Mean first-passage time of a bistable kinetic model driven by cross-correlated noises

Dongcheng Mei,[1,2] Guangzhong Xie,[1,2] Li Cao,[3] and Dajin Wu[3]
[1]*Yunnan Astrophysics Center, Yunnan University, Kunming 650091, China*
[2]*Yunnan Observatory, Chinese Academy of Sciences, Kunming 650011, China*
[3]*Department of Physics, Huazhong University of Science and Technology, Wuhan 430074, China*
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The transient properties of a bistable system driven by cross-correlated noises are investigated; the correlation times of the correlations between the noises are nonzero. The mean first-passage time (MFPT) is calculated. From numerical computations we find the following: (1) The MFPT of the system is affected by both the correlation time τ and the correlation strength λ; (2) τ and λ play opposing roles in the MFPT; (3) when λ or α/D(α and D are the additive and multiplicative noise intensities respectively) are far away from 1, the MFPT as a function of τ is monotonic; however, when both α/D and λ approach 1, the MFPT as a function of τ is nonmonotonic; (4) for the case of perfectly correlated noises ($\lambda=1$), the MFPT corresponding to $\alpha>D$ and $\alpha<D$ exhibit the same behaviors and the MFPT for $\alpha=D$ is continuous, which is very different from the case of the $\tau=0$ [Phys. Rev. E **53**, 5764 (1996)]. [S1063-651X(99)14203-X]

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I. INTRODUCTION

On the level of a Langevin-type description of a dynamical system, the presence of correlation between noises change the dynamics of the system [1–7]. Recently the steady-state statistical properties of a bistable kinetic model with correlations between additive and multiplicative noise for case of zero correlation time ($\tau=0$) are investigated in Ref. [3]. They showed that in the α-D parameter plane, the critical curve separating the unimodal and bimodal regions of stationary probability distribution (SPD) of the model is shown to be affected by λ, the strength of correlations between additive and multiplicative noise terms, the area of the bimodal region in the α-D plane is contracted as λ is increased; When α and D are fixed, the form of SPD changes from a bimodal to a unimodal structure as λ is increased. For the case of perfectly correlated noises ($\lambda=1$), the SPD's corresponding to $\alpha/D>1$ and $\alpha/D<1$ exhibit a very different shape of divergence, and $\alpha/D=1$ plays the role of a critical ratio. However, when $\tau\neq 0$, there is not the phenomenon of critical ratio ($\alpha/D=1$) on the steady-state statistical properties of the model [5]. The transient properties of the model for the case of $\tau=0$ are discussed in Ref. [4]. They showed that the mean first-passage time (MFPT) of the system is affected by λ. For case $\lambda=1$, the MFPT corresponding to $\alpha/D>1$ and $\alpha/D<1$ exhibit very different behaviors, and the MFPT for $\alpha/D=1$ diverges to infinity. However, the transient properties of the model for the case of $\tau\neq 0$ are not investigated. A natural question is whether the presence of nonzero correlation time changes the transient properties of the system.

In this paper, the transient properties of the model for the case of nonzero correlation times of correlations between additive and multiplicative noise sources are investigated. In Sec. II the analytic expressions of SPD and MFPT of the system are presented. In Sec. III discussion and conclusions end the paper.

II. DISTRIBUTION AND MEAN FIRST-PASSAGE TIME OF A BISTABLE KINETIC MODEL

Consider a one-dimensional bistable kinetic system

$$\frac{dx}{dt}=x-x^3+x\xi(t)+\eta(t), \tag{1}$$

where $\xi(t)$ and $\eta(t)$ are Gaussian white noises with zero mean, and

$$\langle\xi(t)\xi(t')\rangle=2D\delta(t-t'), \tag{2}$$

$$\langle\eta(t)\eta(t')\rangle=2\alpha\delta(t-t'). \tag{3}$$

Here α and D are the additive and multiplicative noise intensities, respectively. Assume that the correlation times of the correlations between $\xi(t)$ and $\eta(t)$ are nonzero [5–7]. Here, assume

$$\langle\xi(t)\eta(t')\rangle=\langle\eta(t)\xi(t')\rangle=\frac{\lambda\sqrt{\alpha D}}{\tau}\exp[-|t-t'|/\tau]$$
$$\to 2\lambda\sqrt{\alpha D}\,\delta(t-t') \quad \text{as} \quad \tau\to 0, \tag{4}$$

where τ is the correlation time of the correlations between $\xi(t)$ and $\eta(t)$, and λ denotes the strength of correlation between the $\eta(t)$ and $\xi(t)$. The potential

