Surrogates for finding unstable periodic orbits in noisy data sets

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Recently, searches for unstable periodic orbits in biological and medical applications have become of interest. The motivations for this research range, in order of ascending complexity, from efforts to understand the dynamics of simple sensory neurons, through speculations regarding neural coding, to the hopeful development of new diagnostic and/or control techniques for cardiac and epileptic pathologies. Biological and medical data are, however, noisy and nonstationary. Findings of unstable periodic orbits in such data thus require convincing assessments of their statistical significance. Such tests are accomplished by comparison with surrogate data files designed to test an appropriate null hypothesis. In this paper we test surrogates generated by three different algorithms against correlated noise as well as stable periodic orbits. One of the surrogates is new, and has been specifically designed to preserve the shape of the attractor. We discuss the suitability of these surrogates and argue that the simple shuffled one correctly tests the appropriate null hypothesis. [S1063-651X(99)00505-X]

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I. INTRODUCTION

Searches for the signatures of unstable periodic orbits (UPOs) date from early analyses of nonlinear physical systems and were motivated by the development of new techniques for the control of chaos [1]. Later the control of chaos was demonstrated in biology using a rat brain slice preparation [2], but those findings were not tested against surrogate data. Surrogate testing and concurrent assessments of the statistical significance of the results for typical biological substrates are essential, because high-dimensional behavior, or ''noise'' invariably contaminates the dynamics, and because such substrates are essentially nonstationary. Thus to be certain that UPOs have been detected in such experiments, a running assessment of the statistical precision of the findings is necessary.

A simple, statistically based recurrence method for counting the signatures of encounters of the general trajectory with an UPO of specific period p in noisy data files has been developed. It was demonstrated in a noise contaminated, periodically forced Van der Pol oscillator [3] and later in the hydrodynamically forced crayfish caudal photoreceptor system [4,5]. Moreover, the method has recently been shown to be effective for detecting transient appearances and disappearances of UPOs [6–8], and thus is effective for analyses of nonstationary systems. Later a more complex method based on a dynamical transform of the data and a regrouping of the encounters with a fixed point of specific period emerged [9–11]. Other methods have more recently been put forth [12,13] but the questions we address here relate to the former two. They both operate on time series in the form of sequences of time intervals, $T_1, T_2, \ldots T_n, T_{n+1}, \ldots$. The particular signature which they both search for is, in the first instance, evidence of *crossings* of the line of periodicities, $T_{n+p} \equiv T_n$. We confine this discussion to cases of three-dimensional motion projected onto a two-dimensional surface of section. Such crossings may be represented by a *specific* sequence of points in the phase space T_n, versus T_{n+p}.

In the case of the simple method [3–5] the signature of an encounter is usually represented by a sequence of several points as shown by the example given below. The transform method begins by searching for groups of points near the line of periodicities, thus signaling the possible existence of a periodic fixed point. What is important to note is that both the simple recurrence and the transform methods depend upon the recognition in the data of short sequences of time intervals *that exhibit a very specific behavior*. These sequences are therefore highly correlated over short times. But they carry much more information about the dynamical object of which they are the signature than simple exponential temporal correlations.

The primary experimental observable is the number N of times the general trajectory of the system encounters a signature. The problem is to accurately assess the statistical confidence level associated with any measurement of N. Normally the statistical confidence level can be assessed by testing the findings using suitable surrogates constructed from the original data files [14] or from random number sets. Surrogate data are a widely used tool in testing null hypotheses. They are applied for rejecting hypotheses about the structure of a given set of data, most often, for example, about the type of correlations that may be inherent within the data. Ideally surrogate data retain all or most of the properties of the original data, but are randomized with respect to signatures which indicate the presence of the dynamics sought, in this case the signatures of encounters with UPOs. The simplest surrogates are obtained by randomly shuffling (SS) the locations of the data points in the original file. The amplitude adjusted, Fourier transformed (AAFT) surrogates developed by Theiler *et al*. [14] are another commonly used algorithm. Below we have developed a third type that can be called the attractor surrogate (AS).

