Coherence properties of multimode incoherent spatial solitons in noninstantaneous Kerr media

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We investigate the coherence characteristics of multimode incoherent spatial solitons in noninstantaneous Kerr-like nonlinear media. Other properties of these incoherent solitons are also discussed as a function of their modal composition. [S1063-651X(99)04001-5]

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I. INTRODUCTION

In general, an optical spatial soliton is known to occur when diffraction effects are exactly balanced by light-induced waveguiding [1]. Until recently, optical beam self-focusing in nonlinear media has been investigated only with the aid of coherent sources [2–8]. In 1996, however, incoherent spatial solitons were observed in noninstantaneous nonlinear media such as biased photorefractive crystals [9,10]. This was accomplished via the saturable drift photorefractive nonlinearity, which is also known to support coherent photorefractive spatial solitons [11,12]. In order to explain this recently observed behavior, two complementary approaches have been developed. The first one is the so-called coherent density method [13]. In this theory, the underlying evolution equation takes the form of a nonlinear Schrödinger-like integro-differential equation provided initially the coherent density is appropriately weighted with respect to the angular power spectrum of the incoherent source [13]. For a specific type of nonlinearity (logarithmic), this same approach was found to lead to closed form solutions [14]. The second approach is a self-consistent multimode method [15]. This latter model seeks, in a self-consistent fashion, multimode soliton solutions whose total intensity can be obtained via intensity superposition of all the modes guided within the nonlinear induced waveguide. This method is capable of identifying incoherent spatial soliton states, their range of existence, and coherence properties [15]. The equivalence of these two methods was later established in saturable logarithmic nonlinear media by means of exact results [16]. Another approach for describing broad incoherent bright solitons was also suggested by Snyder and Mitchell [17]. This ray model is to some extent related to the Vlasov transport description previously suggested by Hasegawa in the theory of random-phase solitons in plasmas [18]. Very recently incoherent dark solitons were predicted in photorefractive crystals [19]. The existence of planar dark and vortex solitons was subsequently confirmed experimentally by Chen *et al.* [20]. Their modal composition and coherence properties were later explained and analyzed [21].

It is worth noting that incoherent solitons, as viewed from the perspective of the self-consistent modal theory, are in fact related to the so-called incoherently coupled solitons in photorefractive crystals [22,23] or to the vector solitons in Kerr media [24]. Within the context of incoherently coupled solitons [22], Vysloukh *et al.* [25] have shown that multicomponent coupled soliton modes can be incoherently superimposed in weakly saturating photorefractive crystals. We would like to emphasize, however, that there is a subtle difference between self-trapping an incoherent beam and creating a multicomponent soliton. This difference is due to the statistically varying modal weights of the incoherent beam [15,16]. Consider first a situation where self-trapping occurs from an incoherent light source. During self-trapping, the beam continuously excites several modes of the jointly induced waveguide, established by the time averaged intensity of the speckled beam itself. If the time averaged intensity is decomposed into the modes of the induced waveguide, each mode will have a certain coupling amount or weight. On the other hand, when multiple laser beams are incoherently superimposed (engineering the beam profiles), the relative modal weights are time independent and only the relative phase between each pair of modes changes with time. As concerns the crystal, the two situations are exactly the same. The difference lies in the time dependence of the mode occupancies. In what follows we examine the properties of incoherent spatial solitons in Kerr media. This is of importance since thus far analytical results for incoherent bright spatial solitons have been obtained for logarithmic nonlinear media only [14,16].

In this paper we demonstrate that incoherent spatial solitons are possible in noninstantaneous Kerr-like media. Closed form solutions are obtained using the self-consistent multimode approach [15]. It is shown that the intensity profile of these incoherent soliton states is of the $\mathrm{sech}^2(x/x_0)$ type. Moreover, our analysis indicates that, in this case, the peak intensity and spatial width of these incoherent spatial

