Information operations with an excitable field

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(Received 19 November 1998)

It is well established that a traveling wave can be generated on an excitable field, which is described with a pair of partial differential equations for an activator and inhibitor. In the present paper, we use a numerical simulation to show that the traveling wave, or signaling pulse, can be transmitted from an excitable field to an opposing excitable field via an intervening passive diffusion field in a characteristic manner depending on the spatial geometry of the excitable fields. Using such characteristics, it is possible to design various kinds of logic gates together with a time-sequential memory device. Thus, these functions can perform time-sensitive operations in the absence of any controlling clock. It may be possible to accomplish these computations with excitable fields in an actual system, or to create a ''field computer'' composed of electronic active and passive units. [S1063-651X(99)14805-0]

PACS number(s): 05.45.−a, 05.70.Ln

I. INTRODUCTION

How do living organisms perform computations to maintain their lives in response to changes in the outer environment? The notion of Neumann-type computation may have originated in the search for an answer to this question [1]. One of the most important devices for realizing modern Neumann-type computations was certainly the vacuum tube, which made possible asymmetric signal propagation. Later, the vacuum tube was replaced by the transistor, which has since become an indispensable component of modern computers. Transistors are generally composed of diode contacts. Thus, modern computers rely on the characteristics of diodes, which are asymmetrical with regard to the direction of signal propagation. Another essential component of modern computers is step operation controlled by a ruling clock. Recently, parallel computation through the ''parallel'' connection of unit computers has attracted a considerable amount of interest. However, parallel operations with many unit computers unavoidably require a ruling clock. In contrast, there should exist no ruling clock in the parallel operations in the human brain. It is obvious that living organisms perform parallel operations without a ruling clock.

Over the past several decades, in addition to efforts to develop a faster and more reliable Neumann-type computer, various ideas about so-called neural computation have also been proposed. The McCulloch-Pitts model [2] is a typical model of an artificial neural network. In this model, individual neurons have the function of performing nonlinear transformations, such as step or sigmoidal functions, from input to output. The essence of such transformation is to sum up the input and to carry out a threshold operation on the summation.

In contrast to the above-mentioned framework in current studies on artificial neural networks, actual individual neurons in living matter exhibit the characteristics of nonlinear oscillators. The ability of neurons to generate limit-cycle oscillations implies that the breaking of time-reflection symmetry is an essential element in their function. Unfortunately, these important characteristics of actual neurons have been largely ignored in previous studies on artificial neural networks. A few studies have provided interesting examples of information processing with an excitable field, such as in image processing [3] and logic operations [4–6]. Okamoto, Sakai, and Hayashi reported the concept of chemical logic gates by considering a cyclic enzyme system [7]. Hjelmfelt and Ross extended this concept to bistable and/or excitable, chemical media, and proposed simple logic gates and circuits in a network of nonlinear chemical reactions. They also suggested a chemical neural network model with a chemical reaction in a continuous-flow stirred tank reactor (CSTR) [8].

In these studies, due to their inability to create a diode function, the signal propagates through the network of excitable fields in a ''reversible'' manner, i.e., the direction of signal propagation has no uniqueness, and the signal undergoes back-propagation along the same route between the input and output.

In the present study, we examined the possibility of constructing a ''parallel logic computer'' based on the characteristics of an oscillatory and/or excitable spatial field [9]. Recently, we found that unidirectional signal transmission with an excited propagating wave can be generated with a spatially asymmetric connection between excitable fields separated by a diffusion field, and revealed that such a diode function is created both in an actual experiment with an excitable-chemical system, the Belousov-Zhabotinsky (BZ) reaction, and in computer simulations [10,11].

As has been mentioned already, the diode is an indispensable fundamental element in modern electronics, including computers. This suggests that the realization of unidirectional information flow is essential for computation in general. It is to be noted that a similar situation exists for computations in living systems. Synaptic junctions between nerve cells exhibit unidirectional information flow, and thus act as a kind of diode.

In this study, using such diode characteristics, we extended our idea to develop various kinds of logic gates with an excitable field. We stress that parallel computation can be

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performed with a suitable spatial arrangement of excitable fields in the absence of any ruling clock.

