Control of harmonic generation in a free-electron laser with a quasiperiodic wiggler configuration

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A quasiperiodic wiggler configuration made from two periodic wigglers of irrational period ratio is considered for application to free-electron lasers (FEL's). With an analytical formulation of its spontaneous emission, spectral behavior of a quasiperiodic wiggler is studied. It is found that the nonlinear coupling between the two constituent wigglers of the quasiperiodic wiggler configuration leads to significant spontaneous emission at both the usual FEL harmonic frequencies and new frequencies that are irrational multiples of the fundamental FEL radiation frequency. By adjusting the relative field strengths of the two constituent wigglers and their period ratio, the usual FEL harmonics can be suppressed considerably. This unique spectral characteristic of quasiperiodic wigglers may be used to relieve the problem of mirror damage in some free electron lasers operating in ultraviolet and x-ray wavelengths.
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I. INTRODUCTION

For many electron beam devices, electron interaction with a single-frequency electromagnetic field is relatively well understood. New interaction mechanisms may be produced when the electron beam is strongly coupled to an electromagnetic field at different frequencies. In the case of free-electron lasers (FEL's), such new interaction mechanisms are often enhanced for their exploration by altering the characteristics of the wiggler magnet. For instance, it has been suggested to use an auxiliary harmonic wiggler to enhance radiation at higher harmonics when the field strength of the main wiggler is only modest [1,2]. A different harmonic wiggler configuration has also been shown to be capable of enhancing the gain of optical klystron devices at the fundamental radiation frequency [3]. For high gain Compton FEL's, however, it has been found useful to employ a magnet system comprising two wigglers of similar periods to control the generation of sidebands [4,5]. In addition, a wiggler system of a double period structure has been proposed for mode selection purpose in low gain waveguide FEL's [6].

In a recent paper, we have studied an unusual FEL interaction mechanism in which the electron beam is premodulated at two different frequencies [7]. We have shown that if the two modulation frequencies are not integrally but irrationally related then considerable radiation is generated at frequencies that are irrational multiples of the fundamental FEL radiation frequency. The generation of these irrationally related harmonics modifies the basic FEL interaction mechanism and as such they may be utilized to manipulate the FEL spectrum. In this paper, we extend our previous results and consider the realization of electron beam modulation at two irrationally related frequencies by means of a quasiperiodic wiggler configuration consisting of two conventional periodic wigglers. In order to understand the characteristics of this quasiperiodic wiggler configuration, spontaneous emission of a general dual-wiggler structure is formulated. Number theory arguments [8] are then used to derive the resonant condition for an FEL interaction in an infinitively long quasiperiodic wiggler. Subsequently numerical examples are used to demonstrate that significant radiations can be generated simultaneously at both the usual FEL harmonics, which are integer multiples of the fundamental FEL radiation frequency, and the irrationally related harmonics. When the interference between the two constituent wigglers of the quasiperiodic wiggler configuration is appropriately adjusted and enhanced, the integrally related harmonics are suppressed considerably. This may be used to relieve the problem of mirror degradation for some FEL's operated at ultraviolet and x-ray wavelengths [9,10].

It is worth pointing out that in subjects such as solid-state physics and crystallography, studies of which have, in the past, been centered around periodic structures, the latest development has been the study of quasiperiodic systems, for instance, quasicrystals [11]. Motivated by this, our work aims to study possible benefits of quasiperiodic modulation of electron beams in free-electron laser devices.

II. SPONTANEOUS EMISSION IN A DUAL-WIGGLER MAGNET

We consider a magnet system composed of two different conventional wigglers of periods λ_{w1} and λ_{w2}, respectively. For the simplicity, we assume $\lambda_{w1}>\lambda_{w2}$ and that the on-axis magnetic field of this dual-wiggler configuration may be approximated as

$$\vec{B}_w=\hat{y}(B_{w1}\cos k_{w1}z+B_{w2}\cos k_{w2}z), \tag{1}$$

where $k_{wn}=2\pi/\lambda_{wn}(n=1,2)$. Suppose the lengths of the two constituent wigglers are $L_1=N_1\lambda_{w1}$ and $L_2=N_2\lambda_{w2}$, respectively, where N_1 and N_2 are integers. We further assume $0\leqslant L_2-L_1\leqslant\lambda_{w2}$ such that $N_2=1+\mathrm{Int}[L_1/\lambda_{w2}]$. Therefore Eq. (1) is valid for $0<z\leqslant L_1$ whereas $\vec{B}_w=\hat{y}B_{w2}\cos k_{w2}z$ in the small region of $L_1<z\leqslant L_2$.

Under the perturbation of the alternating magnetic field of Eq. (1), an energetic electron beam traveling along the wiggler axis radiates and the induced electromagnetic radiation may be represented by its on-axis electric field component of

$$\vec{E}=\hat{x}E_0\sin\Phi=\hat{x}E_0\sin(\omega t-kz+\phi). \tag{2}$$

The trajectory of an electron beam in the combined field of the wiggler magnet and the electron induced radiation is in general governed by its equation of motion,

$$\frac{d(\gamma m\vec{v})}{dt}=-e(\vec{E}+\vec{v}\times\vec{B}_w). \tag{3}$$

In the limit of small radiation field, the transverse velocity of an electron entering the dual wiggler along the magnet axis may be easily obtained from Eq. (3) as

$$\beta_x=\frac{1}{\gamma}(a_{w1}\sin k_{w1}z+a_{w2}\sin k_{w2}z), \tag{4}$$

where $a_{wn}=eB_{wn}/mck_{wn}$ is the dimensionless field strength of the nth constituent wiggler $(n=1,2)$. Once again with the small radiation field assumption, the electron energy exchange with the radiation field may be considered to be negligible in the calculation of electron trajectory and as such the longitudinal electron velocity can be derived from $\beta_0^2=\beta_x^2+\beta_z^2$ ($\beta_0 c$ being the initial electron velocity) as

$$\beta_z=\left[1-\frac{1+(a_{w1}\sin k_{w1}z+a_{w2}\sin k_{w2}z)^2}{\gamma^2}\right]^{1/2}. \tag{5}$$

