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Particle injection into the wave acceleration phase due to nonlinear wake wave breaking

S. Bulanov,[1,2] N. Naumova,[1,3] F. Pegoraro,[4] and J. Sakai[5]
[1]General Physics Institute RAS, Moscow, Russia
[2]Scuola Normale Superiore, Pisa, Italy
[3]Forum for Theoretical Physics, INFM, Pisa, Italy
[4]Dipartimento di Fisica, Università di Pisa and INFM, Pisa, Italy
[5]Laboratory for Plasma Astrophysics, Faculty of Engineering Toyama University, Toyama, Japan

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We present analytical results and computer simulations of the nonlinear evolution of wake field waves in inhomogeneous plasmas. The wake wave break that occurs due to the inhomogeneity of the Langmuir frequency makes it possible to inject electrons into the acceleration phase of the wave. Particle-in-cell simulations show that stable beams of energetic electrons are formed. These beams are well bunched in coordinate and velocity space and contain a considerable fraction of the pulse energy [S1063-651X(98)50311-X]

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Among the charged particle accelerators that use collective electric fields in plasmas [1], the laser wake-field accelerator (LWFA) provides one of the most promising approaches to high-performance compact electron accelerators [2]. Acceleration of electrons in electric fields of up to 100 GV/m has been observed in LWFA experiments during the interaction of high-intensity laser pulses with plasmas [3]. The production of accelerated electron beams with low-energy spread requires a very precise injection of extremely short electron bunches in the appropriate phase of the wake field. Since the typical length of the wake-field plasma wave is of the order of $2\pi c/\omega_{\rm pe}\approx 10$–$100\ \mu$m, the length of the injected electron bunch must be less than 2–20 μm. Recently an optical method for injecting electrons in the accelerating phase of the wake field has been proposed [4]. This method uses two laser pulses: the first pulse (the driver) generates the wake field, while the second intersects the wake some distance behind the driver pulse. The ponderomotive force $\sim\nabla a^2$ of the second pulse can accelerate a portion of the electrons so that they become trapped. Here $a^2=(eE/m\omega c)^2=7\times10^{-19}\lambda^2\ [\mu\mathrm{m}]I\ [\mathrm{W/cm^2}]$, $\lambda=2\pi c/\omega$ is the laser radiation wavelength, and I its intensity. As was shown with one-dimensional particle-in-cell (PIC) simulations, high intensities ($I>10^{18}$ W/cm^2, corresponding to $a\approx 2$) are required in both the driver and injecting pulses. A scheme of optical injection which uses three laser pulses, one high-intensity driver, and two counterpropagating injecting laser pulses with moderate intensities, has been proposed in Ref. [5]. In this model the colliding laser pulses excite a slow phase velocity beat wave that injects electrons into the accelerating phase of the fast wake wave. Such all-optical electron injectors would be important in reducing both the size and the cost of the laser-driven accelerators.

However, a problem arises from the requirement to tune the wake field and the injecting laser pulses very precisely. In the range of relativistic intensities of the laser radiation with $a>1$, the wavelength of the wake field depends on the laser pulse amplitude and frequency, which in turn change due to such nonlinear effects as self-focusing, pulse energy depletion, and down-shifting of the carrier frequency. To overcome these synchronization problems, we propose using for electron injection the wake-field breaking of the wake wave of a single laser pulse.

The basic properties of nonlinear wave breaking are well known (see [6] and [7]). In the case of nonlinear Langmuir waves, the Akhiezer-Polovin constraint [8] limits the amplitude of a stationary wave and consequently the acceleration rate in the LWFA: $E_m=(mc\omega_{\rm pe}/e)[2(\gamma_{\rm ph}-1)]^{1/2}$, with $\gamma_{\rm ph}\equiv(1-\beta_{\rm ph}^2)^{-1/2}\approx\omega/\omega_{\rm pe}$, $\beta_{\rm ph}\equiv v_{\rm ph}/c$, and $v_{\rm ph}$ the plasma wave phase velocity which is equal to the group velocity of the laser pulse.

