

## Thermal radiation of a semibounded medium with a transition layer

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We obtained the thermal radiation spectra of a semibounded nonisothermal plasmalike medium separated by a homogeneous layer from the external region. The results are written in the form of the Kirchhoff law, generalized to the case of account for the proper thermal field of the external medium and different temperatures of the radiating media. We note the possibility of the dominant contribution of the proper thermal radiation field of the external medium to the one-sided energy flux. We study the conditions of frequency and angular enlightening for a transparent layer. The influence of the transition layer and the thermal radiation field of the external medium on the frequency spectra of thermal radiation of the system under consideration is investigated in detail. [S1063-651X(98)15710-0]

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### I. INTRODUCTION

Theoretical studies of the radiation spectra of heated bodies are important for an interpretation of the experimental data that allow one to determine the parameters of radiating bodies without a direct contact of these bodies with recording equipment. The Kirchhoff law is the foundation of the classical theory of equilibrium radiation [1]. This law gives a simple relation between the radiating and absorbing properties of bodies by means of a universal function of the frequency and the temperature, namely, the radiation intensity of a blackbody defined by the Rayleigh-Jeans or Planck distributions, which correspond, respectively, to the classical or quantum description.

At the same time, the approach based on the Kirchhoff law is restricted by the approximation of the geometric optics. One can remove this restriction by using an alternative approach proposed by Rytov [2], which is based on the general correlation theory of thermal fluctuations of electromagnetic field. Numerous investigations [2–5] of thermal radiation spectra for bodies of various geometry carried out on the basis of this approach by Rytov and his followers permitted them to construct a theory of electromagnetic radiation of heated bodies without restrictions on the sizes of the body and the wavelength. It turns out that expressions for thermal radiation spectra can be presented in the same form as those in the case of the classical Kirchhoff law; moreover, the results of the theory are also generalized to the case of gyrotropic media.

Henceforward, many authors developed the theory of thermal radiation in different directions: extension of radiating bodies under study to the case of plasma [6–10] and plasma-molecular [11,12] media, an account of spatial dispersion [13,14], nonequilibrium state [15], inhomogeneity of radiating bodies [16–19], etc. It is worth noting that the study of thermal radiation emitted by plasma systems is of particular interest to researches in various fields of physics, because the results on thermal radiation spectra of plasma are intensively used for solving many problems of radiative

transfer in laboratory plasma, astrophysics and inertial fusion, physics of the ionosphere, plasma diagnostics, radio-physics, spectroscopy, plasma technology including processing and depollution of materials, laser welding, etc.

However, the conventional approaches used for calculating thermal radiation spectra (based either on a calculation of the losses of electromagnetic fields in absorbing media produced by some auxiliary sources [3–5] or on the representation of thermal fields as the radiation of random sources distributed in the volume occupied by the body [2,12–15]) do not take into account the proper thermal radiation field of the external transparent medium. The approach proposed in Refs. [20–24] generalizes the results of the correlation theory for a semibounded medium to the case of a noncold transparent external medium and an account of the zero-point oscillations of the field in it. We derived conditions when the contribution of the proper thermal field of the transparent external medium to the correlation functions of the electromagnetic field (including the thermal radiation spectra) was significant.

Since, as a rule, there exists a transition (in the general case, inhomogeneous) region between a radiating body and the external medium, it is of interest to use the approach proposed for studying the radiation spectra of inhomogeneous systems. According to the results obtained by simulating the transition layer by an arbitrary number of homogeneous [16] or inhomogeneous [17] layers, the presence of a transition layer can lead to a considerable rearrangement of the radiation spectra of semibounded homogeneous media. However, just as for homogeneous media, the radiation spectra of inhomogeneous bodies were studied in the case of the cold external medium. In addition, the expressions for the thermal radiation spectra turn out to be so complicated [16,17] that even in the simplest case of a single transition layer, one failed in presenting results in a form similar to the Kirchhoff law.

In the present work, we calculate the thermal radiation spectra of a system consisting of a semibounded nonisothermal plasmalike medium with a homogeneous layer on it, taking into account the proper thermal field of the external transparent medium (Sec. II). The thermal radiation spectra are presented in the form of a generalization of the Kirchhoff

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law to a case that takes into account the inhomogeneity of the radiating system, different temperatures of the media of the system, and plasma nonisothermality. We found the contribution of the proper thermal field of the external medium to the energy density of the one-sided thermal radiation flux. The general relation for the thermal radiation intensity is shown to reduce to the Kirchhoff law in a particular case of a uniformly heated spatially inhomogeneous radiating system. We established conditions for which the contribution of the proper thermal field of the external medium to the energy density of the one-sided thermal radiation flux was dominant. In the particular case of a cold plasma approximation, the results of a numerical analysis for the frequency and angular dependences of the thermal radiation intensity for various thicknesses of the transition layer are given in Sec. III.

## II. THERMAL RADIATION SPECTRA OF A SYSTEM WITH A HOMOGENEOUS TRANSITION LAYER

### A. General relations

Let us find the thermal radiation of a piecewise homogeneous system that occupies the  $z < L$  region into the external transparent  $z > L$  medium with the dielectric permittivity  $\epsilon_3$ . We restrict ourselves to the simplest case when the radiating system consists of two homogeneous media: a half-space of a quasineutral, in the general case multicomponent, plasma-like system (the third medium) that occupies the  $z < 0$  region, and a dielectric layer (the second medium) with the dielectric permittivity  $\epsilon_2(\omega) \equiv \epsilon_2$  occupying the  $0 < z < L$  region (Fig. 1). We consider the case of a nonisothermal plasma when every species  $\sigma$  of charged particles is defined by its temperature  $T_\sigma$ . We take into account the spatial dispersion of the plasma system by using the model of specular reflection of all free charged plasma particles from the  $z = 0$  boundary. The temperatures of the first ( $z > L$ ) and second media are, respectively,  $T_1$  and  $T_2$ . The origin of the Cartesian coordinate system is taken in the plane between the second and third media.

The thermal radiation energy density through a unit surface area oriented in parallel to the boundary  $z = L$  plane is

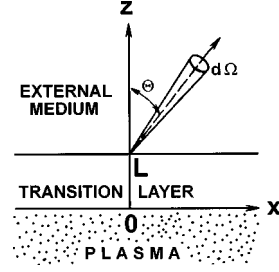


FIG. 1. Geometry of the system under investigation.

defined as the statistical average of the normal component of the Umov-Poynting vector

$$\langle P_z \rangle = \frac{c}{4\pi} e_{zij} \langle E_i(\vec{r}, t) B_j(\vec{r}, t) \rangle \equiv \int_0^\infty d\omega P(\omega), \quad (1)$$

where

$$P(\omega) = \frac{c}{4\pi^2} \text{Re} e_{zij} \langle E_i B_j^* \rangle_{r\omega} \quad (2)$$

is the spectral energy density of the thermal radiation flux taken at  $z > L$ ,  $\vec{E}(\vec{r}, t)$  and  $\vec{B}(\vec{r}, t)$  are the electromagnetic fields generated by random sources distributed in the whole of space, the brackets  $\langle \dots \rangle$  imply the statistical averaging, and  $e_{zij}$  is the third rank absolutely antisymmetric unit tensor. We substitute the expressions for the fluctuating electromagnetic field in the system at hand taken at  $z > L$  [25] in Eq. (2), and carry out statistical averaging by using the correlation functions of the random sources [25] calculated with regard to the boundedness of the media and the spatial dispersion of the plasmalike medium. By omitting cumbersome intermediate calculations, we present the spectral energy density of the thermal radiation flux  $P(\omega) \equiv P(\omega, T_1, T_2, \mathcal{T})$  that describes the radiant heat transfer between the inhomogeneous ( $z < L$ ) and transparent external ( $z > L$ ) media in the following form:

$$P(\omega, T_1, T_2, \mathcal{T}) = \int_{\theta \leq \pi/2} d\Omega \cos \theta I(\omega, \theta, T_1, T_2, T_p^{\text{eff}}(k_\perp, \omega), T_s^{\text{eff}}(k_\perp, \omega)). \quad (3)$$

Here, the variable  $\mathcal{T}$  implies a set of temperatures of all subsystems that form the plasma system,  $I(\omega, \theta, T_1, T_2, T_p^{\text{eff}}(k_\perp, \omega), T_s^{\text{eff}}(k_\perp, \omega))$  is the thermal radiation intensity into the unit solid angle  $d\Omega = \sin \theta d\theta d\varphi$  ( $\theta$  and  $\varphi$  are the polar and azimuth angles giving the direction of the axis of the solid angle  $d\Omega$  that are measured, respectively, from the  $Z$  and the  $X$  axes) that is equal to the sum of thermal radiation intensities of independent  $p$  and  $s$  polarizations,

$$I(\omega, \theta, T_1, T_2, T_p^{\text{eff}}(k_\perp, \omega), T_s^{\text{eff}}(k_\perp, \omega)) = \sum_{\alpha=p,s} I_\alpha(\omega, \theta, T_1, T_2, T_\alpha^{\text{eff}}(k_\perp, \omega)), \quad (4)$$

where  $k_\perp = k_1 \sin \theta$ ,  $k_n^2 = \omega^2 \epsilon_n / c^2 = k_0^2 \epsilon_n$ ,  $n = 1$  and  $2$ .

$$I_\alpha(\omega, \theta, T_1, T_2, T_\alpha^{\text{eff}}(k_\perp, \omega)) = I_\alpha(\omega, \theta, T_1, T_2) + I_\alpha(\omega, \theta, T_1, T_\alpha^{\text{eff}}(k_\perp, \omega)), \quad (5)$$

$$I_\alpha(\omega, \theta, T_1, T_2) = \frac{1}{2} [I_0(\omega, T_2) - I_0(\omega, T_1)] \Gamma_\alpha^{(2)}(k_\perp, \omega), \quad (6)$$

$$I_\alpha(\omega, \theta, T_1, T_\alpha^{\text{eff}}(k_\perp, \omega)) = \frac{1}{2} [I_0(\omega, T_\alpha^{\text{eff}}(k_\perp, \omega)) - I_0(\omega, T_1)] \Gamma_\alpha^{(3)}(k_\perp, \omega), \quad (7)$$

$$I_0(\omega, T) = \frac{\omega^2 \varepsilon_1}{4\pi^3 c^2} \frac{\hbar \omega}{\exp(\hbar \omega / T) - 1} \quad (8)$$

is the radiation intensity of a blackbody with the temperature  $T$  which is measured in energy units into a transparent medium with the dielectric permittivity  $\varepsilon_1$ .  $\Gamma_\alpha^{(n)}(k_\perp, \omega)$  is the absorption coefficient by the layer ( $n=2$ ) or the plasma half-space ( $n=3$ ) of a plane electromagnetic  $\alpha$ -polarized wave incident at the angle  $\theta$  from the first region upon the piecewise homogeneous system

$$\Gamma_\alpha^{(2)}(k_\perp, \omega) = \frac{1}{|d_\alpha(k_\perp, \omega)|^2} \{ \Gamma_\alpha^{(1,2)}(k_\perp, \omega) [1 + \mathcal{R}_\alpha^{(2,3)}(k_\perp, \omega) \exp(-y)] [1 - \exp(-y)] + 4 \exp(-y) \operatorname{Im} R_\alpha^{(1,2)}(k_\perp, \omega) \operatorname{Im} [(1 - \exp(ix)) R_\alpha^{(2,3)}(k_\perp, \omega)] \}, \quad (9)$$

