Dissipative drift waves in partially ionized plasmas containing high-Z impurities or dust

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The theory of drift waves in plasmas containing highly charged impurities (dust) is developed to include such specific features as attraction of dust grains as well as collisions between dust and neutral particles. Nonlinear equations taking into account variations of drift potential together with plasma electron, ion, and neutral densities, as well as dust charges, are obtained. The range of plasma parameters where the effects introduced by dust can trigger the dissipative drift-wave instability is found. [S1063-651X(98)13007-6]

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I. INTRODUCTION

Plasmas containing highly charged impurities, or dust, are currently the subject of increasing interest [1–12]. It is well established that the presence of dust can considerably change collective properties of a plasma. For typical dusty plasmas, such as those in material processing devices [3] or in space [4], the dust grains are highly negatively charged, mainly by plasma currents [5] collecting large numbers of plasma particles. The dust charging process introduces new physics by modifying plasma dielectric properties [6,7]. The most affected are plasma modes with frequencies on the order of or less than the dust charging frequency. The dust charging therefore appears as an important dissipative process leading to recombination of plasma electrons and ions on the dust grain surfaces. The dust-plasma system is in general an open dissipative system, with a sink and (if stationary) a source of electrons and ions. This character of a dusty plasma strongly affects the development of dissipative instabilities (such as drift wave instability), often leading to their considerable enhancement.

The study of wave propagation in a dusty plasma is important for many applications (e.g., the spread of heat flux to the walls in the scrape-off layers of tokamaks) as well as for an understanding of processes in the Earth's ionosphere, space, and cometary plasmas [8–12]. The presence of a relatively high-density dust component (with densities sometimes orders of magnitudes higher than was previously expected) in the lower ionosphere and the upper atmosphere demands a detailed investigation of the wave processes there. Accounting for the presence of dust in the near-wall regions of tokamaks is emphasized in connection with problems of high power loading in future machines. The accumulation of grains from disruptions can create as much as 1 ton of radioactive dust per year in ITER, and dust can considerably influence the heat fluxes and power absorption in the edge tokamak plasmas [11].

There are also indications that the dust particles in a plasma can be self-contracted in dust clouds. The formation of such dust clouds can be supported by various mechanisms of dust attraction [1]. The attraction forces can affect collective dissipative processes in dusty plasmas, e.g., the development of instabilities. Thus one of the aims of the present paper is to investigate the influence of the dust-dust attraction forces on the development of the drift wave instabilities. This is of special interest for plasma physics of nuclear fusion devices as well as for ionosphere physics; we note here that in the Earth's atmosphere and lower ionosphere the presence of dust is due to man-made pollution as well as other processes such as volcanic activity and meteoroid impacts. Other applications include plasma-assisted material processing, e.g., etching experiments, in the presence of an external magnetic field.

Another effect important for applications is dust–neutral-particle collisions. These collisions can be either direct ion–neutral-particle collisions influencing drift waves [13] or dust–neutral-particle collisions affecting drift wave instabilities in the presence of dust. The second effect has not, to our knowledge, been investigated before. Thus here we aim to study the drift wave instabilities in the presence of the dust–neutral-particle collisions. We note that in the case where the drift frequency is much less than the neutral-particle–dust collision frequency, the dust grains can follow the neutral particles in their motion. Since the drift neutral vortices can have significant negative or positive potential, the dust can be trapped in there because of the neutral-particle–dust collisions. Theoretically, such a trapping is possible since the potential difference V created by plasma particles means a potential well $Z_d V$ for dust particles, where Z_d, which can exceed unity by orders of magnitude, is the dust charge in units of the elementary electron charge. Thus the solitary drift vortices can trap highly charged ($Z_d \sim 10^3 - 10^4$, as in many dusty plasmas) dust grains even when V is relatively small. Therefore it is of great interest to derive an equation for nonlinear vortices, taking into account the effects of dust–neutral-particle collisions.

