Evolution of Langmuir waves in a plasma contaminated by variable-charge impurities

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The propagation of Langmuir waves in nonisothermal plasmas contaminated by fine dust particles with variable charge is investigated for a self-consistent closed system. Dust charge relaxation, ionization, recombination, and collisional dissipation are taken into account. It is shown that the otherwise unstable coupling of the Langmuir and dust-charge relaxation modes becomes stable and the Langmuir waves are frequency downshifted. [S1063-651X(98)08612-7]

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Charged impurities or dust grains are often found in space and laboratory plasmas [1,2]. The highly charged dust particles can significantly affect the system since they carry a considerable percentage of the total negative charge of the plasma. In fact, most plasma waves are to some degree affected by the dusts [3–7]. The variable dust charge also leads to a new plasma mode, usually referred to as the charge relaxation mode (CRM), involving dust-charge fluctuations [3,8] originating from the dust-charge variation determined by the instantaneous local electrostatic potential. In most earlier investigations on waves in dusty plasmas, the fact that dusty plasma models are almost always thermodynamically open was sidestepped by invoking unspecified sources or sinks, whose details are nevertheless important for a rigorous treatment of the problem.

In the absence of ionization, recombination, and collisional dissipation, it was found [4] that Langmuir waves can be unstable because of a coupling to the CRM. In this Brief Report we reconsider the problem of linear Langmuir wave propagation in a nonisothermal dusty plasma with dust-charge variation as well as the collisional effects. The latter may be due to collisions between electrons and ions, neutral atoms, or other electrons, as well as the elastic and inelastic (the dust-charging) collisions between the electrons and the dust. These processes exist in most dusty plasmas, which are usually of low temperature and partially ionized. In fact, the dust-charge relaxation process is itself closely associated with ionization and recombination, which maintain the averaged background particle number densities self-consistently during the perturbations by acting as sources and sinks. They also define the equilibrium or steady state. Here we show that these dissipative processes lead to a net damping of the Langmuir waves in typical dusty plasma systems.

We consider the propagation of linear Langmuir waves in a nonisothermal ($T_e \gg T_i$, where T_e and T_i are the electron and ion temperatures) plasma. The size of the dust grains is assumed to be much less than the intergrain distance, the electron Debye radius, and the wavelength of the waves, so they can be treated as heavy point masses. The charge of a dust grain varies because of the microscopic electron and ion currents flowing into the grain according to the potential difference between the dust surface and the adjacent plasma. The dusts are treated as an immobile background since the time scale of charge variation is much smaller than that of the dust motion [8].

The equations describing the propagation of Langmuir waves are

$$\partial_t n_e + \nabla \cdot (n_e \mathbf{v}_e) = S, \tag{1}$$

$$\partial_t \mathbf{v}_e + \nu_{\rm eff} \mathbf{v}_e = (e/m_e) \nabla \varphi - \gamma V_{Te}^2 \nabla n_e / n_{0e}, \tag{2}$$

$$\nabla^2 \varphi = -4\pi e (Z_i n_i - n_e - Z_d n_d), \tag{3}$$

where φ is the electrostatic potential and m_j, n_j (including the stationary value n_{0j}), and $\mathbf{v}_j$ are the mass, density, and fluid velocity of the species $j=e$, i, and d for electron, ion, and dust, respectively. Furthermore, $Z_i e$ and $-Z_d e$ are the charges of the ions and dusts, γ is the adiabatic constant of the electrons, and $V_{Te} = (T_e/m_e)^{1/2}$ is the electron thermal velocity. We have also defined $S = -\nu_{ed} n_e + \nu_i n_e - \beta n_e^2 + \beta_{si} n_e^2 + \nabla \cdot (D_a \nabla n_e)$, where ν_{ed} is the collection rate of plasma electrons by the dust grains, ν_i is the ionization rate, β is the volume recombination rate, β_{si} is the stepwise ionization rate, and D_a is the ambipolar diffusion coefficient. We have also defined the effective electron collision frequency $\nu_{\rm eff} = \nu_e + \nu_e^{\rm el} + \nu_e^{\rm ch}$, where ν_e is the rate of electron collisions with the neutral atoms and plasma particles, $\nu_e^{\rm el}$ is the rate of elastic (Coulomb) electron-dust collisions, and $\nu_e^{\rm ch}$ is the effective rate of collection of plasma particles by the dust.