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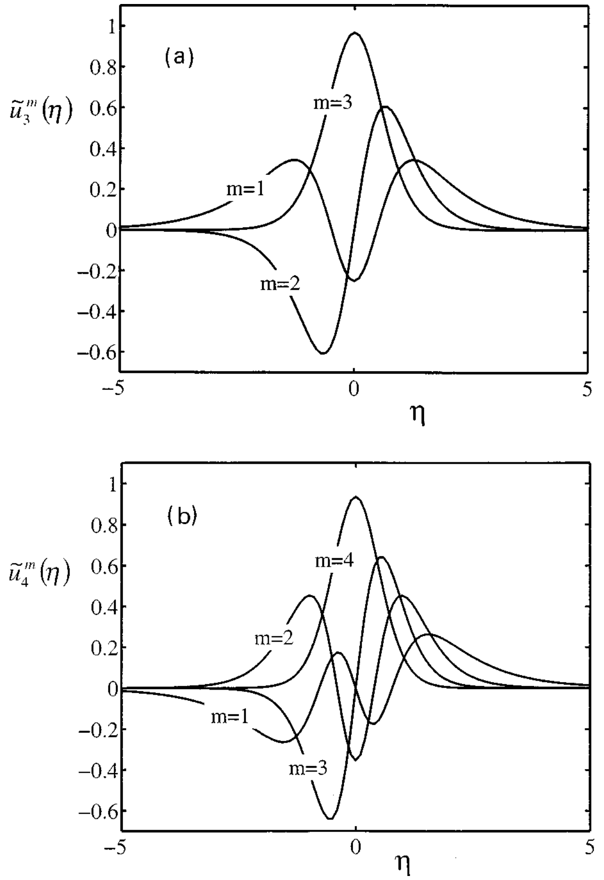

FIG. 1. Normalized mode profiles as a function of η for (a) $n=3$ and (b) $n=4$.

solitons are related to the number of allowed modes. The coherence properties of these soliton states are investigated in detail. Relevant examples are provided.

II. THEORETICAL FORMULATION

We consider a spatially partially incoherent optical beam that propagates in a nonlinear self-focusing Kerr-like medium along the z axis. For simplicity we assume that this beam is planar, that is, it diffracts only along the x direction. The refractive index of this Kerr-like material varies linearly with the optical intensity I, i.e., $n^2=n_0^2+n_2I$, where n_0 is the linear part of the refractive index and n_2 is the nonlinear Kerr coefficient. We also make the important assumption that the nonlinearity responds much slower than the characteristic phase fluctuation time across the incoherent beam so as to avoid speckle-induced filamentation instabilities [26]. In this regime, the material will experience only the time averaged beam intensity. For example, such noninstantaneous Kerr nonlinearities can be encountered in biased photorefractive crystals in the low-intensity regime (i.e., when the so-called dark irradiance is much larger than the intensity of the optical beam [11,12]) or in materials with thermal nonlinearities [27]. Furthermore, let the total electric field E of this spatially incoherent beam be written in terms of a slowly varying envelope U, that is, $E=U\ \exp(ikz)$, where the wave vector k is given by $k=k_0n_0=(2\pi/\lambda_0)n_0$ and λ_0 is the free-space wavelength. In this case, it can be readily shown that the envelope U evolves according to

$$i\frac{\partial U}{\partial z}+\frac{1}{2k}\frac{\partial^2 U}{\partial x^2}+\frac{k_0^2n_2I}{2k}U=0. \tag{1}$$

Let us now assume that the incoherent spatial soliton in this Kerr-like medium has a $\mathrm{sech}^2(x/x_0)$ intensity profile, that is,

$$I=I_0\ \mathrm{sech}^2(x/x_0), \tag{2}$$

where I_0 represents the peak intensity of the optical incoherent multimode beam and x_0 is associated with its spatial extent. In this case, Eq. (1) takes the form

$$i\frac{\partial U}{\partial \xi}+\frac{1}{2}\frac{\partial^2 U}{\partial \eta^2}+\frac{\alpha^2}{2}\mathrm{sech}^2(\eta)U=0. \tag{3}$$

In this equation we have used normalized coordinates and quantities, i.e., $\eta=x/x_0$, $\xi=z/kx_0^2$, and $\alpha^2=k_0^2x_0^2n_2I_0$. The incoherent spatial soliton solutions of this equation can then be obtained by expressing the optical envelope U through a superposition of all the modes involved, that is, $U\propto\Sigma_m c_n^m u_n^m(\eta)\exp(i\beta_m\xi)$, where c_n^m are the mode-occupancy coefficients that vary randomly with time, $u_n^m(\eta)$ is the profile of the mth-order mode, and β_m is its phase constant. The discrete index n stands for the total number of modes allowed in the system. By substituting this form of U into Eq. (3), we obtain, for each mode, the ordinary differential equation

$$\frac{d^2u_n^m}{d\eta^2}+[\alpha^2\ \mathrm{sech}^2(\eta)-2\beta_m]u_n^m=0. \tag{4}$$