Each of these surrogates preserves some property of the original data set, while more-or-less effectively randomizing the signature being sought. The SS surrogates preserve the time interval histogram, but increase the disorder as indicated by an increase of the bandwidth of the power spectrum. AAFT surrogates preserve short time correlations which may exist within the data; that is, they preserve the power spectrum. This surrogate was designed to protect analyses from being deceived by the presence of exponential correlations in otherwise random data sets; that is, fooled by ‘‘colored noise.’’ Finally, our new surrogate, AS, preserves both short time correlations and the *shape* of the attractor in the phase space; that is, the shape of the cloud of time intervals plotted in the T_{n+p} versus T_n plane. Since both algorithms [3,9] search for signatures based on phase space topology, that is, sequences of time intervals which trace a specific shape indicating the presence of an unstable periodic fixed point, the AS surrogate is appropriate for testing the null hypothesis when using these two methods from this point of view.

The purpose of this paper is to test the effectiveness of these three surrogate types and their immunity to the effects of colored noise when used with the simple recurrence method [3–5]. It is important to perform these tests, since many research groups are now using the simple recurrence method for analyzing a wide variety of biological and medical data. These include findings of UPOs in thermally sensitive sensory neurons [6,7,15] and hypothalamic neurons [7,8], immature hippocampal networks from rabbit brain slices [16], human epileptic [17] and cardiac [18] activities, synaptic discharges from a central neuron [19], and human coordinated movements [20]. In all of these experiments the simple recurrence method was adopted. Moreover, finding UPOs and estimating their eigenvalues rapidly in real time is critical to applications involving the control of, for example, pathological cardiac or epileptic dynamics. Thus the simplest algorithm which consumes the least CPU time, but remains effective and accurate, will be advantageous. We conclude below that the SS surrogates in conjunction with the simple recurrence method best fulfill this requirement.

This paper is organized as follows. In Sec. II, we define the signature of an encounter of the general trajectory with a period-1 UPO. We show an example encounter taken from rat facial cold receptor data [6]. Having defined the encounter, only then is it possible to state the null hypothesis. In Secs. III and IV, we test the three surrogates against one- and two-dimensional Ornstein-Uhlenbeck (OU), or colored, noise and, in Sec. V, against a noise driven FitzHugh-Nagumo (FN) dynamics [21]. Except for the FN dynamics, these data files were of a length typically found in biology. We test the null hypothesis using a known number of encounters inserted into the random data sets. In Sec. VI, we outline the algorithm for generating the new AS surrogates. Finally, in Sec. VII, we summarize our findings and conclude with a brief discussion. Our results indicate that all three surrogates are equally effective and that the method used with any one of the three surrogates is not deceived by colored noise or by noisy *stable* periodic orbits (SPOs). Only the SS surrogate used with the simple recurrence algorithm is, however, effective in distinguishing SPOs in low noise data sets.

II. THE SIGNATURE, THE NULL HYPOTHESIS, AND STATISTICAL SIGNIFICANCE TESTING

All the data with which we are concerned are in the form of sets of time intervals. In the case of our previous experiments, the data are sets of interspike time intervals obtained from extracellular recordings of the activity of sensory neurons. However, any set of time intervals or any sequence of points from an embedding of a continuous dynamics are appropriately analyzed with this algorithm. An example data set, from the rat facial cold receptor [6], is shown in Fig. 1(a), where we have plotted the first return map of the attractor, T_{n+1} versus T_n. The *signature* of an encounter of the general trajectory with the period-1 UPO is defined as follows: *any set of three points that approach the line of periodicities* (45° *line) with sequentially decreasing perpendicular distances, followed by a set of three points that depart from it with sequentially increasing perpendicular distances*. An example signature obtained from the data set is shown in Fig. 1(c). One point, number 3, is common to the approaching and departing sequences. The specific set of interspike time intervals that makes this example is given by

$$[T_n, T_{n+6}] = [50,108,72,87,73,121] \quad \text{ms.} \tag{1}$$

Since the signature has been defined, we can now state the null hypothesis: *Random files, including those with temporal correlations, contain encounters with the defined signature in numbers statistically indistinguishable from those found in data files containing the signatures of UPOs*. The statistical significance is assessed with a well-known measure:

$$K = \frac{N - \langle N_s \rangle}{\sigma}, \tag{2}$$

where N is the number of encounters with the defined signature found in the original data file, $\langle N_s \rangle$ is the mean number ensemble averaged over the surrogate files, and σ is the standard deviation. Assuming Gaussian statistics, $K \geqslant 3$ indicates that the finding is significant with greater than 99% confidence; that is, the probability p that the finding in the original data set is a random result is $p \leqslant 0.01$ [22].