$$\Gamma_\alpha^{(3)}(k_\perp, \omega) = -|T_\alpha^{(2)}(k_\perp, \omega)|^2 \operatorname{Re} r_\alpha^{(1,3)}(k_\perp, \omega), \quad (10)$$

where  $x = 2L \operatorname{Re} k_{z2}$ ,  $y = 2L \operatorname{Im} k_{z2}$ ,  $k_{zn} = (k_n^2 - k_\perp^2)^{1/2}$ ,  $\operatorname{Im} k_{zn} \geq 0$ ,  $n = 1$  and  $2$ ,

$$d_\alpha(k_\perp, \omega) = 1 + R_\alpha^{(1,2)}(k_\perp, \omega) R_\alpha^{(2,3)}(k_\perp, \omega) \exp(i2Lk_{z2}), \quad (11)$$

$$T_\alpha^{(2)}(k_\perp, \omega) = \frac{T_\alpha^{(1,2)}(k_\perp, \omega) T_\alpha^{(2,3)}(k_\perp, \omega)}{d_\alpha(k_\perp, \omega)} \exp(iLk_{z2}) \quad (12)$$

is the transmission coefficient of a plane electromagnetic  $\alpha$ -polarized wave through the layer, where  $T_\alpha^{(n,m)}(k_\perp, \omega) = 1 + R_\alpha^{(n,m)}(k_\perp, \omega)$ ,  $\mathcal{R}_\alpha^{(n,m)}(k_\perp, \omega) = |R_\alpha^{(n,m)}(k_\perp, \omega)|^2$  ( $n, m = 1, 2, 3$ ,  $n \neq m$ ),

$$R_\alpha^{(n,m)} = -\frac{1 + r_\alpha^{(m,n)}(k_\perp, \omega)}{1 - r_\alpha^{(m,n)}(k_\perp, \omega)}, \quad (13)$$

$$\Gamma_\alpha^{(n,m)}(k_\perp, \omega) = 1 - \mathcal{R}_\alpha^{(n,m)}(k_\perp, \omega). \quad (14)$$

If the  $n$ th medium is transparent, the quantities  $\mathcal{R}_\alpha^{(n,m)}(k_\perp, \omega)$  and  $R_\alpha^{(n,m)}(k_\perp, \omega)$  have the meaning of the energy and the amplitude Fresnel reflection coefficients from the  $m$ th medium of homogeneous (for  $k_\perp < k_n$ ) or inhomogeneous (for  $k_\perp > k_n$ ) plane electromagnetic  $\alpha$ -polarized waves incident on it from the adjacent transparent half-space with the dielectric permittivity  $\varepsilon_n$ : the quantity  $R_\alpha^{(n,m)}(k_\perp, \omega)$  is the ratio of the tangential electric components of reflected and incident electromagnetic waves; and  $T_\alpha^{(n,m)}(k_\perp, \omega)$  and  $\Gamma_\alpha^{(n,m)}(k_\perp, \omega)$  are, respectively, the Fresnel transmission and absorption coefficients of the  $m$ th semibounded medium of a plane electromagnetic  $\alpha$ -polarized wave incident on this medium from a transparent semibounded medium with the dielectric permittivity  $\varepsilon_n$ .

As it follows from Eqs. (9)–(11), the quantities  $\Gamma_\alpha^{(n)}(k_\perp, \omega)$  that determine the absorbing properties of the inhomogeneous radiating system under consideration are completely defined by the quantities  $r_\alpha^{(n,m)}(k_\perp, \omega)$ , which can be presented as

$$r_\alpha^{(n,m)}(k_\perp, \omega) = -\frac{Z_\alpha^{(n)}(k_\perp, \omega)}{Z_\alpha^{(m)}(k_\perp, \omega)}, \quad n, m = 1, 2, 3, \quad (15)$$

where  $Z_\alpha^{(n)}(k_\perp, \omega)$  is the surface impedance of the  $n$ th medium for fields with  $\alpha$  polarization. For the system at hand, these quantities are [15,26]

$$r_s^{(2,1)}(k_\perp, \omega) = -\frac{k_{z1}}{k_{z2}}, \quad r_p^{(2,1)}(k_\perp, \omega) = -\frac{\varepsilon_1 k_{z2}}{\varepsilon_2 k_{z1}}, \quad (16)$$

$$r_s^{(3,n)}(k_\perp, \omega) = -\frac{i}{\pi} \frac{c^2 k_{zn}}{\omega^2} \int_{-\infty}^{\infty} dk_z \frac{\exp(ik_z z)}{\Delta_T(k, \omega)}, \quad (17)$$

$$r_p^{(3,n)}(k_\perp, \omega) = -\frac{i}{\pi} \frac{\varepsilon_n}{k_{zn}} \int_{-\infty}^{\infty} dk_z \left[ \frac{k_\perp^2}{\varepsilon_L(k, \omega)} + \frac{k_z^2}{\Delta_T(k, \omega)} \right] \times \frac{\exp(ik_z z)}{k^2}, \quad (18)$$

where  $n = 1, 2$ ,

$$\Delta_T(k, \omega) = \varepsilon_T(k, \omega) - \frac{c^2 k^2}{\omega^2}, \quad (19)$$

and  $\varepsilon_{L,T}(k, \omega)$  are the longitudinal and transverse dielectric permittivities of an unbounded plasma:

$$\varepsilon_{L,T}(k, \omega) = \varepsilon_0 + 4\pi \sum_{\alpha=p,s} \kappa_{L,T}^\sigma(k, \omega). \quad (20)$$

$\varepsilon_0 \equiv \varepsilon_0(\omega)$  is the dielectric permittivity of the background (the dielectric permittivity of the medium in which free charged particles are placed in the case of a gaseous plasma or the dielectric permittivity of the lattice in the case of a

solid-state plasma),  $\kappa_{L,T}^\sigma(k, \omega)$  are the longitudinal and transverse electric susceptibility of free charged particles of species  $\sigma$  [9,10], and

$$\begin{aligned}\kappa_L^\sigma(k, \omega) &= \frac{1}{4\pi} \frac{\omega_{p\sigma}^2}{k^2 \omega} \int_{-\infty}^{\infty} d\vec{v} \frac{(\vec{k}\vec{v})^2 \cdot \partial f_{0\sigma}(v)/\partial v}{\omega - \vec{k}\vec{v} + i\nu_\sigma}, \\ \kappa_T^\sigma(k, \omega) &= \frac{1}{8\pi} \frac{\omega_{p\sigma}^2}{k^2 \omega} \int_{-\infty}^{\infty} d\vec{v} \frac{[\vec{k}\vec{v}]^2 \cdot \partial f_{0\sigma}(v)/\partial v}{\omega - \vec{k}\vec{v} + i\nu_\sigma}.\end{aligned}\quad (21)$$

$\omega_{p\sigma} = (4\pi e_\sigma^2 n_{0\sigma}/m_\sigma)^{1/2}$ ,  $e_\sigma$ ,  $m_\sigma$ ,  $n_{0\sigma}$ , and  $f_{0\sigma}(v)$  are the plasma frequency, charge, mass (or effective mass in a case of a degenerate plasma), mean density, and unperturbed distribution function that is assumed to be isotropic for charged particles of species  $\sigma$ , and  $\nu_\sigma$  is the effective collision frequency of charged particles of species  $\sigma$  with neutral particles (the simplest relaxation model is used here for describing collisions of charged particles with neutral particles).

The quantities  $T_\alpha^{\text{eff}}(k_\perp, \omega)$  have the meaning of the effective temperatures for the polarization  $\alpha$  and are defined as

$$\begin{aligned}\exp[\hbar\omega/T_s^{\text{eff}}(k_\perp, \omega) - 1]^{-1} &= -\left(\frac{2c}{\omega}\right)^2 \frac{1}{\text{Re}(k_{z2}^{-1} r_p^{(3,2)}(k_\perp, \omega))} \sum_{\beta=\sigma, m} [\exp(\hbar\omega/T_\beta - 1)]^{-1} \\ &\times \int_{-\infty}^{\infty} dk_z \frac{\text{Im} \kappa_T^\sigma(k_\perp, \omega)}{|\Delta_T(k, \omega)|^2}, \\ \exp[\hbar\omega/T_p^{\text{eff}}(k_\perp, \omega) - 1]^{-1} &= -\frac{4\varepsilon_2}{\text{Re}[k_{z2} r_p^{(3,2)}(k_\perp, \omega)]} \sum_{\beta=\sigma, m} [\exp(\hbar\omega/T_\beta - 1)]^{-1} \int_{-\infty}^{\infty} dk_z \frac{1}{k^2} \\ &\times \left[ \frac{k_\perp^2 \text{Im} \kappa_L^\sigma(k_\perp, \omega)}{|\varepsilon_L(k_\perp, \omega)|^2} + \frac{k_z^2 \text{Im} \kappa_T^\sigma(k_\perp, \omega)}{|\Delta_T(k, \omega)|^2} \right],\end{aligned}\quad (22)$$

where, for the sake of the uniform representation of the relations, we introduce the notation  $\kappa_{L,T}^m(k_\perp, \omega)$ ; however, for a subsystem of bound charged particles (the background) that enters into the plasma medium and has the temperature  $T_m$ , only the frequency dispersion is taken into account, i.e.,  $\kappa_{L,T}^m(k_\perp, \omega) \equiv \kappa_m(\omega) = (\varepsilon_0(\omega) - 1)/4\pi$ .

It is worth noting that the introduction of external random sources of radiation only in the region occupied by the radiating system [2,3] leads to the appearance of the component of the energy flux of the zero-point oscillations of the field in the spectra of heated bodies, which then is discarded by assuming that the energy flux of the zero-point oscillations of the field in any direction is suppressed by the opposing flux. Since expression (3) for the spectral energy density of the thermal radiation flux does not contain a term connected with the zero-point oscillations of the field, this means that the introduction of external random sources in the whole of space automatically ensures the compensation of opposing energy fluxes of the zero-point oscillations of the field.