Even in the one-dimensional (1D) case, wave breaking can either completely destroy the regular structure of the wave, or it can develop quite gently, with only a small portion of the wave involved in the break. Even when the crash of the wake field occurs at the plasma-vacuum interface and destroys the wave pattern locally, it may serve the purpose of injecting a portion of the electrons in the accelerating phase in the wake behind the laser pulse far from the plasma boundary [9]. In a homogeneous plasma the wake wave breaks when the square of the laser radiation amplitude a^2 exceeds $\gamma_{\rm ph}$. This regime has attracted attention since it provides both the injection of electrons into the acceleration phase and, at the same time, a high rate of acceleration [10].

In the 1D case the Langmuir wave break occurs when the quiver velocity of electrons, v, becomes equal to the phase velocity of the wave. We use the Lagrange variables (x_0,t), with the Euler coordinate x expressed as $x=x_0+\xi(x_0,t)$, where $\xi(x_0,t)$ is the displacement of an element of the electron fluid from its initial position x_0 during time t. The wave-breaking singularity appears when the Jacobian $J(x_0,t)=|\partial_{x_0}x|=|1+\partial_{x_0}\xi|$ of the transformation from the Euler to the Lagrange coordinates vanishes. For nonrelativistic electron oscillation velocities, the time evolution equation of Langmuir waves is linear in Lagrangian coordinates. For $\xi(x_0,t)=\xi_m\cos(kx_0-\omega_{\rm pe}t)$, the Jacobian vanishes when $k\xi_m=1$. This condition can be reexpressed as $v\equiv\partial_t x=v_{\rm ph}$, where v is the electron velocity and $v_{\rm ph}=\omega_{\rm pe}/k$ is the phase velocity of the wave. In a plasma with inhomogeneous density, the Langmuir wave wave number depends on time

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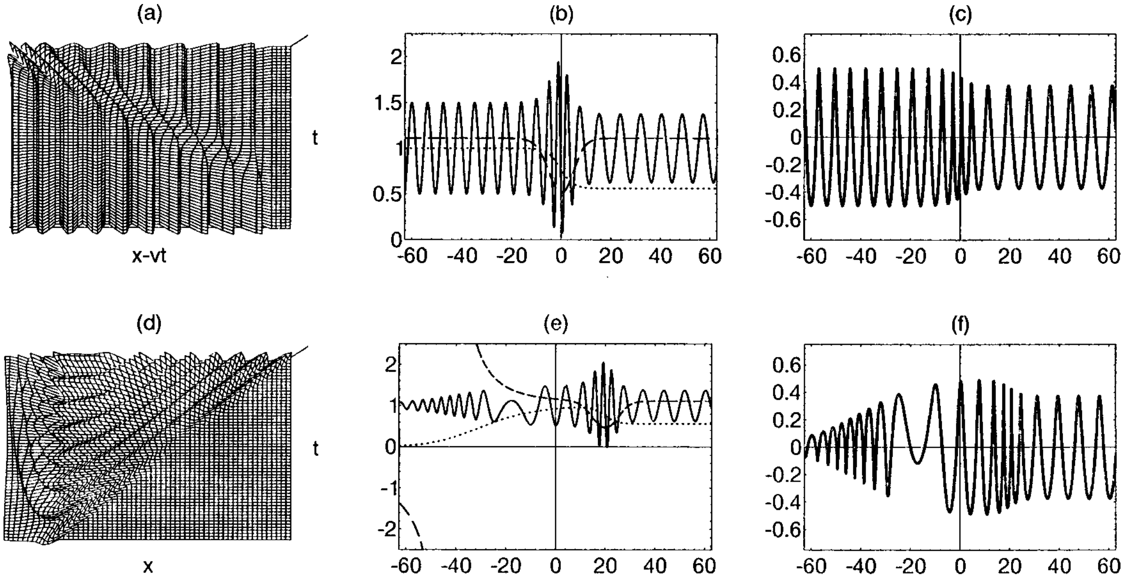

FIG. 1. (a) Pattern of the electric field in the wake wave in the $(x-v_g t,t)$ plane. (b) Plasma density distribution (dotted line), dependence of the Jacobian (solid line), and of the wave phase velocity (dashed line) on the Lagrange coordinate. (c) Electron momentum versus the Euler coordinate in the breaking wave. (d) Pattern of the electric field in the wake wave in the (x,t) plane. (e) and (f) The same quantities as in frames (b) and (c) in the case when the plasma-vacuum interface is taken into account.