III. ORNSTEIN-UHLENBECK NOISE

We now test the susceptibility of the three surrogates to deception by linearly correlated, colored noise as well as their effectiveness in detecting the signatures of known UPOs. First, we generate this noise using the one-dimensional Ornstein-Uhlenbeck process [23],

$$\dot{x} = -\frac{1}{\tau} x + \frac{1}{\tau} \sqrt{2D}\, \xi(t), \tag{3}$$

where τ is the correlation time, and $\xi(t)$ is a Gaussian, δ-correlated, random process with zero mean and intensity D. Having generated $x(t)$, we then ‘‘threshold’’ it by tabulating the sequence of time intervals between its positive going zero crossings [24,25]. There is a problem with this process in the case of one-dimensional OU noise in that the threshold crossing rate is theoretically divergent as has been pointed out by Jung [26]. This is related to the fact that the variance of the derivative $\dot{x}(t)$ is unbounded. The correlation function of $x(t)$ is exponential and is given by

$$\langle x(t) x(s) \rangle = \frac{D}{\tau} \exp\left[-\frac{|t-s|}{\tau} \right]. \tag{4}$$

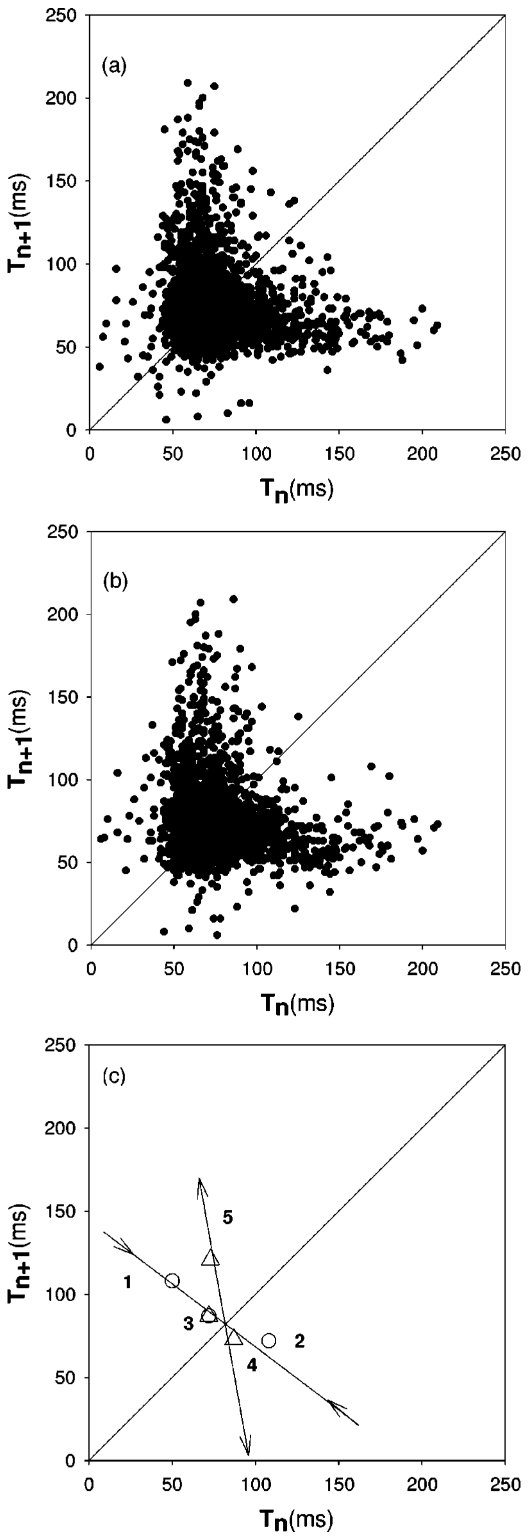

FIG. 1. (a) Interspike time intervals from the rat facial cold receptor [6]. Data are plotted as a return map (T_n versus T_{n+1}). Note the asymmetric shape of the data, indicating short term correlations. (b) Surrogate data from above figure, using AS algorithm. The shape of the return map is nearly identical to that of the original data, whereas the SS and AAFT surrogates destroy the return map distribution. (c) A typical UPO encounter from the rat facial cold receptor data. Three points converge towards the line of periodicity (circles) followed by three diverging points (triangles).

