Structure of Yukawa dusty plasma mixtures

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Parameters characterizing the structure of a confined Yukawa system are estimated for ''dusty plasmas,'' clouds of charged macroscopic particles formed near the boundary between a plasma and the sheath, and levitated by a negatively biased electrode. When we have dust particles with different charge-to-mass ratios, they form a two-dimensional Yukawa mixture or separate two-dimensional one-component Yukawa systems, depending on the charge density in the sheath and the number density of dust particles. Pointed out is the possibility that dust particles with a larger charge-to-mass ratio have layered structures in the domain of neutral plasma, being supported by those with a smaller charge-to-mass ratio in the domain of sheath.
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I. INTRODUCTION

The physics of dusty plasmas, assemblies of macroscopic particles immersed in plasmas, has attracted the keen interest of researchers as an important practical problem in plasma processes of semiconductor manufacturing, and also as a subject of basic statistical physics [1]. The observation of crystal-like structures in dusty plasmas [2–5] has added a new example to the classical Coulomb lattice which was predicted more than 60 years ago [6].

In our recent works [7–12], we regarded dust particles as interacting via the isotropic repulsive Yukawa potential

$$\frac{q^2}{r}\exp(-\kappa r), \tag{1.1}$$

where $-q$ is the (negative) charge on a dust particle, and being trapped in a one-dimensional potential well of the form

$$v_{\rm ext}(z)=\tfrac{1}{2}kz^2. \tag{1.2}$$

We have analyzed the phase diagram for the structure of this confined Yukawa system by molecular dynamics simulations and theoretical approaches.

As for the interaction between dust particles, it has been pointed out that there exists an anisotropic interaction coming from the ion flow in the sheath [13–17]. An important result of this anisotropic potential may be the phenomenon of alignment of dust particles along the flow often observed in experiments. However, there also exist experiments where such an alignment is not apparent [3]. The anisotropy of the interaction may also influence the spacing and orientation of dust crystals, and the absence of alignment does not directly imply the absence of anisotropy. We may, however, expect to have cases where the isotropic part of the interaction potential plays the central role in determining the overall structure in the z direction, even if configurations in the xy plane relative to adjacent layers are affected by the anisotropic part.

Defining the mean distance between dust particles a by

$$a=(\pi N_S)^{-1/2} \tag{1.3}$$

from the surface number density of dust particles N_S, we express the strength of screening by surrounding plasma by a parameter

$$\xi=\kappa a. \tag{1.4}$$

We have also introduced a parameter η, defined by

$$\eta=\frac{\pi^{1/2}}{2}\frac{\frac{1}{2}ka^2}{q^2/a}, \tag{1.5}$$

to describe the strength of the confining potential relative to mutual repulsion. Structures at low temperatures are determined by a competition between these two forces, and are expressed as a phase diagram in the (ξ,η) plane [8]. When $\eta\gg1$, we have the two-dimensional Yukawa system. With the decrease of η, the number of layers increases discretely. We have shown that this phase diagram is reproduced by a rather simple theoretical model [8].

It has been found in experiments that the radius of dust particles of the same kind appearing in plasma processes has a rather small dispersion [18]. However, there may be cases where different kinds of macroscopic particles coexist in plasmas, and their separation is necessary. In this paper, we clarify the origin of the one-dimensional confining potential, and discuss various possibilities of structures for mixtures of Yukawa particles.

II. PARAMETERS CHARACTERIZING CONFINING POTENTIAL

We consider the case where our dusty plasma is formed above a horizontal plane electrode which is wide enough to regard the system under consideration as one-dimensional. A typical example of environment of our dusty plasma is shown in Fig. 1. Dust particles are under the vertical gravitational field, and are levitated by the electric field between the negatively biased electrode and the bulk part of the plasma.

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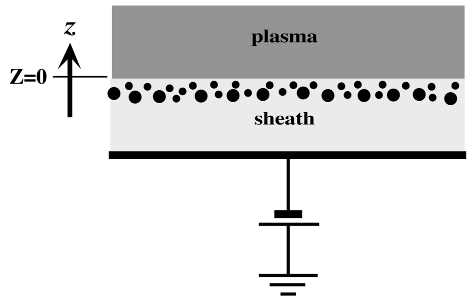

FIG. 1. Dusty plasma confined near the boundary of the sheath and plasma bulk.

