

Propagating front in an excited granular layer

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A partial monolayer of $\sim 20\,000$ uniform spherical steel beads, vibrated vertically on a flat plate, shows remarkable ordering transitions and cooperative behavior just below $1g$ maximum acceleration. We study the stability of a quiescent disordered or “amorphous” state formed when the acceleration is switched off in the excited “gaseous” state. The transition from the amorphous state back to the gaseous state upon increasing the plate’s acceleration is generally subcritical: An external perturbation applied to one bead initiates a propagating front that produces a rapid transition. We measure the front velocity as a function of the applied acceleration. This phenomenon is explained by a model based on a single vibrated particle with multiple attractors that is perturbed by collisions. A simulation shows that a sufficiently high rate of interparticle collisions can prevent trapping in the attractor corresponding to the nonmoving ground state. [S1063-651X(99)13105-2]

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I. INTRODUCTION

Granular materials exhibit phases that resemble yet are distinct from ordinary solids, liquids, and gases. (For reviews see Refs. [1,2].) Granular materials at rest behave much like solids (e.g., they can sustain stress), but when excited they can behave as a dense liquid or a more dilute gas, with varying degrees of correlated motion. Many phenomena observed in granular materials involve more than one phase simultaneously. For example, in sheared granular materials and in avalanches both a solid phase and a liquid or gas phase are present. The coexistence of phases and the mechanisms of transitions between phases are of great interest. Although there are no attractive interparticle forces, it is provocative to compare and contrast these phenomena to their condensed matter analogs.

A critical difference is the presence of inelastic collisions and dissipation of energy in a granular medium [2]. Energy has to be added continuously in order to maintain granular material in a fluid state. Dissipation can also render the excited state unstable against density fluctuations. Regions of elevated density experience an elevated collision rate, which decreases average particle speeds and leads to further local density increases through the granular equivalent of mass diffusion. The excited state collapses locally and clusters of particles at rest appear. This phenomenon has been studied numerically [3–7] for the cooling of a granular gas without energy input [8]. Clustering has been predicted to persist in the presence of excitation [9], whether the energy is provided homogeneously [10,11] or at the boundary [12]. Clustering due to inelasticity has been studied experimentally by several groups [13,14].

Granular materials can be excited conveniently with a controlled energy input through vertical vibrations of the container. Since energy is transferred into horizontal motion through interparticle collisions, the (mean) vertical kinetic energy is always larger than the horizontal kinetic energy [15–17]. Olafsen and Urbach [14], in a study of a vertically vibrated partial layer of steel spheres, found spontaneous clustering into a two-dimensional ordered crystal at rest sur-

rounded by a sea of vibrating particles, as the peak acceleration was reduced below $1g$. Both phases were found to coexist in steady state.

In this article, we study a triggered phase transition that transforms a quiescent disordered phase to an excited granular gas via a propagating front. Starting with a partial two-dimensional (2D) layer of spherical beads at rest on a vibrating flat plate below a threshold vibration amplitude, we have observed that a small external perturbation of only one bead can induce a phase transition into a rapidly moving gaseous state that spreads to all beads. We determine the frequency-dependent range of vibration amplitudes where the phase transition can be triggered, and observe a subcritical-to-supercritical transition at a threshold frequency. We also measure the growth rate and interface shape of the gaseous region. We account for our observations using numerical studies of a single periodically forced bouncing particle subjected to random perturbations arising from collisions. The simulation explains the origin of the propagating front and may also help to explain the crystal-fluid coexistence discovered earlier [14].

Other metastable states are known in granular matter. For example, a small perturbation suffices to trigger avalanches and a small (tapping) perturbation can start the flow of granular material from a hopper. The propagating front described here is different from these examples; it arises from the coexistence of two dynamical attractors.

