Nonlinear dynamics of an ordinary electromagnetic mode in a pair plasma

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Nonlinear generation of an ordinary electromagnetic mode (o mode), which has high phase velocity ($v_{\mathrm{ph}} \gg c$), and at the same time, is almost longitudinal for long wavelengths, is discussed in an electron-positron plasma. The solution of the problem of increasing energy (''plasmon condensate'') of the long-wavelength o mode based on modulations of the wave by the beat wave of two higher frequency transverse electromagnetic waves (propagating along the external magnetic field) is proposed. The system of equations describing three-dimensional nonlinear dynamics of this ''superluminal'' o mode is derived and analytical solution for the modulated wave is found. The generated waves can have components propagating obliquely to the magnetic field. Important consequences of the effect to processes in pulsar's magnetospheres, in particular, the pulsar radio emission, are discussed. [S1063-651X(99)05404-5]

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I. INTRODUCTION

Recently, relativistic electron-positron e^+e^- plasma has attracted much attention [1]. Collective processes in the plasmas are especially important for the physics of many astrophysical objects such as pulsars, active galactic nuclei, etc. Also, there is recent interest in laboratory electron-positron plasmas [2–4]. The literature on nonlinear waves and wave-particle interactions in a relativistic pair plasma can be loosely divided into two classes. In the first class are studies related to particular astrophysical objects such as those based on the well-developed model of pulsar magnetospheres, which apply the results in attempts to explain the observed properties of pulsar radio emission [5–9]. The second class involves investigations of basic collective properties of e^+e^- pair plasmas, linear [1,10–13] or nonlinear (modulational instabilities, soliton formation, etc.) [9,14–17].

The pulsar radio emission is believed to originate in the pulsar magnetosphere, which is populated by a relativistic e^+e^- plasma. Since the pulsar magnetic field is extremely strong, $B_0 \sim 10^{12}$ G, plasma particles lose their perpendicular momentum very fast through synchrotron radiation, and their distribution function is essentially one dimensional. Moreover, because of the equal masses of electrons and positrons, there is more pairing symmetry in an electron-positron plasma than in an electron-ion plasma (e.g., there is a gyromotion in a magnetic field in opposite directions at the same frequency), and the spectrum of collective modes in the plasma contains fewer branches of propagating waves; in particular, the low-frequency modes associated with motions of ions (e.g., ion-acoustic and ion-cyclotron waves) are absent. The different low-frequency dispersion properties of an electron-positron plasma complicates consideration of nonlinear mechanisms of wave-wave and wave-particle interactions that, as is widely believed, are responsible for the pulsar radio emission.

In Refs. [14,15], attempts were made to describe modulational processes in a pair plasma, leading eventually to formation of Langmuir solitons. However, in order to obtain closer analogies with the well-known modulational effects in an electron-ion plasma [18], these models included artificial assumptions as either the presence of an additional hot (electron or positron) component or the absence of a magnetic field in the presence of a rarefied ion component. However, it is unclear how the additional component with a different temperature and/or density can be created in a pulsar magnetosphere.

The possibility of modulation of a longitudinal wave by the beat of (also longitudinal) Langmuir waves, propagating along the magnetic field, was studied in works [6–8]. The solitons, derived in [6–8], moving along the magnetic field lines, were supposed to act as additional sources of radiation. However, this type of four-plasmon interaction is limited to a narrow frequency range where the modes have phase velocity close to the speed of light. Moreover, as was demonstrated in [9,17], the soliton solutions obtained are unstable with respect to transverse perturbations. Thus the problem of the mechanism for reradiation of the energy of fast (with velocities exceeding the speed of light), almost longitudinal ordinary modes into waves with slower (sub)luminal velocities (which can leave the magnetosphere plasma) remains open.

The aim of this paper is to investigate the possibility of a nonlinear (modulational) instability of an ordinary (o)mode having phase velocity in the broad range of velocities exceeding the speed of light. We propose the mechanism of the nonresonant generation of the o mode by two high frequency transverse waves t and t', propagating along the external magnetic field in the opposite directions. The electric field vectors $\mathbf{E}$ and $\mathbf{E}'$ of these waves are perpendicular to the magnetic field, and intensive interaction with parallel (longitudinal) perturbations is possible because of nonlinear drift motions of plasma particles in the fields $\mathbf{E}$, $\mathbf{E}'$, and $\mathbf{B}_0$. This