Let us adopt an ion matrix sheath model, and assume that the density of charges in the sheath (except for those of dust particles) is given by $en_{\rm sh}$, e being the elementary charge, and is nearly constant in the domain where dust crystals are formed. Since it is well known that the charge density is not uniform even in the simplest model of the self-consistent sheath [19], this assumption may be an oversimplification. The main purpose of this paper, however, is the analysis of the complexity due to the dispersion of dust particles, and we simplify other factors as much as possible.

When we take the z axis in the opposite direction to the gravitation, the gravitational and the electrostatic potentials for a dust particle of mass m and charge $-q$ are written as mgz and $2\pi qen_{\rm sh}z^2$, respectively. Thus, in the domain $z<0$, dust particles are in the potential well:

$$\phi_{\rm ext}(z<0)=mgz+2\pi qen_{\rm sh}z^2$$

$$=\phi_{\rm ext}(z_0)+2\pi qen_{\rm sh}(z-z_0)^2, \qquad (2.1)$$

where

$$z_0=-\frac{mg}{4\pi qen_{\rm sh}}=-\frac{g}{4\pi en_{\rm sh}}\frac{m}{q}<0. \qquad (2.2)$$

In this case, the confining potential is given by (Eq. 1.2), with

$$k=4\pi qen_{\rm sh}, \qquad (2.3)$$

and η is calculated as

$$\eta=\left(\frac{e}{q}\right)\left(\frac{n_{\rm sh}}{N_S^{3/2}}\right). \qquad (2.4)$$

Note that η is the ratio of the charge densities due to the space charge in the sheath $en_{\rm sh}$ and the charge due to dust particles, if the latter is extended to three dimensions.

In the domain $z>0$, we have

$$\phi_{\rm ext}(z>0)=mgz. \qquad (2.5)$$

In our model, the external potential $\phi_{\rm ext}$ cannot keep dust particles afloat in the domain of neutral plasma.

When we have only one species of dust particles, the structure at low temperatures is completely determined by parameters ξ and η [8]. In the case where there are two or more species of dusts, we also have to take the dependence of z_0 on species into account. We thus define a parameter δ by

$$\delta=-\frac{z_0}{a}=\frac{1}{2}\frac{mga}{2\pi qen_{\rm sh}a^2}=\frac{g}{4\pi en_{\rm sh}a}\frac{m}{q} \qquad (2.6)$$

to represent the separation in the z direction: The equilibrium position z_0 which is proportional to the charge-to-mass ratio q/m is compared with the mean distance a.

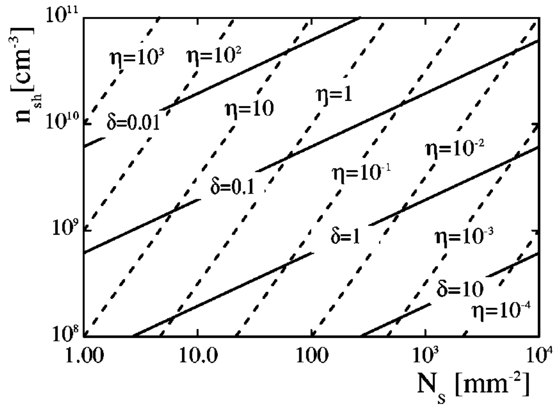

FIG. 2. Values of characteristic parameters η and δ for dust particles with $-q=-10^4e$ and $m=10^{-10}$ g.

For a typical case of dust particle with the charge $q=10^4e$ and $m_0=10^{-13}$ kg$=10^{-10}$ g, parameters η and δ are given by

$$\eta=10^2\times\left(\frac{10^4e}{q}\right)\left(\frac{n_{\rm sh}}{10^9\ {\rm cm}^{-3}}\right)\left(\frac{1\ {\rm mm}^{-2}}{N_S}\right)^{3/2}, \qquad (2.7)$$

$$\delta=6.0\times10^{-2}\times\left(\frac{10^4e}{q}\right)\left(\frac{m}{m_0}\right)\left(\frac{N_S}{1\ {\rm mm}^{-2}}\right)^{1/2}\times\left(\frac{10^9\ {\rm cm}^{-3}}{n_{\rm sh}}\right). \qquad (2.8)$$