In the present paper, we derive the generalization of the Hasegawa-Wakatani [14] nonlinear equations for drift waves, taking into account the neutral-particle–dust interactions and dust-dust attraction. Both effects can play an im-

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portant role in the dust-cloud vortices. A detailed investigation of these vortices, including their evolution and trapping of self-contracting (due to the attraction mechanisms) clouds of dust grains, needs a complicated numerical calculation and is a subject for future studies. The most important point is that dust plays a part in the motion of both vortices, namely: (1) in the neutral component (such as usual air) vortices, when the rate of dust–neutral-particle collisions is sufficiently high, and the dust grains follow the vortex rotation; and (2) in the drift vortices (appearing due to the presence of inhomogeneity of plasma density), when the dust charge is sufficiently high, and the grains are trapped in the potential well of the drift wave. Such a combined vortex, as was also discussed before [15], can in principle be responsible for the energy transfer from drift towards air vortices in the lower ionosphere and the upper atmosphere. This effect competes with the heating of the upper atmosphere by drift vortices, and, being a complementary process, needs dust grains to be present in the region of interaction (note that the local heating of the upper atmosphere can also be responsible for air convection, turbulence, and solitary vortices).

The paper is organized as follows. In Sec. II, the forces between dust particles are analyzed; collisions of dust with plasma particles and neutral particles are considered in Sec. III; in Sec. IV, we present the main continuity and momentum equations describing dynamics of the plasma species, including neutral particles and dust; the second-order set of equations for drift waves is derived in Sec. V; the drift wave instabilities are analyzed in Sec. VI; and in Sec. VII we briefly discuss the results and remaining problems.

II. FORCES BETWEEN DUST GRAINS

The main difference between dust grains and other plasma particles is that the charge of the grains (being very large) is not fixed and depends on the surrounding plasma parameters. The presence of another dust particle near a test grain changes the plasma electron and ion distributions and fluxes on its surface, affecting dust charging and therefore changing the dust-dust interactions. The interactions are different, depending on whether the interdust distance is less or more than the plasma Debye screening length. For the latter case, the Coulomb interaction is effectively screened out, and the interactions due to mutual shadowing of plasma fluxes onto the grain surfaces become important [1]. For distances less that the Debye length, only the shadowing of the neutral plasma particle bombardment can compete with electrostatic repulsion forces. The bombardment force is determined by the solid angle of the shadow; and therefore, for distances larger than the size of the dust particles, the force is inversely proportional to the square of the distance between the dust particles (i.e., has the same dependence on the distance as the Coulomb forces). It is also clear that the bombardment force should be of attractive character, increasing proportionally to the surface area of the dust particles, i.e., proportionally to their radius squared. Note that this conclusion is independent of whether the bombardment is due to neutral or charged plasma particles. The presence of the plasma fluxes on a dust particle also changes the shielding of its electrostatic field, since the shadow effect creates additional charge distribution around the particle with the potential proportional to the solid angle of the shadow. This leads to an electrostatic unscreened repulsion inversely proportional to the cube of the interparticle distance (which is the space derivative of the additional potential that is inversely proportional to the square of the distance). At large distances, the repulsion is small compared with the attraction, since the latter force is inversely proportional to the squared interdust distance.

For the non-Debye screening at distances much larger than the Debye radius, the electrons are practically Boltzmann distributed (the relative role of their flux on the dust particle is small), and their density is given by

$$n_e = n_{0e}\left(1 + \frac{e\phi}{T_e}\right), \tag{1}$$

where ϕ is the electrostatic potential around the dust grain and T_e is the electron temperature. For plasma ions, due to the shadowing, the ion density contains the additional contribution, which for distances $r \gg \lambda_D$, where λ_D is the plasma Debye length, is given by

$$n_i = n_{0i}\left(1 - \frac{e\phi_0}{T_i}\right) - n_{0i}\frac{a^2}{4r^2}\left(1 + \frac{2e^2 Z_d}{aT_i}\right), \tag{2}$$

where ϕ_0 is the potential on the surface of the grain, a is the grain radius, $-Z_d e$ is the grain charge, and T_i is the ion temperature. The quasineutrality condition allows us to write the dust repulsion and attraction potentials, which for distances much larger that the Debye length is given by (for simplicity, we assume ions to be single charged, $Z_i = 1$, and the dust size to be much smaller than the ion Debye length, $a \ll \lambda_{Di}$)

$$U_{ra} = \eta_r \frac{Z_d^2 e^2 a}{2r^2} - (\eta_b + \eta_c)\frac{a^2}{\lambda_D^2}\frac{Z_d^2 e^2}{r}. \tag{3}$$