The mode profiles $u_n^m(\eta)$ and their phase constants β_m can then be found by solving Eq. (4). In order to find the mode occupancy coefficients, the self-consistency of this solution must be satisfied, i.e., the total intensity of this incoherent beam should be given by Eq. (2). The self-consistency method relies on the basic concept that a soliton itself is a mode of the self-induced (via the nonlinearity) waveguide. This was suggested by Askar'yan [28] and later developed into a self-consistency methodology by Snyder *et al.* [29] to analyze coherent solitons. For incoherent bright solitons it was used in [15] and subsequently in [21] for incoherent dark beams.

III. RESULTS AND DISCUSSION

To find the modal structure of these multimode incoherent spatial solitons, we first adopt the transformation $t=\tanh(\eta)$, in which case Eq. (4) takes the form

$$(1-t^2)\frac{d^2u_n^m}{dt^2}-2t\frac{du_n^m}{dt}+\left[\alpha^2-\frac{2\beta_m}{1-t^2}\right]u_n^m=0. \tag{5}$$

It can now be easily shown that, when $\alpha^2=n(n+1)$, where $n=1,2,...$ (an integer determined by the amount of nonlinearity) and $m=1,2,...,n$, the allowed modes can be expressed in terms of associated Legendre functions [30], i.e.,

$$u_n^m(\eta)=P_n^m(\tanh\ \eta), \tag{6}$$

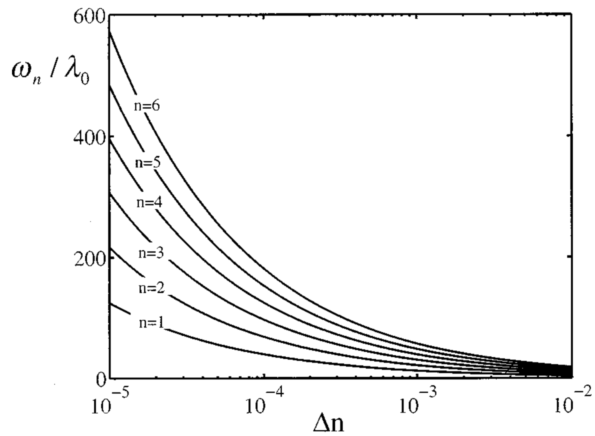

FIG. 2. Normalized intensity FWHM of Kerr-like incoherent spatial solitons versus nonlinear index change Δn for $n=1,2,...,6$.

and $\beta_m = m^2/2$, where $P_n^m(x)$ are associated Legendre functions of the first kind. In general, $P_n^m(x) = (1-x^2)^{m/2} d^m P_n(x)/dx^m$, where $P_n(x)$ denote Legendre polynomials. The value of the integer n (associated with α^2 or the degree of nonlinearity) represents the number of allowed modes in the self-induced waveguide and m the mode index for this case. Note that the lowest-order mode occurs when $m=n$, whereas the highest occurs at $m=1$. For a given value of n, the statistically varying optical field of the incoherent spatial soliton is given by

$$U(\eta,\xi) \propto \sum_{m=1}^{n} c_n^m P_n^m(\tanh\ \eta)\exp(im^2\xi/2).$$

Note that the self-consistency condition also requires that the mode-occupancy coefficients also depend on the number of modes n. The time averaged intensity of this beam can then be evaluated from

$$\langle |U|^2\rangle \propto I = \sum_{m=1}^{n} \langle |c_n^m|^2\rangle [P_n^m(\tanh\ \eta)]^2, \tag{7}$$

where we made use of the fact that under incoherent excitation the time average of cross-interference terms among modes is zero $[\langle c^i(c^j)^*\rangle \propto \delta_{ij}]$ [15,16]. Finally, Eq. (7) can be used to obtain the mode-occupancy coefficients c_n^m that

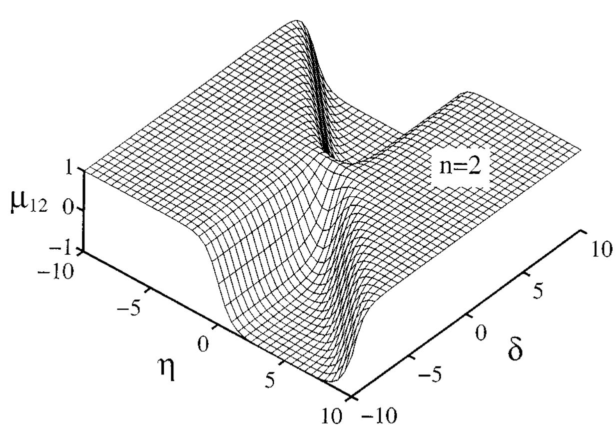

FIG. 3. Spatial coherence function as a function of η and δ for $n=2$.