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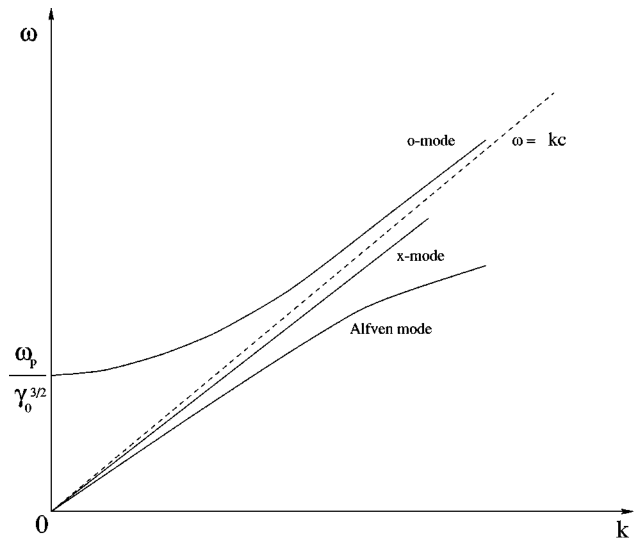

FIG. 1. Spectre of waves in an electron-positron plasma for oblique propagation.

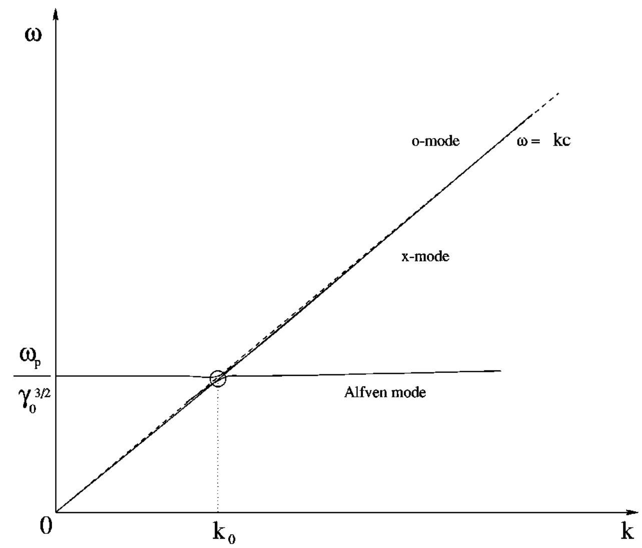

FIG. 2. Spectre of waves in an electron-positron plasma for parallel propagation.

formulation is entirely different from that of Refs. [6–9,17].

We consider here a strongly magnetized electron-positron plasma moving with relativistic velocity along the magnetic field lines. In our study, we use (unless the opposite is specified) the reference frame connected with the moving plasma. This assumption does not mean that the perpendicular motions of plasma particles (appearing as a result of interactions) are nonrelativistic. Furthermore, we calculate the nonlinear current using the assumptions of small amplitudes of the interacting waves as well as small ratios of plasma kinetic pressure to the magnetic pressure and plasma electron frequency to the frequencies of the two electromagnetic pump waves (the beat wave appearing as a result of their interaction modulates the considered longitudinal *o* mode). We also assume that the amplitudes of the electromagnetic pump waves considerably exceed the amplitude of the longitudinal mode that is justified by our assumption that mostly transverse waves are generated in pulsar magnetosphere through (anomalous) cyclotron resonance [19–21]. Using the above small parameters, we obtain a system of nonlinear three-dimensional equations, which is solved analytically neglecting back reaction of the modulations on the pump waves.

The paper is organized as follows: In Sec. II, we consider linear theory of waves in a relativistic electron-positron plasma; motion of a test particle in the field of the incident and scattered waves is studied in Sec. III; dynamics and instability of the *o* mode is investigated in Sec. IV.

II. WAVES IN A RELATIVISTIC ELECTRON-POSITRON PLASMA

The linear collective properties of an electron-positron plasma are now well established [1,10–13]. According to the theory, for oblique propagation with respect to the external magnetic field (directed along *z* axis), there are three normal modes, see Fig. 1. One is the purely transverse extraordinary *x* mode with dispersion in the laboratory frame [12],

$$\omega_x = kc\left(1-\frac{1}{8}\frac{\omega_p^2}{\omega_B^2}\frac{1}{\gamma_0^3}\right), \tag{1}$$

where $\omega_p = [4\pi(n_e+n_i)e^2/m]^{1/2} = [8\pi n e^2/m]^{1/2}$ is the ''combined'' plasma frequency (i.e., taking into account contributions of plasma electrons and positrons), $\omega_B = eB/mc$ is the cyclotron frequency, and γ_0 is the Lorentz factor of plasma particles moving along the field lines. Below, we consider a strongly magnetized plasma, $\omega_p \ll \tilde{\omega}_B \equiv \omega_B/\gamma_0$. The second and third modes are of mixed longitudinal-transverse character. The lower-frequency mode is analogous to the Alfvén wave, and the higher-frequency mode is the fast (superluminal), $v_{\rm ph} > c$, ordinary mode (*o* mode). Analytical expressions for dispersion of these modes are available in some limits. We consider the case $kc \ll \sqrt{2}\omega_p$ for waves propagating almost parallel to the magnetic field $|\mathbf{k}_\perp| \ll k_z$. For the *o* mode we have [12]

$$\omega_o^2 \simeq \frac{\omega_p^2}{\gamma_0^3} + 3k_z^2c^2 + |\mathbf{k}_\perp|^2c^2. \tag{2}$$