through the relationship [7] $\partial_t k = -\partial_x \omega$. The resulting growth over time of the wave number results in the break of the wave even when the initial wave amplitude is below the wave-break threshold. We can use the fact that in this case only a relatively small part of the wave is involved, to inject the electrons gently into the acceleration phase. The Langmuir wave frequency may depend on the coordinate also due to relativistic effects that cause the wake wave amplitude to vary [11]. This appears naturally due to the laser pulse depletion as it is accompanied by the downshifting of the laser pulse frequency [12], which in turn changes the wake field amplitude.

We assume the plasma frequency to vary between the two values ω_{p1} and ω_{p2} with a dependence on the Lagrange coordinate x_0 of the form

$$\omega_{pe}(x_0) = \frac{\omega_{p1}+\omega_{p2}}{2} - \frac{\omega_{p1}-\omega_{p2}}{2}\tanh\frac{x_0}{L_1}, \tag{1}$$

with L_1 the width of the region where the plasma is inhomogeneous. The displacement and the velocity in the plasma wave behind the laser pulse are given by

$$\xi(x_0,t) = \xi_m \cos(\psi(x_0,t)),$$
$$v(x_0,t) = -\xi_m \omega_{pe}(x_0)\sin(\psi(x_0,t)), \tag{2}$$

where the phase of the wave is defined as $\psi(x_0,t) = \omega_{pe}(x_0)(t - x_0/v_g)$, and we neglected the dependence of the group velocity v_g of the pulse on x_0.

The resulting wave pattern in the (x,t) plane is shown in Fig. 1(a), where the longitudinal electric field in the wake wave is shown in the frame moving with the laser pulse. Behind and ahead of the inhomogeneity region, the wavelength of the wake field does not depend on time, while in the inhomogeneous region it decreases with time. In Fig. 1(b) the dependence of the Jacobian and of the wave phase velocity $v_{ph}(x_0,t) = \omega/k = -(\partial_t\psi)/(\partial_{x_0}\psi)$ on the Lagrange coordinate is shown. In Fig. 1(c) the electron velocity in the wave is plotted as a function of the Euler coordinate at the breaking time $t_{break} \approx 2L/\xi_m \Delta\omega_{pe}$. In the inhomogeneous region, the wake phase velocity decreases until it becomes equal to the quiver velocity of the electrons.

Since the parameters of the nonlinear Langmuir wave approach wave breaking gradually, its pattern in phase plane (p,x) takes specific features. Assuming that the inhomogeneity is weak compared to the plasma wave wavelength, we describe the wave structure in the co-moving frame taking a "frozen" dependence on time. In a relativistically strong Langmuir wave, the electron quiver velocity v and the electron density n depend on the coordinate $X = k_p(x - v_{ph}t)$ through the relationships [8]

$$2^{1/2}X = \int_\beta^{\beta_0} \frac{(\beta_{ph}-s)}{(1-s^2)^{3/2}[(1-\beta_m^2)^{-1/2}-(1-s^2)^{-1/2}]^{1/2}}ds$$
$$\equiv \int_\beta^{\beta_0} g(s)ds, \tag{3}$$

$$n = \frac{n_0\beta_{ph}}{\beta_{ph}-\beta}.$$

Here $k_p = \omega_{pe}/c$, $\beta = v/c$, $\beta_m = v_m/c = k_p\xi_m$, with $v_m = \omega_{pe}\xi_m$ the maximum value of the electron quiver velocity in the wake wave. Close to the wave-breaking limit we have $\beta_m \simeq \beta_{ph}$. We assume that the wave break occurs at the point $X=0$ and set $\beta_0 = \beta_m$ as the upper limit of integration in Eq. (3). By expanding the integrand $g(s)$ in Eq. (3) in series of $\beta_m - s \ll 1$ for $\beta_m < \beta_{ph}$, and for $\beta_m = \beta_{ph}$, and integrating Eq. (3), we obtain for $\beta_m < \beta_{ph}$

$$v(X)=v_m-c\frac{\beta_m(1-\beta_m^2)^{3/2}}{2(\beta_{\rm ph}-\beta_m)^2}X^2+\cdots \quad (4)$$

and

$$n(X)=\frac{2n_0\beta_{\rm ph}(\beta_{\rm ph}-\beta_m)^2}{(\beta_{\rm ph}-\beta_m)^3-\beta_m(1-\beta_m^2)^{3/2}X^2}+\cdots . \quad (5)$$