If the proper thermal field of the external (the first) medium is regarded as the radiation field generated by some infinitely far sources [2,22], then we can present the spectral energy density of the thermal radiation flux (3) as a superposition of two one-sided fluxes directed in mutually opposite directions

$$P(\omega, T_1, T_2, T) = P^\mu(\omega, T_1, T_2, T) + P^d(\omega, T_1) \quad (23)$$

away from the  $z=L$  boundary ( $P^\mu$ ) and toward it ( $P^d$ ), equal to

$$P^\mu(\omega, T_1, T_2, T) = \int_{\theta \leq \pi/2} d\Omega \cos \theta I^\mu(\omega, \theta, T_1, T_2, T_p^{\text{eff}}(k_\perp, \omega), T_s^{\text{eff}}(k_\perp, \omega)), \quad (24)$$

$$P^d(\omega, T_1) = \pi I_0(\omega, T_1), \quad (25)$$

where

$$\begin{aligned}I^\mu(\omega, \theta, T_1, T_2, T_p^{\text{eff}}(k_\perp, \omega), T_s^{\text{eff}}(k_\perp, \omega)) &= \sum_{\alpha=p, s} I_\alpha^\mu(\omega, \theta, T_1, T_2, T_\alpha^{\text{eff}}(k_\perp, \omega)), \\ I_\alpha^\mu(\omega, \theta, T_1, T_2, T_\alpha^{\text{eff}}(k_\perp, \omega)) &= I_\alpha^\mu(\omega, \theta, T_1, T_2, T_\alpha^{\text{eff}}(k_\perp, \omega)) \\ &= I_\alpha^{(2)}(\omega, \theta, T_2) + I_\alpha^{(3)}(\omega, \theta, T_\alpha^{\text{eff}}(k_\perp, \omega)) \\ &\quad + \frac{1}{2} I_0(\omega, T_1) \mathcal{R}_\alpha(k_\perp, \omega)\end{aligned}\quad (26)$$

is the thermal radiation intensity for the polarization  $\alpha$  of the

one-sided energy flux directed away from the  $z=L$  boundary into the unit solid angle  $d\Omega$ , and  $I_\alpha^{(2)}(\omega, \theta, T_2)$  and  $I_\alpha^{(3)}(\omega, \theta, T_\alpha^{\text{eff}}(k_\perp, \omega))$  are, respectively, the thermal radiation intensities for the polarization  $\alpha$  of the second and third media into the external  $z>L$  medium; moreover, the expressions for them are defined by the Kirchhoff law for every independent polarization

$$I_\alpha^{(2)}(\omega, \theta, T_2) = \frac{1}{2} I_0(\omega, T_2) \Gamma_\alpha^{(2)}(k_\perp, \omega), \quad (28)$$

$$I_\alpha^{(3)}(\omega, \theta, T_\alpha^{\text{eff}}(k_\perp, \omega)) = \frac{1}{2} I_0(\omega, T_\alpha^{\text{eff}}(k_\perp, \omega)) \Gamma_\alpha^{(3)}(k_\perp, \omega), \quad (29)$$

and the absorption coefficients  $\Gamma_\alpha^{(2)}(k_\perp, \omega)$  and  $\Gamma_\alpha^{(3)}(k_\perp, \omega)$  are defined by Eqs. (9) and (10) obtained with regard to the influence of inhomogeneity of the radiating  $z<L$  system on the absorption properties of each of its homogeneous components.

Here  $\mathcal{R}_\alpha(k_\perp, \omega) = |R_\alpha(k_\perp, \omega)|^2$ , where

$$R_\alpha(k_\perp, \omega) = \frac{R_\alpha^{(1,2)}(k_\perp, \omega) + R_\alpha^{(2,3)}(k_\perp, \omega) \exp(i2Lk_{z2})}{d_\alpha(k_\perp, \omega)} \quad (30)$$

is the reflection coefficient of a plane electromagnetic  $\alpha$ -polarized wave from the piecewise homogeneous system ( $z<L$ ) under consideration.

Equations (24), (26), and (27) generalize the classical Kirchhoff law in the Levin-Rytov form for the energy flux density of the thermal radiation of a uniformly heated body to the case of a radiating system consisting of two differently heated media, and account for the contribution of the proper thermal field of the external medium to the one-sided thermal radiation flux. The first two terms in Eq. (27) correspond to the radiation of the piecewise homogeneous system ( $z<L$ ), and the last describes the thermal radiation of the external medium into the solid angle  $d\Omega$ .

The presentation of the thermal radiation intensity [Eqs. (26)–(29)] in the form of the Kirchhoff law that connects the radiating and absorbing abilities of the body allows us, in some cases, to make quantitative conclusions about features of the radiation spectra, without recourse to numerical calculations, by using only the well-known results of the theory on the propagation of electromagnetic waves. For example, in a particular case of the transparent second medium ( $\text{Im}\epsilon_2 = 0$ ), the radiation of the homogeneous system is independent of the temperature  $T_2$  of this medium. Indeed, if the dissipation of the medium is absent, electromagnetic waves are not absorbed by it [ $\Gamma_\alpha^{(2)}(k_\perp, \omega) = 0$  for any thickness of the layer]. For the quantities that define the contribution of the second medium to the thermal radiation intensity spectra of the inhomogeneous system ( $z<L$ ), it yields  $I_\alpha(\omega, \theta, T_1, T_2) = 0$  and  $I_\alpha^{(2)}(\omega, \theta, T_2) = 0$ . Therefore, the thermal radiation intensities of two- [Eq. (4)] and one-sided [Eq. (27)] energy fluxes of a semibounded body separated by a transparent layer from the external medium have the same form as the corresponding quantities [21,22] found for a semibounded system without a transparent layer. This means that the presence of a transparent dielectric layer has an effect only on the absorption properties of the semibounded

medium due to multiple reflections of electromagnetic waves from the  $z=0$  and  $z=L$  boundaries.

### B. Isothermal plasma

In the case of an isothermal plasma ( $T_\sigma, T_m = T_3$ ), the effective temperatures for  $p$ - and  $s$ -polarized fields are the same:  $T_p^{\text{eff}}(k_\perp, \omega) = T_s^{\text{eff}}(k_\perp, \omega) \equiv T_3$ . For this reason, there is no need for dividing the thermal radiation flux into components corresponding to the independent polarizations. The expressions for the thermal radiation intensities of two- [Eq. (4)] and one-sided [Eq. (27)] fluxes are simplified to the forms

$$I(\omega, \theta, T_1, T_2, T_3) = \sum_{n=2}^3 I(\omega, \theta, T_1, T_n), \quad (31)$$

$$I^u(\omega, \theta, T_1, T_2, T_3) = \sum_{n=2}^3 I(\omega, \theta, T_n) + I_0(\omega, T_1) \mathcal{R}(k_\perp, \omega), \quad (32)$$

where

$$\begin{aligned} I(\omega, \theta, T_1, T_n) &= \sum_{\alpha=p,s} I_\alpha(\omega, \theta, T_1, T_n) \\ &= [I_0(\omega, T_n) - I_0(\omega, T_1)] \Gamma^{(n)}(k_\perp, \omega), \end{aligned} \quad (33)$$

$$\begin{aligned} I(\omega, \theta, T_n) &= \sum_{\alpha=p,s} I_\alpha(\omega, \theta, T_n) \\ &= I_0(\omega, T_n) \Gamma^{(n)}(k_\perp, \omega), \end{aligned} \quad (34)$$

$$\Gamma^{(n)}(k_\perp, \omega) = \frac{1}{2} \sum_{\alpha=p,s} \Gamma_\alpha^{(n)}(k_\perp, \omega) \quad (35)$$

comprise the absorption coefficient of a plane unpolarized wave by the  $n$ th medium that is a part of the inhomogeneous  $z<L$  system, and

$$\mathcal{R}(k_\perp, \omega) = \frac{1}{2} \sum_{\alpha=p,s} \mathcal{R}_\alpha(k_\perp, \omega) \quad (36)$$

is the energy reflection coefficient of a plane unpolarized wave from the piecewise homogeneous  $z<L$  system.

By introducing external random sources of the fluctuating electromagnetic field into the whole of space, on the basis of direct calculations, we obtained the representation of the thermal radiation intensity of the one-sided flux as the Kirchhoff law in the Levin-Rytov form [Eqs. (27) and (32) for a nonisothermal and isothermal plasma, respectively] in the case of an inhomogeneous system consisting of two homogeneous media. Since every term in Eqs. (27) and (32) describes the thermal radiation of a separate homogeneous medium that is a part of the inhomogeneous system under consideration, and is determined by the Kirchhoff law with the temperature of this medium, we can assume that in a more general case, when the transition layer between the plasma and external media is inhomogeneous and can be simulated by an arbitrary number  $N$  of homogeneous layers,

each being determined by its temperature  $T_n$  and dielectric permittivity  $\varepsilon_n(\omega)$ , the expression for the thermal radiation intensities of two- and one-sided fluxes hold the forms similar to Eqs. (31) and (32) [or Eqs. (5) and (27) in the case on nonisothermal plasma], namely,

$$I(\omega, \theta, T_1, T_2, \dots, T_{N+1}, T_{N+2}) = \sum_{n=2}^{N+2} I(\omega, \theta, T_1, T_n), \quad (37)$$

$$I''(\omega, \theta, T_1, T_2, \dots, T_{N+1}, T_{N+2}) = \sum_{n=2}^{N+2} I(\omega, \theta, T_n) + I_0(\omega, T_1) \mathcal{R}(k_{\perp}, \omega). \quad (38)$$

The summation in Eqs. (37) and (38) extends over all media of the transition layer ( $n=2, 3, \dots, N+1$ ) and the plasma ( $n=N+2$ ) and quantities  $I(\omega, \theta, T_1, T_n)$  and  $I(\omega, \theta, T_n)$  are defined by laws (33) and (34), where  $\Gamma_{\alpha}^{(n)}(k_{\perp}, \omega)$  is the absorption coefficient of an unpolarized wave by the  $n$ th medium that is a part of the inhomogeneous  $z < L$  system at hand, and  $\mathcal{R}(k_{\perp}, \omega)$  is the energy reflection coefficient of an unpolarized wave from the  $z=L$  boundary. In addition, it seems possible to assume that representations (37) and (38) for the thermal radiation intensity are valid in the case of the simulation of the transition region by an arbitrary number of inhomogeneous layers (with constant temperatures) without defining concretely the character of inhomogeneity within each of the layers.

It follows from this that in a particular case of a transparent  $m$ th medium ( $2 \leq m \leq N+1$ ) of the transition layer, the thermal radiation intensities of two- and one-sided fluxes are independent of  $T_m$ , due to the absence of radiation intensities corresponding to the  $m$ th medium:  $I(\omega, \theta, T_1, T_m) = 0$  and  $I(\omega, \theta, T_m) = 0$ .

### C. Uniformly heated medium

In the case of a uniformly heated ( $T_2 = T_3 \equiv T$ )  $z < L$  system, Eqs. (31) and (32) are reduced to the forms

$$I(\omega, \theta, T_1, T) = [I_0(\omega, T) - I_0(\omega, T_1)] \Gamma(k_{\perp}, \omega), \quad (39)$$

$$I''(\omega, \theta, T_1, T) = I_0(\omega, T) \Gamma(k_{\perp}, \omega) + I_0(\omega, T_1) \mathcal{R}(k_{\perp}, \omega), \quad (40)$$

where  $\Gamma(k_{\perp}, \omega) = \sum_{n=2}^3 \Gamma^{(n)}(k_{\perp}, \omega) = 1 - \mathcal{R}(k_{\perp}, \omega)$  is the absorption coefficient of an unpolarized wave by the inhomogeneous semibounded system.

The first term in Eqs. (39) and (40) corresponds to the thermal radiation intensity of the piecewise homogeneous  $z < L$  medium with the temperature  $T$  into the external region. It is a form of representation of the thermal radiation intensity when the dielectric properties of the radiating system at hand have an effect only on the absorption coefficients  $\Gamma(k_{\perp}, \omega)$  that corresponds to the classical Kirchhoff law.