For most short-wavelength free-electron lasers driven by rf linacs and electron storage rings, the electron energy is highly relativistic [12]. Thus $\gamma^2\gg 1$ and the above equation may be approximated as

$$\begin{aligned}\beta_z&\approx 1-\frac{1}{2\gamma^2}(a_{w1}\sin k_{w1}z+a_{w2}\sin k_{w2})^2\\&=1-\frac{1}{4\gamma^2}\{(1+a_{w1}^2+a_{w2}^2)-a_{w1}^2\cos 2k_{w1}z\\&\quad-a_{w1}^2\cos 2k_{w1}z-2a_{w1}a_{w2}\cos(k_{w1}+k_{w2})z\\&\quad+2a_{w1}a_{w2}\cos(k_{w2}-k_{w1})z\}.\end{aligned} \tag{6}$$

It is seen that due to the nonlinear dependence of β_z on the two wiggler fields in Eq. (5) the electron's longitudinal velocity contains oscillations at not only the externally imposed wave numbers of k_{w1} and k_{w2} but also the sums and differences of these wave numbers. More specifically, we obtain terms dependent upon $2k_{w1}, 2k_{w2}, (k_{w1}+k_{w2})$, and $(k_{w2}-k_{w1})$. These wave numbers play a very significant role in determining the FEL spectrum as will be shown in Sec. IV. Note that the high-order terms neglected in deriving Eq. (6) contain many other wave numbers, which have very small amplitudes for highly relativistic electron beams.

The energy exchange of an electron with its radiation field is governed by the energy conservation law

$$\frac{d(\gamma mc^2)}{dt}=-e\vec{E}\cdot\vec{v}. \tag{7}$$

Substitution of Eq. (2) into the above equation gives

$$\frac{d\gamma}{dt}=-\frac{eE_0}{2\gamma mc}\sum_{n=1}^{2}a_{wn}[\cos(\Phi-k_{wn}z)-\cos(\Phi+k_{wn}z)]. \tag{8}$$

As illustrated in Eq. (8), the longitudinal electron velocity of Eq. (6) affects crucially the beam-wave interaction described by Eq. (7) via the phase angles of $\Phi\mp k_{wn}z=\omega t-(k\pm k_{wn})z+\phi$ in which the electron spatial location z needs to be expressed in terms of t. To this end, the electron average velocity in the longitudinal direction, $\beta_{z0}c$, needs to be formulated first. Integration of Eq. (6) gives the average longitudinal velocity

$$\beta_{z0}\approx 1-\frac{1}{2\gamma^2}(1+\tfrac{1}{2}a_{w1}^2+\tfrac{1}{2}a_{w2}^2+\xi a_{w1}a_{w2}), \tag{9}$$

where

$$\xi=\frac{\sin(2\pi N_1\lambda_{w1}/\lambda_{w2})}{N_1\pi(\lambda_{w1}^2/\lambda_{w2}^2-1)}. \tag{10}$$

Note that $\xi=1$ when $\lambda_{w1}=\lambda_{w2}$. However, $\xi\ll 1$ if the two wiggler periods differ from each other appreciably. With β_{z0} formulated in Eq. (9), Eq. (6) becomes

$$\begin{aligned}\beta_z&=\beta_{z0}+\frac{1}{4\gamma^2}[a_{w1}^2\cos 2k_{w1}z+a_{w2}^2\cos 2k_{w2}z]\\&\quad+\frac{a_{w1}a_{w2}}{2\gamma^2}[\xi+\cos(k_{w1}+k_{w2})z-\cos(k_{w1}-k_{w2})z].\end{aligned} \tag{11}$$

If we approximate z with the nominal electron position $z_0=c\beta_{z0}t$ on the right-hand side of the above equation, a straightforward integration of β_z in Eq. (11) leads to the formulation of the electron spatial location as

$$z=c\int_0^{z_0/\beta_{z0}c}\beta_z(\tau)d\tau=z_0+\frac{1}{k}[p(z_0)+q(z_0)], \tag{12}$$

where

$$p(z_0)=\frac{ka_{w1}^2}{8\gamma^2\beta_{z0}k_{w1}}\sum_{n=1}^{2}\left[\frac{a_{wn}}{a_{w1}}\right]^2\frac{\lambda_{wn}}{\lambda_{w1}}\sin 2k_{wn}z_0, \tag{13a}$$

$$q(z_0)=\frac{ka_{w1}a_{w2}}{2\gamma^2\beta_{z0}k_{w1}}\left[\xi k_{w1}z_0+\frac{\sin(k_{w1}+k_{w2})z_0}{1+\lambda_{w1}/\lambda_{w2}}-\frac{\sin(k_{w1}-k_{w2})z_0}{1-\lambda_{w1}/\lambda_{w2}}\right]. \tag{13b}$$

The terms $p(z_0)$ and $q(z_0)$ are related to the nonlinear coupling between the two wiggler components of the dual-wiggler structure, and as will be shown in Sec. IV they affect crucially the FEL spectrum. Using Eq. (12), the phase angles in Eq. (8) become

$$\Phi \mp k_{wn} z \approx \left[\frac{\omega}{c\beta_{z0}} - (k \pm k_{wn}) \right] z_0 + \phi - [p(z_0) + q(z_0)]$$
$$= \Delta k_n^{\pm} z_0 + \phi - [p(z_0) + q(z_0)], \qquad (14)$$

where $\Delta k_n^{\pm} z = [\omega/(c\beta_{z0}) - (k \pm k_{wn})] z$ and the approximation of $(k \pm k_{wn})/k \approx 1$ has been assumed.