In this paper, we do not consider the Alfvén mode, and so do not specify its dispersion. However, we note that for parallel propagation there is a coupling point $\omega_p \approx \omega_0 = k_0 c$, see Fig. 2, where all the three modes are indistinguishable (in a cold plasma), and proper consideration of their nonlinear properties must take this into account. The electric field of the *x* mode is perpendicular to the plane of vectors **k** and **B**; the electric fields of the *o* mode and Alfvén mode are in the plane. The oblique subluminal Alfvén mode is strongly suppressed due to Landau damping if its phase speed, effectively $v_A/(1+v_A^2/c^2)^{1/2}$, is less than the speed of the bulk of the particles; in the opposite limit Alfvén waves are weakly damped.

Low-frequency modes analogous to the ion-acoustic wave in an eletron-ion plasma, are absent in an electron-positron plasma. Thus when considering nonlinear effects in the wave propagation, the only possibility for amplitude modulations of the *o* mode is due to nonresonant excitation of a beat wave. This was pointed out in Ref. [6], see also [7,8]. In the cited papers, the possibility of amplitude modulation of the purely parallel *o* mode (which is often called the Langmuir mode) was considered for $kc \ll \omega_p$; the beat wave is generated as a result of interaction of two Langmuir waves with close frequencies $|\omega_L - \omega_L'| \ll \omega_p$. However, in an electron-positron plasma, with equal densities of electrons and positrons (as in [6–8]), this second-order process cancels because of the equal masses and opposite charges of the plasma particles. This is because the second-order nonlinear current is proportional to the charge cubed, and the electron and positron contributions are equal and opposite. Furthermore, when considering interaction of waves under the condition $|\omega_L - \omega_L'| \ll \omega_p$, the beat wave cannot be generated in the superluminal $|\omega_L - \omega_L'| \gg |k_z - k_z'| c$ range of phase velocities because of the wave dispersion. And, finally, when considering waves near the coupling point $\omega_L \approx \omega_0 = k_0 c$, one needs to invoke nonlinear interactions with the *x* mode and the Alfvén mode. This possibility was studied in Ref. [17], where self-similar unstable solutions satisfying the nonlinear Schrödinger equation were found. An analogous problem was considered in Ref. [9] where it was demonstrated that small transverse parturbations lead to unstable solutions.

Consider the possibility of modulations of the fast *o* mode by transverse waves. Note that by ''transverse'' we imply not only the *x* mode, but also the high-frequency (compared with ω_p) *o*-mode, where its dispersion is close to the vacuum case. The difference between these modes in this case is only in their polarization. In the interaction, we are interested in the longitudinal superluminal component of the perturbation appearing as a result of the interaction of two transverse waves:

$$\left| \frac{\omega^t - \omega'^t}{k_z - k_z'} \right| > c. \tag{3}$$

Substituting Eq. (1) into Eq. (3), and using $|\mathbf{k}| \simeq k_z(1 - \mathbf{k}_\perp^2/2k_z^2)$, this inequality implies

$$1 - \frac{1}{2}\left[\frac{\mathbf{k}_\perp^2}{k_z(k_z - k_z')} - \frac{\mathbf{k}_\perp'^2}{k_z'(k_z - k_z')} \right] > 1. \tag{4}$$

To satisfy Eq. (4) for $k_z > k_z'$, we require $|\mathbf{k}_\perp'| > |\mathbf{k}_\perp|$ as well as

$$\frac{|\mathbf{k}_\perp'|}{k_z} \frac{|\mathbf{k}_\perp'| - |\mathbf{k}_\perp|}{k_z - k_z'} > 1. \tag{5}$$

We note that there is an important qualitative difference between a pair plasma and the more familiar (and more studied) electron-ion plasma when wave-wave and wave-particle interactions are considered. In an electron-ion plasma, the density fluctuations associated with Debye shielding can produce electric dipole radiation when forced to oscillate. This effect has no counterpart in a pair plasma because the electrons and positrons oscillate out of phase. As a consequence, the nonlinear shielding [1], which tends to dominate wave-wave interactions in electron-ion plasmas is absent, and Thomson scattering, which is the same for electrons and positrons, becomes the dominant nonlinear effect. Thus the process of reradiation of a wave by a particle in such a plasma can be considered on an isolated electron (positron), similar to Thomson scattering when an electromagnetic wave *t* forces oscillations of the particle generating a wave *t*′.

