Transition free-electron laser (amplifier) driven by an electron bunch

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A theory of transition free-electron laser (FEL) driven by an electron bunch of finite longitudinal length is presented. It is shown that one can introduce concepts of a long, a short, and a very short bunch depending on the degree of correlation between the bunch effective length and the transition radiation formation length. It is obtained that a long Gaussian bunch produces an electromagnetic pulse with a double-exponent-type envelope $\exp[\exp(-\tau^2)]$, whereas in the short bunch case the latter has an oscillating nature. Both analytic and numerical studies show that the long bunch compression into the short one improves the bunch-field coupling and results in the enhancement of the logarithm of a gain at the pulse center by nearly one order. The further bunch compression leads to the gain vanishing. It is noted that a similar picture is probably possible for an undulator FEL. The results are generalized for a bunch train. [S1063-651X(99)06103-6]

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Electromagnetic waves produced by the passage of a charged particle through the interface between two media with different dielectric constants are known as transition radiation (TR) [1]. In contrast to the Cherenkov effect [when the radiated frequencies ω are limited by $n(\omega)>1$ condition] and to the Compton (or undulator) effect (where the frequencies are bounded both by the wavelength of a pump wave and by the energy E of an electron beam), in the device there are no restrictions on frequency values (of course, except for an obvious condition $\hbar\omega<E-mc^2$). Hence the transition radiation seems to be a very suitable source for a tunable free-electron laser (FEL). An analysis of single-particle spontaneous TR has been done in a great number of both theoretical and experimental works and generalized in monographs [2–4]. Coherent TR, produced by an electron bunch (e bunch) with a length l, which is of the order of or smaller than the emission wavelength λ, was studied in [5]. Reference [6] marked the beginning of studies of the stimulated transition effect. A number of both linear and nonlinear effects considered in our papers (including a charged particle acceleration, an e beam modulation and polarization, and an electromagnetic wave amplification, for such configurations as a single interface, a dielectric plate, a resonant medium) were summarized in Ref. [7], Sec. III. Here one can find a number of references in the field as well. Our analysis showed [7] that a notable gain in a transition laser (TL) driven by an ordinary e beam could be achieved in a low-frequency range. Interesting ways of the FEL gain increasing in a high-frequency range have been discussed in [8,9]. Note that there is a well-known approach for the transition effect increasing based on a resonant medium [2,3]. The possibilities for such a device application for the ultraviolet and x-ray FEL were considered in [10–12].

As was shown in [7], in a space-heterogeneous medium one may consider a monochromatic amplified signal as a superposition of plane waves which have the same frequency, but different wave vectors [see Eqs. (1) and (1a) below]. The amplification mechanism for such fields is based on the fact that the wave vectors of photons, which take part in the radiation and absorption processes (and, as a result, the corresponding photon numbers), are different [7]. By varying an electron beam velocity, one can reach the wave vector region where the probability of a photon-stimulated emission dominates over the probability of its stimulated absorption, i.e., where the device operates in the laser regime. (Of course, there is a difference between this amplification mechanism and an inversion population concept in ordinary [13] and free-electron [14] lasers.) Since in our device a source of energy is the free electron, one can then speak about a transition FEL operating in the amplifier regime.

In all the papers quoted above, it has been presumed that e beams are spatially uniform. However, real beams always have a certain finite length. In this paper, we consider operation regimes of a TL driven by an e bunch of arbitrary length l. The concepts of a long, a short, and a very short bunch are introduced. It is shown that a long-bunch-length decreasing improves the bunch-field coupling and, as a result, the TL gain is enhanced at the electromagnetic pulse center. It reaches its maximum in the short bunch case and vanishes rapidly when the bunch length vanishes. Thus there is a possibility to optimize the TL operation.