In Eq. (14), all phase angles of relevance to Eq. (8) are expressed in terms of the nominal electron position only. This allows Eq. (8) to be integrated directly to give the electron energy change over the entire length of the dual wiggler, $\Delta\gamma$. According to Madey's theorem, on the other hand, the spontaneous emission of the electron beam is proportional to $\langle(\Delta\gamma)^2\rangle$ where $\langle\cdots\rangle$ denotes an average over the initial electron phase. Without the angular dependence of the spontaneous emission taken into account, this relationship in SI units is given by [13,14]

$$\frac{d^2W}{d\omega d\Omega} = \frac{m^2 c \omega^2}{8\pi^2 \epsilon_0 E_0^2} \langle(\Delta\gamma)^2\rangle. \qquad (15)$$

Therefore $\langle(\Delta\gamma)^2\rangle$ needs to be formulated from $\Delta\gamma$. From Eqs. (8) and (14), we obtain

$$\Delta\gamma = -\frac{eE_0 L}{2\gamma\beta_{z0} m c^2} \sum_{n=1}^{2} a_{wn} (\mathcal{T}_n^+ - \mathcal{T}_n^-), \qquad (16)$$

where

$$\mathcal{T}_n^{\pm}(\phi) = \mathrm{Re}\left\{ \frac{1}{L} \int_0^L \exp i(\Delta k_n^{\pm} z_0 - p(z_0) - q(z_0) + \phi) dz_0 \right\}$$
$$= \mathrm{Re}\{ e^{i\phi} [a_n^{\pm} + i b_n^{\pm}] \} \qquad (17)$$

and

$$a_n^{\pm} = \frac{1}{L} \int_0^L \cos[\Delta k_n^{\pm} z_0 - p(z_0) - q(z_0)] dz_0, \qquad (18a)$$

$$b_n^{\pm} = \frac{1}{L} \int_0^L \sin[\Delta k_n^{\pm} z_0 - p(z_0) - q(z_0)] dz_0. \qquad (18b)$$

Thus the summation term in Eq. (16) becomes

$$\sum_{n=1}^{2} a_{wn} (\mathcal{T}_n^+ - \mathcal{T}_n^-)$$
$$= \mathrm{Re}\left\{ e^{i\phi} \left[\sum a_{wn}(a_n^+ - a_n^-) + i \sum a_{wn}(b_n^+ - b_n^-) \right] \right\}$$
$$= \left\{ \left[\sum a_{wn}(a_n^+ - a_n^-) \right]^2 + \left[\sum a_{wn}(b_n^+ - b_n^-) \right]^2 \right\}^{1/2}$$
$$\times \cos(\phi + \theta), \qquad (19)$$

where

$$\theta = \tan^{-1} \left(\frac{\sum a_{wn}(a_n^+ - a_n^-)}{\sum a_{wn}(b_n^+ - b_n^-)} \right). \qquad (20)$$

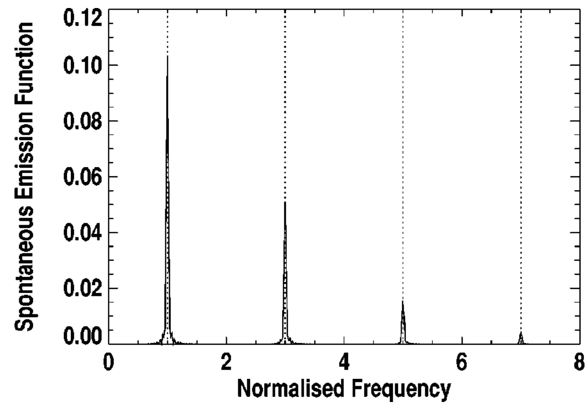

FIG. 1. Spontaneous emission as a function of the radiation frequency normalized to the fundamental frequency for a single wiggler of $a_w = 1$.

The relationship in Eq. (19) can then be used to derive the electron energy change in Eqs. (15) and (16). Introducing

$$\mathcal{F}^2 = \left[\sum a_{wn}(a_n^+ - a_n^-) \right]^2 + \left[\sum a_{wn}(b_n^+ - b_n^-) \right]^2 \qquad (21)$$

we have

$$(\Delta\gamma)^2 = \left(\frac{eE_0 L}{2\gamma\beta_{z0} m c^2} \right)^2 \mathcal{F}^2 \cos^2(\phi + \theta) \qquad (22a)$$

$$\langle(\Delta\gamma)^2\rangle = \frac{1}{2} \left(\frac{eE_0 L}{2\gamma\beta_{z0} m c^2} \right)^2 \mathcal{F}^2. \qquad (22b)$$

Consequently the spontaneous emission in Eq. (15) is formulated from Eq. (22b) to give

$$\frac{d^2W}{d\omega d\Omega} = \frac{1}{4} \sqrt{\frac{\mu_0}{\epsilon_0}} \left[\frac{eN_1}{2\gamma(1-\beta_{z0})} \right]^2 \left(\frac{\omega}{\omega_{r1}} \frac{L}{L_1} \right)^2 \mathcal{F}^2, \qquad (23)$$

where $\omega_{r1} = k_{w1} c \beta_{z0} / (1 - \beta_{z0})$ is the fundamental resonant frequency in the first constituent wiggler. To emphasize the spectral features of spontaneous emission, we introduce a spontaneous emission function of

$$\mathcal{S} \equiv \frac{\omega^2}{\omega_{r1}^2} \frac{\mathcal{F}^2}{a_{w1}^2}. \qquad (24)$$

In Fig. 1, we plot the spontaneous emission function against the normalized radiation frequency ω/ω_{r1} under the condition of $a_{w1} = 1$ and $a_{w2} = 0$. When normalized to the value of the spontaneous emission at the fundamental frequency, the values at the third, fifth, and seventh harmonics are found to be 0.4866, 0.1485, and 0.0424, respectively. These figures compare very well with those predicted with the following relationship [15]:

$$\frac{d^2W}{d\omega d\Omega} \propto h^2 [J_{(h-1)/2}(f\zeta) - J_{(h+1)/2}(f\zeta)]^2, \qquad (25)$$

where $\zeta = (a_w^2/4)/(1 + a_w^2/2)$ and h is the number of harmonic. In other words, the formulation in Eqs. (21) and (23) developed for dual wigglers reproduces the well known results of Eq. (25) for the single-wiggler case.