II. METHODOLOGY

In multicellular living organisms, cells are arranged in three-dimensional space and separated by an intercellular field, such as the intercellular medium in neural systems. To make a simple model of interacting excitable cells, we consider here a geometrically asymmetric connection between different excitable fields, or active fields, to represent excitable-cell space, separated by a diffusion field (intercellular space), or passive field. The diffusion field corresponds to the synaptic junction between neurons. In the current studies on the artificial neural networks, the asymmetry in the signal propagation has been incorporated as the characteristics given by the individual researchers. Contrary to this, in the present study we have not introduced the asymmetric character for the direction of the signal propagation in *a priori* manner. It is noted that our model is not limited to networks of neurons, but can also be applied to other kinds of networks with excitable cells, such as the heart, excitable systems in plants, etc. [12,13]. We adapt the FitzHugh-Nagumo–type equation [14] to a spatially two-dimensional system to represent an excitable field:

$$\tau\frac{\partial u}{\partial t}=-\gamma\{ku(u-\alpha)(u-1)+v\}+D_u\nabla^2u,$$

$$\frac{\partial v}{\partial t}=\gamma u \tag{1}$$

where u is a variable that corresponds to the membrane potential in excitable cells and v is related to feedback ion currents through the membrane. It has been established that this set of equations can represent excitable and oscillatory behavior just as in nerve cells [14]. Note that the FitzHugh–Nagumo equation is a type of reaction-diffusion equation, where u represents the activator and v represents the inhibitor. As an additional benefit of this, using Eqs. (1) one can also compare the results obtained with a numerical simulation to those with an actual experimental system, i.e., with a chemical reaction (the BZ reaction [15]). The manner of wave propagation in the BZ reaction is interpreted with a set of partial differential equations, the so-called Oregonator [15]. The Oregonator exhibits almost the same mathematical structure as that found in Eq. (1), except for the existence of the diffusion term of the inhibitor, $D_v\nabla^2v$, in the Oregonator. In the usual experiment with the BZ reaction, the diffusions of both the activator and the inhibitor are alive. When the metal catalyst is immobilized in the medium by using a suitable reaction matrix such as a cation-exchange region, diffusion of the inhibitor becomes negligible [16]. This indicates that Eq. (1) can be examined in an actual reaction-diffusion system. Indeed, we have been performing this type of experiment with the BZ reaction and have confirmed the unique characteristics of wave propagation that were represented as numerical simulations in our last study [17]. Details of these experimental studies with the BZ reaction will be reported elsewhere.

The FitzHugh-Nagumo equation retains space-reflection, or space-inversion symmetry, but disrupts time-inversion symmetry. This implies that excitable fields have a dissipative nature, which is one of the essential aspects of life, i.e., biological phenomena on Earth are maintained under nonequilibrium open conditions. When we consider symmetry in the spatiotemporal structure with regard to propagation of the excited wave, the propagating wave should exhibit asymmetry with regard to the direction of propagation. This is because the time inversion is broken in the equation, and the wave propagation has a dimension of space divided by time. In the following numerical simulations, we will show that logic gates (AND, OR, NOT) and memories can be created by suitable geometric asymmetric arrangement of excitable and diffusion fields. These characteristics of logic operations and memory exhibit significant dependence on the time sequence.

Numerical simulations were carried out on Eq. (1) using the alternating direction implicit (ADI) method [18], with differential calculus and the Euler method. The boundary condition at the edge of the frame is taken to be no flux (Neumann condition), while that between the excitable and diffusion fields is taken to be free. For simplicity, the parameter γ is taken to be unity for the excitable field. In the diffusion field, γ is taken to be zero, i.e., only the diffusion term remains in Eq. (1), and $v=0$. In addition, $\tau=0.03$, $k=3.0$, $\alpha=0.02$, and $D_u=0.015$. The grid size is 250×250 points in a square lattice, and the time interval is taken to be $\triangle t=0.005$. The mesh size is taken to be $\frac{1}{4}-\frac{1}{6}$ of the minimum width of the diffusion field between the excitable fields.

III. RESULTS OF THE SIMULATION

In an excitable (reaction-diffusion) field, there exists a threshold value for parameter u: if the magnitude of the activator u is less than the threshold, no traveling wave is generated, while a traveling wave occurs if the activator becomes larger than the threshold. When one pulse signal propagates along an excitable field and arrives at the boundary between the excitable and diffusion fields, activator u diffuses into the diffusion field.

Figure 1 shows the geometrical dependence of the manner of diffusion of the activator u as an aftereffect of the arrival of the propagating waves on the interface between the active excitable and passive diffusion fields, as calculated using Eq. (1) with the conditions described in the methodology. In the left column the gray band is the traveling wave shown in a binary representation, where the magnitude of the inhibitor $v\geqslant0.23$. Hereafter, we adopt the spatial profile of the inhibitor v as a representation of the excited wave in the figures, since the manner of wave propagation with inhibitor v is more clearly represented in the pictures than the profile with activator u. In the right-hand column, the gray (diffusion) area indicates the region where the maximum u is above the critical value ($u_c\simeq0.184$) for inducing excitation in the opposing excitable field (see, e.g., Fig. 2). Thus, when the neighboring excitable field is located within this gray area, the traveling pulse can transmit through the passive diffusion field to the opposing excitable field. Based on these data, we can adjust the gap between the opposing excitable fields in a