$$U(x)=-\tfrac{1}{2}x^2+\tfrac{1}{4}x^4 \tag{5}$$

corresponding to Eq. (1) has two stable states $x_1=-1$, $Fx_2=1$ and an unstable state $x_0=0$. By virtue of the Novikov theorem [8], Fox's approach [9], and the ansatz of Hanggi *et al.* [10], the approximate Fokker-Plank equation corresponding to Eq. (1) with Eqs. (2), (3), and (4), is obtained [5]:

$$\frac{\partial P(x,t)}{\partial t}=-\frac{\partial}{\partial x}A(x)P(x,t)+\frac{\partial^2}{\partial x^2}B(x)P(x,t), \tag{6}$$

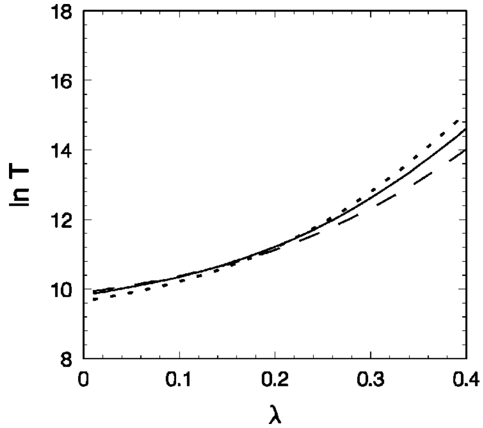

FIG. 1. The MFPT of the bistable system (12) as a function of λ for $\tau=0.2$ with $\alpha=0.1$ and $D=0.1$ (straight line), $\alpha=0.12$ and $D=0.1$ (dotted line), $\alpha=0.1$ and $D=0.11$ (dashed line).

where

$$A(x)=x-x^3+Dx+\frac{2\lambda\sqrt{\alpha D}}{1+2\tau} \tag{7}$$

and

$$B(x)=Dx^2+\frac{2\lambda\sqrt{\alpha D}}{1+2\tau}x+\alpha. \tag{8}$$

The stationary probability distribution of system can be obtained from Eq. (6) with Eqs. (7) and (8) and is given by

$$P_{st}(x)=NB(x)^{-1/2}\exp\left(-\frac{\vec{U}(x)}{D}\right) \quad \text{for} \quad 0\leqslant\lambda\leqslant 1, \tag{9}$$

where the generalized potential is given in terms of a quadrature by

$$\vec{U}(x)=-\int^x \frac{(z-z^3)dz}{z^2+\frac{2\lambda}{1+2\tau}\sqrt{\frac{\alpha}{D}}z+\frac{\alpha}{D}}, \tag{10}$$

and N is the normalization constant of Eq. (9). It must be pointed out that the correlation time τ must be zero when the strength of the correlation between noises λ is zero; however, Eq. (9) is valid when $\tau=0$. The form of the stationary probability had been investigated previously in Ref. [5]. Our prime concern here is the transient properties of the system, i.e., the MFPT of a particle escapes from a stable state.

We now proceed with the MFPT. The exact expression for the MFPT for a particle to reach the final point x_2, from the initial point x_1 is given by [11–13]

$$T(x_1\to x_2)=\int_{x_1}^{x_2}\frac{dx}{B(x)P_{st}(x)}\int_{-\infty}^{x}P_{st}(y)dy. \tag{11}$$

In particular, we choose $x_1=-1$ as the initial point, and $x_2=0$ as the final point. Substituting Eqs. (8) and (9) into Eq. (11), the expression of MFPT is obtained:

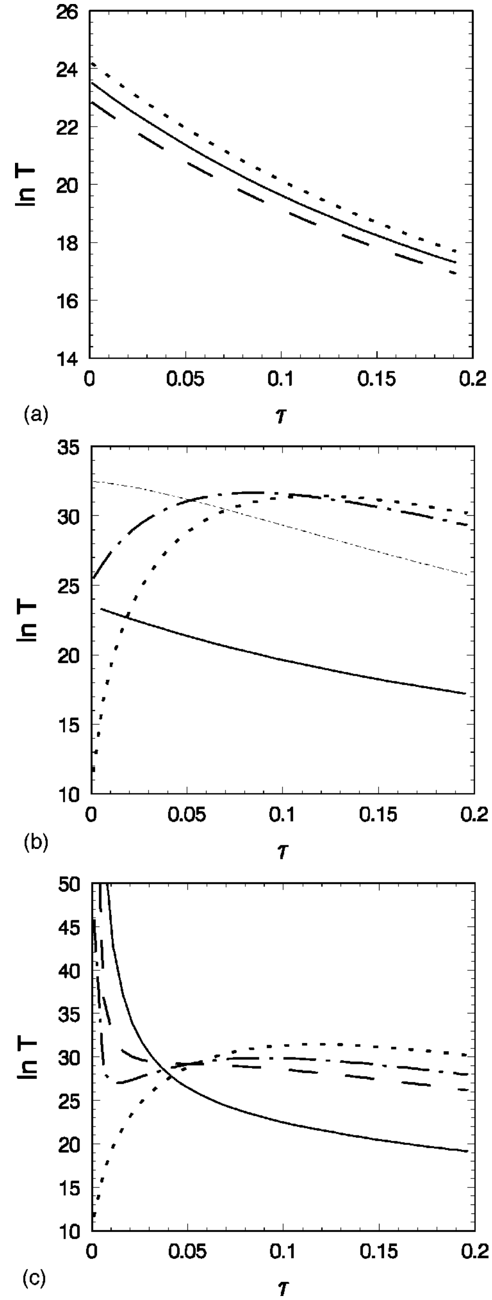

FIG. 2. (a) The MFPT of the bistable system (12) as a function of τ for $\lambda=0.5$ with $\alpha=0.1$ and $D=0.1$ (straight line), $\alpha=0.11$ and $D=0.1$ (dotted line), $\alpha=0.09$ and $D=0.1$ (dashed line). (b) The MFPT of the bistable system (12) as a function of τ for $\alpha=D=0.1$, with $\lambda=0.5$ (straight line), $\lambda=0.8$ (dashed line), $\lambda=0.95$ (dash-dotted line), $\lambda=1$ (dotted line). (c) The MFPT of the bistable system (12) as a function of τ for $\lambda=1$ with $\alpha=0.1$ and $D=0.2$ (straight line), $\alpha=0.1$ and $D=0.12$ (dashed line), $\alpha=0.1$ and $D=0.105$ (dash-dotted line), $\alpha=D=0.1$ (dotted line).

$$T(-1\to 0)=D^{-1}\int_{-1}^{0}dx\; F(x)\exp[\vec{U}(x)/D]$$

$$\int_{-\infty}^{x}dy\;\; F(y)\exp[-\vec{U}(y)/D], \tag{12}$$

where

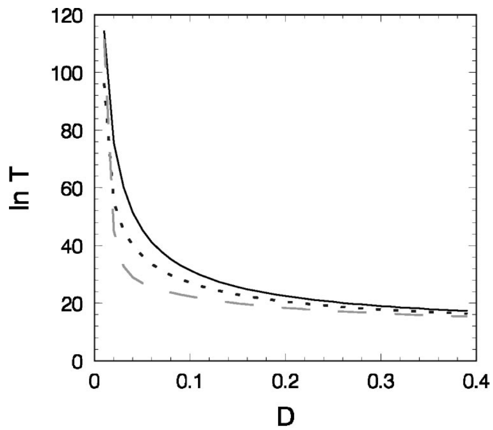

FIG. 3. The MFPT of the bistable system (12) as a function of D for $\lambda=1$ and $\tau=0.1$ with $\alpha=0.1$ (straight line) $\alpha=0.18$ (dotted line), $\alpha=0.3$ (dashed line).

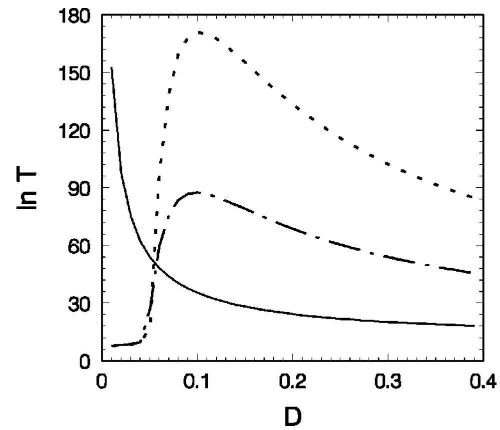

FIG. 4. The MFPT of the bistable system (12) as a function of D for $\lambda=1$ and $\alpha=0.05$ with $\tau=0.1$ (straight line), $\tau=0.02$ (dash-dotted line), $\tau=0.01$ (dotted line).

$$F(x)=\left(x^2+\frac{2\lambda}{1+2\tau}\sqrt{\frac{\alpha}{D}}x+\frac{\alpha}{D}\right)^{-(1/2)}. \tag{13}$$

From Eq. (13), we can see that when $\tau\neq 0$, as $x\rightarrow -1$ the value of the $F(x)$ for the case of $\lambda=1$ and $\alpha=D$ does not diverge to infinity. However, for the case of that, the value of the $F(x)$ approaches infinity as well as $\vec{U}(x)$ (10) diverges to infinity as $\tau\rightarrow 0$ and $x\rightarrow -1$, therefore, the value of the MFPT [Eq. (12)] approaches to infinity. The transition from $x=-1$ to $x=0$ of a particle is suppressed.