II. EXPERIMENTAL PROCEDURE AND QUALITATIVE OBSERVATIONS

The experiments are conducted in a rigid 32 cm diameter circular container made of anodized aluminum that is oscillated vertically with a single frequency between 40 and 140 Hz using a VTS500 vibrator from Vibration Test Systems Inc. A computer controlled feedback loop keeps the vibration amplitude constant and reproducible. Spherical (grade 100) nonmagnetic 316 stainless steel beads with 1.59 ± 0.02 mm diameter are used as granular material. Most experiments were carried out with a single layer of $\sim 18\,400$ beads, which

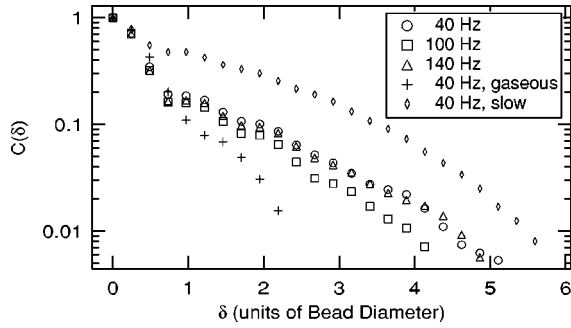


FIG. 1. Radial autocorrelation function $C(\delta)$ of images of the amorphous state taken after sudden cooling from initial states forced at different frequencies (triangles, circles, squares). Slow cooling (diamonds) produces stronger correlations. Correlations are much weaker in the gaseous state (crosses).

corresponds to a fractional coverage of $c = 0.50$ (half that of a hexagonal close packed layer). The particles are illuminated from four sides and images are captured with a 512×512 pixel variable scan charge-coupled-device (CCD) camera (CA-D2, Dalsa Inc.) at 2 frames/s. In some experiments a 512×480 pixel fast CCD camera (SR-500, Kodak Inc.) operated at 30–60 frames/s was used. The coefficient of restitution for collisions with the aluminum plate (i.e., the ratio of velocities after and before a collision), measured from images of successive vertical bounces taken with the fast camera, is $\alpha_{\text{plate}} = 0.95 \pm 0.02$ at commonly observed particle speeds. This result is similar to $\alpha_{\text{bead}} = 0.93 \pm 0.02$, which was measured in Ref. [13]. (A similar value, $\alpha_{\text{bead}} = 0.95 \pm 0.03$, was obtained in Ref. [18].) We found that α_{plate} decreases with increasing impact velocity (in agreement with Refs. [19,20]). When rotational motion or a tangential velocity component is present during collisions, the effective restitution coefficient decreases.

Conducting nonmagnetic 316 stainless steel was chosen for the beads. We found that the phase transitions occur at the same accelerations when the shaker's magnetic field was reduced by 70%, and that the growth rate of the excited phase is unchanged. These facts indicate that the external magnetic field generated by the shaker does not influence the results presented here. We also did not find any significant electrostatic effects.

A. Quiescent disordered (amorphous) state

In order to create reproducible initial conditions for all experiments, the container is vibrated for more than 20 s at a peak acceleration $a \geq 2g$. Then the vibration is abruptly turned off and the particles come to rest through free cooling (i.e., decrease of particle energies through inelastic collisions). Figure 1 shows the resulting radial autocorrelation function $C(\delta)$ of images of the amorphous state after sudden cooling starting from different forcing frequencies; the horizontal axis is scaled by the bead diameter. The correlation function is calculated from the greyscale intensity I after subtraction of the average greyscale intensity as $C(\delta) = \langle I(r)I(r+\delta) \rangle_r / 2\pi\delta \langle I(r)^2 \rangle_r$. We note a sharp peak near $\delta=0$ corresponding to the image of a single bead, and a tail that is insensitive to the vibration frequency and amplitude prior to the shut off. This tail is not present in the gaseous

state, also shown. The clustering of beads during sudden cooling, which was anticipated numerically, appears to be insensitive to the frequency at which the beads were excited. When the amplitude is lowered slowly, clustering becomes measurably stronger, as is evident from the upper curve.

B. Propagating front

The focus of the experiments presented in this article is the transition to a moving gaseous state, starting from the reproducible quiescent amorphous state produced by sudden cooling. If the peak acceleration is much larger than $1g$, all beads start moving immediately when the vibration is switched on. For peak accelerations much smaller than $1g$, the beads remain quiescent on the plate indefinitely and return into the quiescent state when perturbed externally. In contrast, for a frequency dependent intermediate range of peak accelerations just below $1g$, the beads remain at rest when the vibration is switched on, but a transition to the moving gaseous state can be triggered by setting at least one bead into motion. Figure 2 shows the typical evolution of a portion of the layer from the amorphous state to the gaseous state when a region of several beads is perturbed. The peak acceleration a of the vibrated plate has been set below, but close to, a threshold a_c that is dependent on driving frequency f ; the beads remain essentially at rest on the plate [Fig. 2(a)]. An external perturbation is then applied to a few beads, either by rolling an additional bead through an inclined cylinder onto the surface or by pushing a few beads manually. The perturbed bead(s) start to bounce and move in the horizontal direction, transmitting energy to their neighbors, which in turn start to move. An area within which all beads are moving develops quickly, surrounded by a denser area that marks the interface between the granular gas and the essentially static amorphous state [Fig. 2(b)]. The area of the moving gaseous phase increases rapidly as the dense front propagates into the amorphous phase [Fig. 2(c)], until all beads are moving. For accelerations where a crystalline phase can exist (see below), we observe the development of a crystal during the transition and its coexistence with the gaseous phase in the steady state.