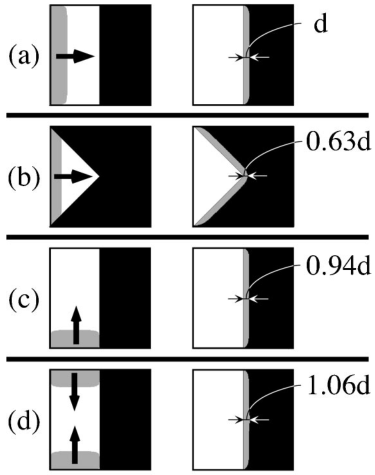

FIG. 1. Diffusion distance of activator u as calculated using Eq. (1), where the parameters and method are described in the methodology. In the left-hand column, the dark and white regions correspond to the diffusion and excitable fields, respectively, and the gray region indicates the propagating wave in binary representation with $v \geqslant 0.23$. In the right-hand column, the gray (in diffusion field) part represents the area where the maximum activator u is above the threshold value $u_c \simeq 0.184$, at which excitation is induced in the opposing excitable field. The diffusion distance in the planar boundary is taken as d; $d = 0.16$. In (a), (b), and (c), a single traveling wave is generated. In (d), collision of a pair of traveling waves is shown. The diffusion distance changes dramatically depending on the geometry of the excitable and diffusion fields and also on the manner of wave propagation.

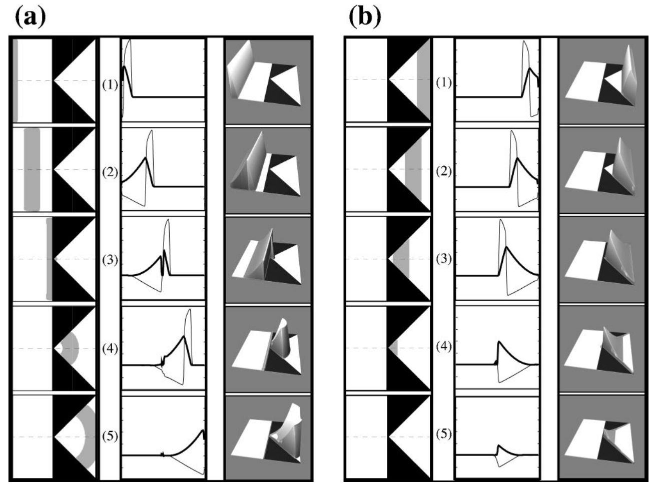

FIG. 2. A spatial arrangement exhibiting the characteristics of a diode, consisting of asymmetric excitable fields separated by a diffusion field. The gap width is taken to be 0.12 $(0.75d)$. The left-hand columns in (a) and (b) indicate the time series (from top to bottom) of the traveling wave with a time interval of 1.7 in (a) and 0.9 in (b) with binary representation for $v \geqslant 0.23$. The middle columns show the spatial profiles of activator u (thin line) and inhibitor v (thick line) along the direction of wave propagation (broken line in the left columns) in the field. The right-hand columns show the corresponding profiles of the inhibitor v in a quasi-three-dimensional representation. The dark and white regions are the diffusion and excitable fields, respectively. Unidirectional flow of the propagating wave is noted: in (a), the wave propagates through the diffusion field, whereas in (b), the wave fails to transmit through the junction.

quantitative manner, to generate the desired functions, as shown below.

A. Diode with excitable/diffusion fields

Figure 2 shows examples of the time series (from top to bottom) of the manner of propagation of a traveling wave with asymmetrical arrangement of excitable fields separated by a diffusion field. The left-hand columns in (a) and (b) give the binary representation with $v \geqslant 0.23$. The middle columns show profiles of the activator and inhibitor along the center (broken line in the left-hand columns) of the field. The right-hand columns show the corresponding profiles of the inhibitor v with a quasi-three-dimensional representation. In the geometrical arrangement with plane and wedge boundaries on the excitable field, the propagating wave transmits from the left to the right [Fig. 2(a)], whereas the inverted wave fails to cross the junction in the other direction [Fig. 2(b)].

Thus, it is clear that the characteristics of a diode are produced with suitable asymmetric spatial arrangement of the excitable fields [10,11]. The origin of the diode character is obvious, by referring to the critical region of the diffusion of the activator shown in Fig. 1. Note that the appropriate distance for generating the diode character can be easily evaluated from the results of the calculation regarding the diffusion area of the activator.