III. RESONANT CONDITION IN A DUAL WIGGLER

With the eight spatial integrals defined in Eq. (18), Eq. (23) may be used to calculate numerically the spontaneous emission in a dual-wiggler configuration for different combination of wiggler fields and periods. To aid such a study, however, it is desirable to identify first conditions under which there is a strong spontaneous emission.

It is well known that an electron beam radiates most strongly when it is in resonance with its radiating electromagnetic field. For the case of a dual wiggler, this resonance condition is satisfied when one or more spatial integrals in Eq. (18) become maximized. However the analytical complexity of Eq. (18) makes it difficult and ineffective to formulate exactly the resonant condition in a dual wiggler of finite length. To simplify the mathematical derivation, we approximate the finite dual-wiggler configuration with an infinite dual wiggler and consider the following integral:

$$\mathcal{J}=\lim_{L\to\infty}\frac{1}{L}\int_0^L \exp i(\Delta k_n^{\pm}z_0-p(z_0)-q(z_0)). \tag{26}$$

Here $p(z_0)$ and $q(z_0)$ may be reexpressed from Eq. (13) as

$$p(z_0)=-u_1\sin 2k_{w1}z_0-u_2\sin 2k_{w2}z_0, \tag{27a}$$

$$q(z_0)=-u_0k_{w1}z_0-u_3\sin(k_{w1}+k_{w2})z_0 -u_4\sin(k_{w2}-k_{w1})z_0, \tag{27b}$$

where

$$\chi=a_{w1}^2/[8\gamma^2(1-\beta_{z0})], \tag{28a}$$

$$u_0=-4(a_{w2}/a_{w1})(\omega/\omega_{r1})\xi\chi, \tag{28b}$$

$$u_1=-(\omega/\omega_{r1})\chi, \tag{28c}$$

$$u_2=(a_{w2}/a_{w1})^2(\lambda_{w2}/\lambda_{w1})u_1, \tag{28d}$$

$$u_3=4(a_{w2}/a_{w1})u_1/(1+\lambda_{w1}/\lambda_{w2}), \tag{28e}$$

$$u_4=4(a_{w2}/a_{w1})u_1/(1-\lambda_{w1}/\lambda_{w2}). \tag{28f}$$

Note that

$$e^{i(\eta\sin\alpha z)}=\sum_{n=-\infty}^{+\infty}J_n(\eta)e^{in\alpha z}, \tag{29}$$

thus the phase angle in Eq. (26) becomes

$$\begin{aligned}
&\exp i(\Delta k_n^{\pm}z_0-p(z_0)-q(z_0))\\
&=\exp i[\Delta k_n^{\pm}z_0+u_0k_{w1}z_0+u_1\sin 2k_{w1}z_0+u_2\sin 2k_{w2}z_0\\
&\quad+u_3\sin(k_{w1}+k_{w2})z_0+u_4\sin(k_{w2}-k_{w1})z_0]\\
&=\exp[i(\Delta k_n^{\pm}z_0+u_0k_{w1}z_0)]\\
&\quad\times\sum_{n_1}J_{n_1}(u_1)e^{i2n_1k_{w1}z_0}\sum_{n_2}J_{n_2}(u_2)e^{i2n_2k_{w2}z_0}\\
&\quad\times\sum_{n_3}J_{n_3}(u_3)e^{in_3(k_{w1}+k_{w2})z_0}\sum_{n_4}J_{n_4}(u_4)e^{in_4(k_{w2}-k_{w1})z_0}\\
&=\sum_{n_1}\sum_{n_2}\sum_{n_3}\sum_{n_4}J_{n_1}(u_1)J_{n_2}(u_2)J_{n_3}(u_3)J_{n_4}(u_4)\\
&\quad\times\exp[i(\Delta k_n^{\pm}z_0+(2n_1+n_3-n_4+u_0)k_{w1}z_0\\
&\quad+(2n_2+n_3+n_4)k_{w2}z_0)].
\end{aligned} \tag{30}$$

As illustrated in their definition in Eq. (28), u_0, u_1, u_2, u_3, and u_4 are independent of z_0. Hence the infinite integral of Eq. (26) may be reduced to

$$\begin{aligned}
\mathcal{J}=&\sum_{n_1}\sum_{n_2}\sum_{n_3}\sum_{n_4}J_{n_1}(u_1)J_{n_2}(u_2)J_{n_3}(u_3)J_{n_4}(u_4)\\
&\times\lim_{L\to\infty}\frac{1}{L}\int_0^L\{\exp[i(\Delta k_n^{\pm}+(2n_1+n_3-n_4+u_0)k_{w1}\\
&+(2n_2+n_3+n_4)k_{w2})z_0]\}dz_0.
\end{aligned} \tag{31}$$

Note that

$$\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}e^{i\Omega t_0}dt=\begin{cases}0 & \text{if}\quad \Omega\neq 0,\\ 1 & \text{if}\quad \Omega=0.\end{cases} \tag{32}$$