In the case of an inhomogeneous system, Eqs. (39) and (40) have the same form as the expressions for the thermal radiation intensities of two- and one-sided fluxes in the case of a homogeneous half-space [22], and differ from the last only by a specific form of the absorption  $[\Gamma(k_{\perp}, \omega)]$  and

reflection  $[\mathcal{R}(k_{\perp}, \omega)]$  coefficients. By assuming the validity of representations (37) and (38) for the thermal radiation intensities of an inhomogeneous system consisting of an arbitrary number of variously heated media, we obtain the same relations [Eqs. (39) and (40)] (with the corresponding absorption  $[\Gamma(k_{\perp}, \omega) = \sum_{n=2}^{N+2} \Gamma_{\alpha}^{(n)}(k_{\perp}, \omega) = 1 - \mathcal{R}(k_{\perp}, \omega)]$  and reflection  $[\mathcal{R}(k_{\perp}, \omega)]$  coefficients) for the thermal radiation intensities of two- and one-sided fluxes in the case of an arbitrary inhomogeneous body with the temperature  $T$ . The same representation of the thermal radiation intensities for inhomogeneous and homogeneous systems seems to be quite natural because, by virtue of the Kirchhoff law, the ratio of the radiating power of a body to its absorption ability depends neither on the properties of the body nor on its composition and form.

For the quasiclassical approximation ( $\hbar \omega \ll T, T_1$ ), Eqs. (39) and (40) are reduced to the forms

$$I(\omega, \theta, T_1, T) = I_0(\omega, T - T_1) \Gamma(k_{\perp}, \omega), \quad (41)$$

$$I''(\omega, \theta, T_1, T) = I_0(\omega, T) \left\{ \Gamma(k_{\perp}, \omega) + \frac{T_1}{T} \mathcal{R}(k_{\perp}, \omega) \right\}, \quad (42)$$

where

$$I_0(\omega, T) = \frac{\omega^2 \varepsilon_1}{4 \pi^3 c^2} T \quad (43)$$

is the radiation intensity of a blackbody with the temperature  $T$ , defined within the framework of the classical approximation.

As seen from Eq. (41), the intensity of heat transfer between the transparent external and inhomogeneous media is defined by the same expression as the expression for the thermal intensity radiation of a uniformly heated system consisting of a semibounded plasma and a homogeneous transition layer into the external cold medium, if in the last expression we substitute  $T - T_1$  for  $T$ . This allows us to use directly the results [16] concerning the radiation of piecewise-homogeneous media relative to problems on heat transfer between homogeneous and inhomogeneous media, provided that the temperature of the radiating system is measured from the temperature of the external medium. Equations (38), (40), and (42) for the thermal radiation intensity of the one-sided flux generalize the results for a semibounded plasma [13] and for a plasma with a coating [16] to the case where the proper thermal field of the transparent external medium is taken into account.

Let us study the role of the proper thermal field of the external medium in the total flux. This may be useful for analyzing the experimental data which are used for evaluating the temperature of the radiating body, as well as conclusions about the composition and properties of the plasma systems. It is clear that an account of the proper thermal field of the external medium may be significant if the temperatures of the external and radiating media are of the same order. At the same time, even in the case of media with significantly different temperatures, neglect of the contribution of the radiation of the external medium may considerably affect the results. This is most simply seen from Eq.

(42). Indeed, in the case of slightly absorbing media, we cannot ignore the second term in Eq. (42) despite the fact that  $T_1/T \ll 1$ . In addition, if the condition

$$\frac{T_1}{T} \gg \frac{\Gamma(k_\perp, \omega)}{\mathcal{R}(k_\perp, \omega)} \quad (44)$$

is satisfied, the one-sided radiation flux is, mainly, defined by the radiation of the external medium, i.e.,

$$I^u(\omega, \theta, T_1, T) \approx I_0(\omega, T_1). \quad (45)$$

To evaluate the possibility of the realization of condition (44), here we consider the simplest case corresponding to the radiation of a homogeneous electron semibounded plasma without a transition layer disregarding the spatial dispersion of the plasma. In this case,

$$\varepsilon(\omega) = \varepsilon_0 - \frac{\omega_{pe}^2}{\omega(\omega + i\nu_e)}. \quad (46)$$

For frequencies lower than the cutoff frequency  $\omega_{\text{cut}} = \omega_{pe}/(\varepsilon_0 - \varepsilon_1 \sin^2 \theta)^{1/2}$  ( $\varepsilon_0$  is assumed to be real), the value of  $\mathcal{R}(k_\perp, \omega)$  is close to unity. In the frequency range  $\nu_e \ll \omega \ll \omega_{\text{cut}}$ , when the condition  $|\varepsilon_1 \sin^2 \theta - \text{Re } \varepsilon(\omega)| \gg \text{Im } \varepsilon(\omega)$  is valid, condition (44) is reduced to the form

$$\frac{T_1}{T} \gg \frac{Y}{(X + \sin^2 \theta)^{1/2}} \frac{\cos \theta}{X + 1} \frac{3 \sin^2 \theta + (1 + \cos^2 \theta)X}{\sin^2 \theta + X \cos^2 \theta}, \quad (47)$$

where  $X = \text{sgn}(\omega_{pe}/\varepsilon_0^{1/2} - \omega) |\text{Re } \varepsilon(\omega)|/\varepsilon_1$ , and  $Y = \text{Im } \varepsilon(\omega)/\varepsilon_1$ .

For frequencies lower than  $\omega_{pe}/\varepsilon_0^{1/2}$  at  $X \gg 1$ , Eq. (47) has the form

$$\frac{T_1}{T} \gg \frac{Y}{X^{1/2}} \frac{(1 + \cos^2 \theta) \cos \theta}{\sin^2 \theta + X \cos^2 \theta}. \quad (48)$$

For angles when  $\tan \theta \ll X^{1/2}$ , we obtain a simple criterion for the temperatures and the plasma parameters

$$\frac{T_1}{T} \gg \left( \cos \theta + \frac{1}{\cos \theta} \right) \frac{\nu_e}{\omega_{pe}} \varepsilon_1^{1/2}. \quad (49)$$

As an example, we consider the radiation of a semi-bounded gaseous plasma with the temperature  $T = 10^4$  K into the external medium at room temperature  $T_1 = 300$  K. If the radiation is recorded along the normal to the plasma boundary, at  $\varepsilon_1 = 1$ , condition (49) holds at  $\nu_e/\omega_{pe} \ll 0.015$ . For typical values of the ratio  $\nu_e/\omega_{pe}$  equal to  $10^{-3}$  for a gaseous plasma, we have that, at  $\theta = 0$ , the radiation of the plasma accounts only about 6% of the total radiation intensity of the measured one-sided flux.

At angles when  $\tan \theta \gg X^{1/2}$  (angles which are close to the grazing angle of a wave), condition (47) can be written as follows:

$$\frac{T_1}{T} \gg \frac{\cos \theta}{\varepsilon_1^{1/2}} \frac{\nu_e}{\omega_{pe}} \left( \frac{\omega_{pe}}{\omega} \right)^2. \quad (50)$$

In the low-frequency range  $\omega \ll \nu_e$ , for angles  $\cos \theta \gg Y^{-1/2}$ , the criterion for the temperatures

$$\frac{T_1}{T} \gg \left( \cos \theta + \frac{1}{\cos \theta} \right) \frac{\nu_e}{\omega_{pe}} \left( 2\varepsilon_1 \frac{\omega}{\nu_e} \right)^{1/2} \quad (51)$$

turns out to be weaker than Eq. (49). Therefore, as the frequency decreases, the contribution of the plasma radiation to the total one-sided thermal radiation flux reduces. In particular, for the case at hand, it is less than 1% of the one-sided thermal radiation flux. In the case of well conducting metals, criteria (47)–(51) can be more easily realized and, hence, the thermal radiation of the external medium is dominant in the one-sided thermal radiation flux.

#### D. Transparent transition layer

In the case of a transparent layer, the thermal radiation intensity is defined by the temperatures of external and plasma ( $T_3 \equiv T$ ) media, and, in accordance with Eqs. (32) and (34), the thermal radiation intensity of the one-sided flux for the polarization  $\alpha$  has the form

$$I_\alpha^u(\omega, \theta, T_1, T) = \frac{1}{2} I_0(\omega, T) \Gamma_\alpha(k_\perp, \omega) + \frac{1}{2} I_0(\omega, T_1) \mathcal{R}_\alpha(k_\perp, \omega), \quad (52)$$

where  $\Gamma_\alpha(k_\perp, \omega) = 1 - \mathcal{R}_\alpha(k_\perp, \omega)$  is the absorption coefficient of a plane  $\alpha$ -polarized electromagnetic wave by the inhomogeneous semibounded system.

Let us turn our attention to the behavior of the energy reflection coefficient  $\mathcal{R}_\alpha(k_\perp, \omega)$  that determines the thermal radiation spectra of the system under consideration. Since  $k_\perp = k_1 \sin \theta \leq k_1$ , the quantity  $k_{z2} = (k_2^2 - k_\perp^2)^{1/2} = k_0(\varepsilon_2 - \varepsilon_1 \sin^2 \theta)^{1/2}$  is either real [for any angle  $\theta$  in the case where  $\varepsilon_2 > \varepsilon_1$  or at  $\theta < \theta_{\text{cut}}$ , where  $\theta_{\text{cut}} = \arcsin(\varepsilon_2/\varepsilon_1)^{1/2}$  is the total internal reflection angle if  $\varepsilon_2 < \varepsilon_1$ ] or imaginary (at  $\theta > \theta_{\text{cut}}$  if  $\varepsilon_2 < \varepsilon_1$ ). We consider the case when the quantity  $k_{z2}$  is real, which corresponds to homogeneous waves propagating in the layer. It follows from definition (30) of the quantity  $R_\alpha(k_\perp, \omega)$  that in the case where the double thickness of the layer is greater than the wavelength in it,  $\lambda_2 = \lambda_0/\varepsilon_2^{1/2}$  ( $\lambda_0 = 2\pi c/\omega$  is the wavelength in a vacuum), for fixed values of the thickness of the layer and the wavelength of the incident wave, there exist  $N = [2\tilde{L}] > 1$ , where  $\tilde{L} = L/\lambda_2$  is the normalized thickness of the layer and  $[a]$  is the integer part of the number  $a$ , values of the quantity  $k_\perp$  for which the layer enlightening occurs:

$$R_\alpha(k_{\perp n}, \omega) = R_\alpha^{(1,3)}(k_{\perp n}, \omega), \quad n = 1, 2, \dots, N. \quad (53)$$

The quantities  $k_{\perp n} = k_2(1 - (n/2\tilde{L})^2)^{1/2}$  are determined in such a way that the optical wavelength in the layer ( $\tilde{L}p$ ) is equal to an integer of half-waves in it,

$$2\tilde{L}p = n, \quad n = 1, 2, \dots, N, \quad (54)$$

where  $p = (1 - (k_\perp/k_2)^2)^{1/2}$ .

Therefore, regardless of polarization, when an electromagnetic wave of a given frequency reflects from a transparent layer whose thickness is greater than a half-wavelength

in it, there exist  $N$  incident angles  $\theta_n$  ( $\theta_1 > \theta_2 > \dots > \theta_N$  and  $\theta_1 < \theta_{\text{cut}} \equiv \theta_0$ ) in the case where  $\varepsilon_1 > \varepsilon_2$ ,

$$\theta_n = \arcsin \left( \frac{\varepsilon_2}{\varepsilon_1} \left( 1 - \left( \frac{n}{2\tilde{L}} \right)^2 \right) \right)^{1/2}, \quad n = 1, 2, \dots, N, \quad (55)$$

for which the layer enlightening occurs. In the case where  $\varepsilon_1 < \varepsilon_2$ , the condition  $k_\perp < k_1$  decreases the number of these angles, because  $n$  can take on only the values  $n = M, M + 1, \dots, N$ , where  $M$  is the minimum integer that exceeds the quantity  $2\tilde{L}(1 - \varepsilon_1/\varepsilon_2)^{1/2}$ .