B. Logic operation

From the perspective of Boolean algebra, all logic functions can be represented as the sum of product terms or the product of sum terms, and consist of the fundamental logic functions: logic sum (OR), negation (NOT), and logic product (AND). Thus, any kind of logic function can be attained if one can realize OR, NOT, and AND gates. We found that these three gates can be created by varying the spatial geometry of excitable/diffusion fields.

Logic sum (OR). Figure 3 depicts the operation of logic summation, with binary representation (left) and a quasi-three-dimensional inhibitor profile (right). The field geometry is arranged so that two different inputs come from the left and the top, and the output after the sum operation goes to the lower right. The truth value 1 or 0 is represented by the presence or absence of an excitable wave, respectively. Figure 3(c) shows the operation $(1 \vee 1 \rightarrow 1)$, whereas Fig. 3(a) shows $(1 \vee 0 \rightarrow 1)$, and Fig. 3(b) shows $(0 \vee 1 \rightarrow 1)$. The time difference between the two input signals must fall within a certain range. When this difference is greater than a certain threshold (in the present case, $\triangle t_c = 4.055$), two output signals are generated, in accordance with the time difference between the two input signals. Time reflection can never occur, i.e., when the input comes from the lower right, the signal fails to propagate beyond the narrow gap. With time inversion, a pulse from the lower right fails to transmit to either of the pathways in the other direction (not shown). The existence of the narrow gap in the diffusion field inhibits back-propagation of the signal.

Negation (NOT). Figure 4 shows the arrangement needed to perform the NOT operation, where the input signal from the left is the reference for the NOT operation. When an

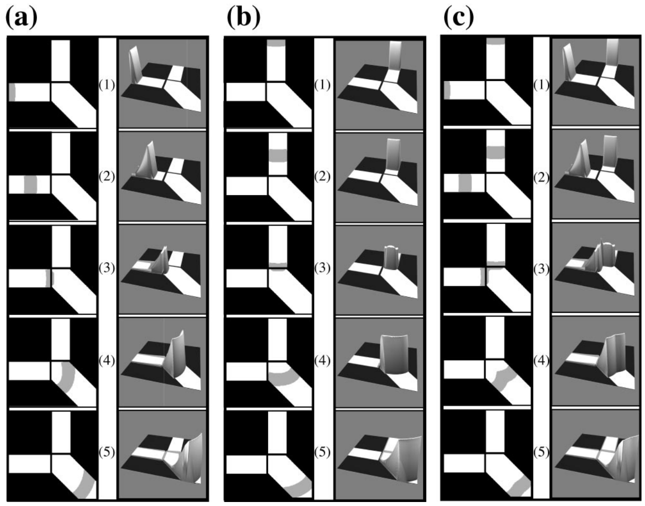

FIG. 3. Logic sum operation (OR). The width of both gaps is 0.156 ($0.975d$). The representation is similar to that in Fig. 2. The time interval is 2.1. In (c), a pair of input signals arrives at the device simultaneously ($1\vee 1\rightarrow 1$). In (a), a signal arrives only from the top ($1\vee 0\rightarrow 1$), and in (b), a signal arrives only from the left ($0\vee 1\rightarrow 1$).

input signal comes from the top, as in Fig. 4(a), the input and reference signals collide and negate each other ($\neg 1\rightarrow 0$). When there is no input, as in Fig. 4(b), the reference signal propagates through the junction and induces an output signal ($\neg 0\rightarrow 1$). To create the NOT operation in the excitable field, a reference signal is essential. If we denote t_i as the transit time of the input signal to the junction and t_r as that of the

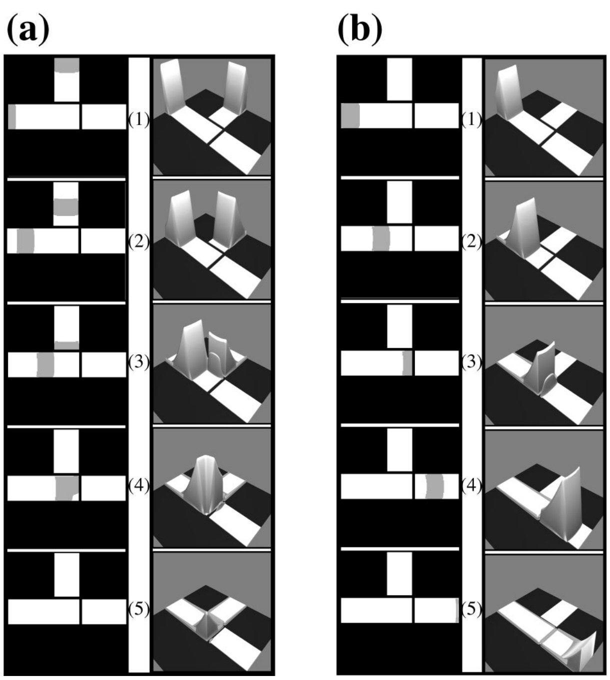

FIG. 4. Negation operation (NOT). The width of both gaps is 0.156 ($0.975d$). The representations are the same as in Fig. 2 The time interval is 1.4 in (a) and 2.2 in (b). The signal from the left is the reference for the timing for the negation operation. In (a), the input signal disappears upon collision with the reference signal ($\neg 1\rightarrow 0$), whereas in (b), the reference signal transmits through the gap and induces an output signal ($\neg 0\rightarrow 1$).