Therefore $\mathcal{J}$ is finite only when

$$\Delta k_n^{\pm}+(2n_1+n_3-n_4+u_0)k_{w1}+(2n_2+n_3+n_4)k_{w2}=0 \tag{33}$$

is satisfied. Since ξ is small when $\lambda_{w1}\neq\lambda_{w2}$,

$$|u_0|=\frac{\xi a_{w1}a_{w2}(\omega/\omega_{r1})}{1+a_{w1}^2/2+a_{w2}^2/2+\xi a_{w1}a_{w2}}\ll\frac{\omega}{\omega_{r1}}. \tag{34}$$

Also from Eq. (14), we have

$$\Delta k_n^{\pm}=\frac{\omega}{c\beta_{z0}}-k\mp k_{wn}=k_{w1}\frac{\omega}{\omega_{r1}}\mp k_{wn}. \tag{35}$$

Using the condition of Eqs. (34) and (35), Eq. (33) becomes

$$\frac{\omega}{\omega_{r1}}-l-n\frac{\lambda_{w1}}{\lambda_{w2}}=0, \tag{36}$$

where l,n are integers. This is the resonant condition for a strong spontaneous emission in dual wigglers of infinite length.

We first consider the case where the ratio of the two wiggler periods is a rational number. This implies that there exist a pair of integers h_1 and h_2 such that $\lambda_{w1}/\lambda_{w2}=h_1/h_2$. As a result, Eq. (36) becomes

$$\frac{\omega}{\omega_{r1}}=l+n\frac{h_1}{h_2}. \tag{37}$$

The above equation suggests that if the wiggler periods are of rational ratio, the radiation frequency of a dual wiggler is also a rational multiple of the fundamental resonant frequency of its first constituent wiggler (and also of the second wiggler). In other words, the induced radiation in the dual wiggler occurs at one or more integral harmonics of ω_{r1}. In the special case of $l=0,n=1$ and $h_2=1$, Eq. (37) reduces to

$$\lambda_{w1}/\lambda_{w2}=h_1, \quad \omega/\omega_{r1}=h_1, \tag{38}$$

suggesting that if the period of the second wiggler is a harmonic of that of the first then the radiation of the dual wiggler is at the same harmonic of the resonant frequency of its first constituent wiggler. Therefore Eq. (40) is essentially the resonant condition in the two harmonic wiggler configuration [1,2,16]. As an extension from the conventional wiggler configuration, the two harmonic wiggler concept has been proposed to enhance FEL radiation at a particular harmonic frequency [1]. However, these two different wiggler configurations are similar in the sense that their spontaneous emissions are both induced at integer harmonics of the fundamental radiation frequency of one of the constituent wigglers.

On the other hand, if the wiggler period ratio is an irrational number, then there exist no two integers h_1 and h_2 that satisfy Eq. (37) [8]. In other words,

$$\frac{\omega}{\omega_{r1}} \neq l+n\frac{h_1}{h_2}. \tag{39}$$

For such cases, the spontaneous emission peaks at a frequency that is not rationally related to ω_{r1} and is said to occur at an ''irrational harmonic'' of the fundamental radiation frequency of the first constituent wiggler [7]. It is conceivable that based on such an unique spectral property new techniques may be derived to relieve the long-standing problem of mirror damage by harmonic radiation in x-ray free-electron laser and synchrotron devices.

IV. APPLICATION

The resonant condition of Eq. (33) is derived for a dual wiggler of infinite length. For dual wigglers of a finite length, their spontaneous emission needs to be assessed from Eq. (23) through the eight finite spatial integrals in Eq. (18). We first consider the case of a harmonic wiggler configuration with $N_1=20, a_{w1}=1, a_{w2}=0.4$, and $\lambda_{w2}=\lambda_{w1}/3$. The spontaneous emission is calculated from Eqs. (18), (23), and (24), and plotted as a function of the radiation frequency in Fig. 2(a). Compared to the single wiggler case of Fig. 1, Fig. 2(a) suggests that the addition of a second wiggler of $\lambda_{w2}=\lambda_{w1}/3$, a third harmonic of the period of the original wiggler, leads to an enhanced spontaneous emission at the third and ninth harmonics of the fundamental radiation frequency ω_{r1}, both of which are also odd harmonics of the second wiggler. This agrees well with the conclusions drawn in previous studies of harmonic wigglers [1,2] and thus confirms that the spontaneous emission is enhanced at certain harmonics of ω_{r1} when the magnetic field is increased at these harmonics of λ_{w1} with the addition of the second wiggler. On the other hand, it is of interest to note that also enhanced is the radiation at the fifth and seventh harmonics of ω_{r1}, neither of which are harmonics of the second wiggler. This suggests that radiation enhancement observed at the fifth and seventh harmonics may have resulted from the cross terms (the nonlinear terms), through $p(z_0)$ and $q(z_0)$ in Eq. (13), between magnetic fields of the first and the second wigglers.

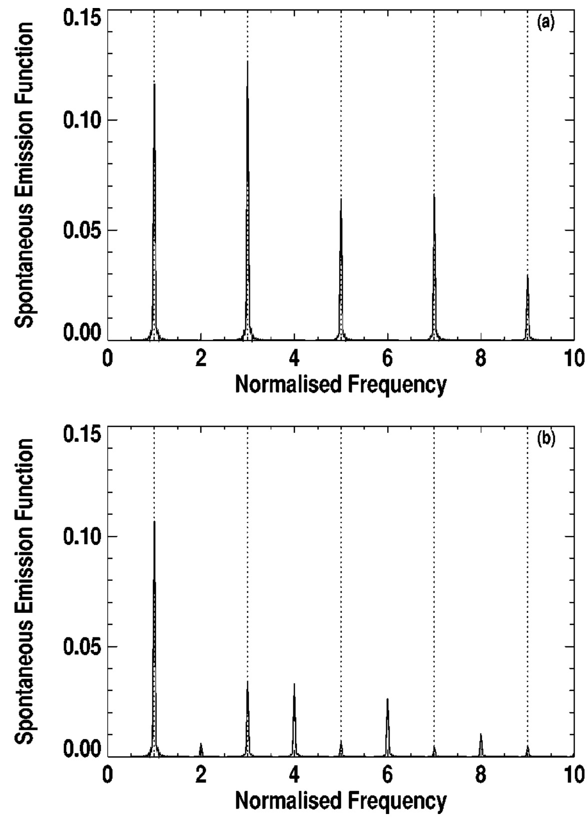

FIG. 2. Spontaneous emission function against (ω/ω_{r1}) in a two harmonic wiggler of $N_1=20, a_{w1}=1$, and $a_{w2}=0.4$ with (a) $\lambda_{w1}/\lambda_{w2}=3$ and (b) $\lambda_{w1}/\lambda_{w2}=2$.

