Hybrid solitary waves in quadratic nonlinear media

Antonio Picozzi and Marc Haelterman
Service d'Optique et d'Acoustique, Université Libre de Bruxelles, 50 Avenue F.D. Roosevelt, Code Postal 194/5, B-1050 Brussels, Belgium
(Received 30 October 1998)

Considering nondegenerate, backward quasi-phase-matched parametric interaction, we show that quadratic media support two dimensional ''hybrid'' solitary waves. Their structure results from the combination of two distinct mechanisms which, in isolation, are at the origin of the two classes of quadratic solitary waves considered in nonlinear optics. In the transverse dimension the structure results from a balance between diffraction and quadratic nonlinearity while the longitudinal structure results from net energy exchanges between the three interacting velocity-mismatched waves. The hybrid solitary waves can propagate at arbitrarily small velocity, a feature that should make them easy to observe experimentally. [S1063-651X(99)05803-1]

PACS number(s): 42.65.Tg, 42.65.Yj, 42.65.Re

Nonlinear localized waves and solitons are ubiquitous in physics. They can be found in such diverse fields as hydrodynamics, plasma physics, and nonlinear optics where they are grouped in numerous classes according to their originating mechanism. In nonlinear optics two main classes of solitary waves are distinguished that are of fundamentally different nature. On the one hand, one finds solitary waves that arise from the interplay of nonlinearity and diffraction (or dispersion in the temporal case). These localized waves are well known in cubic nonlinear media where they can take the form of bright or dark solitons. They have been generalized to quadratic nonlinear media through the concept of parametric solitary wave, for the bright [1] as well as for the dark [2,3] structures. In view of their potential applications to all-optical switching, parametric solitary waves have attracted growing attention in the last few years both from theoretical [4] and experimental view points [5]. The second class of solitary waves of nonlinear optics gathers the solitary waves that originate from energy exchanges between interacting waves of different velocities [6]. Their structure is determined by an exact balance between the energy exchange rates and the velocity mismatch between the interacting waves. This type of solitary wave also occurs in other branches of nonlinear science, such as plasma physics, hydrodynamics, or acoustics [7,8]. Several solitary waves of this class, but with different originating mechanisms, have been extensively investigated in the field of nonlinear optics [9–11]. In particular, the similarity between three-wave interaction solitary waves and the self-induced transparency soliton [12] governed by the sine-Gordon equation has been established [8]. Besides this latter case, energy-exchange-induced solitary waves have been observed experimentally in the context of stimulated Raman [10] and Brillouin [11] scattering. In the particular case of backward interaction in quadratic nonlinear media, their spontaneous formation has been predicted in both the amplifier [13] and the cavity [14] configurations.

We consider here the backward phase-matching configuration of the nondegenerate three-wave interaction in quadratic media. We report, for the first time to our knowledge, on a hybrid solitary wave that arises from the combined action of two distinct mechanisms corresponding to the two classes of solitary waves in nonlinear optics. The hybrid solitary wave is two dimensional. Its transverse structure results from a balance between diffraction and nonlinearity (first class) while its longitudinal structure is due to a net energy exchange between the three interacting waves (second class). This longitudinal structure was recently shown to form a solitary-wave attractor in the backward configuration [13]. The backward configuration was chosen because it confers robustness to the solitary wave especially as regards the onset of modulational instability. This is important in the present context since the transverse structure of the hybrid solitary wave is of the dark type that was shown to be always modulationally unstable in diffractive quadratic solitary waves [3,15,16]. Here, owing to the robustness of the hybrid solitary wave against modulational instability the dark transverse structure is stable .

We consider a quadratic material in which nondegenerate parametric interaction takes place through backward quasi-phase-matching so that one of the daughter waves (i.e., the signal) counterpropagates with respect to the pump field. The idea of the quasi-phase-matching technique is to modulate periodically the nonlinear susceptibility in order to introduce an additional wave vector that compensates for the natural phase mismatch between the counterpropagating fields [17]. Under these conditions the slowly varying field envelopes A_i at frequency ω_i and wave number k_i, obey the coupled partial differential equations:

$$\frac{\partial A_1}{\partial t} - \frac{\partial A_1}{\partial z} + \mu_1 A_1 = A_3 A_2^* + i\kappa_1 \frac{\partial A_1}{\partial y^2}, \tag{1a}$$