Near its maximum the density distribution has a parabolic shape, with maximum $n_m=n_0\beta_{\rm ph}/(\beta_{\rm ph}-\beta_m)$ and width $(2(\beta_{\rm ph}-\beta_m)^3/[\beta_m(1-\beta_m^2)^{3/2}])^{1/2}$. When $\beta_m\to\beta_{\rm ph}$, the maximum density tends to infinity while the width of the density spike tends to zero. For $\beta_m=\beta_{\rm ph}$, from Eq. (3) we obtain

$$v(X)\simeq v_m-c(9\beta_m/2)^{1/3}(1-\beta_m^2)^{1/2}X^{2/3}+\cdots , \quad (6)$$

the electron density in the spike tends to infinity as

$$n(X)=\frac{n_0\beta_m}{(9\beta_m/2)^{1/3}(1-\beta_m^2)^{1/2}X^{2/3}}+\cdots \quad (7)$$

for $X\to 0$, and a characteristic cusplike pattern $p'\propto x'^{2/3}$ appears in the phase plane

$$\frac{p(X)}{mc}\simeq\frac{\beta_m}{(1-\beta_m^2)^{1/2}}-\frac{(9\beta_m/2)^{1/3}}{(1-\beta_m^2)}X^{2/3}+\cdots . \quad (8)$$

However, integrating the electron density (7) in the neighborhood of the singularity, we find that the total number of particles in the density spike is finite.

As the result of the break, fast electrons from the wave crest are trapped by the wave and are preaccelerated into the region where the phase velocity increases and the wake field has a regular and steady structure. In this way we obtain a gentle injection of electrons into the acceleration phase in the wake far from the breaking region.

The breaking leads to the local decay of the wake wave. Its energy is transported away by the fast electrons. From the energy balance we estimate the fast electron density in the breaking region to be equal to $n_{\rm inj}=n_0\xi_m/L$.

In order to study the long time evolution of the breaking wake wave, we performed numerical simulations using the 1D PIC code described in Ref. [12]. In these simulations a circularly polarized laser pulse is initiated in the vacuum region and then interacts with a weakly inhomogeneous plasma. The laser pulse length is $12\lambda=24\pi c/\omega$, and its amplitude is $a=eE/m\omega c=2$. Ions are assumed to be immobile. Asymptotically, as $x\to\infty$, the plasma is homogeneous with a density $n/n_{\rm cr}=1/625$ that corresponds to $\omega/\omega_{\rm pe}=25$. The plasma density varies smoothly from zero at $x=32\lambda$ to $1/547n_{\rm cr}$ at $x=96\lambda$ to avoid the distortion of the plasma wave due to wave break at the vacuum-plasma interface discussed in Ref. [9]. The plasma is homogeneous in the domain $96<x/\lambda<128$ and its density decreases gradually from $1/547n_{\rm cr}$ to $1/625n_{\rm cr}$ in the domain $128<x/\lambda<152$. To illustrate the difference in the wake-field evolution that arises because of the finite width of the vacuum-plasma interface, we present the electric field distribution in the plane (x,t) [Fig. 1(d)], the Jacobian and phase velocity [Fig. 1(e)], and the wake wave pattern [Fig. 1(e)] for $\omega_{\rm pe}(x_0)=[(\omega_{p1}+\omega_{p2})-(\omega_{p1}-\omega_{p2})\tanh(x_0/L_1)][1+\tanh(x_0/L_2)]$. In the vacuum-plasma interface we see the formation of wake breaking toward the vacuum region. This process is similar to the ''electron vacuum heating'' discussed in Ref. [13].