In other words, the nonlinear coupling, or the interference, between the two wiggler components of a dual wiggler is responsible for the emission enhancement at frequencies that are not common harmonics of the two wiggler components.

In Fig. 2(b), the spontaneous emission is plotted as a function of radiation frequency for a similar harmonic wiggler with $\lambda_{w2}=\lambda_{w1}/2, a_{w1}=1, a_{w2}=0.4$, and $N_1=20$. Thus the second wiggler is effectively a second harmonic wiggler of the first constituent wiggler. As shown in Fig. 2(b), the introduction of this second harmonic wiggler results in additional radiation peaks at even harmonics of ω_{r1}. Also shown in Fig. 2(b) is that many of the original odd harmonics of ω_{r1} experience a reduction in their emission strength except for the fundamental frequency. Since the odd harmonics of the second wiggler are not integrally related to that of the first, the spectral characteristics in Fig. 2(b) cannot be explained by the argument of simple superposition of odd harmonics of the two constituent wigglers. The aforementioned observation of Fig. 2(b) thus supports the suggestion that the interference between magnetic fields of the two constituent wigglers affect strongly the strength of spontaneous emission, especially at frequencies that are not common harmonics of these two constituent wigglers.

In order to understand the characteristics of quasiperiodic wigglers, particularly their similarity to and difference from harmonic wigglers, we consider an example of $a_{w1}=1, a_{w2}=0.4$, and $\lambda_{w1}/\lambda_{w2}=\sqrt{5}$. The spontaneous emission is plotted in Fig. 3 and exhibits radiation peaks at the fundamental frequencies as well as around their odd harmonics of both the first and the second wiggler components of the quasi-

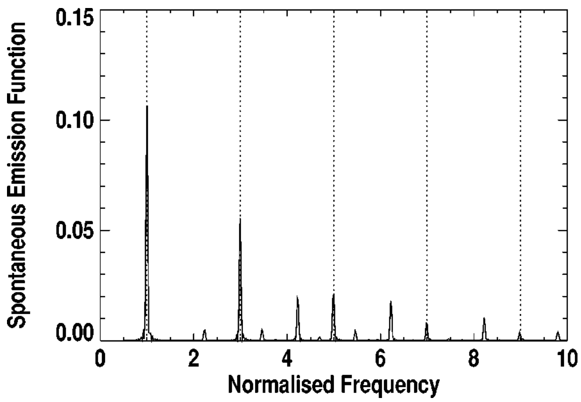

FIG. 3. Spontaneous emission in a quasiperiodic wiggler of $a_{w1}=1, a_{w2}=0.4, N_1=20$, and $\lambda_{w1}=\sqrt{5}\lambda_{w2}$.

periodic wiggler. In addition, radiation is also seen to be significant at frequencies that are not harmonics of either wiggler component. These additional radiation peaks are again a consequence of the interference between the two wiggler components.

It is evident from Figs. 2 and 3 that there are more radiation peaks in the spontaneous emission spectrum of a quasiperiodic wiggler than in that of a comparable harmonic wiggler. In the case of harmonic wigglers, strong radiation occurs only at integer harmonics of the fundamental radiation frequency ω_{r1}, whereas for a quasiperiodic wiggler spontaneous emission can be significant at integer harmonic frequencies of the fundamental radiation frequencies of both constituent wigglers as well as other frequencies. These additional radiation frequencies are not integer multiples of either of the two fundamental radiation frequencies, and thus are referred to as ''spurious harmonics'' of the quasiperiodic wiggler. Because of the generation of these spurious harmonics, the spontaneous emission spectrum of a quasiperiodic wiggler is more complex than that of a comparable harmonic wiggler. It is our main motivation to examine whether new physics realized with the quasiperiodic wiggler configuration may be used to improve the performance of free-electron lasers and similar devices.

To demonstrate clearly the generation of spurious harmonics, we consider spontaneous emission at different ratios of wiggler periods of the two wiggler components in a quasiperiodic wiggler. In Fig. 4(a), the spontaneous emission is plotted as a function of radiation frequency and wiggler period ratio for a quasiperiodic wiggler of $a_{w1}=1, a_{w2}=0.8$, and $N_1=20$. It is seen from Fig. 4(a) that although strong spontaneous emissions are indeed induced at the usual radiation frequencies of the first wiggler component (at ω_{r1} and its odd harmonics), they are suppressed considerably by the presence of a large number of smaller radiation peaks scattered in areas where the ratio of wiggler periods is close to unity. Detailed inspection of Fig. 4(a) suggests that most of them occur at frequencies that are not integer multiples of the fundamental frequencies of either wiggler components. Hence we extrapolate that these new radiation peaks are induced by the interference between the two wiggler components. Examination of Eqs. (13), (18), and (21) indicates that the interference between two wiggler components is contained mainly in $q(z_0)$ but more importantly that $q(z_0)$ is indeed significant when $\lambda_{w1}/\lambda_{w2}\approx 1$ as it contains a