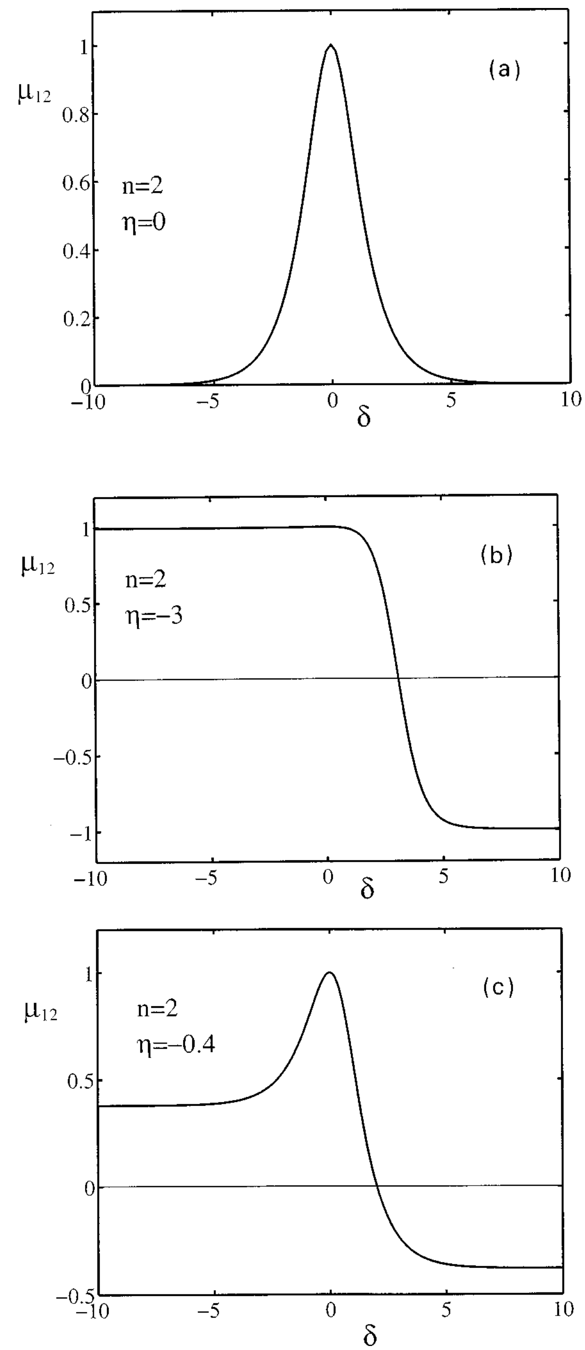

FIG. 4. Cross section of the coherence function for $n=2$ and (a) $\eta=0$, (b) $\eta=-3$, and (c) $\eta=-0.4$.

self-consistently lead to the incoherent spatial soliton $I = I_0\ \mathrm{sech}^2(\eta)$ assumed in the very beginning of this analysis. It proves convenient to normalize the eigenmodes in the following fashion: $\tilde{u}_n^m(\eta) = [\langle |c_n^m|^2\rangle / I_0]^{1/2} u_n^m(\eta)$ or, equivently, $I_0 \Sigma_{m=1}^n [\tilde{u}_n^m(\eta)]^2 = I_0 \mathrm{sech}^2(\eta)$. The functional form of the normalized eigenfunctions $\tilde{u}_n^m(\eta)$ is given in Appendix A. Figure 1(a) depicts the normalized mode profiles $\tilde{u}_n^m(\eta)$ for a three-component $(n=3)$ multimode incoherent soliton, whereas Fig. 1(b) is for a four-component structure $(n=4)$. Having found closed form solutions for these spatial incoherent solitons, the question naturally arises as to what factors contribute to their spatial full width at half maximum (FWHM). To answer this question one has to consider the