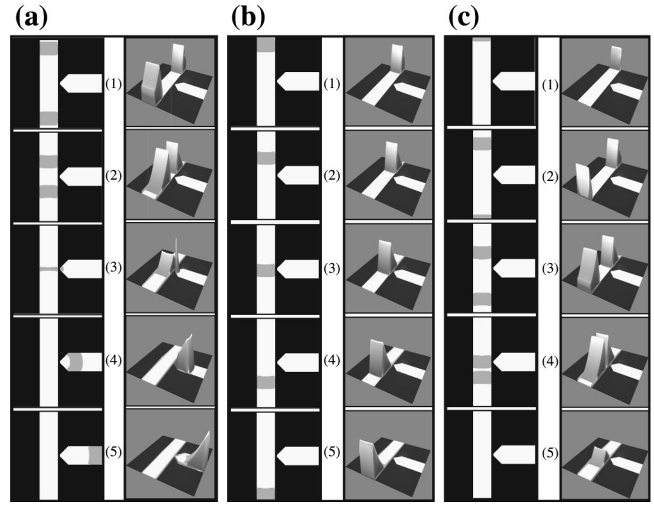

FIG. 5. Time-coincidence detector. The gap width is 0.156 ($0.975d$). The representations are the same as in Fig. 2. The time interval is 1.9 in (a), 1.8 in (b), and 1.5 in (c). In (a), the two input signals arrive at the same time and induce an output signal to give an AND logic operation ($1\wedge 1\rightarrow 1$). (b) shows ($1\wedge 0\rightarrow 0$). In (c), input signals arrive with a time difference $\triangle t=1.5$ and cause no output signal ($1\wedge 0\rightarrow 0$).

reference signal, the condition $t_i<t_r$ should be satisfied to perform the NOT operation. It is also to be noted that the time difference t_r-t_i should be less than a certain value, being similar to the effect of the time difference in the *AND* operation, as will be described below. Time sensitivity is, thus, an essential aspect of this NOT operation in an excitable field.

Logic product (AND). Figure 5 shows the arrangement for the AND operation. When the two inputs arrive at the center almost simultaneously, as shown in Fig. 5(a), a new wave is generated and propagates through the output channel, ($1\wedge 1\rightarrow 1$). Figure 5(b) shows the operation ($1\wedge 0\rightarrow 0$). When the two inputs arrive with a time difference, as in Fig. 5(c), no wave is induced at the output channel; $1(t_1)\wedge 1(t_2)$ produces 1 when $|t_1-t_2|\leqslant\triangle t_c$ and 0 when $|t_1-t_2|>\triangle t_c$. In the system given in Fig. 5, $\triangle t_c=0.235$. Here, the relative timing of the arrival of the input pair determines the operation of the system. The time difference at which the AND operation is achieved can be adjusted by changing the gap and geometry in the device. Again, back-propagation of the signal is inhibited due to the presence of a gap in the passive field.

C. Detection of a time difference

As an additional example of time-sensitive operation, Fig. 6 exemplifies the discrimination of a time-difference between a pair of input signals. The input signals arrive from the top and the bottom of the excitable field. On the right, three excitable fields are arranged to detect the time difference. It is clear in Fig. 6 that the time difference between this pair of input signals can be measured with this device. If one could arrange many detectors along a pathway where the input signals arrive as traveling waves, it would be possible to detect greater time differences with a finer time resolution. Interestingly, a similar array is found in the auditory nerves in the owl [19,20].