$$\frac{\partial A_2}{\partial t} + r_2 \frac{\partial A_2}{\partial z} + \mu_2 A_2 = \rho_2 A_3 A_1^* + i\kappa_2 \frac{\partial A_2}{\partial y^2}, \tag{1b}$$

$$\frac{\partial A_3}{\partial t} + r_3 \frac{\partial A_3}{\partial z} + \mu_3 A_3 = -\rho_3 A_2 A_1 + i\kappa_3 \frac{\partial A_3}{\partial y^2}, \tag{1c}$$

with $\omega_3 = \omega_2 + \omega_1$ and $k_3 = k_2 + K - k_1$, where $K = 2\pi/\Lambda$, Λ being the spatial period of the grating. For definiteness we call A_1, A_2, A_3 the signal, idler, and pump

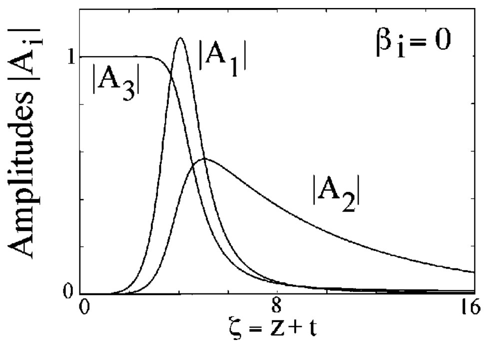

FIG. 1. Typical solitary-wave solution in the pure one-dimensional case. Parameters are $\kappa_i=0$, $\mu_1=0.3$, $\mu_2=0.35$, $\mu_3=0$.

waves, respectively. For convenience, the field amplitudes, the time t, the space coordinates (z,y), and the damping rates γ_i are normalized with respect to the pump amplitude E_0 at the input of the crystal and with respect to the parametric coupling coefficient $\sigma_1=2\pi d v_1/\lambda_1 n_1$ (where n_i, v_i, and d, respectively, are the refractive index, the group velocity at frequency ω_i, and the effective nonlinear susceptibility), i.e., $A_i/E_0\rightarrow A_i$; $t\sigma_1 E_0\rightarrow t$; $(z,y)\sigma_1 E_0/v_1\rightarrow(z,y)$; $\gamma_i(\sigma_1 E_0)^{-1}\rightarrow\mu_i$. In these units, the diffraction coefficients are $\kappa_i=v_i\sigma_1 E_0/2v_1^2 k_i$, while the nonlinear susceptibility and group velocity parameters are $\rho_i=\sigma_i/\sigma_1$ and $r_i=v_i/v_1$ (i=1,2). From now on, we will assume for simplicity and without loss of generality that $\rho_3=2\rho_2=2$ and $r_3=r_2=1$.

In a recent work we investigated Eqs. (1) in the pure one-dimensional case ($\kappa_i=0$) and found a family of solitary-wave solutions [13] whose characteristic shape is represented in Fig. 1. The solution is reached starting from any initially localized profile of the signal envelope in the presence of a counterpropagating continuous pump. In order to investigate the existence of hybrid solitary waves, we introduced here the transverse dimension through the diffraction terms in Eqs. (1) ($\kappa_i\neq 0$). Noting the particular symmetry of Eqs. (1) which are invariant under the transformation $(A_1,A_2,A_3)\rightarrow(-A_1,-A_2,A_3)$, we can easily anticipate the existence of dark topological structure across the transverse profile of the solitary wave. Indeed, the sign indetermination of A_1,A_2 in Eqs. (1) should allow for the parametric growth of the signal and idler modes with a phase difference of π in two distinct regions of the transverse space. Such a phase defect would form a dark topological solitary wave if it could be stabilized through a mutual compensation of diffraction and nonlinearity. Due to the nontrivial energy exchange mechanism that forms the longitudinal solitary-wave structure in the counterpropagating waves, this transverse stabilization mechanism is not obvious and should be checked numerically with great care.

In order to check the existence and spontaneous formation of such a hybrid dark solitary wave, we consider here the numerical simulation of the backward amplification process of a signal field that exhibits a π-phase shift in its transverse profile and that is localized in time. Note that since we are looking for a solitary-wave structure induced by the energy transfer from the pump to the signal and idler waves, we have to assume zero loss for the pump ($\mu_3=0$). It is the only way to keep constant the energy transfer in order to generate stationary field structures. This approximation is usual for solitary waves that belong to the second class and will be discussed later.