In the presence of an external magnetic field the generation of the wave *t*′ can have a different origin. In particular, when the incident wave is in anomalous Doppler-resonance $\omega - k_z v_z + \tilde{\omega}_B = 0$ with a plasma particle, the particle, in the process of radiation, increases its radius of gyration [19–21]. The frequency of the emitted *t*′ wave is close to ω_B/γ, where γ is the Lorentz factor of the particle. In this case the particle energy is the source for the radiation. Parallel radiation damping in the case $\gamma^2 v_\perp^2 \gg c^2$ (here, $v_\perp$ is the perpendicular component of the particle velocity), implies a damping force given by [22]

$$\mathbf{f} = -\frac{2}{3}\gamma^2 \frac{e^2 \omega_B^2}{c^2} \frac{v_\perp^2}{c^2} \frac{\mathbf{v}}{c}. \tag{6}$$

Thus the forces that act on a particle are those due to the electric and magnetic fields of the incident and emitted waves, as well as the external magnetic field and the radiation damping.

III. MOTION OF A TEST PARTICLE

There are well-established methods for calculating higher-order currents in a plasma [1,18]. For a relativistic pair plasma, these methods can be based on the general procedure elaborated in a series of papers [23]. Here, however, we present a simpler more physical calculation based on the hydrodynamic approximation.

The equation of motion for a test particle moving together with the plasma (i.e., in our reference frame $p_{0z} = 0$) in the external magnetic field and the fields of the incident and scattered waves, taking into account the radiation damping force, is

$$\frac{d\mathbf{p}}{dt} = e\left[\mathbf{E} + \mathbf{E}' + \frac{1}{c}\mathbf{v} \times (\mathbf{B}_0 + \mathbf{B} + \mathbf{B}') \right] + \mathbf{f}, \tag{7}$$

where $\mathbf{f}$ is given by Eq. (6). There are two small parameters in the problem. First, there is the smallness of the wave fields as compared to the external magnetic field, $(\mathbf{E}, \mathbf{E}', \mathbf{B}, \mathbf{B}') \ll \mathbf{B}_0$. Second, there is the smallness of the wave energies compared to the plasma particle thermal energy, $(|\mathbf{E}|^2, |\mathbf{E}'|^2) \ll mc^2 n \gamma_T$. Furthermore, we split the test particle momentum into three parts:

$$\mathbf{p} = \mathbf{p}_{0\perp} + \mathbf{p}_1 + \mathbf{p}_2, \tag{8}$$

where $\mathbf{p}_{0\perp}$ corresponds to the unperturbed motion of the particle in the external magnetic field $\mathbf{B}_0$, $\mathbf{p}_1 \ll \mathbf{p}_{0\perp}$ is the linear perturbation of $\mathbf{p}_{0\perp}$ due to the waves, and $\mathbf{p}_2 \ll \mathbf{p}_1$ is the nonlinear perturbation of the particle motion.

Expansion of the Lorentz factor $\gamma = [1 + (\mathbf{p}_{0\perp} + \mathbf{p}_1)^2 / m^2 c^2]^{1/2}$ to first order gives

$$\frac{\mathbf{v}}{c} \simeq \frac{\mathbf{p}_{0\perp}}{mc\gamma_{0\perp}} + \frac{\mathbf{p}_1}{mc\gamma_{0\perp}} - \frac{(\mathbf{p}_0 \cdot \mathbf{p}_1)\mathbf{p}_0}{mc\gamma_{0\perp}^3}, \tag{9}$$

where $\gamma_0=[1+\mathbf{p}_{0\perp}^2/m^2c^2]^{1/2}$. After substitution of this expression into Eq. (7), which is expanded in the same way, we find a system of coupled equations for $\mathbf{p}_{0\perp}$, $\mathbf{p}_1$, and $\mathbf{p}_2$. In the zeroth approximation, we have

$$\frac{d\mathbf{p}_{0\perp}}{dt}=\frac{e}{mc}\mathbf{p}_{0\perp}\times\mathbf{B}_0. \qquad (10)$$

If the test particle is an electron, the solution of Eq. (10) is

$$p_{0x}=p_{0\perp}\cos\tilde{\omega}_Bt, \quad p_{0y}=-p_{0\perp}\sin\tilde{\omega}_Bt, \qquad (11)$$

where $\tilde{\omega}_B=\omega_B/\gamma_{0\perp}$. For a positron, the sign of the y component of the momentum is opposite.