Here, the coefficient η_r is

$$\eta_r = \frac{T_e}{T_e + T_i}\left(1 + \frac{aT_i}{2Z_d e^2}\right) \approx 1. \tag{4}$$

The last approximation is written for $T_i \ll T_e$; note also that $\lambda_D \approx \lambda_{Di} = (T_i/4\pi n_i e^2)^{1/2}$ for $T_i \ll T_e$, n_i is the ion density here. The attraction part of the potential is found by calculating the change in the ion momentum in the process of direct bombardment of the dust particle (subscript b) and the Coulomb scattering by the dust particle (subscript c) [1]. We have

$$\eta_b = \frac{1}{2\sqrt{\pi}}\left(\frac{aT_i}{Z_d e^2}\right)^2 \int_{y_{\min}}^{\infty}\left(1 + \frac{Z_d e^2}{aT_i y^2}\right)^{5/2} y^4 \exp(-y^2)\, dy \tag{5}$$

and

$$\eta_c = \frac{1}{4\sqrt{\pi}}\int_{y_{\min}}^{\infty}\left(1 + \frac{Z_d e^2}{aT_i y^2}\right)\ln L\ \exp(-y^2)\, dy, \tag{6}$$

where

$$y_{\min}=\frac{a}{\lambda_D}\sqrt{\frac{Z_d e^2}{aT_i}},$$

$$L=\left(\frac{\lambda_D^2}{a^2}+\frac{Z_d^2 e^4}{4a^2T_i^2y^4}\right)\left(1+\frac{Z_d e^2}{aT_i}+\frac{Z_d^2e^4}{4a^2T_i^2y^4}\right)^{-1}. \quad (7)$$

For free dust particles, the corresponding part of the linear dielectric permittivity is given by

$$\epsilon^d_{\mathbf{k},\omega}=1-\frac{\omega_{pd}^2}{\omega^2}, \quad (8)$$

where $\omega_{pd}=(4\pi n_d e^2 Z_d^2/m_d)^{1/2}$ is the dust plasma frequency and m_d is the grain mass. Note that the charge Z_d is usually large, and therefore, although the dust plasma frequency is small as compared to electron and ion plasma frequencies, it is not negligible. Taking into account the dust attraction and repulsion, we now obtain

$$\epsilon^d_{\mathbf{k},\omega}=1+\frac{\epsilon^{0d}_{\mathbf{k},\omega}-1}{1+[(\pi/4)\eta_r ka-(\eta_b+\eta_c)(a^2/\lambda_D^2)](\epsilon^{0d}_{\mathbf{k},\omega}-1)}$$

$$=1-\frac{\omega_{pd}^2}{\omega^2-(\pi/4)\eta_r\omega_{pd}^2 ka+(\eta_b+\eta_c)\omega_{pd}^2a^2/\lambda_D^2}, \quad (9)$$

where $\epsilon^{0d}_{k,\omega}$ is the usual kinetic dust plasma response without long range repulsion and attraction effects.

III. COLLISIONS OF DUST WITH PLASMA PARTICLES AND NEUTRALS

In general, the full electron (ion)–dust collision frequency must take into account the Coulomb elastic as well as charging collisions. While the former are typical for any plasma component, the latter are specific only for the macrosize (comparing with the sizes of electrons and ions) dust grain, which can effectively absorb plasma particles on its surface. The presence of the absorbing charging collisions leads to the mentioned openness of the dust-plasma system, where a stationary state can only be maintained by an external ionization source compensating the dust charging loss of plasma particles. Another consequence of the charging process is that the effective collision frequencies entering continuity and momentum equations for plasma species are different.

The standard [16] calculation of the Coulomb elastic electron (ion)–dust collision rate (which enters the Euler equation for the plasma component) gives us (see, e.g., [9,11])

$$\nu^{el}_{e(i)d}=\frac{4\sqrt{2\pi}Z_d^2 n_d e^4\Lambda}{3m_{e(i)}^2\overline{v}^3_{e(i)}}, \quad (10)$$

where n_d is the dust density, $m_{e(i)}$ is the electron (ion) mass, and $\overline{v}_{e(i)}$ is the average velocity of plasma particles. When the equilibrium distribution is Maxwellian (which in general is not necessarily the case in the presence of dust impurities), we have $\overline{v}_{e(i)}=v_{Te(i)}$, where $v_{Te(i)}=[T_{e(i)}/m_{e(i)}]^{1/2}$ is the electron (ion) thermal velocity. The Coulomb logarithm Λ in Eq. (10) is defined by the ratio of the maximum impact parameter, which is of the order of the plasma Debye length λ_D, to the minimum impact factor, which we assume to be the radius a of the dust grains. We have $\Lambda=\ln(\lambda_D/a)$ and assume $a\ll\lambda_D$.