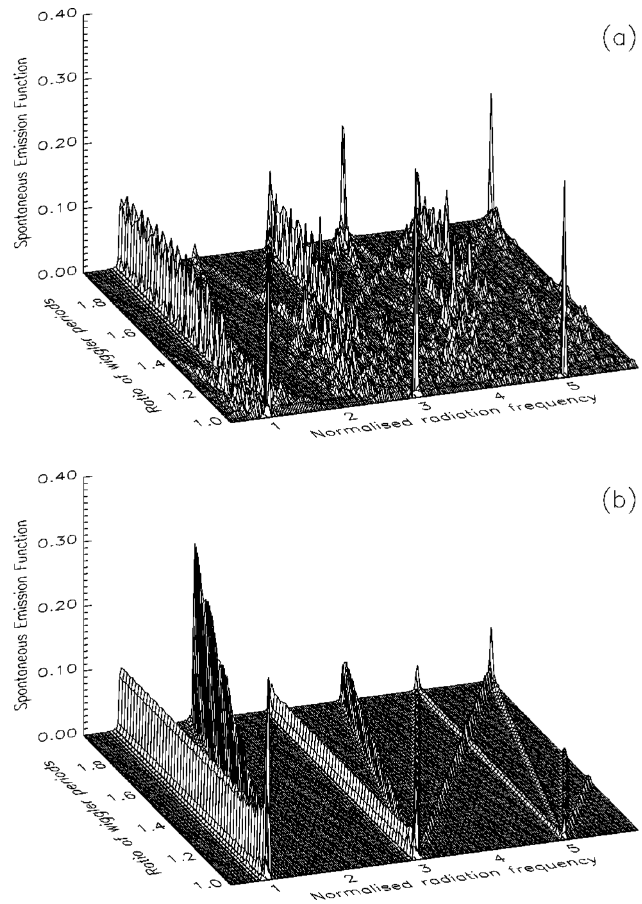

FIG. 4. Spontaneous emission function in a quasiperiodic wiggler of $a_{w1}=1, a_{w2}=0.8$, and $N_1=20$ with (a) $q(z_0)$ given in Eq. (13b) and (b) $q(z_0)$ set to zero.

$\sin(k_{w1}-k_{w2})z_0/(1-\lambda_{w1}/\lambda_{w2})$ term. As a result, it is reasonable to deduce that the interference between the two wiggler components affects the FEL mechanism predominately through $q(z_0)$ of Eq. (13b). To confirm this quantitatively, the spontaneous emission of Fig. 4(a) is recalculated using Eqs. (13), (18), (21), and (24) with the contribution of $q(z_0)$ set to zero and the results are plotted in Fig. 4(b). It is shown in Fig. 4(b) that the majority of the small radiation peaks in Fig. 4(a) are now absent and the spontaneous emission spectrum now contains predominately the fundamental and odd harmonics of both wiggler components.

Also present in Fig. 4(b) is a small group of radiation peaks at frequencies that are not the usual odd harmonics of either wiggler component of the quasiperiodic wiggler but nevertheless appear to be related to these harmonics in a simple manner. These are also due to the interference between the two wiggler components through $p(z_0)$ rather than $q(z_0)$. This may be understood with a resonant condition analysis similar to that in the previous section but with $q(z_0)$ set to zero. In this case,

$$n_3=n_4=0, \quad u_0=0 \tag{40}$$

in Eqs. (27)–(33) and thus Eq. (33) becomes

$$\frac{\omega}{\omega_{r1}}k_{w1}\mp k_{wn}+2n_1k_{w1}+2n_2k_{w2}=0 \tag{41}$$

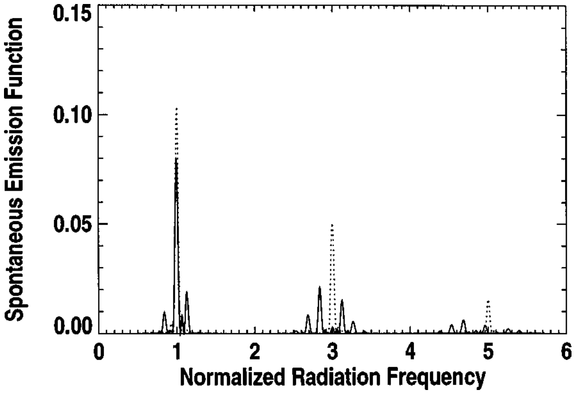

FIG. 5. Spontaneous emission function in a conventional wiggler (dashed curve) and a quasiperiodic wiggler of $a_{w2}=0.2$ and $\lambda_{w1}/\lambda_{w2}=\sqrt{7}-1.5$. For both cases, $a_{w1}=1$ and $N_1=20$.

or

$$\frac{\omega}{\omega_{r1}}=-2n_1-2n_2\frac{\lambda_{w1}}{\lambda_{w2}}\pm\frac{\lambda_{w1}}{\lambda_{wn}}. \tag{42}$$

Here radiation frequencies are simply proportional to the ratio of wiggler periods with different coefficients as suggested in Fig. 4(b). Thus the radiation frequencies additional to the fundamental and odd harmonics of both wiggler components in Fig. 4(b) are also spurious harmonics although their generation is due to $p(z_0)$ rather than $q(z_0)$. Mathematically the addition of $q(z_0)$ in Eqs. (18) and (21) merely permits many more combinations of integers n_1,n_2,n_3, and n_4 that satisfy Eq. (33). This is particularly the case in areas of low wiggler ratios where the difference between the two wiggler periods is small to permit many different values of n to satisfy Eq. (41). As a result, significant overlaps of radiation peaks are developed in areas of $\lambda_{w2}/\lambda_{w1}\approx 1$. Since these interference induced radiations are not rationally related to the usual odd harmonic radiations of the two individual wiggler components, the latter are suppressed considerably.