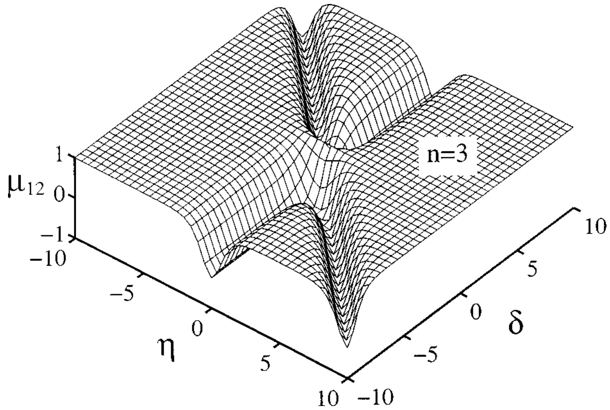

FIG. 5. Spatial coherence function as a function of η and δ for $n=3$.

expression $\alpha^2=n(n+1)$. More specifically, since $\alpha^2=k_0^2x_0^2n_2I_0$, given the number of guided modes n and the peak intensity I_0, the spatial width x_0 can be determined. In turn one quickly finds the result $x_0=\sqrt{n(n+1)/k_0^2n_2I_0}$. Using Eq. (2) and provided n is known, the intensity FWHM of these soliton states ω_n can be calculated from $\omega_n=1.76x_0$ and thus

$$\omega_n=\frac{1.76}{k_0}\sqrt{\frac{n(n+1)}{n_2I_0}}. \tag{8}$$

Note that when $n=1$ (''single-mode'' soliton), this expression (8) is in full agreement with the results previously obtained for coherent Kerr spatial solitons [1]. Equation (8) can also be written in terms of a normalized FWHM (ω_n/λ_0) as function of the maximum induced nonlinear index change ($\Delta n=n_2I_0$), i.e., $\omega_n/\lambda_0=(1.76/2\pi)\sqrt{n(n+1)/\Delta n}$. Figure 2 shows the dependence of the normalized intensity FWHM of these incoherent optical solitons on Δn for different values of n. This figure clearly shows that for a given nonlinear index change, the intensity FWHM tends to increase with the number of modes involved. This should have been anticipated since the incoherence increases with n. It is also important to note that in Fig. 2, the existence curves are discrete as opposed to the continuum range found in Ref. [15]. This is due to the specific $\text{sech}^2(x/x_0)$ intensity shape assumed at the beginning of our analysis.

We now investigate the coherence properties of these spatial solitons. This is done by evaluating the complex coherence factor μ_{12} of these soliton states [31], which is given by

$$\mu_{12}(\eta,\eta+\delta)=\langle U(\eta,\xi)U^*(\eta+\delta,\xi)\rangle/\sqrt{\langle|U(\eta)|^2\rangle\langle|U(\eta+\delta)|^2\rangle},$$

where δ represents the normalized distance between two points. From Eq. (2) and by using the modal expansion of the optical envelope U, for a given n we obtain

$$\mu_{12}(\eta,\eta+\delta)=\frac{\sum_{m=1}^{n}\tilde{u}_n^m(\eta)\tilde{u}_n^m(\eta+\delta)}{\sec h(\eta)\sec h(\eta+\delta)}, \tag{9}$$

where again we made use of the fact that the time average of cross-interference terms is zero under incoherent excitation.