In the first approximation, the equation of motion can be written in the form

$$\frac{dp_{1x}}{dt}=e(E_x+E_x')+\tilde{\omega}_B(p_{1y}\cos\tilde{\omega}_B+p_{1x}\sin\tilde{\omega}_B)\cos\tilde{\omega}_B+f_x, \qquad (12)$$

$$\frac{dp_{1y}}{dt}=e(E_y+E_y')-\tilde{\omega}_B(p_{1x}\sin\tilde{\omega}_B+p_{1y}\cos\tilde{\omega}_B)\sin\tilde{\omega}_B+f_y. \qquad (13)$$

The solution of Eqs. (12) and (13) is given by

$$p_{1x}=\frac{e|\mathbf{E}_\perp|\sin\omega t}{2(\omega+\tilde{\omega}_B)}+\frac{e|\mathbf{E}_\perp'|\sin\omega' t}{2(\omega'+\tilde{\omega}_B)}+\frac{|\mathbf{f}_\perp|}{4\tilde{\omega}_B}\sin\tilde{\omega}_Bt, \qquad (14)$$

$$p_{1x}=-\frac{e|\mathbf{E}_\perp|\sin\omega t}{2(\omega+\tilde{\omega}_B)}+\frac{e|\mathbf{E}_\perp'|\sin\omega' t}{2(\omega'+\tilde{\omega}_B)}+\frac{|\mathbf{f}_\perp|}{4\tilde{\omega}_B}\sin\tilde{\omega}_Bt. \qquad (15)$$

Here, for the radiation damping force we use Eq. (6) together with the unperturbed solution, similar to Eq. (11): $v_{0x}=|\mathbf{v}_{0\perp}|\cos\tilde{\omega}_Bt, v_{0y}=-|\mathbf{v}_{0\perp}|\sin\tilde{\omega}_Bt$. We also assume that

$$E_x=|\mathbf{E}_\perp(\mathbf{r},t)|\cos\omega t, \quad E_y=|\mathbf{E}_\perp(\mathbf{r},t)|\sin\omega t, \qquad (16)$$

as well as

$$E_x'=|\mathbf{E}_\perp'(\mathbf{r},t)|\cos\omega' t, \quad E_y'=-|\mathbf{E}_\perp'(\mathbf{r},t)|\sin\omega' t, \qquad (17)$$

where the wave amplitudes $\mathbf{E}_\perp^{(\prime)}(\mathbf{r},t)$ are slow functions of position and time. For positrons, we have similar solutions with the change $e\to -e$ and, therefore, also $\omega_B\to-\omega_B$.

Taking into account that the parallel component of $\mathbf{p}_2\times\mathbf{B}_0$ is zero, we have

$$\frac{dp_{2z}}{dt}=\frac{e}{mc\gamma_{0\perp}}[\mathbf{p}_1\times(\mathbf{B}+\mathbf{B}')]_z. \qquad (18)$$

From Maxwell's equations, we have

$$B_x=\frac{E_y}{\cos\Theta}, \quad B_y=-\frac{E_x}{\cos\Theta}, \qquad (19)$$

where we use the wave dispersion equation $\omega\simeq|\mathbf{k}|c$ and introduce the angle Θ between the external magnetic field and the wave vector: $\cos\Theta=k_z/|\mathbf{k}|$. Substituting Eqs. (14), (15), and (19) into Eq. (18), we obtain the nonlinear longitudinal perturbation equation,

$$\frac{dp_{2z}}{dt}=\frac{e^2\mathbf{E}_\perp^2}{mc\omega_p}\sin(\Delta\omega t)\left[\frac{\omega_p}{\tilde{\omega}_B+\omega}\frac{1}{\cos\Theta}-\frac{\omega_p}{\tilde{\omega}_B+\omega'}\frac{1}{\cos\Theta'}\right]-\frac{e|\mathbf{f}_\perp|}{mc\tilde{\omega}_B}\left[|\mathbf{E}_\perp|\frac{\sin\Delta\Omega\tau}{\cos\Theta}+|\mathbf{E}_\perp'|\frac{\sin\Delta\Omega'\tau}{\cos\Theta'}\right], \qquad (20)$$

where we introduce $\Delta\omega=(\omega^t-\omega'^t)/\omega_p$, $\Delta\Omega=(\tilde{\omega}_B-\omega')/\omega_p$, $\Delta\Omega'=(\tilde{\omega}_B-\omega)/\omega_p$, and $\tau=\omega_pt$. The electric field E_{2z} is the result of the nonlinear interaction of the waves t and t' with the plasma particles. We note that in the reference frame where the parallel particle momentum is zero, $p_z=0$, the parallel component of the radiation damping force is also zero, $f_z\propto v_z=0$. Thus we have $p_{1z}=p_{2\perp}=0$ and the second-order current density $\mathbf{j}^{(2)}$ has only a parallel component.