The effective frequency of collection of plasma particles by dust, which appears due to their bombardment of the grain surface, for Maxwellian distributions, is given by

$$\nu^{ch}_e=\nu^{ch}_i 3(1+P)(\tau+\mathcal{Z})\frac{4+\mathcal{Z}}{4\tau+2\mathcal{Z}}=\nu^{ch}_d P\frac{\tau+\mathcal{Z}}{1+\tau+\mathcal{Z}}\,\frac{4+\mathcal{Z}}{\mathcal{Z}}. \quad (11)$$

Here, we introduce the standard dimensionless parameters [6,7]

$$P=\frac{n_d Z_d}{n_{0e}}, \quad \tau=\frac{T_i}{T_e}, \quad \mathcal{Z}=\frac{Z_d e^2}{aT_e}, \quad (12)$$

which satisfy the balance equation of the electron and ion currents on the dust grain surface

$$\exp(-\mathcal{Z})=\left(\frac{m_e}{m_i\tau}\right)^{1/2}(1+P)(\tau+\mathcal{Z}). \quad (13)$$

Furthermore, the dust charging frequency in Eq. (11) is given by

$$\nu^{ch}_d=\frac{\omega_{pi}^2 a}{\sqrt{2\pi}v_{Ti}}(1+\tau+\mathcal{Z}), \quad (14)$$

where $\omega_{pi}^2=(4\pi n_{0i}e^2/m_i)^{1/2}$ is the ion plasma frequency.

Under the same assumption of the thermal particle distributions, the elastic collision frequencies (10) can be written as

$$\nu^{el}_{ed}=\frac{2\nu_{ch}Pe^{\mathcal{Z}}}{3\mathcal{Z}(1+\tau+\mathcal{Z})}(\tau+\mathcal{Z})\Lambda \quad (15)$$

and

$$\nu^{el}_{id}=\frac{2\nu_{ch}P}{3\mathcal{Z}(1+\tau+\mathcal{Z})}\,\frac{\Lambda}{\tau(1+P)}. \quad (16)$$

The dust charging collisions lead to the sink term in the continuity equations for plasma electrons and ions. Assuming that in the zeroth approximation, the loss of the plasma particles on the dust grains is compensated for by external sources, we write the continuity equation in the form

$$\partial_t n_{e(i)}+\nabla\cdot(n_{e(i)}\mathbf{v}_{e(i)})=-\overline{\nu}_{e(i)d}n_{e(i)}+\overline{\nu}_{0e(i)d}n_{0e(i)}, \quad (17)$$

where the term $\overline{\nu}_{0e(i)d}n_{0e(i)}$ corresponds to the action of the external sources (such that in equilibrium the number of plasma particles is maintained constant), and for the thermal equilibrium distributions of plasma particles we have [6,7]

$$\overline{\nu}_{ed}=\overline{\nu}_{id}(1+P)=\nu^{ch}_d\frac{P(\tau+\mathcal{Z})}{\mathcal{Z}(1+\tau+\mathcal{Z})}. \quad (18)$$

Note that in the presence of perturbations of dust density, we also have to take them into account in (17) and (18); see below.

IV. EQUATIONS OF MOTION OF PLASMA PARTICLES, NEUTRAL PARTICLES, AND DUST

For plasma electrons, the starting equation is the Euler equation, neglecting the electron inertia,

$$\mathbf{0}=-v_{Te}^2\nabla n_e+v_{Te}^2 n_e\nabla\phi-\Omega_e n_e\mathbf{v}_e\times\hat{\mathbf{z}}-n_e\nu_{ed}\mathbf{v}_e, \quad (19)$$

where the dimensionless electric field potential $\phi=e\varphi/T_e$, $v_{Te}=(T_e/m_e)^{1/2}$ is the electron thermal velocity, and $\Omega_e=eB_0/m_e c$ is the electron gyrofrequency (the magnetic field $\mathbf{B}_0$ is directed along the $\hat{z}$ axis). In the zeroth approximation, the electron diamagnetic drift velocity is given by