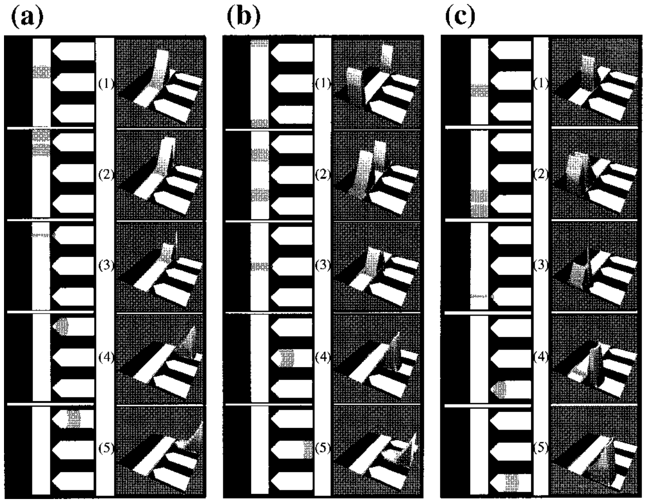

FIG. 6. Time-difference discriminator. The gap width is 0.156 $(0.975d)$ and the distance between the detectors is 0.988. The representations are the same as in Fig. 2. The time interval is 2.2. Two input signals arrive from the top and bottom, and the time difference between the entering signals is $\triangle t = +6,\ 0,\ -6$ in (a), (b), and (c), respectively. Depending on the difference in the timing between the pair of input signals, a detector performs a product operation, $1(t_1) \wedge 1(t_2) \rightarrow 1(\triangle t \leqslant \triangle t_c)$, $\rightarrow 0(\triangle t > \triangle t_c)$, $\triangle t_c = 0.235$.

D. Memory

In any computational machine, memory is indispensable. We show here that a pulse input and time-sequential input can be recorded on a circular device with an excitable field.

Figure 7 shows an example of a memory device consisting of an excitable ring together with junctions at which to write and/or to read. If an input signal arrives from the lower left, the wave follows the circle in a clockwise manner, and the information is stored as a traveling wave. When an input signal enters from the lower right during the clockwise movement of a pulse, the memorized pulse is erased due to collision of the counterpropagating waves. When multiple pulses are used as input signals, the time sequence of the input signals is stored as signals in the circular track. Thus, an excitable field can be used to create ''dynamic memory,'' which can store the input time series, erase old memory, and read out the original time sequence of the input pulses. It is also possible to read out such pulse trains in a repetitive manner when a specific readout channel is situated close to the circular track.

IV. DISCUSSION

Depending on the spatial geometry of the active excitable field and passive diffusion field, three different characteristics, i.e., bidirectionality, unidirectionality (diode), and propagation failure, can be created. With the combination of three characteristic gates, it should be possible to construct a parallel computing machine with no ruling clock. These interesting properties of logic operations and memory can actually be realized in a chemical reaction system, i.e., the BZ reaction. It has been confirmed that diode characteristics are generated in actual experiments with the BZ medium [10]. We have also performed experiments with the BZ medium and found that almost all of the characteristics observed in the present simulation are also observed in the real world, i.e., in the BZ reaction. The problem with the BZ reaction is its fragility; in other words, the wet-reaction medium is not suitable for a practical computing machine.

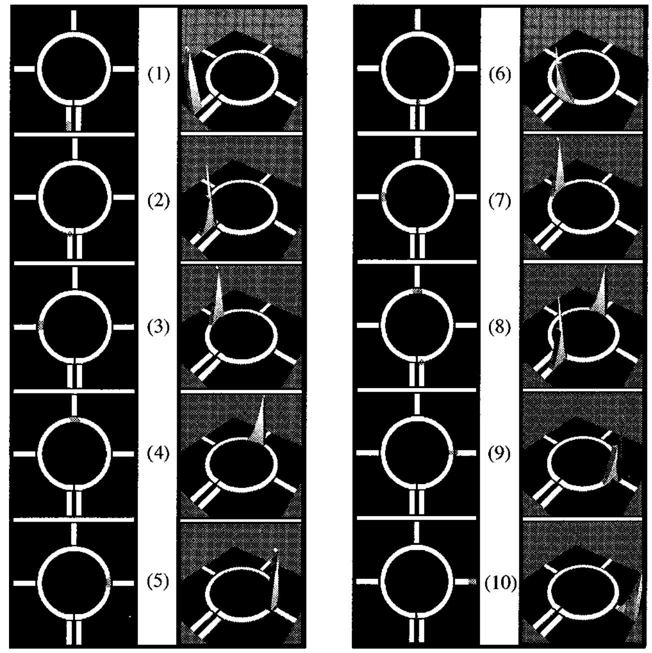

FIG. 7. Example of a memory device with the ability to write, read, and erase. All of the widths are taken to be 0.156. The representations are the same as in Fig. 2. The times of the frames are $t = 0,\ 3,\ 10.25,\ 17.25,\ 24.25,\ 31.75,\ 39.25,\ 46.5,\ 54,$ and 56.75, respectively. The frames progress from top left to bottom right. Input signals arrive from the lower left and follow the ring clockwise, indicating that the input signal is memorized in the circular track. When a control signal arrives from the lower right, the two signals collapse and induce an output signal to the right. Thus, the control signal can be used to write to the output channel and to erase the traveling wave on the circular track.

