Essential parameter in the formation of photonic band gaps

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It is generally believed that the spatial periodic variation of a dielectric constant ϵ or refractive index $\sqrt{\mu\epsilon}$ can give rise to photonic band gaps. However, through microwave experiments of microstrip lines with a periodic array of holes, we demonstrate that the wave impedance, which depends on $\sqrt{\mu/\epsilon}$, is an essential parameter, giving photonic band gaps rather than the refractive index. We explain physically the effect of the magnetic permeability μ on photonic band gaps, already reported. [S1063-651X(99)08904-7]

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Periodic dielectric structures (photonic crystals) that exhibit electromagnetic stop bands (photonic band gaps, i.e., PBG's) have attracted much attention because of their ability to control the propagation of electromagnetic wave [1–3]. It has been known that a strong periodic variation in the dielectric constant or refractive index gives rise to photonic band gaps [3]. The concept of a photonic crystal has been rapidly extended to other materials [4,5]. Recently, there has been an increasing interest in microwave and millimeter-wave applications of PBG structures using microstrips [6–8]. A microstrip line has been used as a waveguide or a transmission line in microwave integrated circuits. It consists of a low-loss insulating substrate sandwiched between a metal ground plane and a metal strip line, as shown in Fig. 1. Microwaves are guided through the substrate along the strip line reflecting between two metal walls. In contrast with a parallel metal plate waveguide, the boundary conditions of microstrip line do not allow pure TEM modes, but quasi-TEM modes; the longitudinal components of electric and magnetic fields are significantly small, but not vanishingly so. The introduction of a periodic array of holes in the strip line can affect the modes of a guided microwave, because the geometry of the waveguide periodically changes along the strip line. Because of the translational symmetry along the strip line, the guided mode can be characterized by wave vector limited to a value between $-\pi/a$ and π/a, where a is the spatial period of the array of holes [9]. Coupling between the modes at π/a and $-\pi/a$ due to the periodic perturbation can create a splitting of the lowest-order quasi-TEM modes at the Brillouin zone edge, and the splitting is called the PBG. A wave cannot propagate along the modulated strip line when its frequency lies within the gap, because there are no such allowed propagating states. The concept of a defect in semiconductors, which gives rise to an isolated energy level within its energy band gap, is also applicable to the artificial periodic structure. A defect state in the PBG can be created by locally breaking the periodicity of the structure [10]. Since the defect state is strongly localized about the local defect, a point defect can act like a microcavity [9]. Furthermore, it is possible to optimize the performance of a single mode waveguide by choosing an appropriate defect size, since the frequency of defect state depends on the defect size [11].

A schematic of a PBG single mode microwave waveguide, employed in this study, is shown in Fig. 1. The starting material was a microstrip line composed of RT/duroid 6010 with dielectric constant of 10.2- and 0.635-mm thicknesses, and 35-μm-thick copper layers coated on both sides of dielectric. A periodic array of square holes, not a periodic array of circular holes, was introduced into the strip line, for the convenience of method of moments (MoM) simulation [12]. The defect was created by increasing the distance d between the centers of the fourth and fifth holes. The holes in the strip line were etched by using the conventional photolithographic method, employing an ultraviolet light source, F_2Cl etchant, and photoresist. A HP 8753 Network Analyzer was used to measure the transmitted spectra through the waveguide. Two 50-Ω microstrip lines at both ends was incorporated in order to contact with the input and the output tip of the network analyzer.

Figures 2(a) and 2(b) show the measured spectra of transmitted microwaves when the microstrip lines have a periodic array of square holes and a defect of $d=7.0$ mm, as shown in the insets of the figure. The strip line with the periodic array exhibited a PBG between 9.85 and 13.54 GHz. The introduction of the defect created a single transmission mode in the PBG. We have observed, as the defect length d increases, that the frequency of the defect mode shifted to a lower frequency from the upper band edge, and that the width of the PBG became wider. When the defect length was 6.5 mm, the frequency of the defect mode was located near the center of the gap. The transmission spectrum through the waveguide of $d=7.0$ mm was simulated with a commercial

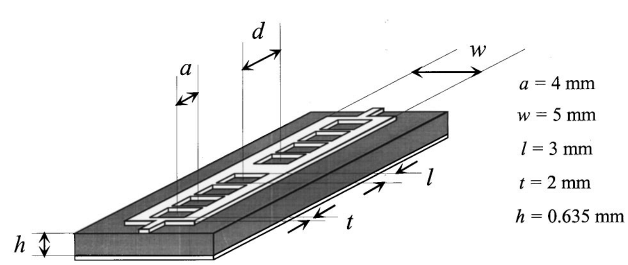

FIG. 1. Schematic of photonic band gap single mode microwave waveguide. a is the hole period, d the defect length, l the width of square hole, w the waveguide width, and h the substrate thickness.