$$\mathbf{v}_{0e}=-\frac{v_{Te}^2}{\Omega_e n_{0e}}\hat{\mathbf{z}}\times\nabla n_{0e}=-\frac{v_s^2}{\Omega_i n_{0e}}\hat{\mathbf{z}}\times\nabla n_{0e}, \quad (20)$$

where $\Omega_i=eB_0/m_i c$ is the ion cyclotron frequency and $v_s=(T_e/m_i)^{1/2}$ is the ion sound velocity. We assume that $\hat{x}$ is the direction of the density gradient, and the drift waves propagate in the direction $\hat{y}$. The characteristic length of the electron density inhomogeneity is given by

$$L_n^{-1}=\frac{1}{n_{0e}}\frac{\partial n_{0e}}{\partial x}. \quad (21)$$

Using Eq. (21), Eq. (20) can be written as

$$v_{0e,y}=-v_s\frac{\rho_s}{L_n}, \quad (22)$$

where $\rho_s=v_s/\Omega_i$ is the ion Larmor radius at the electron temperature. This equation contains the natural small parameter of the drift approximation, namely, the ratio of the ion Larmor radius at the electron temperature to the characteristic inhomogeneity length

$$\epsilon\equiv\frac{\rho_s}{L_n}\ll 1. \quad (23)$$

Furthermore, we use an expansion in this parameter, including linear and nonlinear effects up to the quadratic (in the wave fields) nonlinearities. All zero-order equilibrium values can be inhomogeneous in space (in the present study, we consider only their dependences on x). Due to the quasineutrality in the equilibrium, the ion equilibrium density can be expressed via the electron equilibrium density as

$$n_{0i}=n_{0e}(1+P). \quad (24)$$

A similar relation can be written for small perturbations $\delta n_i/n_{0i}$, assuming that the quasineutrality is maintained (which is the case when the wave length is much larger than the Debye length). We have

$$\delta n_i=\delta n_e+\delta Z_d n_{0d}+Z_{0d}\delta n_d, \quad (25)$$

and therefore

$$\frac{\delta n_i}{n_{0i}}=\frac{\delta n_e}{n_{0e}}\frac{1}{1+P}+\left(\frac{\delta Z_d}{Z_{0d}}+\frac{\delta n_d}{n_{0d}}\right)\frac{P}{1+P}. \quad (26)$$

The further expansion procedure is standard, although the feature is that the new types of linear and nonlinear terms appear due to the variations of the dust charge δZ_d. Using the dust charging equation, we find

$$\partial_t\left(\frac{\delta Z_d}{Z_{0d}}\right)=-\nu_d^{ch}\frac{\delta Z_d}{Z_{0d}}+\frac{1+P}{P}\bar{\nu}_i\left[\frac{\delta n_e}{n_{0e}}-\frac{\delta n_i}{n_{0i}}\right]. \quad (27)$$

In the approximation next to Eq. (20) in the small parameter ϵ, Eq. (23), we find for the perpendicular electron velocity

$$\delta\mathbf{v}_{e,\perp}=\frac{v_{Te}^2}{\Omega_e}\left[\hat{\mathbf{z}}\times\nabla\phi-\hat{\mathbf{z}}\times\delta\left(\frac{1}{n_{0e}}\nabla n_{0e}\right)\right]. \quad (28)$$

In the same approximation, we have the parallel motion of plasma electrons given by

$$v_{1e,z}=\frac{v_{Te}^2}{\nu_{ed}}\partial_z\left(\phi-\frac{\delta n_e}{n_{0e}}\right). \quad (29)$$

The motion of ions is described by the momentum equation

$$\partial_t\mathbf{v}_i+\mathbf{v}_i\cdot\nabla\mathbf{v}_i=-\frac{v_{Ti}^2}{n_i}\nabla n_i-v_s^2\nabla\phi+\Omega_i\mathbf{v}_i\times\hat{\mathbf{z}}-\nu_{id}\mathbf{v}_i. \quad (30)$$

For the unperturbed motion of ions, we have the ion diamagnetic drift velocity

$$\mathbf{v}_{0i,\perp}=\frac{v_s^2}{\Omega_i}\hat{\mathbf{z}}\times\nabla\phi. \quad (31)$$