C. Interface shape

We use the absolute difference of two consecutive frames to calculate the area and the interface shape of the gaseous region. Such a difference image, converted to black and white, produces a white spot at the initial and final positions of a moving bead, leaving all stationary beads and the background black. This allows us to distinguish between the essentially stationary amorphous phase and the moving gaseous phase. Figures 2(d)–2(f) are difference images corresponding to Figs. 2(a)–2(c). (The difference images reveal that a few beads do move even in the amorphous phase, so small spontaneous perturbations are always present.) In order to extract the size and shape of the gaseous region from the difference images, we create a continuous white region using NIH image. The perimeter of the gaseous region, extracted with this method from the absolute difference of two consecutive frames, is shown overlaid onto the second

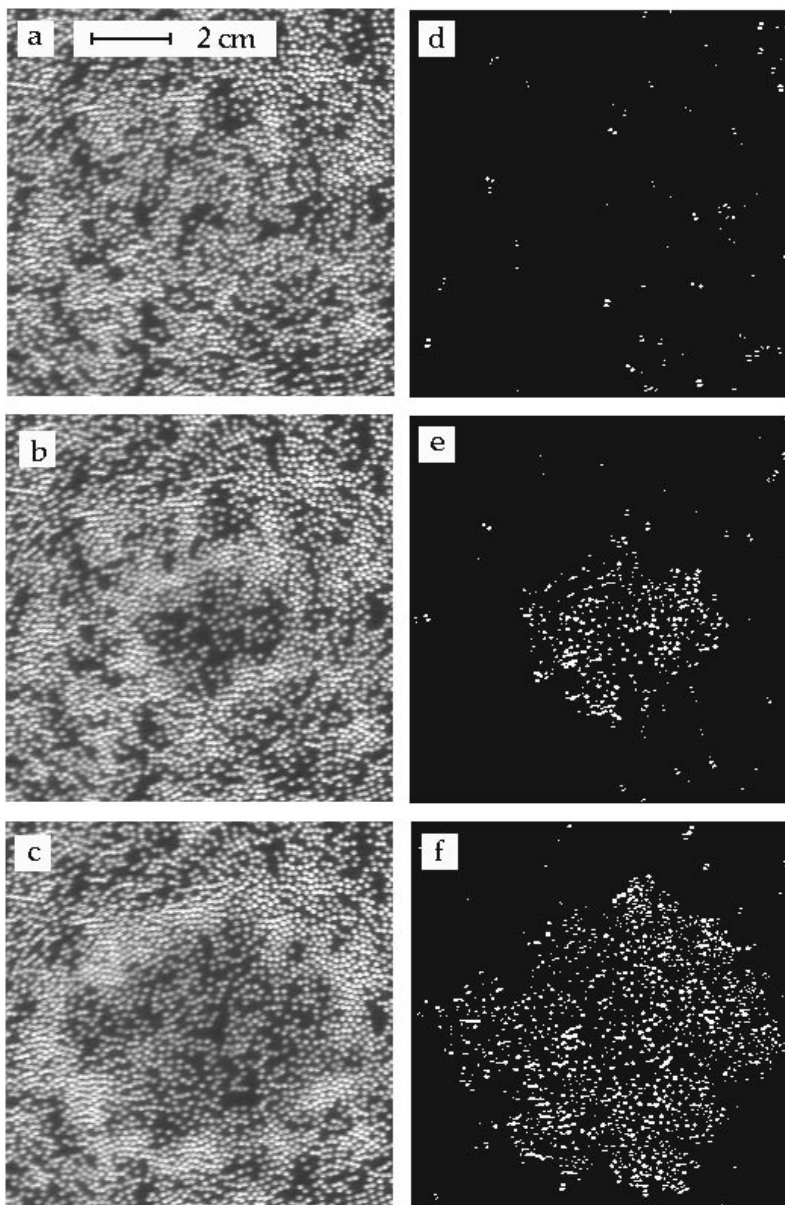


FIG. 2. Front propagation during the amorphous-to-gaseous transition at 80 Hz with $a=0.93g$. Images captured every 3.5 s are shown at left; absolute differences between images taken 0.5 s apart are shown on the right to highlight moving particles. (a),(d) Amorphous initial state; (b),(e),(c),(f) unstable front propagation.