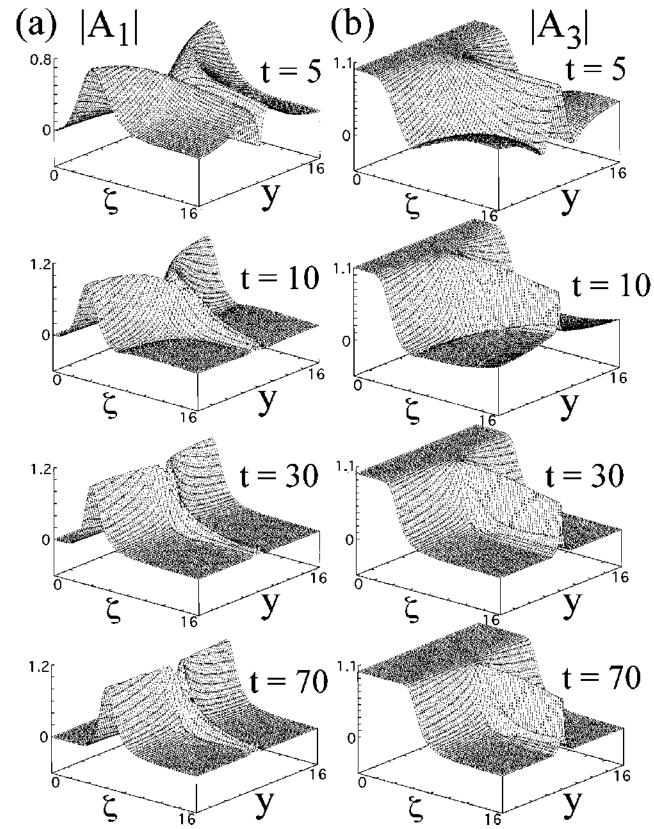

FIG. 2. Hybrid solitary-wave generation: evolution of spatial amplitudes profiles of the signal (a) and pump (b) envelopes [along the longitudinal (z) and transverse (y) axis] in the signal reference frame defined by ($\zeta=z+t,\tau=t$) (amplitudes are given in units of E_0).

Under these conditions we solve numerically Eqs. (1) extending to two dimensions the procedure outlined in Ref. [18]. A typical result is illustrated in Fig. 2 that shows the spatial profile along the longitudinal and tranverse axis of the signal and pump waves in the signal reference frame defined by ($\zeta=z+t,\tau=t$). In this example the damping parameters are $\mu_1=0.3,\mu_2=0.35$ and the diffraction parameters are $\kappa_1=\kappa_2=10^{-3},\kappa_3=0.5\times10^{-3}$. As the initial condition in $t=0$, we took a plane wave $A_3(z,y,t=0)=1$ for the pump. For the signal we considered a transverse dark profile bounded along the longitudinal z axis $A_1(z,y,t=0)\propto\epsilon\tanh[\Delta(y-L/2)]z(L-z)$, where ϵ, Δ are constants, L is the size of the numerical window, and for the idler a zero field $A_2(z,y,t=0)=0$. After a complex transient ($t<10$) the three interacting fields self-structurate in the form of the anticipated hybrid solitary wave ($t>30$). The same solution is reached starting from any signal envelope, provided that it exhibits a transverse π-phase shift. This allows us to consider the hybrid solitary wave as a strong attractor solution of the system.

We plot in Fig. 3 the longitudinal and transverse profiles of the hybrid solitary wave. As expected, the two-dimensional structure is hybrid in the sense that, on the one hand, it is localized in the transverse dimension as a dark

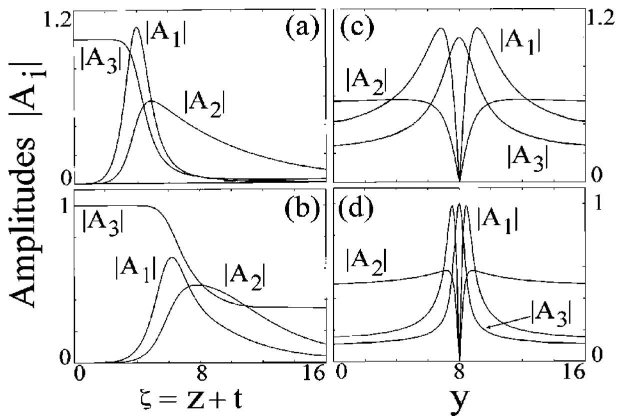

FIG. 3. Typicals longitudinal (a: $y=1$; b: $y=7.5$) and transverse (c: $\zeta=5$; d: $\zeta=6$) profiles of the hybrid solitary wave in its asymptotic regime ($t=30$).