Note that for  $p$ -polarized waves, there exists one more incident angle [ $\theta_B^{(1,2)} = \arctan(\varepsilon_2/\varepsilon_1)^{1/2}$  is the Brewster angle] for which the presence of a dielectric layer of any thickness has no effect on the energy reflection coefficient. At  $\theta = \theta_B^{1/2}$ ,  $\mathcal{R}_p(\theta_B^{(1,2)}, \omega) = \mathcal{R}_p^{(1,3)}(\theta_B^{(1,2)}, \omega)$ , whereas  $\mathcal{R}_p(\theta_B^{(1,2)}, \omega) = \mathcal{R}_p^{(1,3)}(\theta_B^{(1,2)}, \omega) \exp(i4\pi\tilde{L}(\varepsilon_1 + \varepsilon_2)^{1/2})$ .

Thus the reflection coefficient  $R_\alpha(k_\perp, \omega)$  of an electromagnetic wave from a transparent layer is an oscillating

function of the angle of incidence which coincides with the quantity  $R_\alpha^{(1,3)}(k_\perp, \omega)$  at the enlightening angles of the layer  $\theta_n$ . As a result, the angular dependence of the thermal radiation intensity of the system under consideration also has an oscillating behavior.

As the thickness of the layer grows, the number of half-waves which are present along the thickness of the layer increases. This leads to an increase in the number of oscillations of the reflection coefficient. For this reason, in the case of very thick layers, the quantities  $\mathcal{R}_\alpha(k_\perp, \omega)$  and  $I_\alpha^u(\omega, \theta, T_1, T)$  turn out to be fast-oscillating functions of the angle  $\theta$ . Since measurement equipment does not record the exact value of thermal radiation flux at a specified value of the angle  $\theta$  but a certain value of the flux averaged over some interval of angles  $\Delta\theta$  in the vicinity of the angle  $\theta$ , in the case where the thermal intensity has many oscillations within the interval  $\Delta\theta$ , we should average the quantity  $\mathcal{R}_\alpha(k_\perp, \omega)$  over the interval of angles  $\Delta\theta$ . In the limit of an infinitely thick transparent layer ( $L \rightarrow \infty$ ), the result of averaging at  $k_\perp < \min(k_1, k_2)$  is the following:

$$\lim_{L \rightarrow \infty} \mathcal{R}_\alpha(k_\perp, \omega) = \frac{\Gamma_\alpha^{(1,2)}(k_\perp, \omega) + \Gamma_\alpha^{(2,3)}(k_\perp, \omega) - 2\Gamma_\alpha^{(1,2)}(k_\perp, \omega)\Gamma_\alpha^{(2,3)}(k_\perp, \omega)}{\Gamma_\alpha^{(1,2)}(k_\perp, \omega) + \Gamma_\alpha^{(2,3)}(k_\perp, \omega) - \Gamma_\alpha^{(1,2)}(k_\perp, \omega)\Gamma_\alpha^{(2,3)}(k_\perp, \omega)}. \quad (56)$$

In a particular case of the same media adjacent to the layer [in this case, all media are transparent and  $\mathcal{R}_\alpha^{(1,2)}(k_\perp, \omega) = \mathcal{R}_\alpha^{(2,3)}(k_\perp, \omega) \equiv \mathcal{R}_\alpha^{(1,2)}(\theta)$ ], Eq. (56) for the energy reflection coefficient of an electromagnetic wave from an infinitely thick transparent layer is reduced to the form

$$\lim_{L \rightarrow \infty} \mathcal{R}_\alpha(k_\perp, \omega) = \frac{2\mathcal{R}_\alpha^{(1,2)}(\theta)}{1 + \mathcal{R}_\alpha^{(1,2)}(\theta)}. \quad (57)$$

Since  $\mathcal{R}_\alpha^{(1,2)}(\theta) \leq 1$ , by virtue of Eq. (57), an account of the infinitely distant second boundary leads to a natural increase in the energy reflection coefficient of electromagnetic waves as compared with the energy reflection coefficient of electromagnetic waves from a semibounded medium  $\mathcal{R}_\alpha^{(1,2)}(\theta)$ . The ratio of the energy reflection coefficient of electromagnetic waves from an infinitely thick transparent layer to that in the case of a semibounded system does not exceed 2 and the specific value of the ratio is defined by the angle of incidence and the ratio of dielectric permittivities of the adjacent media.

As an illustration, in Figs. 2 and 3, the oscillating behavior of the angular dependence of the energy reflection coefficient  $\mathcal{R}_\alpha$  of the electromagnetic wave from the transparent dielectric layer incident on it from a vacuum ( $\varepsilon_1 = 1$ ) (curves 2) is shown for the simplest case corresponding to a transparent dielectric base ( $\varepsilon_3$  is the dielectric permittivity of the base). Here we also present the angular dependences of the energy reflection coefficients from the semibounded dielectric medium  $\mathcal{R}_\alpha^{(1,3)}(\theta)$  (curves 1) and from the infinitely

thick layer (dashed curves) calculated by Eq. (56). For the given thickness of the layer  $L/\lambda_0 = 10$ , for both polarizations, there exist eight [for  $\varepsilon_2 < \varepsilon_3$  (Fig. 2,  $N = 28$  and  $M = 21$ )] or three [for  $\varepsilon_2 > \varepsilon_3$  (Fig. 3,  $N = 63$  and  $M = 61$ )] enlightening angles of the layer at which the reflection coefficients from the layer reach their local extremal values equal to the values of the reflection coefficient from the base without a coating. These figures visually illustrate well-known theoretical conclusions [27] about the influence of the dielectric coating on the reflection coefficients of electromagnetic waves in the case of a transparent base at  $\varepsilon_1 = 1$ ; that is, for any angle of incidence a one-layer coating may only decrease at  $\varepsilon_2 < \varepsilon_3$  [Fig. 2(a)], or increase at  $\varepsilon_2 > \varepsilon_3$  [Fig. 3(a)], the reflection coefficient of  $s$ -polarized waves, whereas in the case of  $p$ -polarized waves [Figs. 2(b) and 3(b)] the same influence of the coating takes place only for angles less than the Brewster angle  $\theta_B$ . In the interval of angles  $\theta_B^{(1,2)} < \theta < \pi/2$ , the presence of the coating gives the opposite effect, that is, it leads to an increase at  $\varepsilon_2 < \varepsilon_3$  [Fig. 2(b)] or a decrease at  $\varepsilon_2 > \varepsilon_3$  [Fig. 3(b)] in the reflection coefficient of  $p$ -polarized waves, while if the angle of incidence is equal to the Brewster angle, the energy reflection coefficient of  $p$ -polarized waves is independent of the thickness of the layer. The last case corresponds to the intersection of a dashed curve with curves 1 and 2 at a point  $\theta = \theta_B^{(1,2)}$  [ $\theta_B^{(1,2)} \approx 54.7^\circ$  in Fig. 2(b), and  $\theta_B^{(1,2)} \approx 72.5^\circ$  in Fig. 3(b)]. Note that in the case of a transparent base, if the condition  $\varepsilon_2\varepsilon_3/(\varepsilon_1(\varepsilon_2 + \varepsilon_3)) < 1$  is valid for  $p$ -polarized waves, there exists one more enlightening angle. This angle is independent of the thickness of the layer, and equal to the Brewster angle [27]  $\theta_B^{(2,3)} = \arcsin[\varepsilon_2\varepsilon_3/(\varepsilon_1(\varepsilon_2 + \varepsilon_3))]^{1/2}$ , which is



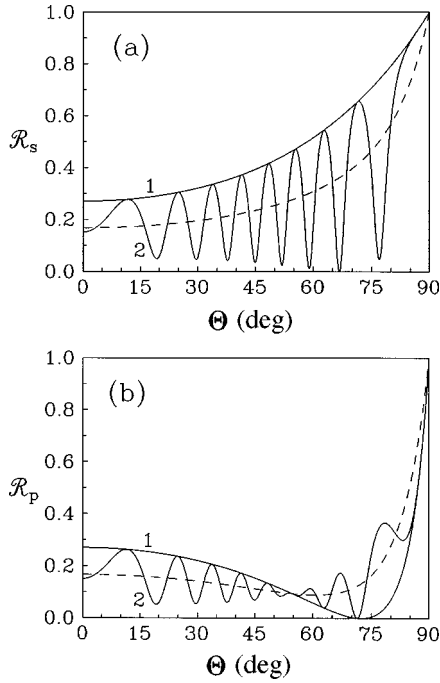


FIG. 2. Angular distributions of the energy reflection coefficients of plane  $s$ - (a) and  $p$ -polarized (b) electromagnetic waves from the dielectric layer at  $L=0$  (1),  $L=10\lambda_0$  (2), and  $L=\infty$  (dashed curves);  $\varepsilon_1=1$ ,  $\varepsilon_2=2$ , and  $\varepsilon_3=10$ .

greater (at  $\varepsilon_3 > \varepsilon_1$ ) or smaller (at  $\varepsilon_3 < \varepsilon_1$ ) than  $\theta_B^{(1,2)}$ . However, for the values of the dielectric permittivities given in the present work, this case cannot be realized because  $\varepsilon_2\varepsilon_3/(\varepsilon_1(\varepsilon_2 + \varepsilon_3)) > 1$ .

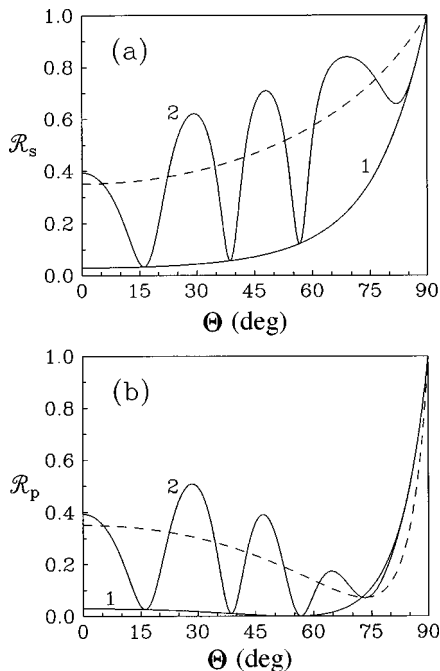


FIG. 3. Angular distributions of the energy reflection coefficients of plane  $s$ - (a) and  $p$ -polarized (b) electromagnetic waves from the dielectric layer at  $L=0$  (1),  $L=10\lambda_0$ , and  $L=\infty$  (dashed curves);  $\varepsilon_1=1$ ,  $\varepsilon_2=10$ , and  $\varepsilon_3=2$ .