frame in Fig. 3. The dense front clearly moves as the gaseous phase expands, so we consider the dense front as part of the gaseous phase [21].

III. QUANTITATIVE RESULTS

A. Phase diagram

The transition between the amorphous and gaseous phases, triggered by external perturbations, can occur only for a small range of vibrator accelerations. Below a low acceleration limit a_l , no perturbation is able to initiate a gaseous region that persists for at least one minute. Above a higher acceleration a_c , the spontaneous perturbations present in the amorphous state [observable in Fig. 2(d)], are sufficiently strong to trigger a propagating front within two minutes without an external perturbation. Figure 4 shows a_l and a_c as a function of the driving frequency f . The hyster-

etic region between a_c and a_l was found to be reproducible to within 1% of the driving acceleration. It decreases with increasing frequency, and a subcritical to supercritical transition takes place at $f_t \approx 120$ Hz. At higher frequencies, spontaneous excitation of the gaseous phase is observed for $a \leq a_c$, but any excitation decays slowly, and even large external perturbations fail to trigger a transition into the gaseous state. For $a > a_c$, on the other hand, areas of local excitation develop (often in numerous spots simultaneously) and propagate quickly throughout the system. The freezing and evaporation points of the crystalline phase are also shown in Fig. 4 as open symbols. At lower frequencies they fall in the middle of the hysteretic region, while at higher frequencies (roughly above f_t) they lie above the amorphous-to-gaseous transition. When the freezing point falls within the hysteretic region, the amorphous-to-gaseous transition leads to a gaseous phase for accelerations above

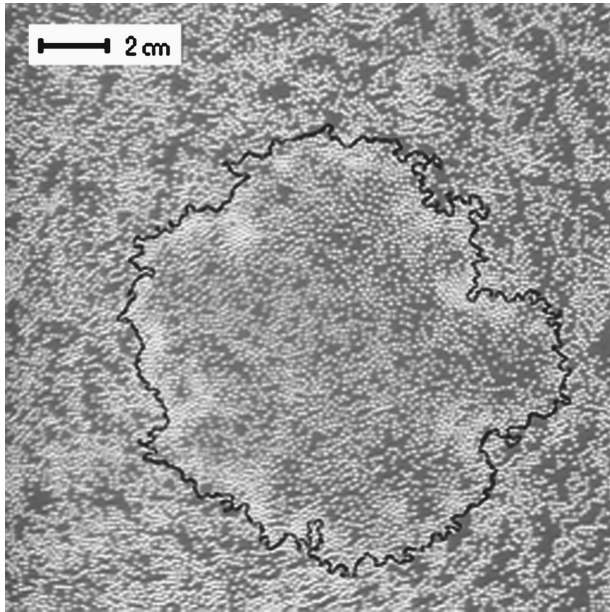


FIG. 3. Perimeter of the gaseous region (black line) superimposed on an image of the beads. The dense front clearly moves as the gaseous region expands.

the freezing point, and to coexisting crystalline and gaseous regions at accelerations below the freezing point. Additional measurements indicate that at a lower bead coverage $c = 0.25$, both a_l and a_c change by less than 5%.

B. Growth rates

The growth rate of the gaseous region can be measured between a_l and a_c (and even somewhat above a_c by applying an external perturbation quickly, before a spontaneous instability occurs). Figure 5 shows a double logarithmic plot of the area A of the gaseous region as a function of time after initiation by a perturbation. The measurements are carried