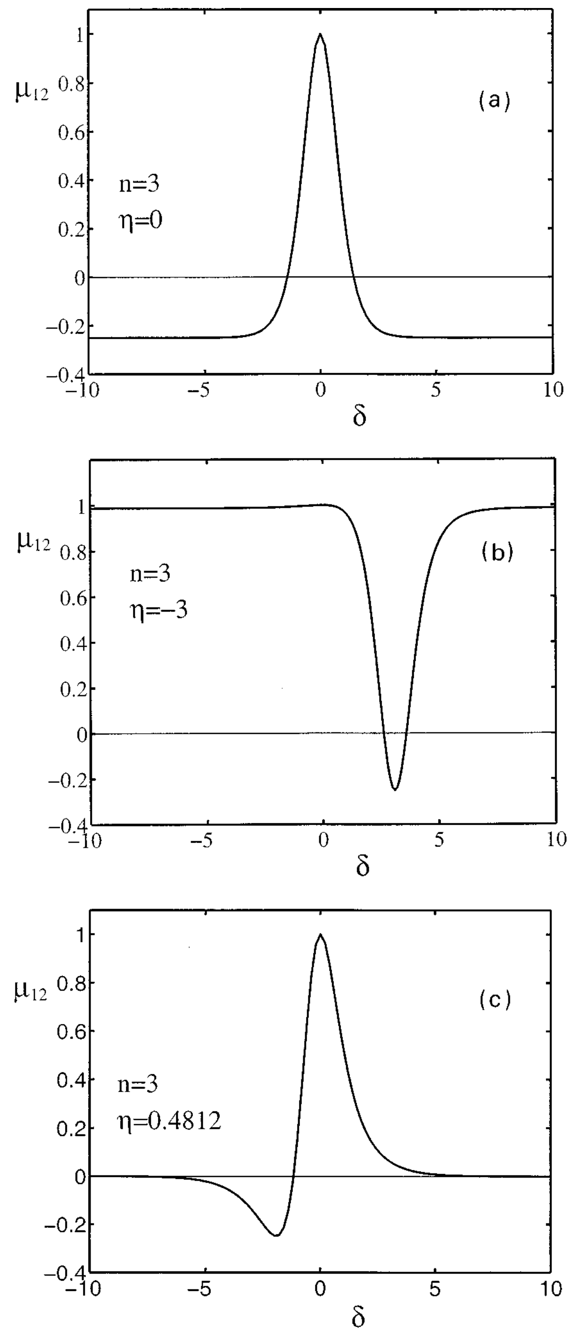

FIG. 6. Cross section of the coherence function for $n=3$ and (a) $\eta=0$, (b) $\eta=-3$, and (c) $\eta=0.4812$.

The complex coherence factors $\mu_{12}(\eta,\eta+\delta)$ up to $n=5$ have been tabulated in Appendix B. When $n=1$, which corresponds to a coherent optical soliton, $\mu_{12}(\eta,\eta+\delta)=1$, as should have been anticipated. This in turn implies an infinite correlation length. As the number of modes n increases, the coherence properties of these incoherent soliton states become more complicated. Figure 3 provides a two-dimensional plot of μ_{12} as a function of normalized position η and point separation δ for $n=2$. As expected, $\mu_{12}=1$ for $\delta=0$ and this is true for every value of n. Note that any cut ($\eta=\text{const}$) of these two-dimensional plots provides the ''correlation'' (magnitude and phase) between the two points located at η and $\eta+\delta$. Moreover, as it is well known [31], the magnitude of the complex coherence factor is always

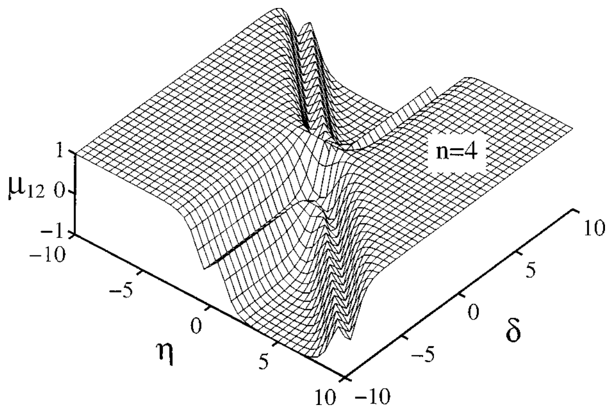

FIG. 7. Spatial coherence function as a function of η and δ for $n=4$.