We shall assume that the wave perturbations behave like $\sim \exp[i(kz - \omega t)]$ and use the probe model for dust charging.

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In the absence of the perturbations, the system is quasi-neutral so that $Z_i n_{i0} = n_{e0} + Z_d n_{d0}$. The dust-charge relaxation process is described by the charge balance equation [8]

$$d_t q_d = I_e(q_d) + I_i(q_d), \tag{4}$$

where q_d is the average charge on the dust grain and $I_e(q_d)$ and $I_i(q_d)$ are the electron and ion grain currents flowing into the grain surface. The quantities q_d, I_e, and I_i involve both steady-state and perturbed components, i.e., $q_d = q_{d0} + q_{d1}$, $I_e = I_{e0} + I_{e1}$, and $I_i = I_{i0} + I_{i1}$, where

$$I_{e0} = -\pi a^2 e (8T_e/\pi m_e)^{1/2} n_{e0} \exp[e\Delta\varphi_g/T_e], \tag{5}$$

$$I_{i0} = \pi a^2 e Z_i (8T_i/\pi m_i)^{1/2} n_{i0}[1 - e\Delta\varphi_g/T_i] \tag{6}$$

are the steady-state electron and ion currents at the grain surface and $q_{d0} = C\Delta\varphi_g$ is the stationary charge of the grain. Here a ($\ll r_{De}$) is the grain radius, r_{De} is the electron Debye radius, $C = a(1 + a/r_{\mathrm{De}})$ is the effective grain capacitance, and $\Delta\varphi_g = \varphi_g - \varphi_0$ is the steady-state potential difference between the grain and the adjacent plasma. The floating potential φ_0 is determined by equating the equilibrium electron and ion currents (5) and (6).

For Langmuir waves, we can neglect I_{i1} with respect to I_{e1} since it is on the slower ion time scale. From Eqs. (4)–(6) we obtain for the perturbed dust charge

$$d_t q_{d1} + \nu_{\mathrm{ch}} q_{d1} = -|I_{e0}| n_{e1}/n_{e0}, \tag{7}$$

where $\nu_{\mathrm{ch}} = a\omega_{pi}^2 \mathcal{A}/\sqrt{2\pi} V_{Ti}$ is the charging rate of the dust particle [8], defined by the equilibrium electron and ion microscopic currents (5) and (6). Here V_{Ti} is the ion thermal velocity, $\tau = T_i/T_e$, $\mathcal{A} = 1 + \tau + \mathcal{Z}$, and $\mathcal{Z} = Z_d e^2/aT_e$. The effective charging rate is $\nu_e^{\mathrm{ch}} = \nu_{\mathrm{ch}} P(4 + \mathcal{Z})(\tau + \mathcal{Z})/\mathcal{A}\mathcal{Z}$ and the electron capture rate at the grain surface is [8,9] $\nu_{ed} = \nu_{\mathrm{ch}} P(\tau + \mathcal{Z})/\mathcal{A}\mathcal{Z}$, $P = Z_d n_{d0}/n_{e0}$. The frequency of elastic electron-dust collisions is [9] $\nu_e^{\mathrm{el}} = 4\sqrt{2\pi} Z_d^2 n_d e^4 \Lambda/3 m_e^2 V_{Te}^3$, where $\Lambda = \ln(r_{\mathrm{De}}/a)$ is the Coulomb logarithm. The expressions for the rate ν_e of electron collisions as well as for ν_i, β, β_{si}, and D_a can be found in Ref. [10]. Equations (1)–(3) and (7) describe the coupling between the high-frequency electrostatic Langmuir waves and the CRM.