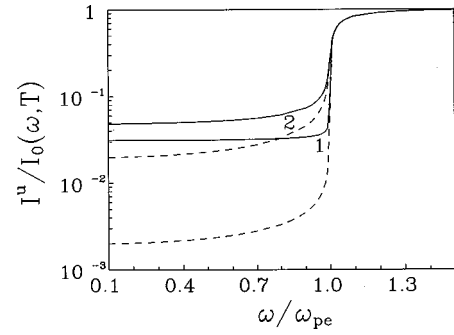


FIG. 4. Frequency spectrum of the normalized thermal radiation intensity of the one-sided energy flux at  $\nu_e/\omega_{pe}=10^{-3}$  (1) and  $\nu_e/\omega_{pe}=10^{-2}$  (2);  $\theta=0$ .

### III. NUMERICAL ANALYSIS OF THERMAL RADIATION SPECTRA OF A SEMIBOUNDED PLASMA WITH A TRANSPARENT COATING

We can study the influence of the temperature, the dielectric properties of the media, the plasma parameters, and the thickness of the transparent layer on angular and frequency distributions of the thermal radiation intensity of the one-sided energy flux, on the basis of the numerical analysis, which can be carried out by using the general relations (27)–(30) or (32) and (34)–(36) for nonisothermal and isothermal plasma, respectively. For the sake of simplicity, in the present work, we restrict ourselves below, mainly, to studying the thermal radiation spectra of a gaseous plasma (except for Fig. 9), without regard to the spatial dispersion of the plasma [the dielectric properties of the plasma is described by the well-known relation (46) at  $\varepsilon_0=1$ , which corresponds to the cold plasma approximation [10]] that is separated by a homogeneous dielectric layer from the external medium. Since, as shown above, the presence of a transparent dielectric coating is reduced to a simple change in the Fresnel reflection coefficients in the radiation spectra due to multiple reflections of waves radiated by the plasma and the external medium, for the radiation spectra we use a more simple relation [Eq. (52)] given at  $\text{Im } \varepsilon_2=0$ .

We consider the thermal radiation of a plasma with the parameters  $T=10^4$  K,  $n_{0e}=10^{12}$  cm<sup>3</sup>, and  $\nu_e/\omega_{pe}=10^{-3}$  [except for Fig. 4(a), where the frequency spectra of the radiation intensity are given for various ratios  $\nu_e/\omega_{pe}$ ], which are typical of a gas-discharge plasma of low pressure [28] into the external medium ( $\varepsilon_1=1$ ) which is kept at room temperature  $T_1=20^\circ\text{C}$ . The significant difference between the temperatures of the plasma and the external medium allows us to illustrate clearly the necessity of an account of the proper thermal field of the external media even under the condition  $T_1 \ll T$ . The solid curves in Figs. 4–8 correspond to the thermal radiation intensity  $I^u$  of the one-sided energy flux that involves the radiation of both the plasma and the external medium. The radiation intensity  $I^u$  is normalized to the radiation intensity of a blackbody  $I_0(\omega, T)$  (in the case of radiation along the normal, Figs. 4 and 7) with the temperature of the plasma or to the radiation intensity of a blackbody for a single polarization  $I_0(\omega, T)/2$  (Figs. 5, 6, and 8) if the radiation is recorded separately for each of the polarizations. For comparison, in the figures we give the normalized distributions of the radiation intensity in the case of the cold ex-

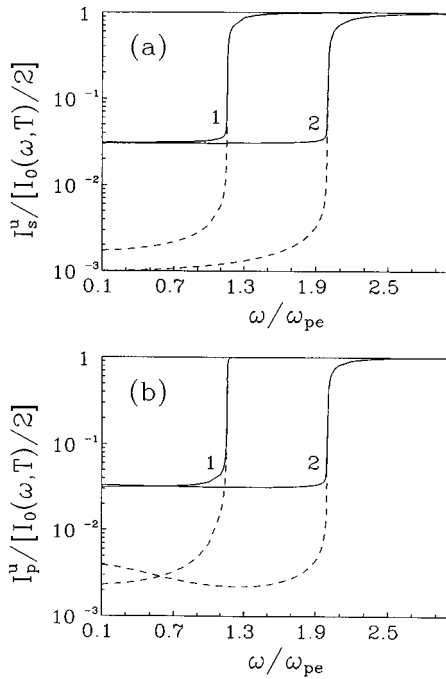


FIG. 5. Frequency spectrum of the normalized thermal radiation intensity of the one-sided *s*- (a) and *p*-polarized (b) energy flux at  $\theta = \pi/6$  (1) and  $\theta = \pi/3$  (2).

ternal medium ( $T_1 = 0$  K, dashed curves), which describe the plasma thermal radiation and, according to the Kirchhoff law, coincide with the absorption coefficient  $\Gamma_\alpha^{(3)}$  of a plane  $\alpha$ -polarized wave by a plasma half-space without (Figs. 4–6) and with (Figs. 7 and 8) a coating incident upon the system under investigation from the external medium at an angle equal to the angle of radiation. A study of the absorption coefficients of electromagnetic waves by plasma and plasma-

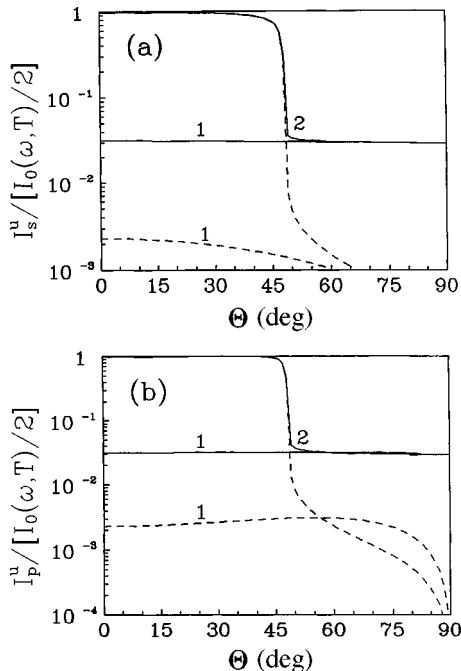


FIG. 6. Angular distribution of the normalized thermal radiation intensity of the one-sided *s*- (a) and *p*-polarized (b) energy flux at  $\omega / \omega_{pe} = 0.5$  (1) and  $\omega / \omega_{pe} = 1.5$  (2).

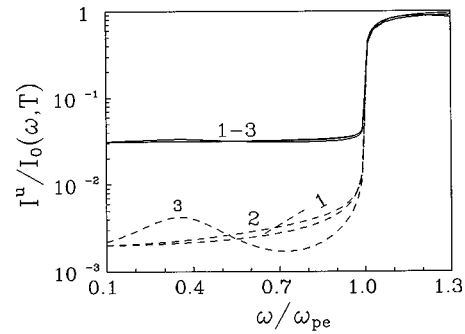


FIG. 7. Frequency distributions of the normalized thermal radiation intensity of the one-sided energy flux in the case of the presence of a transparent dielectric coating for  $L=0$  (1),  $L=0.1$  cm (2), and  $L=1$  cm (3);  $\theta=0$ .

like systems is of interest in its own right due to its numerous applications. In the case of a semibounded plasma, the frequency and angular distributions of the absorption coefficients were studied in detail (see, for example, Refs. [29–34] and references cited therein). Similar investigations, and the numerical analysis of these quantities carried out with regard to the spatial dispersion of the plasma system [14,35–38], showed that the absorption coefficients of electromagnetic waves depended significantly on the mechanism of interaction of charged plasma particles with the boundary surface.

First let us consider, in greater detail, the behavior of frequency (Figs. 4 and 5) and angular (Fig. 6) distributions of the thermal radiation intensities of the one-sided energy flux in the absence of a coating ( $L=0$ ). When radiating along the normal to the boundary (Fig. 4), the normalized radiation intensity is, in fact, constant at frequencies lower than the plasma frequency. For  $\nu_e / \omega_{pe} = 10^{-3}$  (curve 1), a

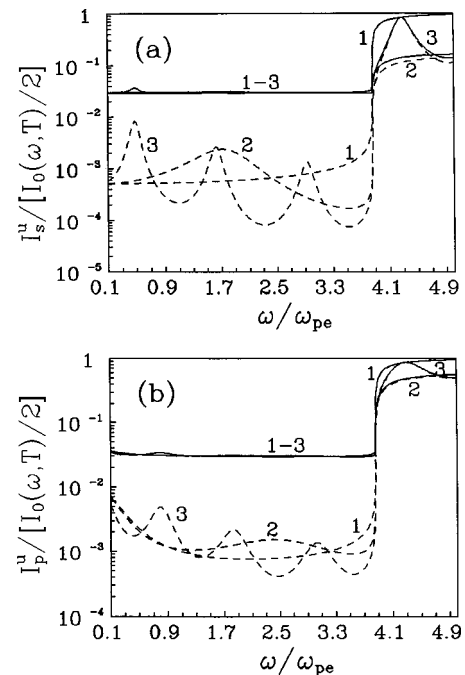


FIG. 8. Frequency distributions of the normalized thermal radiation intensity of the one-sided *s*- (a) and *p*-polarized (b) energy flux in the case of the presence of a transparent dielectric coating for  $L=0$  (1),  $L=0.1$  cm (2), and  $L=1$  cm (3);  $\theta = 5\pi/12$ .

constant level of the normalized radiation intensity is well described by the ratio  $T_1/T \approx 0.029$ , which is in good agreement with the conclusion of the above-mentioned analytical investigations of the dominant role of the proper thermal field of the external medium in one-sided thermal radiation energy flux in the case where estimation (49) is valid. For frequencies  $\omega < 0.8\omega_{pe}$ , the thermal radiation intensity of the plasma (dashed curve 1) accounts for less than 10% of the total thermal radiation intensity of the one-sided energy flux. For this reason, neglect of the proper thermal radiation of the external medium in this frequency range leads to underestimating (more than by an order) the radiation in the direction perpendicular to the boundary of both media. The thermal radiation of the plasma proves to be significant in the one-sided thermal radiation energy flux only near the plasma frequency at  $\omega \leq \omega_{pe}$ , and dominates in it at frequencies  $\omega > \omega_{pe}$ . In the last case, the radiation of the plasma into the external region is produced, in fact, by almost all of its volume, and not only by a thin subsurface plasma layer that is of the order of the depth of field penetration into the plasma as in the case where  $\omega < \omega_{pe}$ . For this reason, at frequencies  $\omega > \omega_{pe}$ , the plasma radiates almost like a blackbody. At  $\omega > \omega_{pe}$ , the proper thermal radiation intensity of the external medium accounts to less than 3% of the total thermal radiation intensity of the one-sided energy flux, and it may be neglected. Therefore, in this frequency range, the thermal radiation intensity is quite well described by the results presented in Ref. [39], which correspond to the thermal radiation intensity of a semibounded plasma into the external cold medium.

As the collision frequency increases, the dissipation of the plasma system grows. The last process is accompanied by an increase in the plasma radiation. However, at  $\nu_e/\omega_{pe} = 10^{-2}$ , at frequencies  $\omega < 0.7\omega_{pe}$ , the thermal radiation intensity of the plasma (dashed curve 2 in Fig. 4) does not exceed the radiation intensity of the external medium. Therefore, in the frequency range  $\omega \leq \omega_{pe}$ , the thermal radiation intensity of the one-sided energy flux is defined by both components (as opposed to the case where  $\nu_e/\omega_{pe} = 10^{-3}$ , the plasma radiation cannot be neglected).