Consider two limits: (1) $\omega^t\gg\tilde{\omega}_B$ and (2) $\omega^t\ll\tilde{\omega}_B$ (note that in both limits we have $\omega_p\ll\tilde{\omega}_B$). In the first limit the test particle does not have time to complete one Larmor cycle, and generation of the wave t' is due to reradiation of the wave t by the particle whose unperturbed motion is effectively rectilinear. For simplicity, we do not consider the possibility of generation on higher cyclotron harmonics. In this case, the frequency of the radiated wave ω'^t is close to the frequency of the incident wave ω^t. For $\cos\Theta\simeq\cos\Theta'\approx1$ and $|\mathbf{E}_\perp|\approx|\mathbf{E}_\perp'|$, we obtain from Eq. (20),

$$\frac{dp_2}{dt}=\frac{\omega_{*0}\tilde{\omega}_B}{\omega^2}\frac{|\mathbf{E}_\perp^t|^2}{\gamma_{0\perp}}\sin(\Delta\omega t). \qquad (21)$$

In the second limit $\omega^t\ll\tilde{\omega}_B$, the wave t' is emitted over many Larmor cycles of the test particle. The frequency ω'^t is then close to the gyrofrequency $\tilde{\omega}_B$, and we have $\omega^t\ll\omega'^t$ for $\tilde{\omega}_B-\omega'^t\ll\omega_p$. We have in this case,

$$\frac{dp_2}{dt}=\frac{\omega_{*0}}{\tilde{\omega}_B}\frac{|\mathbf{E}_\perp'^t||\mathbf{f}_\perp|}{2\gamma_{0\perp}}\sin(\Delta\omega' t). \qquad (22)$$

This is a new type of the ponderomotive force that appears because of synchrotron radiation damping. The forces (21) and (22) are manifestation of the nonlinear coupling between the longitudinal and transverse components.

For further convenience, we rewrite Eqs. (21) and (22) as

$$\frac{dp_2}{dt}=a\sin(\Delta\omega t), \qquad (23)$$

where for $\omega^t\gg\tilde{\omega}_B$ and $\Delta\omega=(\omega^t-\omega'^t)/\omega_{*0}$,

$$a=\frac{\omega_{*0}\tilde{\omega}_B}{\omega^2}\frac{|\mathbf{E}_\perp^t|^2}{\gamma_{0\perp}}, \qquad (24)$$

and for $\omega^t\ll\tilde{\omega}_B$ and $\Delta\omega=(\tilde{\omega}_B-\omega')/\omega_{*0}$,

$$\frac{dp_2}{dt}=\frac{\omega_{*0}}{\tilde{\omega}_B}\frac{|\mathbf{E}'^t_\perp||\mathbf{f}_\perp|}{2\gamma_{0\perp}}. \quad (25)$$

From Eq. (23), we have the solution

$$p_{2z}(t)=\frac{a}{\Delta\omega}(1-\cos\Delta\omega t). \quad (26)$$

IV. NONLINEAR DYNAMICS OF THE FAST ORDINARY MODE

Forces (21) and (22) demonstrate nonlinear coupling of longitudinal components E^l with transverse components E^t. To describe the nonlinear dynamics, we start from Maxwell equations, which imply

$$\frac{\partial^2\mathbf{E}}{\partial t^2}+c^2\nabla\times\nabla\mathbf{E}+\frac{4\pi}{c}\frac{\partial\mathbf{j}}{\partial t}=0, \quad (27)$$

where $\mathbf{j}$ is the nonlinear current of second order in the electric field.

We consider a wave packet propagating at the small angle with respect to the external magnetic field. Separating the low-frequency and high-frequency transverse components, we have

$$\mathbf{E}=\mathbf{E}^l+\mathbf{E}^t, \quad (28)$$

where $E^t_z=0$, and assume $\omega^t\gg\omega^l$. Furthermore, we write

$$E^{l,t}_i=\tfrac{1}{2}E^{l,t}_i(\mathbf{r},t)\exp(i\omega^{l,t}t-i\mathbf{k}^{l,t}\cdot\mathbf{r}), \quad (29)$$

with $i=x,y,z$. For the transverse waves, $\omega^t\simeq k^t c$. For the longitudinal o mode (Langmuir wave), we have from Eq. (29) [see also Eq. (2)],

$$\omega_l^2=(\omega_*)^2(1+3k_z^2c^2+k_\perp^2c^2), \quad (30)$$

where $\omega_*^2=2\omega_p^2\gamma_0^{-3}$. We also introduce the potential perturbations of the background density; thus

$$\omega_*^2=\omega_{*0}^2\left(1+\frac{\delta n}{n_0}\right). \quad (31)$$

For the considered o mode, $\omega_p\gg k^l c$. The choice of the characteristic time scale strongly influences the considered equations. Here, we consider the case $\Delta\omega\ll\omega\sim\partial/\partial t$, which corresponds to the inequality (5). Thus we assume that in the longitudinal direction $k^l c\ll\Delta\omega\ll\omega_p$. For the transverse fields E^t, we have $\omega\ll k^t c$, as well as $k^l\gg|k'^t_\perp|-|k^t_\perp|>|k^t_z|-|k'^t_z|$ [cf. condition (5)], thus $k^l\gg\partial/\partial(x,y,z)$. Because of $\omega^t\gg\omega^l$, it is natural to obtain a system of coupled equations presented as a result of expansion in the parameter $\omega^l/\omega^t\ll 1$.