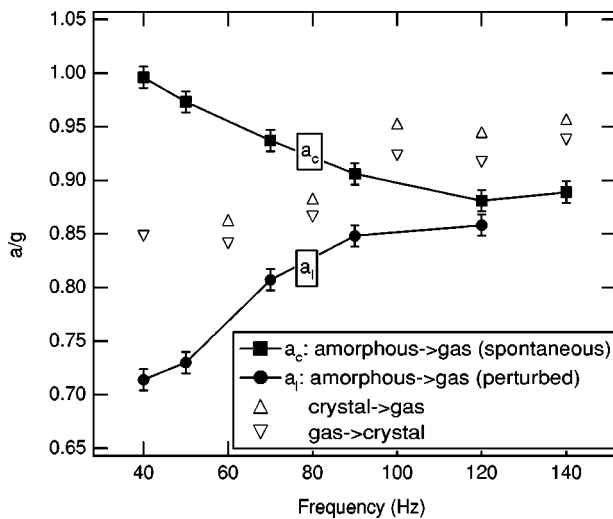


FIG. 4. Hysteresis of the amorphous-to-gaseous phase transition. Peak accelerations (normalized by g) for perturbed and spontaneous amorphous-to-gaseous transitions are shown (closed symbols). The freezing and evaporation points of the crystalline phase are also shown (open symbols).

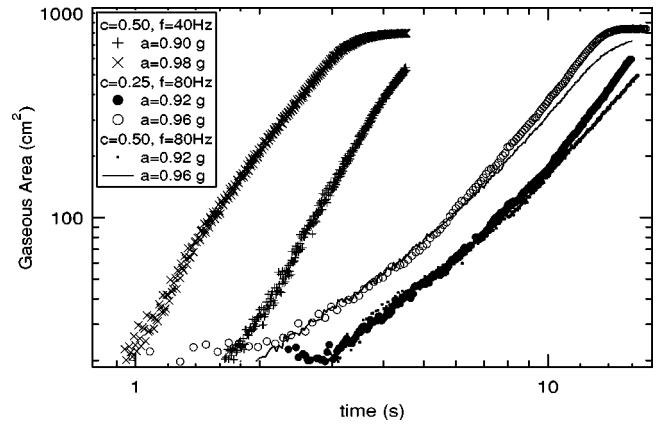


FIG. 5. Growth of the gaseous area A for various peak accelerations, bead coverages, and vibration frequencies. Smaller frequencies and larger accelerations lead to faster melting. Lower coverage leads to faster melting for large areas only.

out at 40 and 80 Hz for several peak accelerations within the hysteresis region and slightly above a_c . A doubling of the driving frequency slows the rate of growth of the gaseous area by approximately a factor of 4. Reducing the coverage does not change the initial growth rate, but leads to faster growth at later times. The total plate area $A_{\max} = 804 \text{ cm}^2$ limits the growth eventually. Figure 5 indicates that the growth, after an initial transient during which the dense front forms, can be approximately described by a power law with an exponent that is independent of acceleration and only slightly dependent on coverage. However, the exponent changes significantly with frequency.

A realistic description of the dynamics of growth of the gaseous region is complicated by the dynamics of the dense layer of beads ahead of the interface, which appears to slow the growth of the gaseous region. The dense layer is pushed uniformly by the advancing gas under some conditions (see Fig. 3), but in other cases crystalline regions form that do not move. The advancing front then grows around those regions, leaving behind slowly melting crystalline regions within the gaseous phase. At small coverages ($c \sim 0.05$) an interface between the amorphous and gaseous phase is not defined, as moving beads can pass many stationary beads between collisions. In this case the rate of increase in the number of moving beads is only limited by the mean interval between collisions with stationary beads.

It is nevertheless possible to describe the growth process approximately by noting that the growth rate dA/dt increases in proportion to the perimeter $P(A)$ between the gaseous and the amorphous regions as shown in Fig. 6 for areas between ~ 50 and 400 cm^2 . We find

$$dA/dt = \beta(a, f)P(A), \quad (1)$$

where the front velocity β depends on a and f . This indicates that the growth rate of the gaseous area is roughly proportional to the rate at which beads collide with the interface once a dense layer has formed. The front velocity β is shown in Fig. 7. It increases approximately linearly with a and goes through zero near $a_l(f)$.