Values of these parameters are shown in Fig. 2. For spherical particles with the mass density ρ, the radius R, and the potential $-V$, these relations are rewritten as

$$\eta=1.4\times10^2\times\left(\frac{1\ \mu{\rm m}}{R}\right)\left(\frac{10\ {\rm V}}{V}\right)\times\left(\frac{n_{\rm sh}}{10^9\ {\rm cm}^{-3}}\right)\left(\frac{1\ {\rm mm}^{-2}}{N_S}\right)^{3/2}, \qquad (2.9)$$

$$\delta=5.4\times10^{-3}\times\left(\frac{\rho}{1.5\ {\rm g/cm}^3}\right)\left(\frac{R}{1\ \mu{\rm m}}\right)^2\left(\frac{10\ {\rm V}}{V}\right)\times\left(\frac{N_S}{1\ {\rm mm}^{-2}}\right)^{1/2}\left(\frac{10^9\ {\rm cm}^{-3}}{n_{\rm sh}}\right), \qquad (2.10)$$

where we have applied the relation $q=RV$.

According to the values of η and δ, we have four cases, and we may expect the behavior of Yukawa mixtures as

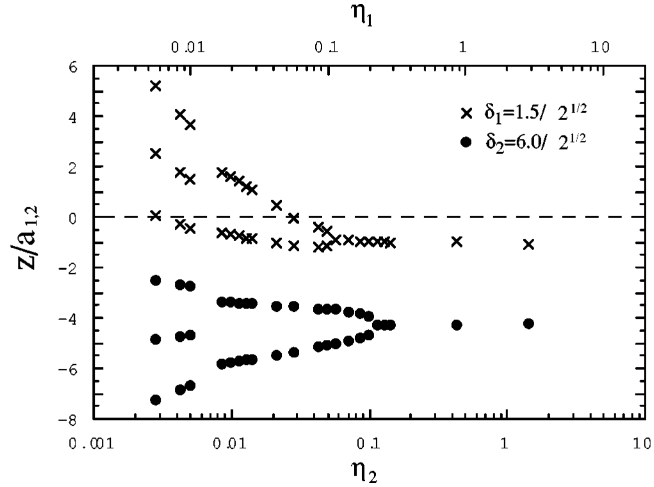

FIG. 3. Positions of layers for $\delta_1=1.5/2^{1/2}$ and $\delta_2=6/2^{1/2}$.

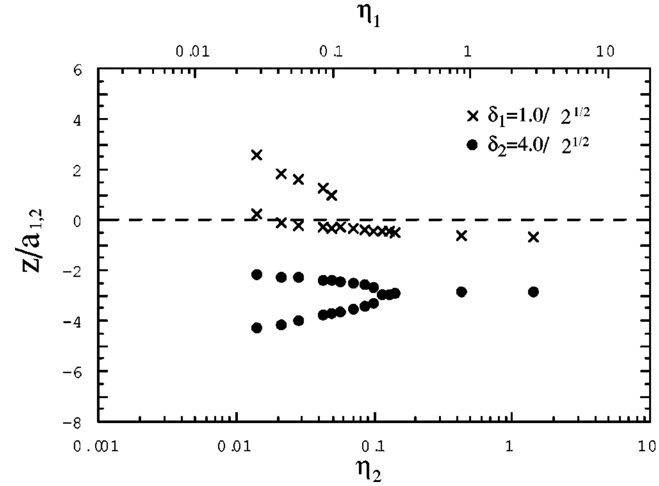

FIG. 4. Positions of layers for $\delta_1=1/2^{1/2}$ and $\delta_2=4/2^{1/2}$.

follows. When $\eta\gg1$ and $\delta\ll1$, the Yukawa mixture forms a two-dimensional system or the two-dimensional Yukawa mixture. When $\eta\gg1$ and $\delta\gg1$, we have separate two-dimensional Yukawa systems, each being composed of one species. When $\eta\ll1$ and $\delta\ll1$, we have a mixture of Yukawa particles with finite thickness. When $\eta\ll1$ and $\delta\gg1$, we have two separate one-component Yukawa systems with finite thicknesses.