We introduce the dimensionless variables,

$$E\rightarrow\frac{eE}{mc\omega_p},\quad \mathbf{r}\rightarrow\frac{\omega_{*0}\mathbf{r}}{c},\quad t\rightarrow\omega_{*0}t. \quad (32)$$

Then, substituting Eqs. (28) and (29) into Eq. (27) and taking into account Eq. (32), we obtain

$$\frac{\partial E^t_{(x,y)}}{\partial t}=0, \quad (33)$$

and

$$2i\frac{\omega^l}{\omega_{*0}}\frac{\partial E^l_{(x,y)}}{\partial t}-i\frac{k^l_0c}{\omega_{*0}}\frac{\partial E^l_z}{\partial(x,y)}=\frac{\delta n}{n_0}E^l_{(x,y)},$$

$$2i\frac{\omega^l}{\omega_{*0}}\frac{\partial E^l_z}{\partial t}-i\frac{k^l_0c}{\omega_{*0}}\left(\frac{\partial E^l_x}{\partial x}+\frac{\partial E^l_y}{\partial y}\right)=\frac{\delta n}{n_0}E^l_z. \quad (34)$$

Equations (33) are written in the zeroth approximation for the expansion of Eq. (27) in the parameter ω^l/ω^t. They reflect the fact that the modulation of Langmuir waves has little effect on the amplitudes of the high-frequency t waves:

$$E^t_\perp\simeq\text{const}. \quad (35)$$

However, the nonlinear terms on the right-hand sides of Eq. (34) are determined by the amplitudes of the high-frequency waves $E^t_\perp$.

If we neglect the right-hand side in the first equation of (34), the resulting expression has a form similar to that found in [17]. However, the difference is in the nonlinear term: in [17], $\delta n/n_0\propto|E^l_z|^2$, whereas in our case the density perturbation is caused by the beating of the two high-frequency transverse waves. The expression for $\delta n/n_0$ can be found by averaging the continuity equation over the high frequency. We find

$$\frac{\partial}{\partial t}\frac{\delta n}{n_0}=\frac{\partial}{\partial z}p_{2z}. \quad (36)$$

Here, we take into account that $\partial\delta n/\partial x=\partial\delta n/\partial y=0$, and p_{2z} is defined by Eq. (26).

Excluding the term with E^l_z, we find from the first equation of (34),

$$\frac{\partial}{\partial t}(\boldsymbol{\nabla}\times\mathbf{E}^l)_z=\frac{\delta n}{n_0}(\boldsymbol{\nabla}\times\mathbf{E}^l)_z. \quad (37)$$

Thus we obtain

$$(\boldsymbol{\nabla}\times\mathbf{E}^l)_z=C\exp\left[\int^t\left(\frac{\delta n}{n_0}\right)dt\right], \quad (38)$$

where C is a constant. Since there is an instability of the curl field, below we assume that $\nabla_\perp\cdot\mathbf{E}^l_\perp=0$. Thus in the presence of the density perturbation there is exponential growth of the transverse fields. We note that in the drift approximation, the density modulation as well as the change of the momentum P_2, caused by the high-frequency fields E^t (when $\mathbf{k}\|\mathbf{B}_0$), are parallel to the axis $\mathbf{z}\|\mathbf{B}_0$. However, the growth of the the fields due to the parallel density modulations may be in any direction: $E^l_{(x,y,z)}\propto\exp(-ik_zz-i\mathbf{k}_\perp\cdot\mathbf{r}_\perp)$.