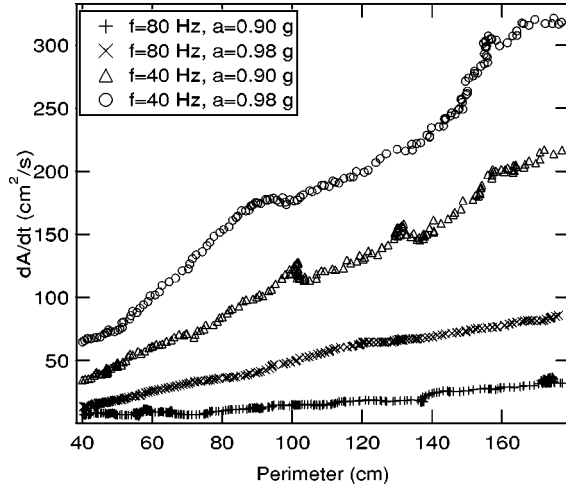


FIG. 6. Growth rate of the gaseous area dA/dt vs perimeter length $P(A)$ for several peak accelerations and vibration frequencies. The dependence is approximately linear. Both dA/dt and $P(A)$ are smoothed by means of a running average over 11 points.

IV. BOUNCING BALL MODEL

A. Model of unperturbed bouncing

The hysteretic nature of both the amorphous-to-gaseous transition (and to a lesser degree the crystalline-to-gaseous transition [14]) below $1g$ peak acceleration indicates that an energy gap exists between the states at rest (amorphous and crystalline) and the lowest energy excited steady state. The simplest model that can be used to explain the observed behavior is based on the dynamics of a single bead bouncing on a plate under sinusoidal vibrations $x_p = X_{\max} \cos(2\pi ft)$. The peak acceleration then is $a = X_{\max} 4\pi^2 f^2$. In a collision with the plate, the vertical bead velocity changes from v to v' :

$$v' = (1 + \alpha_{\text{plate}})\dot{x}_p - \alpha_{\text{plate}}v. \quad (2)$$

The bead at rest on the plate ($v' = v = \dot{x}_p$) always represents a stable steady state solution below $1g$ peak acceleration with an average bead energy

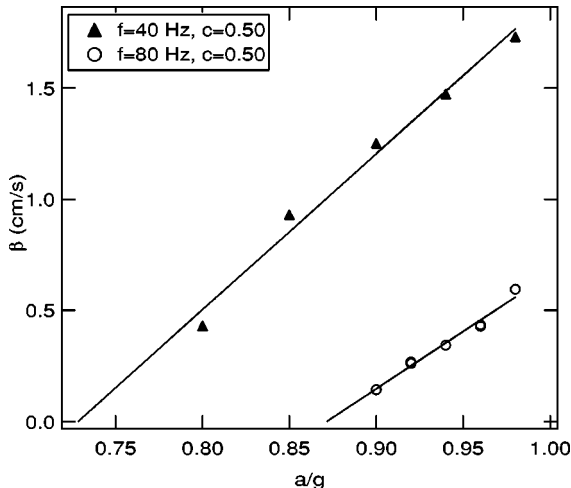


FIG. 7. Front velocity β vs peak plate acceleration (normalized by g) for different frequencies. The dependence is linear; $\beta \rightarrow 0$ near a_1 .

$$E_0 = \frac{a^2}{16\pi^2 f^2} \quad (3)$$

per unit mass. Additional steady states appear as the acceleration is increased. The first nonsticking steady state is a periodic bouncing with speed at the instant of collision given by

$$v' = -v = \frac{1 + \alpha_{\text{plate}}}{1 - \alpha_{\text{plate}}} \dot{x}_p. \quad (4)$$

For periodic bouncing every vibration period this requires $v' = g/(2f)$ and therefore (at the moment of contact)

$$E_1 = \frac{g^2}{8f^2} \quad (5)$$

per unit mass.

Other excited states that are observable under conditions similar to the experiment have higher energy. States with a period n/f , where the bead hits the plate on average once per vibration period and the impact pattern is repeated after n cycles, have an average energy of $E = (g^2/8n)\sum_k \delta t_k^2$. The minimum of E occurs for the simple periodic state $n=1$, where the time intervals between contacts with the plate δt_k are of equal length. Periodic or chaotic states with more than one plate impact per vibration period would have lower energy if they exist, but no such state was found in several calculations of basins of attraction using the bouncing ball program by Tuffillaro *et al.* [22] with parameters similar to the experiment. Those states therefore either do not exist, are unstable, or have a negligible basin of attraction.

For vibrator accelerations below $1g$ an energy gap therefore exists between the lowest energy steady state E_0 , which is the quiescent state, and the first excited state, a periodic bouncing state with energy E_1 . We propose that this energy gap determines the size of the hysteretic region. The measured decrease in the extent of hysteresis with increasing f may reflect the dependence of E_1 and E_0 on $1/f^2$.