TABLE I. Linear, exponentially correlated noise: one-dimensional Ornstein-Uhlenbeck process.

$\tau=$	25 ms	50 ms	75 ms	100 ms
	(a) Noise alone			
N	96	85	92	87
$\langle N_s \rangle$	95	98	98	100
σ	11.3	9.5	9.2	8.7
K(SS)	0.09	−1.37	−0.65	−1.49
K(AAFT)	−0.17	−1.10	−0.55	−1.12
K(AS)	−0.12	−0.96	−0.48	−1.27
	(b) Noise plus added encounters, (E)			
$N+E,(E)$	166 (70)	150 (65)	157 (65)	152 (65)
$\langle N_s \rangle$	107	111	111	113
σ	11.7	10.3	13.9	12.9
K(SS)	5.02	3.78	3.30	3.02
K(AAFT)	4.78	3.12	5.28	3.12
K(AS)	4.16	2.66	2.65	3.28

The correlation function of the generated train of zero crossings has a rather complicated structure [27]: for small time lag it shows an algebraic decay, whereas for large times the decay becomes exponential. Algebraic decay of correlations indicates the existence of self-similarity in the process. For this reason the stochastic sequence generated by the one-dimensional (1D) OU process passed through a threshold is representative of so-called fractal noise [26]. The pathology of $x(t)$ generated in this way is evidenced by the time interval distribution of the thresholded process (not shown here) which is very sharply peaked for time intervals near zero. In spite of this difficulty, we include this process, since it is widely used to generate one-dimensional colored noise.

We have generated data sets of 3000 time intervals in length for four correlation times, spanning the biologically relevant range, with $D=1$. The number of encounters N was obtained for each data set. In this case, these encounters represent ''false positives''; that is, the number of times the signature definition is satisfied simply by chance. We then calculated K using the three surrogate types. In all cases, 100 surrogate files were made from which $\langle N_s \rangle$ and σ were obtained. The results are shown in Table I(a). We note that in no case does the K value indicate statistical significance ($|K|>2$). The algorithm is therefore not deceived by colored noise nor by fractal noise for the correlation times shown, and this is true for all three surrogates.

But are these surrogates effective in detecting statistically significant numbers of encounters with UPOs? In order to explore this question, we inserted the encounter signature specified by Eq. (1) into the noise files used in Table I(a). An encounter signature was inserted midway between every pair of ''false positive'' encounters previously found which bracketed at least ten time intervals. The six existing time intervals were overwritten by those of Eq. (1) in order to maintain the file length constant at 3000 intervals. In this way, N was significantly increased in comparison to the original data file. The results are displayed in Table I(b), where the new number $N+E$, together with the number of encounters inserted, E (in parentheses), are shown in the first

TABLE II. Linear, exponentially correlated noise: two-dimensional Ornstein-Uhlenbeck process.

$\tau=$	25 ms	50 ms	75 ms	100 ms
		(a) Noise alone		
N	180	187	185	190
$\langle N_s \rangle$	174	174	175	174
σ	15.6	15.3	17.5	15.2
K(SS)	0.39	0.85	0.57	1.05
K(AAFT)	0.22	1.01	0.55	0.96
K(AS)	0.15	0.56	0.69	0.82
		(b) Noise plus added encounters, (E)		
$N+E,(E)$	275 (95)	287 (100)	292 (107)	290 (100)
$\langle N_s \rangle$	172	173	177	175
σ	13.9	18.8	14.8	16.5
K(SS)	7.38	6.07	7.75	6.99
K(AAFT)	6.44	12.35	9.72	12.32
K(AS)	5.89	8.77	6.03	6.06

TABLE III. Harmonic noise.