To obtain an equation for the parallel component E^l_z, we find the mixed derivatives $\partial^2E^l_{(x,y)}/\partial t\partial(x,y)$ from the first equation of (34). Thus, differentiating the second equation of (34) with respect to time (i.e., applying $\partial/\partial t$) and substituting the resulting expression for the mixed derivatives, we finally obtain

$$\frac{\partial^2 E_z^l}{\partial t^2}-\frac{1}{4}\left(\frac{k_0^l c}{\omega^l}\right)^2\Delta_\perp E_z^l+i\frac{\partial}{\partial t}\left(\frac{\delta n}{n_0}E_z^l\right)+\frac{i}{4}\frac{k_0^l c}{\omega^l}\frac{\delta n}{n_0}\boldsymbol{\nabla}_\perp\cdot\mathbf{E}_\perp^l=0. \tag{39}$$

As already noted, we neglect the last term on the left-hand side of this equation. The assumption $\boldsymbol{\nabla}_\perp\cdot\mathbf{E}_\perp^l=0$ implies that there are no components of the potential electric field perpendicular to the external magnetic field. We have

$$\frac{\partial^2 E_z^l}{\partial t^2}-K_0^2\Delta_\perp E_z^l+\frac{i}{2}\frac{\partial}{\partial t}\left(\frac{\delta n}{n_0}E_z^l\right)=0, \tag{40}$$

where $K_0=k_0^l c/2\omega_{*0}$. This equation has a form similar to that found in [17]. However, our nonlinear term is proportional to $(E_\perp^t)^2E_z^l$, in contrast to $(E_z^l)^3$ in [17].

Using the continuity equation (36) and Eq. (26), we Fourier transform (40). We also assume that the frequency of the modulations ω far exceeds $\delta\omega$:$\omega\gg\delta\omega$, and expand the electric field $E_z(\omega\pm\delta\omega)$ in the small parameter $\delta\omega/\omega$. Finally, the dispersion equation is given by

$$\omega^2-K_0^2k_\perp^2+\frac{k_z a}{\Delta\omega}=0. \tag{41}$$

From this equation, we can easily see that instability is possible when

$$\frac{k_z a}{\Delta\omega}>K_0^2k_\perp^2. \tag{42}$$

In the approximation considered, the aperiodic growth of the longitudinal potential field is not accompanied by a density modulation since the latter, as the continuity equation (36) implies, is determined by the high-frequency transverse fields. We assume that the energy of the high-frequency t modes is maintained by external sources, which is reasonable for the plasma in a pulsar magnetosphere where excitation of the transverse modes should be very effective [19–21].

From Eq. (36), we also find that

$$\frac{\delta n}{n_0}=-i\frac{k_z a}{\Delta\omega}\int_0^t(1-\cos\Delta\omega t)dt'=-i\frac{k_z a}{\Delta\omega}\left(t-\frac{1}{\Delta\omega}\sin\Delta\omega t\right). \tag{43}$$

Substitution of this equation into Eq. (38) and use of the expansion

$$\exp(i\alpha\sin\Delta\omega t)=\sum_{n=-\infty}^{+\infty}J_n(\alpha)e^{in\Delta\omega t} \tag{44}$$

gives us

$$(\boldsymbol{\nabla}\times\mathbf{E}^l)_z=C\sum_{n=-\infty}^{+\infty}J_n\left(\frac{k_z a}{(\Delta\omega)^2}\right)\exp\left[-i\left(\frac{k_z a}{(\Delta\omega)^2}-n\right)\Delta\omega t\right]. \tag{45}$$

From the latter expression, we see that (averaging over sufficiently long time interval $T\gg1/\Delta\omega$) the transverse perturbation generated by the density modulation is not zero when

$$n=\frac{k_z a}{(\Delta\omega)^2}. \tag{46}$$

In this case, $(\nabla\times\mathbf{E}^l)_z$ is determined by the Bessel function with equal index and argument, i.e., $J_\nu(\nu)\sim\nu^{-1/3}$ [24].

V. CONCLUSION

We conclude that transverse electromagnetic waves generated in a pulsar magnetosphere can create beat density modulations along the magnetic field. When the modulation frequency $\Delta\omega$ is much less than the frequency ω of the generated field pertubations, the growth of a parallel potential field E_z^l is accompanied by the growth of the transverse electromangetic field $\mathbf{E}_\perp$ according to Eq. (38). The results obtained contribute to and develop the theory of nonlinear wave-wave and wave-particle interactions in a pair plasma. Also, the processes studied can be applied to real astrophysical plasmas, in particular, those of pulsar magnetospheres (and the problem of transformation of energy of fast waves with phase velocities exceeding the speed of light into the pulsar radio emission with phase velocities less or equal to the speed of light). Indeed, in the development of the modulational instability, a nonresonant interaction with plasma particles can lead (because of the action of the ponderomotive force or the nonresonant quasilinear diffusion) to nonlinear generation of perpendicular components of particle momenta. If the perpendicular momenta are sufficient for generation of the high-frequency synchrotron radiation, we can expect appearance of an additional high-frequency source of the pulsar radio emission. However, we do not expect considerable change in the particle energies because in the processes of the nonresonant interactions the wave energy is distributed to all particles; at the same time, the change in the energy stored in the fast ''superluminal'' waves can be significant. Some of these mechanisms are now under investigation, and the results will be reported elsewhere.

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