structure of the first class where diffraction is balanced by the nonlinear coupling. On the other hand, its longitudinal profile is localized in the form of a solitary wave reminiscent of the second class. Because of its topological nature, the transverse structure is robust and survives all along the transient and asymptotic dynamics. Far from the phase defect line located in $y=0$, the signal and idler envelopes tend to the profile of the longitudinal solitary wave in the absence of diffraction [this is clearly evidenced by comparing Fig. 2(a) and Fig. 1]. Note that the pump wave is not of the dark type in the transverse dimension, in contrast with the signal and idler modes [Figs. 3(c) and 3(d)]. In this respect, the transverse dark structure of the hybrid solitary wave shares the properties of the spatial topological phase defect found in degenerate optical parametric oscillators [19]. In particular, as in the optical parametric oscillator, the pump envelope in the form of a hump sitting on a constant background is explained by the local frustration of the frequency conversion process due to the zero value of the signal and idler intensities imposed by the phase defect.

The hybrid solitary wave proved to be robust with respect to modulational instabilities. In all our numerical simulations we could not identify any growing modes that might be responsible for modulational instability. This result contrasts with the previously reported quadratic spatial dark solitary wave that was shown to be always modulationally unstable [3]. Let us emphasize that the robustness of the hybrid solitary wave is intimately related to the backward configuration of the parametric interaction considered here. Indeed, due to large wave velocity differences, the backward interaction is responsible for a strong localization of the signal and idler components along the longitudinal axis [20,14]. Under this condition, a given point of the pump carrier wave only interact with the daughter waves over a very short time (of the order of the pulse duration), which prevents the onset of the modulational instability.

Let us remark that the hybrid solitary wave does not propagate with the velocity of light in the quadratic material but rather with a specific subluminous velocity. This is visible in Fig. 2 where we see that the steady-state (i.e., for $t>30$) structure drifts uniformly to the right in the signal reference frame. This is not surprising since we have shown in the one-dimensional case ($\kappa_i=0$) that the solitary waves propagate with a selected subluminous velocity [13]. The two-dimensional hybrid solitary wave considered here can be viewed as being formed by the coupling through diffraction of a continuous set of longitudinal structures. The important point is that, because of the phase defect, these longitudinal solitary waves obviously have different amplitudes as evidenced in Figs. 3(a) and 3(b). As the selected velocity depends on the amplitude of the solitary wave, one could have expected that this nonuniform amplitude distribution results in a certain distribution of the velocities of the longitudinal solitary waves that would have therefore led to a global spreading of the two-dimensional structure. The remarkable and unexpected result is that diffraction coupling locks together all the longitudinal solitary waves of different amplitudes and gives rise to the hybrid solitary wave that propagates with a peculiar subluminous velocity.

To determine the selected velocity, say V_z^*, remark that in the regions far from the phase defect, the wavefront is completely flat and diffraction plays no role. The longitudinal profile of the hybrid solitary wave in these regions thus takes the same shape as that of the pure one-dimensional structure. Therefore, by virtue of the stationarity of the two-dimensional structure, the selected velocity of the hybrid solitary wave is determined by the velocity of the pure one-dimensional one. This velocity has been determined analytically in Ref. [13] following the Kolmogorov-Petrovskii-Piskunov conjecture [21] and reads

$$V_z^* = \frac{\mu_2^2 - \mu_1^2 + 4\sqrt{1-\mu_1\mu_2}}{4+(\mu_1-\mu_2)^2}. \tag{2}$$

We checked by numerical simulation the validity of this theoretical prediction. We found a discrepancy $(V_{z,theor}^* - V_{z,num}^*)/V_{z,num}^*$ between the numerical and theoretical values of V_z^* less than 0.1%.