In studies of spectral thermal plasma distributions, the problem of the necessity of an account of the spatial dispersion of the plasma is of special interest. We can make some quantitative conclusions about the influence of the spatial dispersion of the plasma on the thermal radiation intensity spectra of the one-sided energy flux without recourse to numerical calculations by using the results of Ref. [38], according to which, at  $\nu/\omega_{pe} \geq 10^{-2}$ , the collision mechanism of generation of waves in a plasma is dominant. This fact, together with the numerical results obtained in the present work, allows us to conclude that, at least at plasma temperatures lower than  $10^4$  K, an account of the spatial dispersion of the plasma has a slight influence on the thermal radiation intensity of the one-sided energy flux over all frequency ranges: for small values of the dissipation, when the spatial dispersion is significant, the level of the thermal radiation of the plasma is much lower than the level of the proper thermal radiation of the external medium; however, for values of the dissipation when the radiation intensity of the plasma and the external medium have the same order, we can ignore the

collisionless mechanism of generation of radiation in the plasma.

For the nonzero recording angle of radiation  $\theta$ , at frequencies lower than the cutoff frequency  $\omega_{cut} = \omega_{pe}/\cos\theta$ , the behavior of the thermal radiation intensity of the plasma (dashed curves) is different for  $s$  [Fig. 5(a)] and  $p$  polarizations [Fig. 5(b)]; that is, as the angle  $\theta$  grows, the thermal radiation intensity of the plasma decreases for  $s$ -polarized waves and increases for  $p$ -polarized waves. However, at  $\omega < \omega_{cut}$ , even the  $p$ -polarized thermal radiation intensity of the plasma accounts for not more than 12% of the total radiation intensity of the one-sided flux. As a result, the  $s$ - and  $p$ -polarized thermal radiation intensities of the one-sided energy flux differ only near the cutoff frequency: the more distinct difference between the intensities is observed for smaller values of the angle of radiation [curves 1 in Figs. 5(a) and 5(b)]. Outside this frequency range, the normalized thermal radiation intensity, just as at  $\theta=0$ , is close either to  $T_1/T$  (for  $\omega < \omega_{cut}$ ) or to unity (for  $\omega > \omega_{cut}$ ). This behavior of the thermal radiation intensity of the one-sided energy flux for both polarizations allows us to extend all conclusions mentioned above at  $\theta=0$  concerning the dominant contribution of the plasma or the external medium to the thermal radiation intensity of the one-sided energy flux, as well as allowing the possibility to neglect the spatial dispersion of the plasma in the case of arbitrary angles of radiation.

Angular distributions of the normalized thermal radiation intensity of the one-sided energy flux presented in Fig. 6 show that, at frequencies lower than the plasma frequency, the distributions coincide with each other for both polarizations, and are independent of the angle of radiation (lines 1 which are parallel to the abscissa axis). At  $\omega > \omega_{pe}$  (curves 2), the difference between the polarizations is observed only in the vicinity of the cutoff angle  $\theta_{cut} = \arcsin(1 - \omega^2/\omega_{pe}^2)^{1/2}$  (for the given frequency  $\omega = 1.5\omega_{pe}$ ,  $\theta_{cut} \approx 48^\circ$ ). Except for the vicinity of the angle  $\theta_{cut}$ , the angular distributions of the normalized thermal radiation intensity of the one-sided energy flux have approximately the form of a double step, which takes the values 1 (plasma radiates like a blackbody) at  $\theta < \theta_{cut}$  or  $T_1/T$  (electromagnetic waves, which are incident on the boundary from the plasma, almost totally reflect) at  $\theta > \theta_{cut}$ .

As the plasma temperature increases, the contribution of the proper thermal radiation of the external medium to the one-sided energy flux decreases, on the one hand, and on the other hand, we observe the growth of plasma radiation due to the Cherenkov mechanism of excitation of transverse (in the range of the anomalous skin-effect) and longitudinal (in the frequency range  $0.1\omega_{pe} < \omega < \omega_{cut}$ ) electromagnetic fields. At temperatures of the order of  $T = 10^5$  K, even the thermal intensity radiation of a cold plasma is comparable with the ratio  $T_1/T$ . This means that for such a system, we must take into account contributions of radiations of both the plasma and the external medium to the one-sided energy flux. For such temperatures, and higher temperatures, we must also take the spatial dispersion of the plasma into account. It is worth noting that, in this case, the thermal radiation spectra of a plasma significantly depend on the nature of interaction of charged plasma particles with the boundary surface [14], that is, the radiation intensities of a plasma with the diffusive and specular boundaries can differ more than by an order.

The curves in Figs. 7 and 8 illustrate the influence of a transparent dielectric coating on the frequency distributions of the thermal radiation intensity for various recording angles. As a material of the coating, we take a dielectric with the dielectric permittivity  $\varepsilon_2 = 1.53^2$ , which is typical of a glass in the optical range. In the frequency range below the cutoff frequency ( $\omega_{\text{cut}} = \omega_{pe}$  at  $\theta = 0$  in Fig. 7, and  $\omega_{\text{cut}} = 3.86\omega_{pe}$  at  $\theta = 5\pi/12$  in Fig. 8), the presence of the coating has a noticeable effect only on the level of the plasma radiation. As the coating thickness grows, the difference between the radiation intensities of plasma with (dashed curves 2 and 3) and without (dashed curves 1) a coating increases. For small values of the collision frequency  $\nu_e$ , we can show that at frequencies

$$\omega_n = n \frac{\pi c}{\varepsilon_2^{1/2} L \cos \theta_d} \equiv n \omega_1, \quad (58)$$

where  $n$  are positive integers and  $\theta_d$  is the angle of propagation of an electromagnetic wave in a layer defined by Snell's law  $\varepsilon_2^{1/2} \sin \theta_d = \varepsilon_1^{1/2} \sin \theta$ , the radiation intensities of the plasma with and without a coating coincide. In what follows, for the sake of brevity, frequencies at which the presence of a dielectric coating has no effect on the radiation spectra of the plasma are called "enlightening" frequencies for the layer. The frequencies  $\omega_n$  correspond to the well-known enlightening condition [27,40] (54), according to which  $L = n\lambda_2 / (2 \cos \theta_d)$ . Taking into account that the normalized radiation intensity of the plasma is equal to the absorption coefficient of the electromagnetic wave by the plasma system (in Figs. 4–8, these quantities are shown by dashed curves), for the sake of convenience of comparison with the known results [27,40] on propagation of electromagnetic waves, in what follows we discuss the absorption coefficient of electromagnetic waves by the plasma instead of the thermal radiation intensity of the plasma system. It follows from Eq. (58) that at fixed values of the radiation angle and the thickness of the layer, in the frequency range  $\omega_{\min} \leq \omega \leq \omega_{\max}$ , there exist  $N = N_{\max} - N_{\min}$  enlightening frequencies, where  $N_\gamma = [2L \cos \theta_d / \lambda_2^{(\gamma)}]$ ,  $\lambda_2^{(\gamma)} = 2\pi c / (\varepsilon_2^{1/2} \omega_\gamma)$ , and  $\gamma = \min, \max$ . According to calculations carried out for the given thicknesses of the layer by Eq. (58), below the cutoff frequency we have no frequencies  $\omega_n$  at  $\theta = 0$ , whereas, at  $\theta = 5\pi/12$ , there are two enlightening frequencies ( $\omega_1 \approx 1.42\omega_{pe}$  and  $\omega_2 \approx 2.84\omega_{pe}$ ) only for a layer 1 cm thick, and no frequency  $\omega_n$  at  $L = 0.1$  cm.

However, as seen from Figs. 7 and 8, these conclusions do not agree with the numerical results, because the number of enlightening frequencies in these figures is greater than that defined by Eq. (58), the effect of layer enlightening being observed even in cases where there is no enlightening frequency  $\omega_n$ . For example, dashed curves 1 and 3 in Fig. 7, which correspond, respectively, to a semibounded plasma, without and with a coating 1 cm thick, intersect at the point  $\omega \approx 0.54\omega_{pe}$ , whereas  $\omega_1 \approx 1.1\omega_{pe} > \omega_{\text{cut}} = \omega_{pe}$ . For a layer of thickness  $L = 0.1$  cm, the effect of layer enlightening occurs also at  $\theta = 5\pi/12$  (dashed curve 2 in Fig. 8), though in this case  $\omega_1 > \omega_{\text{cut}}$ .

It is also worth noting that, as opposed to a transparent dielectric base ( $\text{Im } \varepsilon_3 = 0$  and  $\text{Re } \varepsilon_3 > 0$ ), in the case of the

plasma system the reflection coefficient  $\mathcal{R}_\alpha$  at the frequency  $\omega = \omega_n$  does not reach a local extremum equal to the value of the reflection coefficient  $\mathcal{R}_\alpha^{(1,3)}$  of the electromagnetic wave by a semibounded plasma without a coating. Both, these factors (oscillations of the absorption coefficient of the plasma with a coating about the absorption coefficient of the semibounded plasma, and additional enlightening frequencies  $\tilde{\omega}_n$  for a layer) are caused by the condition  $\text{Re } \varepsilon_3 < 0$ , and by dissipation of the plasma system. Unlike the frequencies  $\omega_n$ , at which both amplitudes and phases of the Fresnel reflection coefficients of electromagnetic waves from a semibounded medium with and without a transparent dielectric coating coincide, at frequencies  $\tilde{\omega}_n$  only the equality of amplitudes of these quantities is valid.

Let us estimate the frequencies  $\tilde{\omega}_n$  at  $\text{Im } \varepsilon_3(\omega) / |\text{Re } \varepsilon_3(\omega)| \ll 1$  by restricting ourselves by the simplest case of  $\theta = 0$  and the condition  $\tilde{\omega}_n \ll \omega_{pe}$ . In the frequency range  $\nu_e \ll \omega \ll \omega_{pe}$ , where the dissipation of the plasma system can be neglected, possible enlightening frequencies  $\tilde{\omega}_n$  are less than the corresponding frequencies  $\omega_n$  and, at  $\lambda_{pe}/L < 1$ , they can be defined as follows:

$$\tilde{\omega}_n \approx \omega_n \left( 1 - \frac{\lambda_{pe}}{\pi L} \right), \quad (59)$$

where  $\lambda_{pe} = 2\pi c / \omega_{pe}$ . Note that at  $\theta \neq 0$ , the enlightening frequencies  $\tilde{\omega}_n$  depend on the polarization of an electromagnetic wave, whereas the enlightening frequencies  $\omega_n$  have the same value for both polarizations. In addition, the number of enlightening frequencies  $\tilde{\omega}_n$  for  $p$  and  $s$  polarizations is different. As seen from Figs. 8(a) and 8(b), in the frequency range below the cutoff frequency where the real part of the dielectric permittivity of the base is negative, for a layer 0.1 cm thick (dashed curves 2 in Fig. 8), there are two enlightening frequencies  $\tilde{\omega}_n$  for the  $p$  polarization, and only one  $\tilde{\omega}_n$  for the  $s$  polarization. At  $L = 1$  cm (dashed curves 3), in addition to two enlightening frequencies  $\omega_n$  ( $\omega_1 = 1.42\omega_{pe}$  and  $\omega_2 = 2.84\omega_{pe}$ ), which are independent of polarization, there are, respectively, four and three enlightening frequencies  $\tilde{\omega}_n$  for  $p$  and  $s$  polarizations.