Therefore, we have developed a strategy to construct a computing machine with a dry electric circuit. It is possible to reproduce the essential characteristics of an excitable field and a diffusion field by making a distributed line or field of electric units, as shown in Figs. 8(a) and 8(b). The circuit connecting the above units through a resistor is written using diffusive interaction. The dynamic characteristics of the distributed lines of units (a) and (b) are described with partial differential equations, respectively, Eqs. (2),

$$C\frac{dV}{d\tau} = -i + f(V) - i_0; \quad f(V) = -kV(V-1)(V-\alpha),$$

$$L\frac{di}{d\tau} = V \tag{2}$$

and Eq. (3),

$$C\frac{dV}{d\tau} = -i - i_0, \tag{3}$$

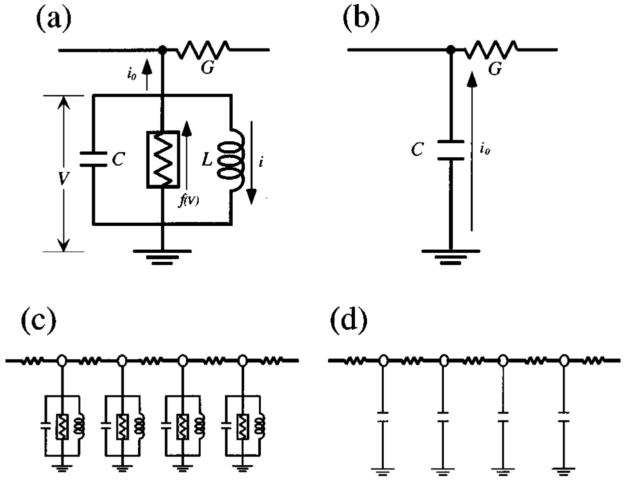

FIG. 8. (a) and (b) Solid electronic circuits, exhibiting the characteristics of active excitable and passive diffusion units, respectively. The active unit consists of a negative resistor, coil, capacitor, and resistor for connection. A tunnel diode can be used as a negative resistor. In our system, the characteristics of a diode are not necessary for the excitable unit. Instead, nonlinearity of the resistor is essential, and units (a) and (b) are not necessarily asymmetric with respect to the two terminals. The one-dimensional series of (a) and (b) constitutes distributed active and passive lines [21], corresponding to excitable and diffusion fields. (c) and (d) Schematic drawings of excitable (active) and diffusive (passive) distributed lines.

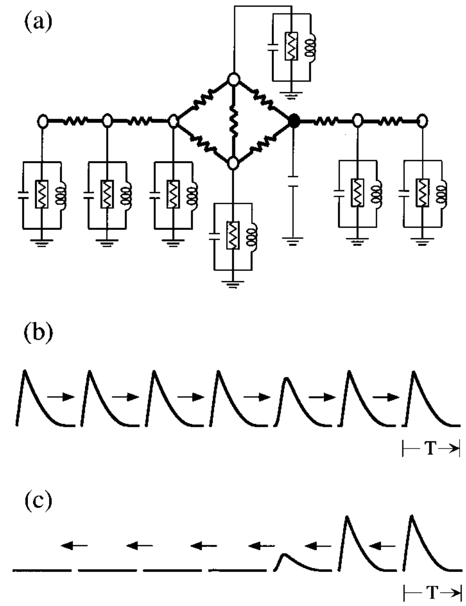

FIG. 9. (a) Circuit with the characteristics of a diode consisting of active and passive units. (b) and (c) Firing patterns in the individual electrical units in the circuit (a), as simulated by coupled ordinary differential equations of the corresponding electric circuit. T shows the time interval, $T=400$. When the left-end unit is excited, the electrical pulse signal propagates toward the right through the passive unit, as shown in (b). When the right-end unit is excited, the signal fails to propagate toward the left, as shown in (c). The parameters of Eqs. (2) and (3) are $C=1.0$, $L=30.0$, $G=0.025$, $k=10.0$, and $\alpha=0.02$.

where parameters in the circuit are those given in Figs. 8(a) and 8(b).

It is clear that Eq. (2) corresponds to the FitzHugh–Nagumo equation [21], and that Eq. (3) represents the diffusion field. The distributed lines of Figs. 8(a) and 8(b) correspond to systems that conform to Eq. (1): $\gamma\neq 0$ in (a) and $\gamma=0$ in (b). The correspondence of excitable and diffusion fields to the continuous distribution lines of Figs. 8(a) and (b), and Figs. 8(c) and 8(d), indicates that one can construct a diode as a discrete connection of such units. In fact, a diode can be created as in the circuit in Fig. 9(a). The ability to create the characteristics of a diode with units as in Figs. 8(a) and 8(b) indicates that unsynchronized parallel operations are also possible with circuits composed of these units. Thus, logic operations together with time-sequential memory could be implemented in a solid circuit by using modern electronic technology, as in large-scale integration (LSI) production. Extension of the idea of continuous-field operations to a circuit with discrete units should promote the study of field computation. In addition, the resulting diode can be controlled with the application of suitable feedback, as in making a transistor from a diode in modern electronics. Detailed investigations of such devices equipped with feedback are under way in our laboratory.