In order to clarify the experimental conditions required for the observation of hybrid solitary waves, let us note that, according to Eq. (2), their velocity V_z^* can be arbitrarily small and even 0. Due to the short typical lengths of available quadratic crystals, the generation of solitary waves of the second class in copropagating phase-matching configuration is not feasible because it requires prohibitive pump powers. Conversely, on the backward configuration, arbitrarily small velocities are possible which makes the hybrid solitary waves observable with relatively low pump intensities. This simply because the time spent in the crystal can be made sufficiently long to allow transient dynamics and complete buildup of the fields to take place within the crystal length. According to Eq. (2), we have $V_z^*=0$ when the pump amplitude E_0 is chosen such that $\mu_1+\mu_2=(\gamma_1+\gamma_2)/\sigma_1 E_0 =2$. Moreover, for this particular case of zero velocity, we may expect to generate the stationary hybrid solitary wave in the presence of pump loss ($\mu_3 \neq 0$). Indeed in this case, the energy transfer from the pump to the signal and idler waves remains constant since the signal and idler envelopes do not move with respect to the exponential pump profile. For nonzero velocity in the case of nonzero pump loss, the solitary wave has to adapt its shape to each value of the pump amplitude and is therefore no longer stationary.

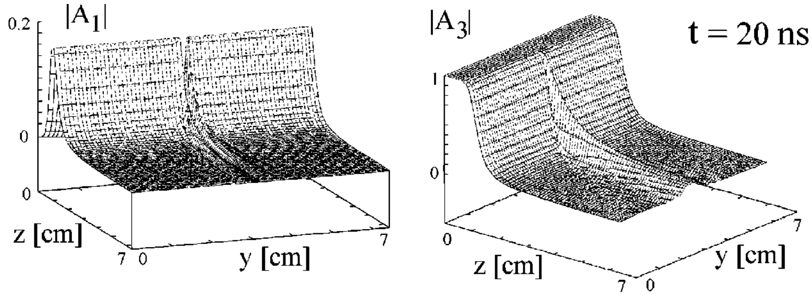

FIG. 4. Signal and pump amplitudes profiles of the zero-velocity hybrid solitary wave (amplitudes are given in units of E_0).

We simulated Eqs. (1) with the following diffraction parameters $\kappa_1 = \kappa_2 = 2\times 10^{-2}$, $\kappa_3 = 10^{-2}$ and the damping parameters $\gamma_1 = 12\ \mathrm{cm}^{-1}$, $\gamma_2 = 3\ \mathrm{cm}^{-1}$, $\gamma_3 = 0.5\ \mathrm{cm}^{-1}$ that corresponds to a pump intensity $I = 30\ \mathrm{MW/cm}^2$ launched in a crystal of length $L = 7$ cm with an effective nonlinear coefficient of $d = 50$ pm/V. The simulation has been performed in the laboratory reference frame starting from the same initial conditions as in Fig. 1. After a complex transient the three interacting fields self-structurate in the form of a zero-velocity hybrid solitary wave whose asymptotic signal and pump envelopes are represented in Fig. 4. Owing to its zero velocity, we have been able to pursue the numerical integration over very long times. In the example of Fig. 2, $t = 2000$ corresponds in dimensional units to 20 ns. Careful checks of the numerical simulations allow us to conclude that the hybrid solitary wave is rigorously stationary and robust against modulational instabilities.

Let us note that a bright counterpart of the dark hybrid solitary wave presented here cannot be expected contrary to what is found in the well-known purely diffractive solitary waves, Refs. [2–4]. Indeed, a transversely limited pump beam cannot see its own diffraction compensated by the nonlinearity all along its propagation since interaction with the signal and idler waves only takes place over a small region of the longitudinal axis.

In summary, we showed that backward nondegenerate parametric interaction in quadratic media sustains a new type of two-dimensional hybrid solitary waves. The structure of these new nonlinear waves results from the interplay of the two mechanisms that are at the origin of two classes of quadratic solitary waves that were up to now considered separately in nonlinear optics. These mechanisms are, on the one hand, the balance between diffraction and nonlinearity that leads to a dark solitary-wave structure in the transverse dimension and, on the other hand, a net energy exchange between the interacting velocity-mismatched waves which leads to longitudinal confinement. Our numerical simulations show that the dark hybrid solitary waves are stable against modulational instability contrary to their purely diffractive counterpart. Moreover, the hybrid solitary waves can have an arbitrarily small velocity, which makes them observable experimentally in quadratic crystals of practical lengths. The experimental observation of this hybrid solitary structure would be of great interest for the fundamental study of spontaneous localization phenomena in nonlinear optics.

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