electron plasma with $\kappa_n = |\nabla n_0/n_0|$ and the density scale length $L_n = 1/\kappa_n$. For deuterium plasma, as an example, $(m/M)^{1/2} \sim 1/60$; therefore $\epsilon \sim O(1/7.5)$. The local theory requires $\kappa_n/k \ll 1$ and therefore we can at most assume $|\kappa_n/k| \sim O(\epsilon)$.

The condition $\nabla \cdot \mathbf{j} = 0$ in the case of zero background velocity implies

$$\mathbf{k} \cdot \mathbf{v}_1 + \kappa_n \cdot \mathbf{v}_1 = 0. \tag{5}$$

We notice again that in the one dimensional case the second term on the left hand side is ϵ times smaller than the first. Therefore, if Eq. (5) is considered to be valid, then both the assumptions of stationary ions and inertialess electrons cannot be used simultaneously. Furthermore, in the second case v_A/c turns out to be of the order of ϵ, which is not a very small number here and hence the relativistic effects can also be important. The third case is more restrictive since $\omega \ll \Omega_e$ is used in EMHD and $\omega_{pi} \ll \omega \ll \Omega_e \ll \omega_{pe}$ cannot be satisfied easily. But this limit is not allowed in the inertialess electron plasma because in this case both compressional magnetic and density perturbations should be ignored.

An additional restriction arises due to the assmption $\omega \ll ck \ll \omega_{pe}$ used in the curl of Ampere's law. This makes all these assumptions in a compact form as $\omega_{pi} \ll \omega \ll ck$

$\ll\omega_{pe}$. Even in the presence of steep density gradient $|\kappa_n/k|\sim O(1/8)$ the local theory does not allow us to ignore ion dynamics completely for inertialess electrons.

Now we try to show how the application of EMHD to describe the so called linear magnetic drift wave suffers from several weaknesses and contradictions. Since electron polarization drift is ignored, in the absence of temperature perturbation the electron equation of motion gives

$$\mathbf{E}=-\frac{1}{c}\mathbf{v}\times\mathbf{B}. \tag{6}$$

The external magnetic field $\mathbf{B}_0$ is assumed to be uniform along the z axis, the density gradient is assumed to be along the x axis, and the propagation of the wave is considered along the y axis in a plasma slab.

The curl of Eq. (6) yields

$$\partial_t\mathbf{B}=-\boldsymbol{\nabla}\times\left(\frac{\mathbf{j}\times\mathbf{B}}{en}\right). \tag{7}$$

Since there is no equilibrium current, Eq. (7) in the linear limit becomes

$$\partial_t\mathbf{B}_1=\frac{1}{en_0^2}\boldsymbol{\nabla}n_0\times(\mathbf{j}_1\times\mathbf{B}_0). \tag{8}$$

The electron equation of motion also gives

$$\mathbf{v}_1=\frac{c}{B_0}\mathbf{E}_1\times\mathbf{z}. \tag{9}$$

For this transverse wave ($\boldsymbol{\nabla}\cdot\mathbf{E}_1=0, n_1=0$), $\mathbf{B}_1$ is along $\mathbf{B}_0$. Then $\mathbf{E}_1$ has to be along the x axis. Using Eqs. (8) and (9), one obtains the linear dispersion relation for this wave as

$$\omega=\lambda^2k^2\left(\frac{\kappa_n}{k}\Omega_e\right)=\frac{kv_A^2}{L_n\Omega_i}. \tag{10}$$

Now we analyze the above assumptions and the equations used and look to see whether they are in accordance with each other or not.

First we note that $\boldsymbol{\nabla}\times(\mathbf{j}_1\times\mathbf{B}_0)=0$ because $\mathbf{k}\perp\mathbf{j}_1,\mathbf{B}_0$. Since $\mathbf{j}_1\times\mathbf{B}_0\neq0$ is assumed in Eq. (8) and $\mathbf{k}\cdot\mathbf{j}_1=0$, while $\mathbf{k}$ is along the y axis, $\mathbf{j}_1=-en_0\mathbf{v}_1$ turns out to be along the x axis, and hence Eq. (6) implies that $\mathbf{E}_1$ has to be along the y axis, suggesting $\boldsymbol{\nabla}\cdot\mathbf{E}_1\neq0$, which is a contradiction of the initially assumed wave geometry discussed above. In this case $n_1\neq0$ must be considered.

Second, we observe that the assumption $\boldsymbol{\nabla}\cdot\mathbf{v}_1\neq0$ is physically a compressible case and hence the electrostatic potential fluctuation should not be ignored. The divergence of Eq. (6) gives

$$\boldsymbol{\nabla}\cdot\mathbf{E}_1=-\frac{1}{c}\mathbf{B}_0\cdot\boldsymbol{\nabla}\times\mathbf{v}_1=-\frac{1}{c}\mathbf{B}_0\cdot\boldsymbol{\nabla}\times\left(-\frac{c}{4\pi n_0e}\boldsymbol{\nabla}\times\mathbf{B}_1\right), \tag{11}$$

which yields

$$\frac{e\phi_1}{T_0}=\frac{1}{\beta}\frac{B_1}{B_0}, \tag{12}$$

where $\beta=c_s^2/v_A^2\ll1$ and c_s is the ion sound speed. According to Eq. (12) the electrostatic energy is not smaller than the magnetic energy associated with such a compressible perturbation.

Third, in Eq. (5) the second term is zero if $\mathbf{v}_1$ is in a direction perpendicular to the density gradient, as it appears to be according to the discussion above. Therefore $\mathbf{k}\cdot\mathbf{v}_1=0$ automatically, which is again a contradiction of the initial assumption.

In summary, we conclude that the approximations used in EMHD are not self-consistent, and the application of this theory to the plasmas quoted in literature are not suitable. However, in some special cases a few of these assumptions if applied carefully to high-Z material plasmas or dusty plasmas may describe some interesting physical phenomena.

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