To determine the stationary electron plasma density we assume that the pressure is not too low, such that recombination losses prevail over diffusion losses. The last term in S can then be ignored. We then obtain from Eq. (1) the lowest-order (steady-state) electron plasma density $n_{e0} = (\nu_i - \nu_{ed})/\beta_{\mathrm{eff}}$, where $\beta_{\mathrm{eff}} = \beta - \beta_{si}$. We note that the ionization rate must be high enough such that $\nu_i > \nu_{ed}$; otherwise no self-consistent stationary state exists.

Linearizing with respect to the wave perturbations, we obtain from Eq. (1)

$$\begin{aligned} \partial_t n_{e1} + n_{e0}\nabla\cdot\mathbf{v}_e = -\nu_{ed}|_{Z_{d0}} n_{e1}, \\ -n_{e0}(\partial_{Z_d}\nu_{ed})_{Z_{d0}} Z_{d1} - 2\beta_{\mathrm{eff}} n_{e0} n_{e1} + \xi_1 \nu_i n_{e1}, \end{aligned} \tag{8}$$

where n_{e1} and Z_{d1} are the perturbations of the electron density and dust charge, respectively, and ξ_1 depends on the model for direct ionization. We shall consider two models, namely, that of a density-fluctuation independent ionization rate, with $\xi_1 = 0$, and that of density-fluctuation-dependent ionization rate, with $\xi_1 = 1$.

From Eq. (3) one can easily obtain the perturbed electron density $n_{e1} = -(1/4\pi e)k^2\varphi + Z_{d1} n_{d0}$. Thus we have for the grain charge and electron density variations

$$q_{d1} = ik^2|I_{e0}|\phi/4\pi e n_{e0}(\omega + i\nu_{\mathrm{ch}}^*) \tag{9}$$

and

$$n_{e1} = \nabla(\tilde{\epsilon}_d \nabla\varphi)/4\pi e, \tag{10}$$

where $\nu_{\mathrm{ch}}^* = \nu_{\mathrm{ch}} + \tilde{\nu}$, $\tilde{\nu} = n_{d0}|I_{e0}|/n_{e0} e$, and $\tilde{\epsilon}_d = 1 - i\tilde{\nu}/(\omega + i\nu_{\mathrm{ch}}^*)$. Furthermore, from Eq. (8) we have

$$\frac{n_{e1}}{n_{e0}} = \frac{i}{\eta}\left[-(\partial_{Z_d}\nu_{ed})_{Z_{d0}} Z_{d1} + \frac{iek^2\varphi}{m_e(\omega + i\nu_{\mathrm{eff}})}\right], \tag{11}$$

where $\eta = \omega - i\nu_{ed} + i\xi_2\nu_i - \gamma k^2 V_{Te}^2/(\omega + i\nu_{\mathrm{eff}})$ and $\xi_2 = 2 - \xi_1$. Finally, equating Eqs. (11) [after substituting of Z_{d1} from Eq. (9)] and (10), we obtain the dispersion relation of the Langmuir waves

$$D(\omega,k) = i\tilde{\nu}\frac{\omega + i\nu_{\mathrm{eff}}}{\omega + i\nu_{\mathrm{ch}}^*}\left[\eta + i\frac{n_{e0}}{n_{d0}}(\partial_{Z_d}\nu_{ed})_{Z_{d0}}\right], \tag{12}$$

where $D(\omega,k) = (\omega + i\nu_{\mathrm{eff}})\eta - \omega_{pe}^2$ and ω_{pe} is the electron plasma frequency. This is the equation describing the linear coupling of the high-frequency Langmuir plasma waves with the CRM mode $\omega = -i\nu_{\mathrm{ch}}'$. It can be solved numerically for any given set of parameters. We also note that if ν_e^{el}, ν_e^{ch}, ν_{ed}, and $(\partial_{Z_d}\nu_{ed})_{Z_{d0}}$ are set to zero in Eq. (12), one recovers the coupling equation of Ref. [4].