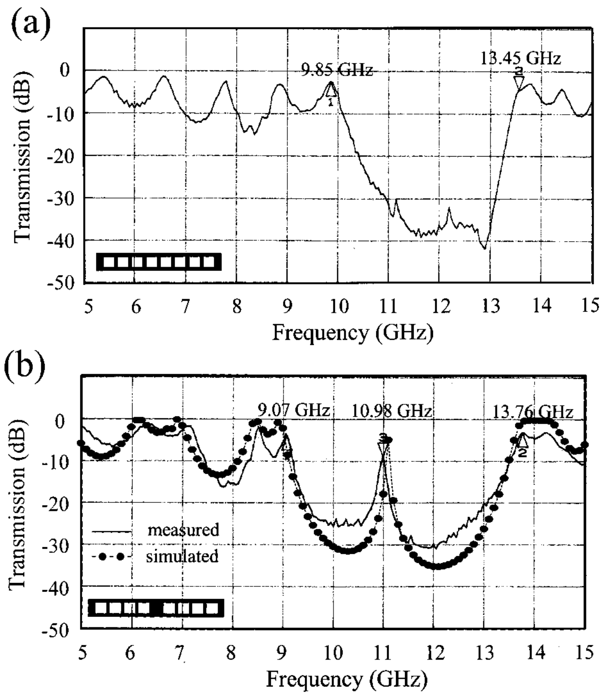

FIG. 2. (a) Spectra of transmitted microwaves when there is no defect. (b) The measured and simulated spectra of transmitted microwaves when the defect length $d=7.0$ mm. The insets denote the corresponding schematics of the strip lines.

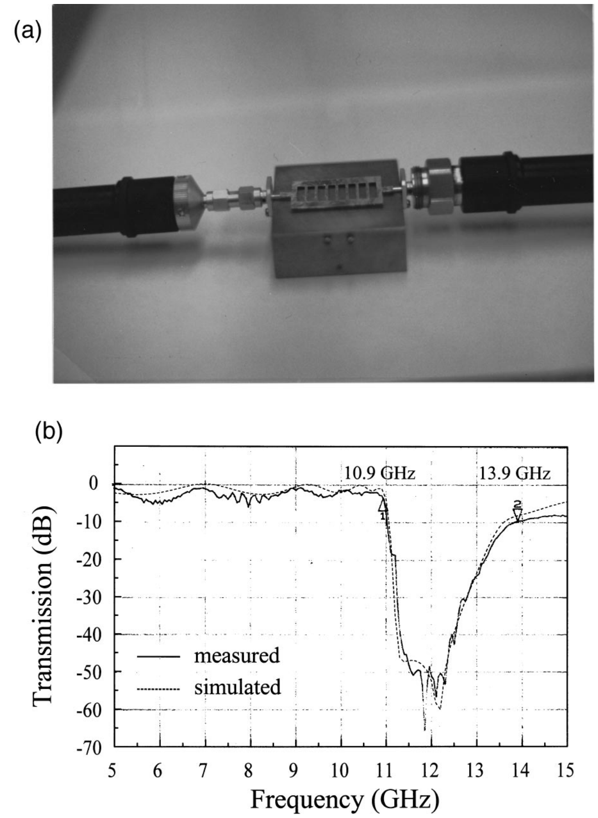

FIG. 3. (a) Photograph of the experimental arrangement. (b) The measured and simulated spectra of the transmitted microwave when the substrate is the air.

MoM simulator of ENSEMBLE Version 4.0, and the result is shown along with the experimental measurement in Fig. 2(b). The results are in excellent agreement, and their discrepancy is within 1%.