At frequencies  $\omega > \omega_{\text{cut}}$ , as in the case of a plasma without a coating, the one-sided thermal radiation energy flux is defined, mainly, by the radiation of the plasma (in Fig. 7, the thermal radiation intensities of the one-sided energy flux and the plasma are, in fact, indistinguishable).

According to a general conclusion [27,40], in the case of transparent media the presence of a coating leads to oscillations of the absorption coefficient  $\Gamma_\alpha$  of an electromagnetic wave by the base which, at  $\varepsilon_2 > \varepsilon_3$ , are below the absorption coefficient  $\Gamma_\alpha^{(1,3)}$  of a semibounded medium without a coating, and which reach its level only at  $\omega = \omega_n$ . In the frequency range  $\omega > \omega_{\text{cut}}$ , this conclusion is clearly confirmed by curves 1 and 3 in Fig. 8, corresponding, respectively, to the absorption coefficient of a semibounded plasma, with and without a transparent dielectric coating. However, a more detailed numerical analysis shows that at the frequency  $\omega = \omega_n$ , the absorption coefficient does not reach its local maximum value, and that there exists a very narrow fre-

frequency range  $\omega_n < \omega < \tilde{\omega}_n$  where a plasma with a coating absorbs electromagnetic waves greater than the plasma without a coating; however at  $\omega = \omega_n$  and  $\omega = \tilde{\omega}_n$ , the presence of a dielectric coating has no effect on the absorption coefficient. As in the case  $\omega < \omega_{\text{cut}}$ , at  $\omega = \omega_n$ , there is a coincidence of both amplitudes and phases of the Fresnel reflection coefficients of electromagnetic waves of a plasma with and without a coating; however at  $\omega = \tilde{\omega}_n$ , only the amplitudes coincide. While at  $\omega < \omega_{\text{cut}}$ , in the case of a small dissipation of a plasma, the appearance of additional enlightening frequencies  $\tilde{\omega}_n$  for a layer is caused, mainly, due to the condition  $\text{Re } \varepsilon_3 < 0$ , in the frequency range  $\omega > \omega_{\text{cut}}$ ,  $\text{Re } \varepsilon_3 > 0$  and frequencies  $\tilde{\omega}_n$  appear only provided that the dissipation of the plasma system is not equal to zero.

For  $\theta = 0$ , the frequencies  $\tilde{\omega}_n$  with  $n > (2L/\lambda_{pe})^2(\varepsilon_2/\varepsilon_0)$  can be approximated in the following way:

$$\tilde{\omega}_n = \omega_n \left[ 1 + \frac{\nu_e}{\omega_{pe}} \frac{8\varepsilon_2^2}{\pi\varepsilon_0 n^4} \left( \frac{L}{\lambda_{pe}} \right)^3 \frac{1}{\varepsilon_2 - \varepsilon_0 + \left( \frac{2L}{\lambda_{pe} n} \right)^2 \varepsilon_2} \right]. \quad (60)$$

As follows from Eq. (60), the additional enlightening frequencies  $\tilde{\omega}_n$  shift to the right of the corresponding frequencies  $\omega_n$  ( $\tilde{\omega}_n > \omega_n$ ), whereas, at  $\omega < \omega_{\text{cut}}$ , by virtue of Eq. (59), there must be another sequence for the enlightening frequencies, namely,  $\tilde{\omega}_n < \omega_n$ . The detuning between the frequencies  $\tilde{\omega}_n$  and  $\omega_n$  ( $\Delta_n = \tilde{\omega}_n - \omega_n$ ) decreases rapidly with an increase in the frequency of the incident wave and a decrease in the effective collision frequency  $\nu_e$ .

The inequality  $\tilde{\omega}_n > \omega_n$  also remains valid at  $\theta \neq 0$ . Usually, the detuning  $\Delta_n$  is very small, and for the given parameters we cannot see it (curve 3 in Fig. 8) even if the scale of the abscissa axis is increased by a factor of 1000. Numerical analysis confirms the conclusion about the growth of the detuning  $\Delta_n$  with an increase in the ratio  $\nu_e/\omega_{pe}$ .

An account of the frequency dispersion of the base also leads to a change in the behavior of the angular dependence of the energy reflection coefficient  $\mathcal{R}_\alpha$  of the electromagnetic wave and, consequently, the thermal radiation intensity of the third medium as compared with those quantities which correspond to the transparent base (Figs. 2 and 3). In the frequency range  $\omega < \omega_{\text{cut}}$ , the dissipation of the system disturbs the conditions  $\mathcal{R}_\alpha \geq \mathcal{R}_\alpha^{(1,3)}$  (for any angle  $\theta$  for  $s$  polarization and for  $\theta < \theta_B^{(1,2)}$  for  $p$  polarization) and  $\mathcal{R}_p \leq \mathcal{R}_p^{(1,3)}$  (at  $\theta_B^{(1,2)} < \theta < \pi/2$ ), which are valid if the dielectric permittivity of the base is less than that for the layer [see Figs. 3(a) and 3(b)]. As a result, at the enlightening angles  $\theta_n$ , the energy reflection coefficient  $\mathcal{R}_\alpha(\theta_n) = \mathcal{R}_\alpha^{(1,3)}(\theta_n)$  is not extremal, and there appear additional enlightening angles  $\tilde{\theta}_n$  which depend on the polarization of the incident wave. As oppose to enlightening angles  $\theta_n$ , there takes place only an equality for amplitudes of the reflection coefficients of electromagnetic waves from the layer and the base, whereas the phase of these coefficients are different.

The angular dependences of the energy reflection coefficients of electromagnetic waves incident from a vacuum

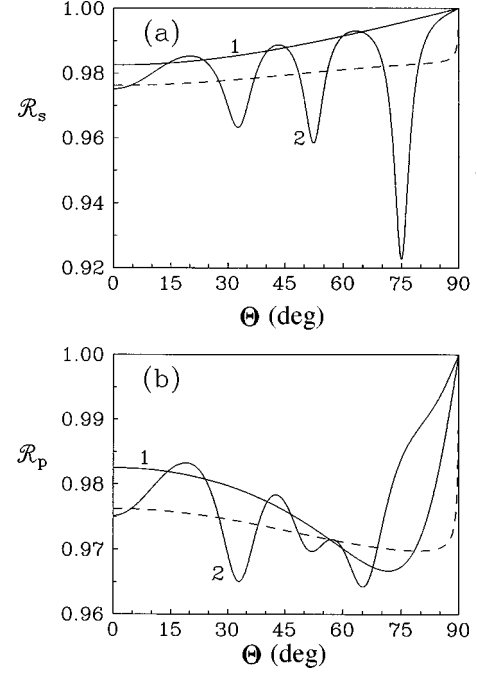


FIG. 9. Angular distributions of the energy reflection coefficients of plane  $s$ - (a) and  $p$ -polarized (b) electromagnetic waves from the dielectric layer applied to the metallic base for  $L=0$  (1),  $L=2 \mu\text{m}$  (2), and  $L=\infty$  (dashed curves).

upon a dielectric layer applied to a metallic base  $\mathcal{R}_\alpha$  (curves 2) and a semibounded metal  $\mathcal{R}_\alpha^{(1,3)}$  (curves 1) given in Fig. 9 can serve as illustrations of the above-mentioned results. The dependences are calculated for  $\lambda_0 = 0.5 \mu\text{m}$ ,  $L = 2 \mu\text{m}$ , and  $\varepsilon_2 = 1.52^2$  in the case of a silver base with the parameters [41]  $\hbar\omega_{pe} = 9.33 \text{ eV}$  and  $\hbar\nu_e = 0.058 \text{ eV}$ , and the typical value of the dielectric permittivity of the background  $\varepsilon_0 = 3.6$  in the optical frequency range. The condition  $\text{Re } \varepsilon_0 \neq 1$  should be taken into account because, for example, for silver, the frequency  $\omega_{pe}$  predicted by the theory of free electrons at  $\varepsilon = 1$  is significantly greater than the experimentally obtained value. A shift of the frequency at which the condition  $\text{Re } \varepsilon_3(\omega) = 0$  holds to a low-frequency range is caused by the contribution of bound electrons in  $\text{Re } \varepsilon_3(\omega)$ . It should be noted that bound electrons also affect the imaginary part of the dielectric permittivity of a metal. In this case, the quantity  $\text{Im } \varepsilon_0(\omega)$  depends significantly on the frequency, which is in agreement with the experimental data [42] obtained for noble metals. In the present work, we neglect the dissipation of the background. As in Figs. 2 and 3, the dashed curve in Fig. 9 corresponds to the energy reflection coefficients from an infinitely thick transparent dielectric layer applied to the base calculated by Eq. (56). For the given parameters, in addition to three enlightening angles ( $\theta_1 \approx 56.7^\circ$ ,  $\theta_2 \approx 40.4^\circ$ , and  $\theta_3 \approx 14.2^\circ$ ), which are the same for both polarizations, and the Brewster angle ( $\theta_B^{(1,3)} \approx 56.7^\circ$ ) at which  $\mathcal{R}_p(\theta_B^{(1,2)}) = \mathcal{R}_p^{(1,3)}(\theta_B^{(1,2)})$ , there are also three additional enlightening angles  $\tilde{\theta}_n$  whose values are different for  $p$  and  $s$  polarizations.

#### IV. CONCLUSIONS

In the present work, we found the thermal radiation spectra of a nonisothermal homogeneous semibounded plasma

into an external medium, taking into account the proper thermal field of the external medium and the homogeneous transition layer between the plasma and external media. The heat transfer intensity due to radiation between the inhomogeneous system and the external medium is presented as the sum every term of which describes heat transfer between the external medium and the corresponding medium, which is a part of the inhomogeneous system under investigation. We found a one-sided thermal radiation energy flux in the direction away from the inhomogeneous system, the intensity of the flux being presented as the generalization of the classical Kirchhoff law in the Levin-Rytov form to the case which takes into account the different temperatures of the media which make up the radiating system, nonisothermality of the plasma, and the proper thermal field of the external medium. The representation of the thermal radiation intensity, in the form where the contribution of the proper thermal radiation of the external medium is given as an isolated term, may be useful for identifying the radiation spectra of heated bodies by using the experimental data.

On the basis of analytical estimations and numerical calculations carried out in the work, we found conditions when

the proper thermal field of the external medium makes a significant (and even predominant) contribution to the thermal radiation intensity spectra of the one-side energy flux. We studied in detail the influence of the transparent dielectric coating on the radiation spectra. The general relations (31)–(34) for the thermal radiation intensity of two- and one-sided energy fluxes can be used for studying the thermal radiation spectra of not only a gaseous plasma, for which a numerical analysis was carried out in this work, but of other media (for example, well conducting metals) as well. Since the temperature of these media is, as a rule, significantly lower than the plasma temperature, an account of the proper thermal field of the external medium may be no less important than in the case of a gaseous plasma.

It is of interest to generalize the results of this work to the case of an inhomogeneous transition layer. In particular, by direct calculations, one should be able to prove the validity of the assumption of representation [Eq. (38)] for the thermal radiation intensity of the one-sided energy flux in the form of the generalized Kirchhoff law, in the case of a simulation of the transition layer by an arbitrary number of homogeneous (or inhomogeneous) layers with different temperatures.

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