III. NUMERICAL SIMULATION AND DISCUSSIONS

A. Results of numerical simulation

In order to clarify the behavior of the mixtures of dust particles in plasmas, we have performed molecular dynamics simulations on dust mixtures. The methods of simulation are the same as our previous ones [7,8,10]: Periodic boundary conditions in the xy plane are imposed, while the deformations of basic vectors of periodicity are taken into account, and the temperature and area in the xy plane are kept constant.

As an example of dust mixtures, we take the one composed of species 1 and 2, where

$$\frac{q_1}{q_2}=\frac{1}{2}, \tag{3.1}$$

$$\frac{m_1}{m_2}=\frac{1}{8}, \tag{3.2}$$

and

$$\frac{N_1}{N_2}=1. \tag{3.3}$$

Here $-q_i$, m_i, and N_i are the charge, mass, and surface number density of the dust of species i. These conditions for charges and masses correspond to the case where both kinds of dust particles are of the same material, the ratio of radii is 2, and the electrostatic potentials are the same. As for the parameters ξ, we assume $\xi=1$ evaluating the mean distance a by the total surface density $N_S=N_1+N_2$.

We define the parameters for species i by

$$\eta_i=\left(\frac{e}{q_i}\right)\left(\frac{n_{\rm sh}}{N_i^{3/2}}\right) \tag{3.4}$$

and

$$\delta_i=-\frac{z_i}{a_i}=\frac{g}{4\pi e n_{\rm sh}a_i}\frac{m_i}{q_i}, \tag{3.5}$$

where

$$a_i=(\pi N_i)^{-1/2}, \tag{3.6}$$

and z_i is the equilibrium position of species i determined by $\phi_{\rm ext}(z)$,

$$z_i=-\frac{g}{4\pi e n_{\rm sh}}\frac{m_i}{q_i}. \tag{3.7}$$

In the above case,

$$\frac{\eta_1}{\eta_2}=\frac{q_2}{q_1}\left(\frac{N_2}{N_1}\right)^{3/2}=2, \tag{3.8}$$

$$\frac{\delta_1}{\delta_2}=\frac{1}{4}. \tag{3.9}$$

The results of simulations are summarized in Figs. 3–6 where the positions of layers normalized by a_i are plotted as functions of η_i: Since the values of η_i depend on species, the abscissa has two scales. When η_i's are sufficiently large, both species are in the one-layer state. With the decrease of the parameters η_i's, formations of multiple layers are also observed. Critical values of η_i for layer formations are given in Table I.

B. Discussions

We first note that, when considered as a separate system of one species, the effective screening parameter is given by

$$\xi_i=\kappa a_i, \tag{3.10}$$

and, in our case,

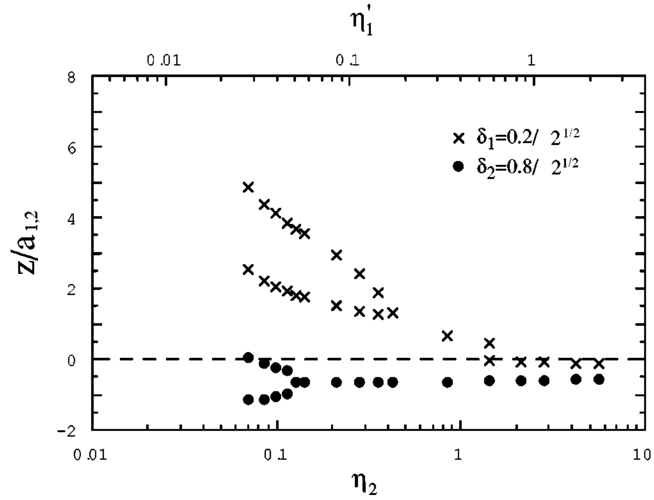

FIG. 5. Positions of layers for $\delta_1=0.2/2^{1/2}$ and $\delta_2=0.8/2^{1/2}$.