It is instructive to estimate the effect of electron capture (by the dust grain) and dissipative collisions on the Langmuir waves. For this purpose it is convenient to make the ansatz $\omega \gg \nu_{\mathrm{eff}}, \nu_{ed}, \nu_{\mathrm{ch}}^*$. Setting $\omega = \omega_1 + \delta_1' + i\delta_1''$, where $\omega_1^2 = \omega_{pe}^2 + \gamma k^2 V_{Te}^2$, we find from Eq. (12)

$$\delta_1' = -(\omega_{pe}^2\tilde{\nu}\omega_1^2 - \mathcal{B})\tilde{\nu}/2\omega_1, \tag{13}$$

where $\mathcal{B} = a^3 n_{i0}\omega_{pi}^2\mathcal{A}/\nu_{\mathrm{ch}}$ and

$$2\delta_1'' = \omega_{pe}^2\tilde{\nu}/\omega_1^2 - \nu_{\mathrm{eff}} - \xi_2\nu_i + \nu_{ed} + \tilde{\nu}\nu_{\mathrm{eff}}\mathcal{B}/\omega_1^2, \tag{14}$$

so that the frequency of the Langmuir waves is down shifted and the waves are damped by most of the collisional processes included here. In the absence of the latter, the first term in Eq. (14) remains and it leads to the Langmuir wave instability discussed earlier [4]. Although the rate ν_{ed} of electron capture by the dust also has a positive sign, it is always smaller than the term $-\xi_2\nu_i$ involving ionization because $\nu_i > \nu_{ed}$ (required by the existence condition for the stationary state) and $\xi_2 > 1$.

For the CRM, we set $\omega = -i\nu_{\mathrm{ch}}^* + i\delta_2$. From Eq. (12) one then obtains

$$\delta_2\omega^2/\tilde{\nu} = 3k^2V_{Te}^2 + (\nu_{\mathrm{ch}}^* - \nu_{\mathrm{eff}})(\nu_{\mathrm{ch}}^* + \xi_2\nu_i - \nu_{ed} - \mathcal{B}), \tag{15}$$

which shows that the dust charging rate is slightly reduced by the coupling with Langmuir waves.

We now estimate the average dust charge and the dissipation parameters $\nu_{\rm eff}$, ν_{ed}, $\nu_{\rm ch}^*$, and $\tilde{\nu}$ for typical dusty plasmas. The factor $e\Delta\varphi_g/T_e$, which defines the average charge on a dust grain and can strongly affect the density ratio n_{d0}/n_{e0} through the quasineutrality condition, can be found from the condition of zero total current flowing into the dust in the absence of the high-frequency perturbations. For a typical dusty argon plasma, we have $T_e \sim 10$ eV, $T_i \sim 1$ eV, $r \sim 5$ μm, $n_{e0} \sim 5\times 10^{10}$ cm^{-3}, and $n_{i0}/n_{e0}=10$. One then obtains $e\Delta\varphi_g/T_e=-1.71$, $Z_{d0}=-6.12\times 10^4$, and $n_{d0}/n_{e0}\approx 1.74\times 10^{-4}$. One can also show that the inequality $\overline{\nu}\ll\omega$, where $\overline{\nu}=\nu_{\rm eff},\nu_{ed},\nu_{\rm ch}^*,\tilde{\nu}$ represents the dissipative effects invoked here, is satisfied. In fact, we find $\overline{\nu}\sim(3\times 10^7)-(5\times 10^8)$ sec^{-1} and $\omega\sim 10^{10}$ sec^{-1}, which validate our ansatz.