Let a linear polarized monochromatic wave fall normally on the vacuum-dielectric interface. For the sake of simplicity we assume that the dielectric refractive index is $n=1+\Delta n$, where $\Delta n\ll 1$. In this case one can write the wave vector potential in the following approximate form:

$$A_y \approx \frac{1}{2}A_0\exp\left(i\omega t - i\frac{\omega}{c}n(z)z\right)+\text{c.c.}$$
$$=\frac{1}{2}A_0\int F_0(q)\exp(i\omega t - iqz)\,dq+\text{c.c.}, \qquad (1)$$

where the Fourier-transformed function

$$F_0(q)=\frac{(i\omega/2\pi c)(n-1)}{(k_1-q)(k_2-q)}. \qquad (1a)$$

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The refractive index $n(z)=n_1=1$ in the $z<0$ range and $n(z)=n_2=n$ in the $z>0$ one, the wave vector $k_r=\omega n_r/c$, and the subscript $r=1,2$. Let an e bunch cross the same interface. We adopt that every portion of the bunch with the same velocity $\mathbf{v}$ has a Gaussian-type shape along the electron trajectory and is perfectly uniform in the transverse plane. Hence one may write the bunch distribution function as $f_0=f_0^{(m)}(\mathbf{p})f_0^{(s)}(\mathbf{r},t)$, where the first factor $f_0^{(m)}$ describes the bunch momentum spread, whereas the second one

$$f_0^{(s)}=\frac{N^{(e)}}{\sqrt{\pi}Sl}\exp\left[-\frac{(\mathbf{r}\cdot\mathbf{n}-vt)^2}{l^2}\right]$$

is responsible for its space configuration; $N^{(e)}$ is the total number of electrons, S the beam area, v the electron velocity, and the unit vector $\mathbf{n}=\mathbf{v}/v$. Note that a parameter $\overline{\rho_0}=N^{(e)}/\sqrt{\pi}Sl$ is, actually, the bunch average density. Assume that the bunch-field coupling is weak and that the amplitude A_0 depends weakly upon t and z variables. In order to determine the laser gain, we make use of the shortened wave and kinetic equations

$$\frac{\partial A_0}{\partial z}+\frac{c}{n_r}\frac{\partial A_0}{\partial t}=\frac{4\pi}{c}\frac{1}{ik_r}j_y\exp\left(i\frac{\omega}{c}n_rz-i\omega t\right), \qquad (2)$$

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}}+e\left(\mathbf{E}_f+\frac{1}{c}[\mathbf{v}\times\mathbf{B}_f]\right)\frac{\partial f}{\partial \mathbf{p}}=0, \qquad (3)$$

where the subscript $r=1,2$ stands for the first and second media, respectively, $\mathbf{E}_f=-c^{-1}\partial\mathbf{A}/\partial t$, and $\mathbf{B}_f=\mathrm{rot}\,\mathbf{A}$. First of all, we calculate the bunch distribution function in linear field approximation. Namely, we write $f=f_0+f_1$, where the first term f_0 is defined above and the second one f_1 is the first-order perturbed part of distribution caused by the field (1). Then, retaining only the oscillating part of the bunch current, one gets

$$j_y=e\int v_yf_1\,d\mathbf{p}$$
$$=\frac{1}{2}e^2cA_0\int d\mathbf{p}\int dq\,\frac{\beta_y^2(\omega^2/c^2-q^2)F_0}{E(\omega-qv_z-i\zeta)^2}f_0e^{i(\omega t-qz)}+\text{c.c.} \qquad (4)$$

(here a vanishing term $i\zeta$ determines the rule of a singular function integration with respect to p_z). If the beam momentum spread $\Delta p/p$ is much less than the field wave vector spread $\Delta q/q$ [the last is determined by the function $|F(q)|^2$, Eq. (1a)], then the beam can be regarded as cold. On substituting $f_0^{(m)}=\delta(\mathbf{p}-\mathbf{p}_0)$ into Eq. (4) and solving Eq. (2), we have

$$A_0(z,t)=A_{00}\exp\left[\int_{-\infty}^{z}R_1\left(\xi,\frac{n_1}{c}\left(\xi-z+\frac{c}{n_1}t\right)\right)d\xi\right] \qquad (5)$$

in the $z<0$ region and

$$A_0(z,t)=D\left(z-\frac{c}{n_2}t\right)\exp\left[\int_0^zR_2\left(\xi,\frac{n_2}{c}\left(\xi-z+\frac{c}{n_2}t\right)\right)d\xi\right] \qquad (6)$$