The suppression of harmonics of ω_{r1} is useful for many coherent radiation sources designed to operate at ultraviolet and x-ray wavelengths. For UV free-electron laser devices, mirror degradation by harmonic radiation and other photons at above the fundamental FEL radiation frequency is a major obstacle for their successful operation [17–19]. Since the quasiperiodic wiggler may be used to control and suppress harmonic generation, it is of interest to examine whether the new wiggler configuration could provide a means to relieve mirror degradation. To this end, we consider a conventional wiggler of $a_{w1}=1$ and $N_1=20$, and a quasiperiodic wiggler constructed from the above conventional wiggler and an auxiliary wiggler of $a_{w2}=0.2$ and $\lambda_{w2}=\lambda_{w1}/(\sqrt{7}-1.5)$. Their spontaneous emissions are plotted in Fig. 5 against the normalized radiation frequency ω/ω_{r1}. As shown in Fig. 5, the significant emission at the third and fifth harmonics from the conventional wiggler is reduced considerably when the auxiliary wiggler is added. The spontaneous emission spectrum of the quasiperiodic wiggler now consists of more but much smaller radiation peaks at frequencies distant from integral harmonics of ω_{r1}. It is known that mirror degradation is wavelength dependent and such a wavelength dependence is also different for different mirror coating materials [18,19]. Thus in cases where the band gap structure of the mirror coating material is particularly suited to absorbing UV photons at harmonic frequencies, the quasiperiodic wiggler provides a useful mechanism to shift the induced photons away from these harmful frequencies. For the new radiation peaks, they are generated at frequencies not integrally related to ω_{r1} and as such they are diffracted much less effectively by the downstream mirror [20]. Consequently when they are reflected to the upstream mirror, their damage to the upstream mirror should be significantly lower.

On the other hand, the extent of mirror degradation is known to be dependent on photon flux intensity [17]. If cavity mirrors' photon absorption is reduced below their saturated absorption level by reducing the intensity of FEL radiation at frequencies suited for significant photon absorption of mirror materials, mirror degradation should be small and could become, once the FEL radiation is turned off, spontaneously recoverable [17]. Figure 5 suggests that the quasiperiodic wiggler produces low intensity photon flux at high-frequencies ($>\omega_{r1}$). For FEL experiments where high-frequency irradiation contributes to mirror degradation differently at different irradiation frequencies [17], it should be possible to relieve mirror degradation by reducing the level of photon absorption at frequencies for which cavity mirrors are particularly vulnerable to high-frequency irradiation. However for cases where high frequency irradiation contributes to mirror degradation uniformly regardless irradiation frequency, it is possible to derive an appropriate quasiperiodic wiggler structure that produces less hard photons than that in a corresponding conventional wiggler. In the example of Fig. 5, the total radiation power of hard photons, calculated from integrating the spontaneous emission power from $2\omega_{r1}$ to $6\omega_{r1}$, is about 1% lower in the quasiperiodic wiggler than that in the conventional wiggler.

Control and suppression of harmonic generation are also important for conventional insertion devices of synchrotron radiation that generate x-ray photons with a wide band of energy or higher harmonics. For these short-wavelength devices, it is necessary to monochromate the generated radiation so as to minimize contamination of experimental data by hard photons [20]. Monochromator crystals may be used to remove hard photons, the energy of which is not integrally related to the fundamental radiation frequency. However, since they diffract simultaneously both the fundamental frequency and higher harmonics to an observer, experimental data are still contaminated by higher harmonics [20,21]. The suppression of harmonic generation achieved in the quasiperiodic wiggler configuration should also be useful for these synchrotron radiation sources.

It should be mentioned that harmonic generation in conventional FEL's may be alternatively controlled by using a different quasiperiodic wiggler configuration, proposed recently as an insertion device of synchrotron radiation [20,21]. In this case, the quasiperiodicity is realized by rearranging permanent magnet bars, in terms of their size and position, within the length of one wiggler period. This quasiperiodicity of one wiggler period is then repeated periodically to form a quasiperiodic wiggler configuration and the eventual magnet system is essentially of a single-wiggler structure. Such a configuration is different from that of the dual-wiggler structure discussed in this paper in that the latter combines two conventional wigglers of different periods to realize quasiperiodicity. Their similarity lies with their function of generating irrationally related harmonics at the expense of integrally related ones. Consequently both may be employed to control emission spectrum of both free-electron lasers and synchrotron radiation sources. It is of interest to note that the quasiperiodicity of the single-wiggler structure was derived by an analogy to the quasiperiodic lattice [22], and how it may be implemented in practice and optimized for better spectral control remain to be addressed. On the other hand, it has been established that spectral control in a quasiperiodic wiggler of the dual wiggler structure is achieved by adjusting and maximizing the interference between its two constituent wigglers and as such its optimization is more straightforward conceptually.

It is equally of interest to compare the proposed quasiperiodic wiggler to a two-frequency wiggler proposed for sideband control in high gain Compton FEL's [4,5]. The sideband control mechanism of the two-frequency wiggler configuration is similar to the spectral control mechanism of the quasiperiodic wiggler configuration, since both explore the interference between two wiggler components to manipulate FEL interaction characteristics. However, their applications are rather different. The former is used for control of sideband generation around a central resonant frequency, whereas the latter is employed for suppression of integrally related harmonics.

V. CONCLUSION

Spectral behavior of a quasiperiodic wiggler configuration has been studied with a formulation of its spontaneous emission. With the aid of number theory, it has been established that such quasiperiodic wigglers produce irrationally related higher harmonics. Their integrally related harmonics can be significantly suppressed by enhancing the interference between the two wiggler components of the quasiperiodic wiggler configuration. Depending on the mirror materials used, this spectral characteristic may be useful to relieve the problem of mirror damage for some free-electron lasers and synchrotron radiation sources operated at ultraviolet and x-ray wavelengths.

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