$\omega=$	251 rad/s	126 rad/s	84 rad/s	63 rad/s
		(a) Noise alone		
N	163	164	196	171
$\langle N_s \rangle$	161	179	182	184
σ	20.1	20.1	20.5	21.0
K(SS)	0.10	−0.75	0.68	−0.62
K(AAFT)	0.46	−0.50	1.21	−0.55
K(AS)	0.86	−0.56	0.90	−0.42
		(b) Noise plus added encounters, (E)		
$N+E,(E)$	234 (71)	226 (62)	253 (57)	226 (55)
$\langle N_s \rangle$	167	179	183	185
σ	16.8	19.0	20.3	12.2
K(SS)	3.98	2.48	3.44	3.36
K(AAFT)	3.32	3.33	3.64	3.12
K(AS)	3.99	3.44	3.28	2.79

row. Now the K values all indicate the presence of UPOs with statistical significance at the 99% level or greater except for the values of K(AS) for the middle two correlation times, for which the confidence levels are somewhat better than 95%.

Next we repeat these tests using two-dimensional OU noise [26] with a single correlation time generated by

$$\dot{y} = -\frac{1}{\tau}(y-x),$$
$$\dot{x} = -\frac{1}{\tau}x + \frac{1}{\tau}\sqrt{2D}\,\xi(t), \tag{5}$$

where the solutions $y(t)$ are thresholded, and all other conditions are the same as described above. The power spectrum and correlation function of $y(t)$ are given by

$$S_{yy}(\omega) = \frac{2D}{(\tau^2\omega^2+1)^2},$$
$$\langle y(t)y(s)\rangle = \frac{D}{2\tau^2}(\tau+|t-s|)\exp\left[-\frac{|t-s|}{\tau}\right], \tag{6}$$

where $y(t)$ was thresholded in the same way as described above. The correlation function of the train of zero crossings generated by this process has no algebraic decay, because the additional differential equation in Eq. (5) destroys the fractal properties of the thresholded process [26].

The results are given in Table II(a) for the noise alone and in Table II(b) for the same noise files but with inserted encounters. We note that as in the one-dimensional case, none of the surrogates are deceived by colored noise ($|K|\ll 2$), and they all are equally effective in detecting UPOs ($K \gg 3$). The detection confidence levels are, however, somewhat higher in this case than for the one-dimensional noise. This is likely due to the fact that the noise files in the one-dimensional case are more disordered (having larger densities of shorter time intervals) owing to the indeterminate threshold crossing rate (which theoretically approaches infinity). Thus the null hypothesis can be rejected with high confidence levels for all three surrogates for the noise generated by both one- and two-dimensional OU processes.

IV. HARMONIC NOISE

Another kind of OU noise which possesses a narrow-band spectrum is the so-called harmonic noise. We generate a sequence of zero crossing times from a damped linear harmonic oscillator driven by additive white noise [28] according to

$$\ddot{x} + \Gamma\dot{x} + \omega_0^2 x = \sqrt{2D\Gamma}\,\xi(t), \tag{7}$$

where Γ is the damping, ω_0 is the natural frequency and $\xi(t)$ is Gaussian, δ-correlated, zero mean noise with unit standard deviation. The power spectrum of harmonic noise is

$$S_{xx}(\omega) = \frac{2D\Gamma}{\omega^2\Gamma^2 + (\omega^2-\Gamma^2)^2}. \tag{8}$$

Again, we thresholded $x(t)$ to make 3000 point time interval files. We generated data files for four natural frequencies corresponding to periods that equal the four correlation times of Tables I and II. In order to maintain the same width of the maximum in the power spectrum of $x(t)$ for these different frequencies, we set the damping $\Gamma = \omega_0/2$ in each case. The results are given in Table III. Table III(a) for harmonic noise alone shows that the algorithm is not deceived by the presence of SPOs in the data and that all three surrogates are equally immune. Again we inserted UPOs as described above. Table III(b) shows that the algorithm continues to detect UPOs within the harmonic noise with statistical significance and that again there is nothing to choose among the surrogates. We can conclude that the null hypothesis can be rejected also in the case of harmonic noise.

TABLE IV. FitzHugh-Nagumo neuron model.