The discussion so far, which is based on a one dimensional idealized model, has neglected perturbations. In the experiments, however, perturbations are important, as each bead is subjected to frequent interparticle collisions (which create and sustain horizontal motion) and to other perturbations (e.g., due to slight nonuniformities of the vibrator surface). Since a perturbation can trigger a transition into a different steady state, we include them in a model that can be simulated numerically.

B. Simulations

We follow the vertical movement of a single bead that is subjected to random perturbations of its vertical speed representing collisions with other beads. Our strategy is to determine the mean energy of the vertical motion and to compare it with that of the first excited state. Therefore, we do not track the horizontal speed of the particle. The time of perturbation events is selected randomly to create an average perturbation rate f_p . The velocity of the bead after a perturbation is $v' = v\alpha_{\text{bead}}r$, where r is a random number with Gaussian distribution and unit variance. To verify that the

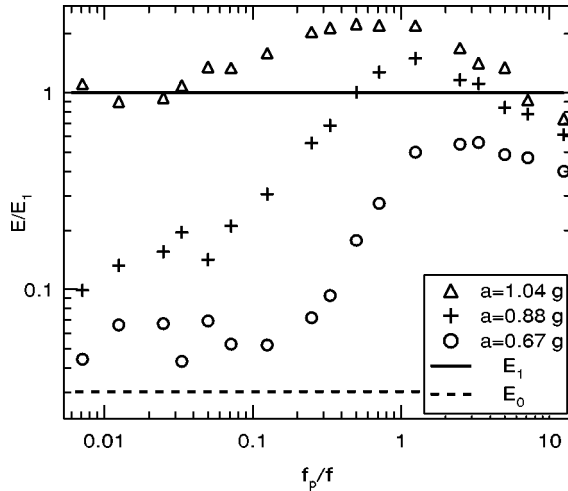


FIG. 8. Simulation: Average energy of a bead vs perturbation frequency, and comparison to energy of the lowest excited state E_1 and quiescent state E_0 (horizontal lines). Above a threshold acceleration $a \approx 0.8g$ the mean energy is larger than E_1 for a range of perturbation frequencies, a necessary condition for maintenance of the excited state in the many particle system.

results do not depend on the choice of the probability distribution, we also tried letting r be either ± 1 (randomly), with similar results.

Energy losses from interparticle collisions are included through α_{bead} . Bead and plate positions are calculated at fixed time intervals δt . The time of bead impact on the plate is calculated to within $O(\delta t^2)$. The bead is considered to be stuck if more than one bounce per time interval occurs. The coefficients used in the simulations were $f = 80$ Hz, $\delta t = 5 \times 10^{-5}$ s, and $\alpha_{\text{plate}} = \alpha_{\text{bead}} = 0.90$ (a rough estimate for the effective coefficient of restitution in the presence of bead rotations during impact). After an initial transient time of 25 s the average behavior of the bead is recorded for 475 s.

Since the calculations are carried out for one bead only, it is not determined *a priori* whether the perturbation rate is sustainable from interbead collisions in a many particle system. Many particles must be in an excited steady state to sustain continued collisions, since two quiescent particles cannot collide. Collisions are only able to maintain or increase the number of excited particles (a necessary condition for sustainability of the collision rate) if the steady state mean energy is comparable to E_1 . Significantly lower computed mean energies would actually not be sustainable in the many particle system, because collisions would lead to a loss of particles from the excited state, and the system would settle into the ground state attractor for all particles.

To check the conditions under which $E \geq E_1$, we show in Fig. 8 the average energy of the particle as a function of f_p . As the perturbation rate is increased, the energy of the particle first increases and then decreases. For comparison, the energies of a single bead in the lowest excited state E_1 and in the quiescent state E_0 of the unperturbed problem are shown as horizontal lines. At $a = 0.66g$, $E < E_1$ for all perturbation rates; therefore the particle will be trapped in the stationary ground state. For $a > 1g$, $E > E_1$ the mean particle energy is larger than E_1 (except for $f_p/f \gg 1$); here the quiescent state attractor does not exist. On the other hand, for $0.8g < a$