$|\mu_{12}|\leqslant 1$. In general, two points are mutually coherent if $\mu_{12}=1$ and mutually incoherent (poor correlation) if $\mu_{12}\approx 0$. Figure 4(a) depicts a cross section of the complex coherence μ_{12} as obtained right at the center ($\eta=0$) of an incoherent soliton when $n=2$. In this case, $\mu_{12}(0,\delta)=\mathrm{sech}(\delta)$, which implies that the correlation between the center and any other point of this soliton decreases with δ. This expression also demonstrates that the tails of this beam ($\delta\rightarrow\pm\infty$) and the center are totally uncorrelated. Figure 4(b), on the other hand, shows how a point at the tails (at $\eta=-3$) correlates with the rest of the beam when again $n=2$. For example, when $\delta\approx 3$, in which case the second point is close to the beam center, $\mu_{12}\approx 0$ (poor correlation), in agreement with Fig. 4(a). As $\delta\rightarrow-\infty$ (i.e., both points are on the left tail of the beam) there is maximum correlation ($\mu_{12}\cong 1$). This high degree of coherence is due to the following fact: At the tails there is essentially only one mode (the highest-order mode $m=1$) that is coherent in itself [15,16]. For $\delta\rightarrow\infty$, $\mu_{12}\cong-1$ and this π phase shift is due to the antisymmetric character of this highest-order mode. Figure 4(c) provides the same information at the intermediate point $\eta=-0.4$ for $n=2$. Note that all these cross sections are very different in character from those found in the case of logarithmically saturable nonlinear media [16]. In logarithmic systems [16], the statistical process is everywhere stationary, i.e., μ_{12} depends only on δ and it is Gaussian in nature. As shown here for the Kerr case, the coherence curves depend also on the position η and do not always go to zero as $\delta\rightarrow\pm\infty$. This feature was also encountered in saturable Kerr media of $I/(1+I)$ type [15,19]. This important difference is due to the finite number of modes associated with the Kerr incoherent solitons. In particular, the induced waveguides in Kerr and Kerr-saturable media exhibit cutoffs (finite number of modes), whereas the logarithmic one does not. Figure 5 depicts the coherence function for $n=3$. Similar cuts are also provided in Fig. 6. Figure 6(a) shows how the center correlates with the rest of the beam for $n=3$. Interestingly enough, in this case, there is a weak correlation between the center ($\eta=0$) and the tails ($\delta\rightarrow\pm\infty$). This is because the highest-order mode ($m=1$), which survives at the tails, happens to be even and it is thus finite at the center. Figure 6(b) gives the correlation between a point at the tails ($\eta=-3$) and the rest of the beam. Note the high degree of coherence ($\mu_{12}\approx 1$) between the two tails. Unlike the case shown in Fig. 4(b), the correlation is in phase since again in this case the highest-order mode is even. Finally, Fig. 6(c) gives the coherence factor at $\eta=\sec h^{-1}(2/\sqrt{5})\approx\pm 0.4812$ for $n=3$. Using the coherence factor given in Appendix B, it can be readily shown that these particular points are completely uncorrelated with the tails of this incoherent soliton. Figures 7 and 8 depict the coherence factors for $n=4$ and 5, respectively. It is interesting to observe the similarity between the μ_{12} surfaces when n is odd or even. Clearly, as the order of the soliton increases, the coherence surfaces become more involved.

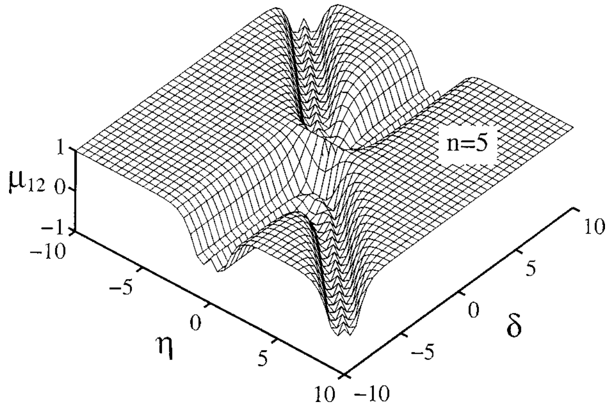

FIG. 8. Spatial coherence function as a function of η and δ for $n=5$.

IV. CONCLUSION

In conclusion, we have shown that multimode incoherent spatial solitons are possible in noninstantaneous Kerr-like media. Closed form solutions were obtained using the self-consistency approach provided their intensity profile is of the $\mathrm{sech}^2(x/x_0)$ type. The coherence properties of these incoherent soliton states were also investigated in detail and explained by means of their modal composition.