Since logic operations and memory are processed using traveling pulses, there should be no bottleneck for connections to usual Neumann-type computers. More than three decades ago, the concept of a neuristor was proposed by Crane [22], indicating that one can perform logic operations with distributed active lines. Although the idea was very interesting, he did not present any in-depth consideration of diode characteristics. He did propose a rather complicated circuit to produce a possible single diode, with consideration of the diffusive effect only with respect to the inhibitor. In the present article, we have shown that suitable geometric arrangements of the active and passive fields can create various useful functions for computations. Further experimental and theoretical studies are needed.

V. CONCLUSION

In the present paper, we have shown that elemental logic devices, as well as diodes and time-difference detectors, can be created with excitable fields by using suitable geometrical arrangements relative to a diffusion field. Time computations, such as detecting a time difference between inputs, can be easily performed in the absence of a governing clock, or without any synchronous operations. Through recent neural research by biological scientists, it is becoming obvious that the spatial geometry of individual neurons is crucial to neural computation [23]. By making a network with logic gates and memory, through a suitable arrangement of excitable and diffusion fields, we propose that a parallel computer without a CPU, or a field computer, may be constructed in the future.

ACKNOWLEDGMENTS

We are grateful for the kind comments of Dr. Péter Érdi, Dr. Ágota Tóth, and Professor Michael Menzinger. The present study was supported in part by the Artificial Intelligence Research Promotion Foundation.

[1] J. Neumann, in *The Computer and the Brain* (Yale University Press, New Haven, 1958), p. 66.
[2] W. S. McCulloch and W. Pitts, Bull. Math. Biophys. **5**, 115 (1943).
[3] L. Kuhnert, K. I. Agladze, and V. I. Krinsky, Nature (London) **337**, 244 (1989).
[4] Á. Tóth and K. Showalter, J. Chem. Phys. **103**, 2058 (1995).
[5] O. Steinbock, P. Kettunen, and K. Showalter, J. Phys. Chem. **100**, 18 970 (1996).
[6] R. Suzuki, IEEE Trans. Biomed. Eng. **14**, 114 (1967).
[7] M. Okamoto, T. Sakai, and K. Hayashi, BioSystems **21**, 1 (1987).
[8] A. Hjelmfelt and J. Ross, Proc. Natl. Acad. Sci. USA **91**, 63 (1994).
[9] K. Yoshikawa, I. Motoike, and K. Kajiya, IEICE Trans. Electron. **E80-C**, 931 (1997).
[10] K. Agladze, R. R. Aliev, T. Yamaguchi, and K. Yoshikawa, J. Phys. Chem. **100**, 13 895 (1996).
[11] T. Kusumi *et al.*, Chem. Phys. Lett. **271**, 355 (1997).
[12] A. T. Winfree, *The Geometry of Biological Time* (Springer, Berlin, 1990).
[13] J. D. Murray, *Mathematical Biology* (Springer, Berlin, 1990).
[14] R. FitzHugh, Biophys. J. **1**, 445 (1961).
[15] *Oscillations and Traveling Waves in Chemical Systems*, edited by R. J. Field, and M. Burger (John Wiley and Sons, New York, 1985).
[16] K. Yoshikawa, R. Aihara, and K. Agladze, J. Phys. Chem. A **102**, 7649 (1998).
[17] I. Motoike and K. Yoshikawa, in *Proceedings of ICONIP'98, The Fifth International Conference on Neural Information Processing*, edited by S. Usui and T. Omori (Ohmsha, Ltd., Tokyo, 1998), p. 720.
[18] *Numerical Recipes in C*, edited by W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery (Cambridge University Press, New York, 1992).
[19] L. A. Jeffress, J. Comp. Physiol. Psychol. **41**, 35 (1948).
[20] M. Konishi *et al.*, in *Auditory Function*, edited by G. M. Edelman, W. E. Gall, and W. M. Cowan (John Wiley and Sons, New York, 1988), p. 721.
[21] J. Nagumo, S. Arimoto, and S. Yoshizawa, Proc. IRE **50**, 2061 (1962).
[22] H. D. Crane, Proc. IRE **50**, 2048 (1962).
[23] H. Agmon-Snir, C. E. Carr, and J. Rinzel, Nature (London) **393**, 268 (1998).