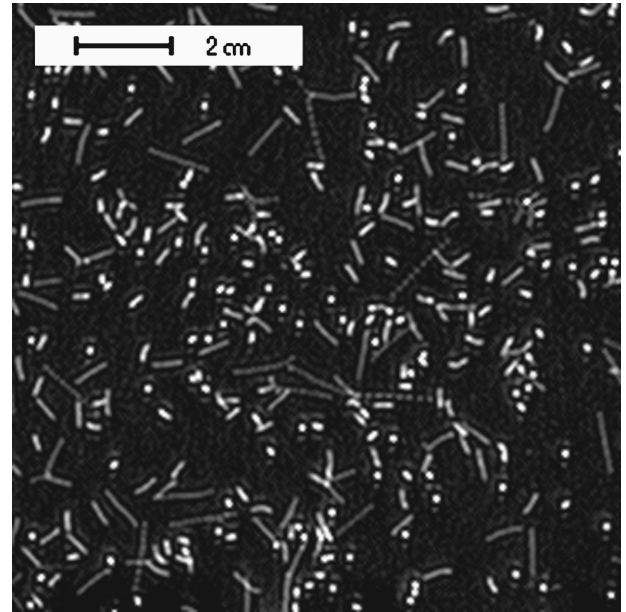


FIG. 9. Coexistence of beads at rest and moving beads in a steady state at low coverage. Ten successive images, taken at 30 frames/s were averaged ($c \approx 0.03$, $a = 0.94g$, $f = 80$ Hz). Moving beads appear as streaks and stationary beads as bright spots. Small clusters of beads at rest are clearly observable.

$< 1g$, $E > E_1$ for a limited range of f_p ; this regime will show hysteresis.

This explains qualitatively the occurrence of hysteresis for a many particle system that is sufficiently dense to provide the required collision rate. The propagating front is basically a self-sustained chain reaction.

V. DISCUSSION AND CONCLUSIONS

In this paper we have described the steady states and phase transitions of a two-dimensional layer of beads subjected to vertical vibrations below $1g$ peak acceleration. We start from a nearly static amorphous state created by temporarily stopping the vibration while all particles are moving rapidly. Free cooling of the granular layer creates a reproducible structure, with a correlation function that is not dependent on the driving frequency or acceleration over a wide range.

Starting from this quiescent disordered (“amorphous”) state, we observe a striking transition to the gaseous state, via a propagating front, as the vibration amplitude is increased. A region of hysteresis exists for which the amorphous phase remains metastable but a perturbation induces a transition. The final state can consist either of gas, or of crystal and gas, depending on the acceleration and coverage. The velocity of the propagating front β , which remains constant during the transition, increases linearly with peak plate acceleration a and decreases with increasing vibration frequency f .

A simple model, based on a single bouncing bead with random perturbations modeling collisions, exhibits behavior that is consistent with the experimental results. An energy gap between the ground and excited states of the single bouncing bead leads to a chain reaction that can sustain the excited state of the many-particle system under the right con-

ditions. The phenomenon is similar in some respects to a flame front.

The coexistence of the ground state and excited state attractor can be seen most clearly at low coverage, where collisions become sufficiently rare that beads can remain in the ground state attractor long enough to become stationary. In Fig. 9 we show a time average image at $c=0.03$. Moving beads appear in this figure as streaks and stationary ones as bright spots. In addition, small clusters of beads at rest are clearly observable.

The crystal-gas coexistence is another potential manifestation of the coexistence of the two attractors, in this case spatially separated by an interface. The simulation therefore suggests a possible origin of the crystalline phase discovered by Olafsen and Urbach [14]. As the acceleration is lowered, the maximum energy provided by the plate decreases until more energy is lost in collisions than can be supplied by the plate. Beads then come to rest in the dense regions of the plate where the collision losses are greatest. A crystal forms as the moving beads apply pressure to the cluster at rest, forcing it into the densest possible configuration. Bouncing

beads that hit the crystal tend to come to rest at the interface, increasing the size of the crystal but simultaneously decreasing the bead concentration in the remaining gaseous phase. Crystal growth stops when the density of the gaseous phase is lowered sufficiently that the (density-dependent) collision rate can be sustained in steady state.

We conclude that the rich variety of steady states that are observed just below 1 g peak vibrator acceleration are related to the coexistence of a ground state attractor that exists up to 1 g and excited state attractors that exist below 1 g. The rapidly propagating front of highly excited beads spreading in a sea of beads at rest, which can be triggered by one moving bead without a change in the acceleration amplitude, is a striking many body consequence of this coexistence.

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