in the $z>0$ one. Here $A_{00}=A_0$ $(z=-\infty)$ is the initial amplitude of the amplified field (1), a function

$$R_r(z,t)=(4\pi/i\omega n_rA_0)j_y\exp(ik_rz-i\omega t).$$

One may determine an unknown function $D(x)$ from the condition of continuity of the vector potential tangential components at the interface $z_1=0$, namely,

$$A_y(z_1-0)=A_y(z_1+0).$$

Since we are interested in the amplified field intensity P in the $z\to\infty$ region, then we must put $z=\infty$ and $n(z\to\infty)=1$ into Eq. (6). Finally, one gets

$$P=P_0\exp(GL_f),$$

where $P_0=\omega^2A_{00}^2/8\pi c$ is the intensity of initial field, L_f the TR formation (and a signal amplification) length, and

$$GL_f=2\pi\overline{\rho_0}r_0\lambda^2\frac{\beta_y^2}{\beta_z^2}\frac{mc^2}{E}\frac{\omega}{c}\mathrm{Re}\frac{d}{dq}\left[\left(q^2-\frac{\omega^2}{c}\right)F_1F_2^*\right]_{q=\omega/v_z} \qquad (7)$$

the TL gain in the natural logarithm scale (further ln gain, for short); $r_0=e^2/mc^2$ is the electron classical radius, E and mc^2 are the electron energy and rest energy, respectively, $\mathrm{Re}\,\varphi$ means the real component of φ, $F_1=F_0$,

$$F_2^*=\frac{1}{4\sqrt{\pi}}\exp(-\tau^2)\sum_{r=1}^{2}l_r\exp(\eta_r^2)[1-\Phi(\eta_r)],$$

$$\eta_r=(-1)^{r+1}\left[\frac{i}{2}(k_r-q)l_r-\tau\right],$$

$$l_r=\frac{l\beta_z}{\beta(1-n_r\beta_z)},\qquad \tau=\frac{(z-ct)\beta}{l},$$

$\beta_z=v_z/c$, $\beta=v/c$, and $\Phi(\eta)$ is the error integral. Note, first, that in the limiting case of a uniform e beam $(l\to\infty)$ and in the $|\tau|\ll1$ region, the function $F_2^*\to F_0^*$ and the formula (7) coincide with the result [7]. Since the wave vector $q=\omega/v_z$, then the first term in η_r is directly proportional to the parameter $g=|(k_r-\omega/v_z)l_r|=\omega l/\beta c$. The latter, actually, is the ratio of the bunch effective length $|l_r|$ to the transition radiation formation length, $L_r=1/|k_r-\omega/v_z|=\lambda\beta_z/2\pi|1-n_r\beta_z|$, in the first $(r=1)$ and second $(r=2)$ media, respectively [2–4]. In other words, the parameter g describes the dependence of the TL efficiency on a (bunch length)$-$(amplification length) correlation degree. To concretize the further analysis, assume [7] that the electron velocity dimensionless components and energy are, respectively,

$$\overline{\beta_z}=1-2\Delta n/3,\quad \overline{\beta_x}=\sqrt{\Delta n/12},\quad \overline{E}=2mc^2/\sqrt{5\Delta n}.$$

On substituting these values into Eq. (7) and recalling that $\Delta n\ll1$, one gets

$$\overline{GL}_f = (\overline{GL}_f)_0 \frac{\sqrt{\pi}}{18} g \exp(-\tau^2)[-2\,\mathrm{Re}(i\overline{Q}_1) + g\,\mathrm{Re}\,\overline{Q}_2], \tag{8}$$

where

$$(\overline{GL}_f)_0 = 1.8\frac{\overline{\rho_0} r_0 \lambda^2}{\sqrt{\Delta n}} \tag{8a}$$

is the TL ln gain from a spatially uniform electron beam,

$$Q_1 = \sum_{r=1}^{2} (-1)^{r+1} r \exp(\overline{\eta}_r^2)[1-\Phi(\overline{\eta}_r)],$$

$$Q_2 = \sum_{r=1}^{2} (-1)^{r} r^2 \left\langle \overline{\eta}_r \exp(\overline{\eta}_r^2)[1-\Phi(\overline{\eta}_r)] - \frac{1}{\sqrt{\pi}} \right\rangle,$$

$$\overline{\eta}_r = (-1)^r\left(\frac{i}{2}g + \tau\right), \quad g = \frac{\omega l}{c}, \quad \tau = \frac{z-ct}{l}. \tag{9}$$