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APPENDIX A

In this appendix we provide the normalized mode functions $\tilde{u}_n^m(\eta)$. Here we define $T=\tanh(\eta)$ and $S=\mathrm{sech}(\eta)$. The $\tilde{u}_n^m(\eta)$ functions appear in accord with their modal order (lowest-order mode first). For $n=1$,

$$\tilde{u}_1^1(\eta)=S.$$

For $n=2$,

$$\tilde{u}_2^2(\eta)=S^2,\quad \tilde{u}_2^1(\eta)=ST.$$

For $n=3$,

$$\tilde{u}_3^3(\eta)=\sqrt{15/16}S^3,\quad \tilde{u}_3^2(\eta)=\sqrt{5/2}S^2T,$$

$$\tilde{u}_3^1(\eta)=\tfrac{1}{4}S(4-5S^2).$$

For $n=4$,

$$\tilde{u}_4^4(\eta)=\sqrt{7/8}S^4,\quad \tilde{u}_4^3(\eta)=\tfrac{3}{4}\sqrt{7}S^3T,$$

$$\tilde{u}_4^2(\eta)=(\sqrt{2}/4)S^2(6-7S^2),\quad \tilde{u}_4^1(\eta)=\tfrac{1}{4}ST(4-7S^2).$$

For $n=5$,

$$\tilde{u}_5^5(\eta)=(\sqrt{210}/16)S^5,\quad \tilde{u}_5^4(\eta)=\tfrac{1}{2}\sqrt{21}S^4T,$$

$$\tilde{u}_5^3(\eta)=(\sqrt{42}/16)S^3(8-9S^2),$$

$$\tilde{u}_5^2(\eta)=(\sqrt{7}/2)S^2T(2-3S^2),$$

$$\tilde{u}_5^1(\eta)=\tfrac{1}{8}S(8-28S^2+21S^4).$$

For $n=6$,

$$\tilde{u}_6^6(\eta)=\tfrac{3}{16}\sqrt{22}S^6,\quad \tilde{u}_6^5(\eta)=\tfrac{5}{16}\sqrt{66}S^5T,$$

$$\tilde{u}_6^4(\eta)=(\sqrt{3}/4)S^4(10-11S^2),$$

$$\tilde{u}_6^3(\eta)=\tfrac{3}{16}\sqrt{10}S^3T(8-11S^2),$$

$$\tilde{u}_6^2(\eta)=(\sqrt{10}/16)S^2(16-48S^2+33S^4),$$

$$\tilde{u}_6^1(\eta)=\tfrac{1}{8}ST(8-36S^2+33S^4).$$

APPENDIX B

In this appendix the functional form of the complex coherence factors μ_{12} is provided up to $n=5$. Again we have defined $T(\eta)=\tanh(\eta)$ and $S(\eta)=\mathrm{sech}(\eta)$. For $n=1$,

$$\mu_{12}(\eta,\eta+\delta)=1.$$

For $n=2$,

$$\mu_{12}(\eta,\eta+\delta)=T(\eta)T(\eta+\delta)+S(\eta)S(\eta+\delta).$$

For $n=3$,

$$\begin{aligned}\mu_{12}(\eta,\eta+\delta)=1+\tfrac{5}{2}\{&S(\eta)S(\eta+\delta)\\ &\times[T(\eta)T(\eta+\delta)+S(\eta)S(\eta+\delta)]\\ &-\tfrac{1}{2}[S^2(\eta)+S^2(\eta+\delta)]\}.\end{aligned}$$

For $n=4$,

$$\begin{aligned}\mu_{12}(\eta,\eta+\delta)=T(\eta)T(\eta+\delta)\{&1-\tfrac{7}{4}[S^2(\eta)+S^2(\eta+\delta)]\\ &+7S^2(\eta)S^2(\eta+\delta)\}+S(\eta)S(\eta+\delta)\\ &\times\{\tfrac{9}{2}-\tfrac{21}{4}[S^2(\eta)+S^2(\eta+\delta)]+7S^2(\eta)\\ &\times S^2(\eta+\delta)\}.\end{aligned}$$

For $n=5$,

$$\begin{aligned}\mu_{12}(\eta,\eta+\delta)=1+7\big(&T(\eta)T(\eta+\delta)S(\eta)S(\eta+\delta)\\ &\times\{1-\tfrac{3}{2}[S^2(\eta)+S^2(\eta+\delta)]+3S^2(\eta)\\ &\times S^2(\eta+\delta)\}-\tfrac{1}{2}[S^2(\eta)+S^2(\eta+\delta)]\\ &+\tfrac{3}{8}[S^4(\eta)+S^4(\eta+\delta)]+S^2(\eta)\\ &\times S^2(\eta+\delta)\{\tfrac{13}{4}-3[S^2(\eta)+S^2(\eta+\delta)]\\ &+3S^2(\eta)S^2(\eta+\delta)\}\big).\end{aligned}$$

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