When a voltage wave of microwave frequency is applied to one end of the microstrip line, it propagates through the strip line toward the other end. Whenever it encounters interfaces between holes and metals perpendicular to the propagation direction, there will be both reflected and transmitted waves. They interfere with each other, forming PBG's. Meanwhile, the electric field wave associated with the voltage wave travels through the dielectric substrate in the same manner as the voltage wave does. That is, the electric field should be reflected (transmitted) at the same moment when the voltage wave is reflected (transmitted), even though there are no holes in the substrate. Thus a PBG is created, as shown above. Let us consider the case of holes drilled in the substrate instead of the strip line. The electric field wave should be reflected and transmitted at the interfaces between holes and dielectric substrate, and the voltage wave in the strip line should follow the electric field wave even though there are no holes in the strip line. We thus anticipate exactly the same phenomena for both cases: holes in the strip line and in the substrate. Each case was confirmed from the results of our experiment and that of Ref. [7], respectively.

The detailed analysis shows that the wavelength of a guided microwave through a microstrip line depends on the effective dielectric constant of the substrate $\epsilon_0\epsilon_{\mathrm{eff}}$, ϵ_0 being the vacuum permittivity. $\epsilon_0\epsilon_{\mathrm{eff}}$ for $w/h \geqslant 1$ (see Fig. 1) is given by [13]

$$\epsilon_{\mathrm{eff}}=\frac{\epsilon_1+\epsilon_2}{2}+\frac{\epsilon_1-\epsilon_2}{2}(1+12h/w)^{-0.5}, \qquad (1)$$

where ϵ_1 and ϵ_2 are the relative permittivities of the substrate and surrounding medium, respectively, with $\mu=1$ the relative permeability. The holes in the substrate periodically change the value of ϵ_1 along the strip line, and so does the wavelength of the microwave. The holes in the metal strip will also periodically vary the effective dielectric constant by means of the metal width w. We have experimentally observed a PBG in a periodic array of holes both in the dielectric substrate and in the metal strip. However, the variation of the effective dielectric constant cannot be achieved by any means when $\epsilon_1=\epsilon_2=1$, i.e., the substrate is the air, so that no PBG's should be expected in this case. The experimental arrangement with holes in the metal strip is shown in the photograph of Fig. 3(a). For the strip line, we used 18-μm-thick commercial aluminum foil cut with a razor blade. The lateral dimensions of the metal strip are four times larger, while the longitudinal ones along the propagation direction are fixed, compared with those shown in Fig. 1. The distance between the metal strip and the ground plane was about 1 mm. It is very surprising that this structure exhibits a PBG, as shown in Fig. 3(b). Again, the measured and simulated spectra of the transmitted wave are in excellent agreement in the gap range. This experimental result cannot be explained in terms of the periodic arrangement of effective dielectric constants alone. As the lateral dimensions decrease, the stop band shifts to a higher frequency. All these results imply that the geometry of the metal strip is intimately related to the occurrence of a PBG. The PBG's for microwaves can be simply and inexpensively made with commercial aluminum foil and a razor blade.

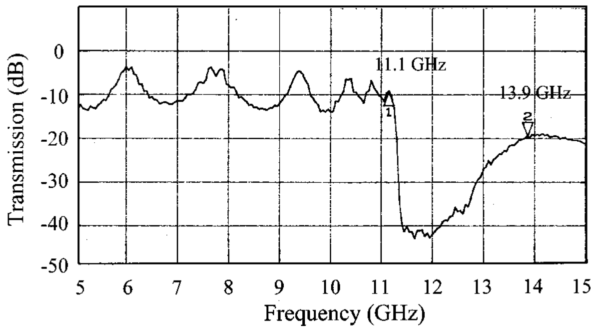

FIG. 4. Measured spectra of a transmitted microwave traveling through a region of the air between the metal strip and the ground plane.

We have considered cases where a voltage wave is applied to a microstrip line. We now consider a microwave traveling through a region of the air between the metal strip and the ground plane. The electric field of the microwave oscillates perpendicularly to the ground plane, so that there will be a voltage wave traveling through a metal strip with holes. Whenever this voltage wave meets interfaces between the metal and hole in the longitudinal direction, it should be reflected and transmitted. Accordingly, the electric field wave would be reflected and transmitted, even though there is not any interface to do so. We thus expect to see a PBG in this case, too, and it is clearly seen in the experimental result shown in Fig. 4. This experiment manifests that a PBG can be caused by a periodic change in metal width.