We say that a bunch is long if the parameter $g \gg 1$ (or if the value $l \gg \lambda/2\pi$). In this limit the asymptotes of Eq. (8) can be written as simple elementary functions, namely,

$$\overline{GL}_f = (\overline{GL}_f)_0 \exp(-\tau^2)\left[1 + \frac{10}{3g^2}(1-2\tau^2)\right] \tag{10}$$

in the $|\tau| \ll g$ region and

$$\overline{GL}_f = (\overline{GL}_f)_0 \frac{1}{12}\left(\frac{g}{\tau}\right)^2 \exp(-\tau^2)\left[1 + \frac{3}{4}\frac{1}{\tau^2}(g^2-2)\right] \tag{11}$$

in the $|\tau| \ll g$ region. As one could anticipate, the bunch produces an electromagnetic pulse of $\tau \sim 1$ (or $l_p \sim l$) length. The bunch length decreasing leads to the ln gain increasing at the pulse center region $|\tau| < 1/\sqrt{2}$ [Eq. (10)] and to the gain decreasing on the pulse wings [Eq. (11)]. Note that one can speak about an electromagnetic wave amplification if the pulse length l_p is of the order of or greater than the wavelength λ, i.e., if the parameter $g \gtrsim 2\pi$. However, it seems useful to consider (purely formally) the very short bunch case, i.e., when the value $g \ll 2\pi$ or $l \ll \lambda/2\pi$. In this limit the ln gain asymptotes are defined by Eq. (11) in the $|\tau| \gg 1$ region and by the equation

$$\overline{GL}_f = (\overline{GL}_f)_0 \frac{1}{6} g^2\left(1 + \frac{5\sqrt{\pi}}{3}\tau\right) \tag{12}$$

in the $|\tau| \ll g$ region. Apparently, function $f(\tau)$ with such asymptotes may achieve, at least, one minimum and one maximum in the $\tau < 0$ and $\tau > 0$ regions, respectively. Therefore, one can assert that in the short bunch case (i.e., when $g \gtrsim 2\pi$ or $l \gtrsim \lambda$) the TL ln gain $\overline{GL}_f(\tau)$ has an oscillating character.

Consider the ln gain dependence upon the bunch length, $\overline{GL}_f(g)$, at the pulse center $\tau = 0$. As follows from Eqs. (10) and (12), the ln gain curve is peaked around the $g \sim 2\pi$ point; it vanishes rapidly if the bunch is very short, i.e., $g \ll 1$, and reaches a saturation value (8a) if the bunch is very long, $g \gg 1$. Apparently, the obtained effect can be used for TL operation improvement.

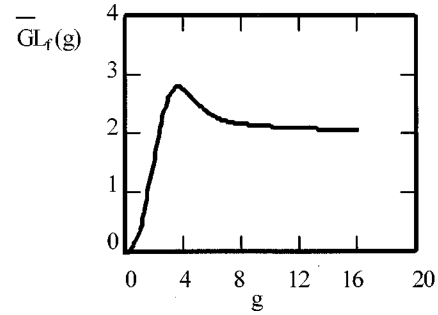

FIG. 1. Gain dependence on the length of an electron bunch in the natural logarithm scale. It is assumed that the bunch average density ρ_0 is an invariant value; the bunch length is normalized by the wavelength of the amplified signal $g = 2nl/\lambda$.