It is also necessary to verify the existence condition $\nu_i > \nu_{ed}$ for the stationary state with the equilibrium density $n_{e0}=(\nu_i-\nu_{ed})/\beta_{\rm eff}$. To estimate the ionization frequency ν_i we use the expression (10.9) of Ref. [11] for the ionization rate (averaged over a Maxwellian distribution) $\langle\sigma_{\rm ion}v_e\rangle$ in hydrogen, where $\sigma_{\rm ion}$ is the ionization crosssection. Accordingly, we have

$$\nu_i=\frac{2\times 10^{-7}N_n}{6.0+T_e/U_i}\sqrt{\frac{T_e}{U_i}}\exp\left(-\frac{U_i}{T_e}\right), \qquad (16)$$

where U_i is the ionization energy and N_n is the number density (in cm^{-3}) of the neutral particles. We see that ν_i is sensitive to the electron temperature. The threshold temperature $T_e^{\rm thres}$ may be estimated by setting $\nu_i=\nu_{ed}$. For the typical hydrogen plasma parameters $n_{i0}\sim 10^{10}$ cm^{-3}, $N_n\sim 10^{14}$ cm^{-3}, $r\sim 1$ μm, $P=1$, and $\mathcal{Z}\sim 2$, we find $T_e^{\rm thres}\sim 7$ eV. A decrease of T_e would lead to a decrease of the ionization frequency and hence a violation of the existence condition for the stationary state. We note that $T_e^{\rm thres}$ depends weakly on the ion density. Furthermore, the threshold is affected by the degree of ionization (an increase of n_{i0}/N_n would lead to a downshift of $T_e^{\rm thres}$), the dust size and charge ($T_e^{\rm thres}$ increases with a and Z_{d0}), and the number of electrons in the system ($T_e^{\rm thres}$ increases when n_{e0}/n_{i0} decreases). It should be pointed out that in using the expression (10.9) of Ref. [11], one should check the relation between T_e and $U_{\rm eff}=(2/3)(U_i-U_*)$, where U_* is the energy of the first excited level. For electron temperatures exceeding $U_{\rm eff}$, multistep ionization dominates and the term U_i in the exponent of Eq. (16) must be replaced by the lower valued U_*. In this case the direct-ionization approximation is no longer valid and the existence threshold for the stationary state may be lowered. For argon plasmas the corresponding expressions for the ionization frequencies are more complicated, but the exponential dependence on $-U_{(*,i)}/T_e$ remains the same [10]. Thus for argon plasmas similar results can be expected.

In conclusion, we have shown that if ionization, recombination, and other collisions are included, the linear coupling of the CRM and Langmuir waves leads to a damping and a frequency downshift of the waves. This result differs considerably from that where a uniform source is invoked to replace the electrons and ions lost to dust charging [4]. Thus the actual ionization and recombination processes that maintain the total charge balance of a dusty plasma system may be important in investigations of instabilities in dusty plasmas.

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[1] C. K. Goertz, Rev. Geophys. **27**, 271 (1989).
[2] A. Bouchoule and L. Boufendi, Plasma Sources Sci. Technol. **3**, 262 (1994).
[3] S. V. Vladimirov, Phys. Rev. E **50**, 1422 (1994).
[4] J. X. Ma and M. Y. Yu, Phys. Rev. E **50**, R2431 (1994).
[5] F. Verheest, Space Sci. Rev. **77**, 267 (1996), and the reference therein.
[6] V. N. Tsytovich, Usp. Fiz. Nauk **40** 53 (1997) [Phys. Usp. **40**, 53 (1997)].
[7] K. N. Ostrikov, M. Y. Yu, and N. A. Azarenkov, Phys. Rev. E **58**, 2431 (1998).
[8] V. N. Tsytovich and O. Havnes, Comments Plasma Phys. Control. Fusion **15**, 267 (1993).
[9] S. V. Vladimirov and V. N. Tsytovich, Phys. Rev. E **58**, 2415 (1998).
[10] L. M. Biberman, V. S. Vorob'ev, and I. T. Yakubov, *Kinetics of Non-Equilibrium Low-Temperature Plasmas* (Nauka, Moscow, 1982).
[11] R. J. Goldston and P. H. Rutherford, *Introduction to Plasma Physics* (IOP Publishing, London, 1995).