Now we can safely say that the reflections and transmissions of a microwave at a periodic array of interfaces give rise to a PBG, so that the boundary conditions at interfaces must be one of the direct reasons for PBG's. For normal incidence, the reflected wave is given by

$$E_{\mathrm{ref}}=\frac{\sqrt{\mu\epsilon'/\mu'\epsilon}-1}{\sqrt{\mu\epsilon'/\mu'\epsilon}+1}E_{\mathrm{inc}}, \qquad (2)$$

where E_{inc} is the incident electric field [14]. The ϵ's and μ's are the relative dielectric constants and relative magnetic permeabilities of the two materials, respectively. We thus believe that the combination of $\sqrt{\mu/\epsilon}$, which has the dimension of wave impedance Z defined as E/H, should be the important factor to be considered here, rather than the refractive index $\sqrt{\mu\epsilon}$. We also know that the geometry of the metal strip affects the PBG. The quantity, which is related to both $\sqrt{\mu/\epsilon}$ and the geometry of the metal strip, is the characteristic impedance $Z_c=E/H$ or V/I of the microstrip line, well known in microwave society. When $\mu=1$, Z_c for $w/h\geqslant 1$ is given by [13]

$$Z_c=\sqrt{\mu_0/\epsilon_0\epsilon_{\mathrm{eff}}}[w/h+1.393+0.667\ln(w/h+1.444)]^{-1}, \qquad (3)$$

where $\sqrt{\mu_0/\epsilon_0\epsilon_{\mathrm{eff}}}$ is the effective wave impedance of the dielectric substrate, and $\sqrt{\mu_0/\epsilon_0}$ the free space wave impedance, i.e., 377 Ω. ϵ_{eff} is given by Eq. (1). Both holes in the strip line and holes in the substrate make a periodic variation in the characteristic impedance by means of w, and ϵ_1, respectively. Also, holes in the ground plane can give the same effect on a guided microwave [8]. As the hole size in the metal strip increases, the variation in the characteristic impedance due to the geometrical factor, w/h, may become larger than that due to the dielectric constant ϵ_1. Then the holes in the metal strip or in the ground plane can cause a stronger modulation in the characteristic impedance than the holes in the substrate. Previous works [7,8] reported that, as the hole radius increases, the PBG's created by holes in the ground plane are wider and deeper than those created by holes in the substrate. This shows that strong periodic variation of the characteristic impedance results in wider PBG's.

The PBG in our experiment is also larger than that generated by the holes in the substrate, and is nearly equal to the one generated by holes of diameter 2.54 mm in the ground plane [8]. When the substrate is the air, Eqs. (1) and (3) show that there is a periodic variation in the characteristic impedance by means of the holes in the metal strip, although there is no periodic variation in the effective dielectric constant. This periodic change in the characteristic impedance was the reason for the PBG's shown in Figs. 3(b) and 4. It can be seen more clearly in magnetic materials ($\mu\neq 1$) that the wave impedance or the characteristic impedance should be an important factor in PBG's. A study of the effect of magnetic permeability on PBG's showed that PBG's tend to disappear in the case where both ϵ and μ have maximum values in the same materials, and become wider in the opposite case where both ϵ and μ have maximum values in different materials [15]. This result cannot be understood in terms of the periodic variation in the dielectric constant or refractive index. They are, however, easily explained by the wave impedance. Although there is a strong periodic variation in the refractive index $\sqrt{\mu\epsilon}$, there is a weak variation in the wave impedance $\sqrt{\mu/\epsilon}$, in the former case, so that the PBG's tend to disappear. Meanwhile, the PBG's become wider in the latter case, since there is a strong periodic variation in the wave impedance, even though there is no variation in refractive index. We thus understand that the wave impedance $\sqrt{\mu/\epsilon}$ must be an essential parameter, rather than the refractive index $\sqrt{\mu\epsilon}$, in the formation of PBG's. When Maxwell's equations are numerically solved, the dielectric constant affects the PBG's by means of $\sqrt{1/\epsilon}$, and the two effects are the same when $\mu=1$. We should note here that the wave impedance plays the essential role by means of the boundary conditions at interfaces, but not through Maxwell's equations, in which the refractive index is important.

In conclusion, we now understand the correct roles of the both dielectric constant and magnetic permeability in the formation of PBG's by means of the characteristic impedance of the microstrip line. In particular, this opens the possibility to use magnetic materials in various PBG structures, and leads to new microwave devices: filters, high quality resonators, frequency selective surfaces, efficient power amplifiers, and antennas. These structures can provide a new method to control the transmission of microwaves in electric circuits, and be incorporated into the design of novel monolithic microwave integrated circuits.

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