$D=$	0.01	0.05
(a) Limit cycle ($a=0.5$)		
N	3631	3817
$\langle N_s \rangle$	3875	3824
σ	99.0	94.9
K(SS)	−2.46	−0.07
K(AAFT)	−1.48	0.89
K(AS)	−1.85	0.66
(b) Noise induced oscillations ($a=1.05$)		
N	3793	3785
$\langle N_s \rangle$	3801	3793
σ	82.2	56.36
K(SS)	−0.58	−0.14
K(AAFT)	−0.49	−0.16
K(AS)	−0.90	−0.16

V. NOISY LIMIT CYCLE

Since biological data can also contain stable limit cycles, it is necessary to test the algorithm and surrogates with this object as well. For this purpose we use a stochastic FitzHugh-Nagumo neuron model [21] governed by

$$\epsilon \dot{x} = x - \frac{x^3}{3} - y,$$
$$\dot{y} = x + a + \sqrt{2D}\,\xi(t), \tag{9}$$

where $\epsilon=0.01$, a is the control parameter, and $\xi(t)$ is Gaussian white noise. For $a<1$ this model possesses a stable limit cycle, while for $a>1$ the spikes appear due to noise only [29]. We study interspike intervals generated by this model in two different regimes: (a) stable limit cycle ($a=0.5$) and (b) subthreshold spike generation due to noise ($a=1.05$). We underline that in this case the threshold is an integral part of the model. The results are presented in Table IV where we have tested the surrogates in these two regimes for two noise levels. Note that all surrogates result in K values indistinguishable from zero. The sole exception occurs in the case of SS surrogates for the low noise, suprathreshold regime [Table IV(a)]. The negative value, where $K(\mathrm{SS})=-2.46$, indicates the existence of a stable limit cycle. These surrogates are thus not confused by the presence of noisy limit cycles, and one of them (SS) can indicate the presence of limit cycles in the low noise case.

VI. A NEW SURROGATE

The method of surrogate data has been developed within the frame of dimensional analysis [35,14]. It is important to emphasize that surrogate testing is designed to distinguish whether a measured observable reflects some characteristic of a low-dimensional, nonlinear dynamical process rather than that of a linear or nonlinear transformation of a purely random process. The main idea is to construct surrogate data that coincide with the given observable with respect to some special properties, e.g., the probability density or the power spectrum, but that are randomized with respect to all other properties. The set of the several realizations of such a process is called the set of surrogate data. The original data and the sets of surrogate data are compared using test statistics, such as the Savit-Green statistic [30], the nonlinear prediction error [31,32], or the Brock-Dechert-Scheinkrann statistic [33].

In this paper surrogate data are constructed for distinguishing purely noisy processes from processes that are noisy but have a dynamical origin. In addition to the well-known SS and AAFT type surrogates, we have designed a new surrogate that preserves short-scale temporal correlations and also maintains the attractor's *shape* (AS) in two dimensions. Since we assume that the large majority of encounters can be caused only by an underlying nonlinear dynamics that contains an UPO, we expect the number of encounters of the trajectories with that UPO to be a powerful test statistic.

The new surrogates are obtained by approximating the experimental data with a second order Markov model. Data sets produced by such processes are completely characterized by their first return map. An example map, taken from actual biological data, is shown in Fig. 1(a).

Technically, the AS surrogates are produced in the following way. Uncorrelated data, which maintain the probability distribution, are created by simple shuffling of the original data. Data which appear to result from a second order Markov process are then generated as follows.

(1) The amplitudes are adjusted in order to conform to a uniformly distributed process.

(2) A desired binning is introduced, meaning that the phase space is tiled by squares with a fixed side length.

(3) The transition matrix T_{ij} is estimated from the relative frequencies that the data visit the squares identified by the coarse-grained (bin) coordinates (i,j).

(4) Symbol sequences, corresponding to the coarse-grained dynamics, are produced by ''iterating'' the transition matrix: The number of symbols is related to the number of bins. Initially an arbitrary symbol s_1 is chosen. The succeeding symbol, s_{t+1}, of the symbol s_t is set to a realization of the discrete distribution $P(s_{t+1}=i)=T_{s_t,i}/\Sigma_i T_{s_t,i}$.