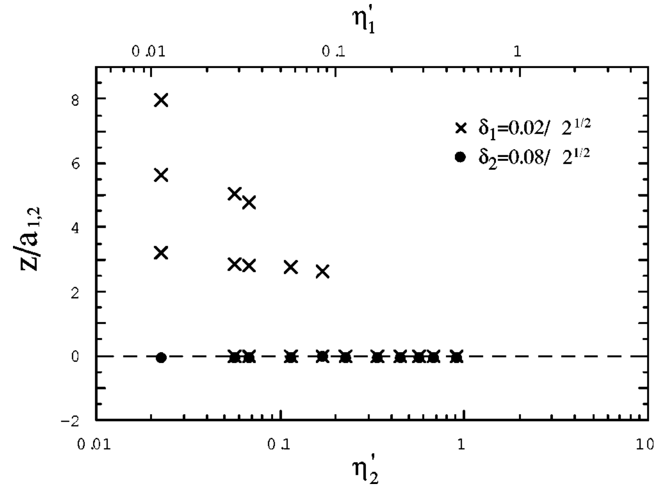

FIG. 6. Positions of layers for $\delta_1=0.02/2^{1/2}$ and $\delta_2=0.08/2^{1/2}$.

$$\xi_1=\xi_2=2^{1/2}\xi=2^{1/2}. \tag{3.11}$$

In the case $\delta_i \gtrsim 1$ shown in Figs. 3 and 4, each species of dust is well separated in the direction of gravity. When η_i's are sufficiently large and both species are in the one-layer state, the positions of layers are almost identical to the values given by δ_i, indicating that the effect of charges in the sheath $en_{\rm sh}$ is dominant for both species.

When the parameters η_i's decrease in Figs. 3 and 4, positions of layers deviate from δ_i: The upper layer goes up and the lower layer slightly goes down. Each system (composed of one species of dust) is influenced not only by the electrostatic potential due to charges in the sheath $en_{\rm sh}$ but also by the potential due to particles in the other system. The effect of the latter, which is small when η_i's are relatively large, is responsible for these deviations. Since the particle charge in the lower layer is larger than that in the upper layer, the downward deviation of the lower is small: Though the magnitude of mutual repulsions are the same for both layers (due to the principle of the action and the reaction), the relative effect of $en_{\rm sh}$ is larger for the lower layer. With a further decrease of η_i's, the upper layer eventually moves into the domain of neutral plasma.

Let us now consider the effect of the sheet of Yukawa particles of species j on the confinement of another Yukawa particles. We may estimate the potential due to Yukawa particles of species j in the layer, regarding them as a uniform distribution with the surface number density N_j and the charge $-q_j$ in the plane $z=z_j(<0)$. The potential due to these particles is given by

$$\psi_j(z)=-\frac{2\pi N_j q_j}{\kappa}\exp[-\kappa|z-z_j|], \tag{3.12}$$

and the total potential for a particle of species i, $\phi_i'(z)$, is given by

$$\phi_i'(z<0)=m_i g z+2\pi q_i e n_{\rm sh} z^2-q_i\psi_j(z) \tag{3.13}$$

or

$$\phi_i'(z>0)=m_i g z-q_i\psi_j(z). \tag{3.14}$$

The equilibrium position z_i' is determined by the condition

$$\frac{d}{dz}\phi_i'(z_i')=0. \tag{3.15}$$

When $\psi_j(z)$ is expanded around $z=z_i'$ as

$$\psi_j(z)=\psi_j(z=z_i')\left[1-\kappa(z-z_i')\mathrm{sgn}(z_i'-z_j)+\frac{1}{2}\kappa^2(z-z_i')^2+\cdots\right], \tag{3.16}$$

we have, when $z_i'<0$,

$$\phi_i'(z)=\phi_i'(z_i'<0)+2\pi q_i e n_{\rm sh}\left[1+\xi_i\frac{z_i'-z_i}{a_i}\mathrm{sgn}(z_i'-z_j)\right](z-z_i')^2+\cdots\ . \tag{3.17}$$

Here we have used condition (3.15). When $z_i'>0$, we have

$$\phi_i'(z)=\phi_i'(z_i'>0)+2\pi q_i e n_{\rm sh}[\xi_i\delta_i\mathrm{sgn}(z_i'-z_j)]\times(z-z_i')^2+\cdots\ . \tag{3.18}$$

The potential $-q_i\psi_j(z)$ has two effects: To change the equilibrium position from z_i to z_i', and to change the confinement around z_i'. Since $-q_i\psi_j(z)$ is repulsive, the equilibrium position of species i moves upward (downward) when $z_i>z_j$ ($z_j>z_i$), or

$$(z_i'-z_i)(z_i-z_j)>0. \tag{3.19}$$

TABLE I. Critical values for layering transitions.