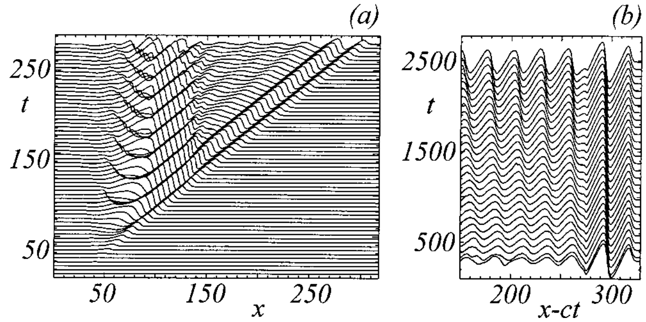

FIG. 2. Pattern of the electric field in the wake wave in the (x,t) plane: (a) in the region of plasma inhomogeneity, (b) behind the laser pulse.

We have performed a long-time run, up to $5000\pi/\omega$, i.e., up to 2500 periods of the electromagnetic wave, in order to study both the injection and the subsequent acceleration of the electrons injected into the wake field. The simulations were made in a ''moving window'' with length 320λ.

In Fig. 2, we show the electric field in the wake in the inhomogeneous plasma region [Fig. 2(a)] in the time interval $0<\omega t/2\pi<280$, and, in Fig. 2(b), behind the laser for $300<\omega t/2\pi<2500$, where it has a regular stationary structure. The local wavelength of the wake field changes, in agreement with Fig. 1(d). In Fig. 3, the phase plane at different times is shown. At $\omega t/2\pi=130$ [frame (a)] we see the formation of the cusp structures that characterize the wave break described by Eq. (8) and correspond to Figs. 1(e) and 1(f). At $\omega t/2\pi=200$ [frame (b)], during the wave break, particles are injected into the accelerating phase of the wake field. Further acceleration is seen for $\omega t/2\pi=300$ [frame (c)] and $\omega t/2\pi=2500$ [frame (d)]. At time $\omega t/2\pi=2500$, the maximum energy of the fast particles is approximately 330 mc^2. The most energetic particles have been accelerated in the first period of the wake wave behind the laser pulse. In Fig. 2(c), the amplitude of the wake wave for periods with

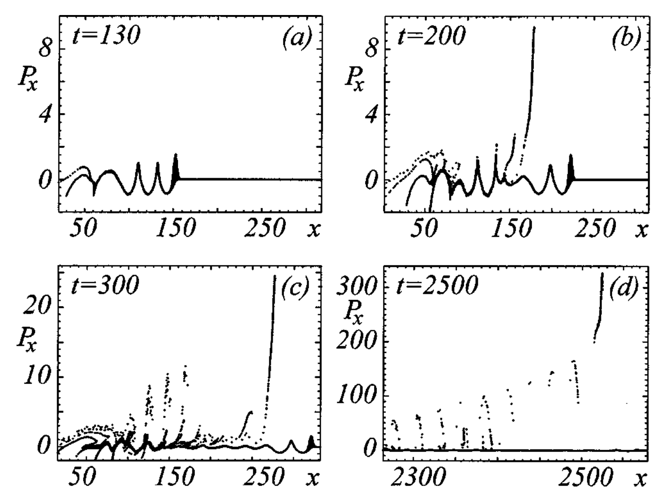

FIG. 3. Electron phase plane: (a) at $t=130$, (b) at $t=200$, (c) at $t=300$, and (d) at $t=2500$.

numbers larger than 3 is much smaller than the amplitude in the first period since the wake wave is loaded by the bunch of accelerated electrons and loses its energy. Thus in the regime presented in Figs. 2 and 3, a significant portion of the energy of the laser pulse is converted into the energy of fast particles.

In conclusion, the main goal of LWF accelerators is to reach the largest possible accelerating field which is limited by wake wave breaking. In a smoothly inhomogeneous plasma (and/or when the amplitude of the wake depends on the coordinates) the wavelength of a relativistically strong Langmuir wave depends on time, and when it becomes of the order of the quiver amplitude of the electrons, the wake starts to break. This provides a mechanism for the injection of electrons into the acceleration phase of the wake field. The resonant wave-particle interaction in the region of homogeneous plasma forms electron bunches that are well localized in coordinate and in energy space, are stable and contain a finite portion of the laser pulse energy.

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