Assume that now an e beam comprises N_b identical bunches. An analysis proceeding as before gives

$$\overline{GL}_f = (\overline{GL}_f)_0 \frac{\sqrt{\pi}}{18} g \sum_{\nu=0}^{N_{b-1}} \exp(-\tau_\nu^2) \times [-2\,\mathrm{Re}(i\overline{Q}_{1\nu}) + g\,\mathrm{Re}\,\overline{Q}_{2\nu}]. \tag{13}$$

Here the functions $\overline{Q}_{r\upsilon}$ and $\overline{\eta}_{r\upsilon}$ are determined by Eqs. (9) after the following substitutions:

$$\eta_r \to \eta_{r\nu} = (-1)^{r+1}\left(\frac{i}{2}g + \frac{z-ct}{l} + \nu\frac{b}{l}\right),$$

$$\tau \to \tau_\nu = \frac{z-ct}{l} + \nu\frac{b}{l}.$$

In arriving at Eq. (13) it has been presumed that the distance between the two nearest bunches is the same value equal to b. If this distance is small enough, $b \ll l/(N_b - 1)$, then the ln gain at the pulse center $z - ct = 0$ is $\overline{GL}_f = N_b(\overline{GL}_f)_0$. Hence, in this limiting case, all bunches operate coherently. In the opposite case $b \gg l$, every bunch produces its own electromagnetic pulse.

Consider these results numerically at the pulse center $\tau = 0$. Adopt that the bunch average density $\overline{\rho_0}$ is an invariant value equal to $2\times10^{11}\,\mathrm{cm}^{-3}$. Let the dielectric refractive index be $n = 1.001$, the amplified radiation wavelength λ

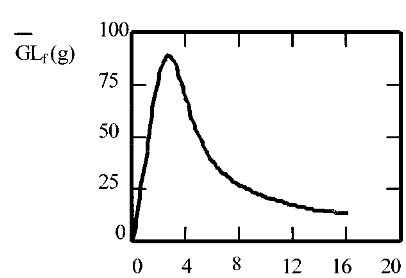

FIG. 2. Graph of the $\overline{GL}_f(g)$ function in the case when the total electron number of the bunch, $N^{(e)}$, is an invariant value during the compression.

$=8\,\mathrm{mm}$, and the electron energy $E=14.45\,\mathrm{MeV}$. (Note that the bunch average current is 1 kA/cm^2; such a high-current application has a specific character [17].) For chosen parameters the uniform-momentum beam approximation, used by us, holds true if the bunch angular and energy spread widths are $\Delta/E<(\Delta n/2)(E/mc^2)=0.4$ and $\delta<\sqrt{\Delta n/3}=1.8\times10^{-2}$, respectively [7]. In the case of a very long bunch, $g_0=100$ or $l_0=12.73\,\mathrm{cm}$, the ln gain $(\overline{GL_f})_0=2.042$. The bunch length decreasing down to 3.82 cm (or $g=30$) leads to the ln gain increasing up to 2.06. The ln gain reaches its maximum equal to 2.8 in the case of a very short bunch, $g=3.6$ or $l=0.5\,\mathrm{cm}$. In the short bunch case, $l_1=1.019\,\mathrm{cm}$ or $g_1=8$, the ln gain is 2.166. Thus the curve $\overline{GL_f}(g)$ has a peak at $g\sim3.6$, vanishes rapidly in the $g\ll1$ region [see Eq. (12)], and tends to the saturating value 2.042 in the $g\gg1$ region (8a) (see Fig. 1).

An essential effect can be achieved in the case when the bunch total electron number N_b is an invariant value (see Fig. 2). The similar bunch compression from $g_0=100$ or $l_0=12.73\,\mathrm{cm}$ down to $g_1=8$ or $l_1=1.019\,\mathrm{cm}$ (i.e., 12.5 times) leads to the ln gain $(\overline{GL_f})_0$ increasing 13.3 times.

In conclusion, consider the possible application of the effect for a Compton (undulator) and Cherenkov FEL. As is known, in these devices in a short length operation regime [15,16] a gain occurs only out of the corresponding synchronism (or resonant) conditions $\omega_1-\omega_2-v(k_1+k_2)=0$ and $\omega-\mathbf{kv}=0$. Meanwhile, namely, these conditions are the prime cause of the spontaneous emission effect in both cases. As far as we know, this dissonance has not been discussed in the literature. It is believed that the operation mechanism in such devices is really connected with the stimulated transition effect at the entrance and at the exit of the systems, and that the gain enhancement effect considered above is, probably, intrinsic to those FEL's as well.

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