δ_i	Transition $1\rightarrow 2$	Transition $2\rightarrow 3$
$\delta_1 = 1.5/2^{1/2}$	$0.099 < \eta_1 < 0.11$	$[0.0099 < \eta_1 < 0.017]$
$\delta_2 = 6.0/2^{1/2}$	$0.099 < \eta_2 < 0.11$	$0.0049 < \eta_2 < 0.0085$
$\delta_1 = 1.0/2^{1/2}$	$0.099 < \eta_1 < 0.11$	
$\delta_2 = 4.0/2^{1/2}$	$0.099 < \eta_2 < 0.11$	
$\delta_1 = 0.2/2^{1/2}$	$0.14 < \eta_1' < 0.17$	
$\delta_2 = 0.8/2^{1/2}$	$0.11 < \eta_2 < 0.13$	
$\delta_1 = 0.02/2^{1/2}$	$[0.085 < \eta_1' < 0.11]$	$[0.034 < \eta_1' < 0.057]$
$\delta_2 = 0.08/2^{1/2}$		
one component		
$(\xi = 2^{1/2})$	0.13	0.011

The parameter controlling the confinement around z_i' changes from k of Eq. (2.3) to k' given by

$$k' = 4\pi q_i e n_{\rm sh}\left[1 + \xi_i \frac{z_i' - z_i}{a_i}\,{\rm sgn}(z_i' - z_j)\right] \quad (z_i' < 0) \tag{3.20}$$

or

$$k' = 4\pi q_i e n_{\rm sh}[\xi_i \delta_i {\rm sgn}(z_i' - z_j)] \quad (z_i' > 0). \tag{3.21}$$

Corresponding to the change to k', the parameter η_i defined by Eq. (3.4) is replaced by η_i', given by

$$\eta_i' = \eta_i\left[1 + \xi_i \frac{z_i' - z_i}{a_i}\,{\rm sgn}(z_i' - z_j)\right] \quad (z_i' < 0) \tag{3.22}$$

or

$$\eta_i' = \eta_i \xi_i \delta_i {\rm sgn}(z_i' - z_j) \quad (z_i' > 0). \tag{3.23}$$

It is clear that the confinement becomes stronger by the effect of ψ_j.

In Table I, critical values for multiple layer formations are compared with our results for the case of one component with $\xi = 2^{1/2}$. As for the transition $1\rightarrow 2$ occurring in the sheath when $\delta_i \gtrsim 1$, we observe that present results of η_i's are a little smaller than our previous one. This may be due to the contribution of the second term in [ ] on the right hand side of Eq. (3.22). The values indicated by [ ] are the case where the upper system is partly in the sheath and partly in the bulk plasma, and the comparison with the case of one component is difficult.

When δ_i's are small as shown in Figs. 5 and 6, upward movements of the upper layers with decreasing η_i's are more remarkable, and upper layers are mostly in the neutral plasma where $\phi_{\rm ext} = mgz$. These layers are thus supported by the repulsion from the underlying layer in the region $z < 0$ with smaller charge-to-mass ratio.

For the case of $\delta_1 = 0.2/2^{1/2}$ and $\delta_1 = 0.8/2^{1/2}$, the critical values of η_1', defined by Eq. (3.23), and η_2 are compared with our previous result for the case of one component with $\xi = 2^{1/2}$. We see that the present results are consistent with the previous ones. The values in [ ] are the case where such a comparison is not possible for the same reason as in the case $\delta_i \gtrsim 1$. The two-dimensional mixture of Yukawa particles is realized for large values of η_i when δ_i's are sufficiently small, as shown in Fig. 6.

IV. CONCLUDING REMARKS

As shown above, mixtures of Yukawa particles have a rich class of structures at low temperatures. We especially note that species with a larger charge-to-mass ratio can be levitated into the domain of neutral plasma by other species with a smaller charge-to-mass ratio which underlies in the domain of sheath. In the neutral domain, the flow of ions is small, and we expect that the isotropic Yukawa potential serves as a more accurate model for interactions between dust particles compared with the domain of the sheath, where the supersonic flow of ions may induce anisotropies in the interaction potential. It may be of particular interest to perform experiments on plasma crystals in dusty plasma with intentionally added heavy dust particles, in order to position the main part of the crystal in the neutral plasma, where a direct comparison with the Yukawa system is possible.

ACKNOWLEDGMENT

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