III. DISCUSSION AND CONCLUSIONS

When the correlation time τ is zero, the MFPT of the bistable kinetic model has been discussed in Ref. [4], and will not be recounted here. Our aim in this paper is to discuss the MFPT of the bistable system when the correlation time τ is nonzero. By virtue of numerical calculation of the MFPT [Eq. (12)], we have plotted the MFPT for the different values of parameters (τ, λ, α, and D) in Figs. 1–4. All quantities plotted are dimensionless as in Ref. [4]. The conclusions that can be drawn from these figures are as follows.

When we fix the value of the correlation time (e.g., $\tau=0.2$) and take the values α and D in the neighborhood of $\alpha=D$, Figures 1(a) and 1(b) show that the MFPT [Eq. (12)] increases as the λ increases. Figure 1(a) shows that the MFPT [Eq. (12)] for $\alpha<D$ is the biggest in the three cases $\alpha>D$, $\alpha=D$, and $\alpha<D$ under the same λ as $\lambda<0.175$. However, the MFPT [Eq. (12)] for $\alpha>D$ is the biggest in these cases under the same λ as $\lambda>0.175$.

When we fix the value λ (e.g., $\lambda=0.5$) and take the values α and D in the neighborhood of $\alpha=D$, Fig. 2(a) shows that the MFPT [Eq. (12)] decreases as the τ increases. It is interesting to point out that the MFPT [Eq. (12)] for $\alpha>D$ is the biggest in the three cases $\alpha>D$, $\alpha=D$, and $\alpha<D$ under the same τ as shown in Fig. 2(a). λ (λ is far away from 1) and τ play an opposing role in the MFPT of the system. However, from Fig. 2(b), we can see that when λ came close to 1, the MFPT [Eq. (12)] for case $\alpha=D$ increases as τ increases and when τ exceeded a certain value (i.e., 0.1), then that decreases as τ incteases. From Fig. 2(c), we also can see that when α/D approached 1, the MFPT [Eq. (12)] for case $\lambda=1$ increases as τ increased and when τ exceeded a certain value (i.e., 0.1) then that decreases as τ increases. Stated briefly, when λ or α/D are far away from 1, the MFPT as a function of τ is monotonic. However, when both α/D and λ approach 1, the MFPT as a function of τ is nonmonotonic.

The MFPT [Eq. (12)] corresponding to $\alpha<D$ and $\alpha>D$ exhibits the same behavior, namely the MFPT [Eq. (12)] decreases with both α and D increasing, and the MFPT [Eq. (12)] for $\alpha=D$ is continuous, which coincides with the conclusions of the case $\lambda=0$ ($\tau=0$) in Ref. [14]. However, when the τ approaches zero ($\tau=0.02$ and $\tau=0.01$), the MFPT [Eq. (12)] exhibits a big peak at position $\alpha=D$ as shown in Fig. 4, that is, the MFPT [Eq. (12)] corresponding to $\alpha>D$ and $\alpha<D$ exhibits very different behaviors [4]: For $\alpha>D$, the MFPT [Eq. (12)] increases with D increasing under the same α ($\alpha=0.05$); However, for $\alpha<D$, the MFPT [Eq. (12)] decreases with D increasing under the same α ($\alpha=0.05$).

The value of the $F(x)$ [Eq. (13)] is finite, the MFPT [Eq. (12)] corresponding to $\alpha<D$ and $\alpha>D$ exhibits the same behavior, namely the MFPT [Eq. (12)] decreases with α and D increasing. However, as $\tau\rightarrow 0$ and $x\rightarrow -1$, for case $\lambda=1$ and $\alpha=D$, $F(x)$ [Eq. (13)] approaches infinity, then the MFPT [Eq. (12)] approaches infinity. Therefore, the MFPT [Eq. (12)] exhibits a big peak at position $\alpha=D$ as the τ approached zero. The MFPT [Eq. (12)] corresponding to $\alpha>D$ and $\alpha<D$ exhibits very different behaviors.

ACKNOWLEDGMENTS

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