(5) A small uniformly distributed noise is added, so that the backwards adjusted amplitudes can be performed.

(6) The amplitude is adjusted backwards.

The amplitude adjustment is a purely technical element required for having equiprobable bins (symbols). Since the whole procedure is sensitive to the number of bins chosen, several such numbers must be tested in order to ensure that the bin number does not influence the main result of the test. One surrogate calculated following this procedure using the original data shown in Fig. 1(a) is shown in Fig. 1(b). The shape of the attractors in the two figures can be compared. The effectiveness of the AS surrogate is demonstrated in the preceding sections.

VII. SUMMARY AND DISCUSSION

In this work we have tested the simple topological recursion method for detecting and counting UPOs in noisy data files against the possibility that it can be deceived by temporally correlated, or colored, noise. In addition, we have tested three different surrogates both for their immunity to deception by colored noise and for their effectiveness in rejecting the null hypothesis using noise files with known numbers of UPOs inserted. We have performed these tests using three different generators of colored noise including two systems which produce noisy SPOs. Our findings can be summarized as follows.

(1) The three surrogates are equally immune to deception by colored noise in data sets generated by the one- and two-dimensional OU process, by the noise driven linear harmonic oscillator, and by the noisy FN system, that is, by all the dynamical systems used in the tests.

(2) The simple topological recursion algorithm is equally effective in detecting known numbers of UPOs inserted into the noise data using any of the three surrogates.

(3) The algorithm and the three surrogates are immune to deception by the presence of SPOs in the data.

It is therefore safe to use the topological recursion algorithm together with SS surrogates. In many diagnostic and possible therapeutic applications, for example those requiring dynamical control, computational speed is essential, therefore the simpler algorithm is advantageous. It is important that this be established satisfactorily within the community [36].

It is worth commenting further on the SS surrogate. Simple shuffling offers the advantage that it realizes a direct replacement of the sequential order, which defines an encounter, with a randomly chosen sequence. We can therefore be certain that *true* encounters with UPOs are not preserved in the SS surrogates. Thus all encounters found in the SS surrogates must be *false* ones. The key to understanding this assertion is the definition of the signature sought in the data files. As defined, the signature is a sequence of time intervals that behave in a very specific way (three points which approach the line of periodicity at sequentially decreasing distances followed immediately by three which depart at sequentially increasing distances). Such sequences are necessarily highly correlated, but they carry far more information than sequences that are simply exponentially correlated. Since the information is contained in the sequential order, a correct surrogate is one which replaces the sequential order with a randomly chosen sequence. The SS surrogate is the only one of the three which does this. Surrogates which preserve correlations without regard to sequential order, such as the AAFT and AS used here, might preserve *true* encounters.

The AAFT put forth by Theiler *et al*. [14] is the most celebrated of the correlation preserving surrogates. However, we must remember that it was designed to test the results of correlation dimension algorithms and their variants. Such algorithms search for nonlinear correlations and can be deceived by the presence of linear correlations in the data. The AAFT surrogate was never designed to test sequentially ordered events as represented here by the defined signatures of UPOs. It was designed, among other things, to preserve the power spectrum of the original data, thus avoiding the introduction of more disorder, or ''whitening'' of the power spectrum. But the ''whitening'' of the power spectrum in surrogates has a certain advantage. As we have shown previously [34], the SS surrogate has the added advantage in this application of being able to distinguish stable limit cycles from UPOs if the noise is not too large. It does so by detecting the *absence* of topological signatures of instability in the data files compared to the surrogates. For data files which are not too noisy, SS surrogates are more disordered than the original data, that is, SS surrogates ''whiten'' the power spectra. One can then find more ''false positives'' in the surrogate compared with the data where, for purely stable periodicities, one often finds zero or insignificantly small numbers of encounters. Thus $N<\langle N_s\rangle$, leading to negative values of K that are the signal of the presence of stable orbits. However, for SPOs accompanied by large inherent noise intensities, it is always true that $K\rightarrow 0$ as is the case here in Sec. V.

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[36] For the convenience of those wishing to test these results for themselves, we have placed all data files used in this work on the web at the URL: http://neurodyn.umsl.edu/colorednoise.html