Similarly to Eqs. (28) and (29), the next approximation gives us the perturbation of the perpendicular ion velocity

$$\delta\mathbf{v}_{i,\perp}=-\frac{v_s^2}{\Omega_i}(\partial_t+\nu_{id})\hat{\mathbf{z}}\times\nabla\phi+\frac{v_s^4}{\Omega_i^2}\hat{\mathbf{z}}\times[(\hat{\mathbf{z}}\times\nabla\phi)\hat{\mathbf{z}}\times\nabla\phi], \quad (32)$$

as well as the velocity for the parallel motion of plasma ions

$$\delta v_{i,z}=\frac{v_{Te}^2}{\nu_{id}}\partial_z\left(\phi-\frac{\delta n_e}{n_{0e}}\right). \quad (33)$$

In the linear approximation, we ignore the effects of the magnetic field on the motion of the dust component and invoke the linearized Euler equation

$$\partial_t\mathbf{v}_d=\frac{T_e Z_d}{m_d}\nabla\phi-\nu_{dn}(\mathbf{v}_d-\mathbf{v}_n)+\frac{\mathbf{F}_d}{m_d}, \quad (34)$$

where $\mathbf{F}_d$ includes the repulsion and attraction forces of the dust-dust interactions, and dust–neutral-particle friction is taken into account. For the neutral particles, we have in the same approximation,

$$\partial_t\mathbf{v}_n=-\nu_{nd}(\mathbf{v}_n-\mathbf{v}_d), \quad (35)$$

where collisions with plasma electrons and ions are neglected. Thus, assuming $\nu_{dn}=\nu_{nd}$, we find for the neutral velocity

$$\mathbf{v}_n=\frac{\nu_{dn}}{\partial_t+\nu_{dn}}\mathbf{v}_d. \tag{36}$$

Equation (34) can then be written as

$$\partial_t\left(1+\frac{\mu\nu_{dn}}{\partial_t+\nu_{dn}}\right)\mathbf{v}_d=\frac{T_eZ_d}{m_d}\nabla\phi+\frac{\mathbf{F}_d}{m_d}, \tag{37}$$

where $\mu=m_nn_n/m_dn_d$.

For the dust-dust interaction forces, we obtain from the pair interaction forces (3) the force on the dust fluid element

$$\mathbf{F}_{ra}=\frac{Z_d^2e^2}{a}\int n_d(\mathbf{r}')d\mathbf{r}'\left[-\eta_r\frac{1}{2}\frac{\partial}{\partial\mathbf{r}}\frac{a^2}{|\mathbf{r}-\mathbf{r}'|^2}+(\eta_b+\eta_c)\frac{a^2}{\lambda_{Di}^2}\frac{a}{|\mathbf{r}-\mathbf{r}'|^2}\right]. \tag{38}$$

Here, we have taken into account that the repulsion force is inversely proportional to the cube of the intergrain distance, while both the attraction forces are inversely proportional to the square of the intergrain distance [17]; this is true for distances larger than the plasma Debye length. Thus we have after Fourier transform

$$i\mathbf{k}\cdot\mathbf{F}_d=m_d\omega_{pd}^2\frac{\delta n_d}{n_{0d}}\left(\frac{\pi}{4}\eta_rka-\eta_a\frac{a^2}{\lambda_D^2}\right). \tag{39}$$

Thus we have derived the closed set of equations fully describing the motion of the plasma electron, ion, neutral particle, and dust components, taking into account the dust charging effects and dust-dust interactions, as well as charging and elastic collisions. In the following sections, on the basis of Eqs. (20), (26)–(29), (31)–(33), (37), and (39), we obtain the second-order nonlinear equations and analyze linear drift wave instabilities that may lead to the generation of various nonlinear structures (e.g., combined drift–neutral-particle vortices in a partially ionized dusty plasma).

V. NONLINEAR EQUATION FOR DRIFT WAVES

Expanding the electron continuity equation in the parameter ϵ, Eq. (23), we find

$$\partial_t\left(\frac{\delta n_e}{n_{0e}}\right)+\overline{\nu}_{ed}\left(\frac{\delta n_e}{n_{0e}}+\frac{\delta n_d}{n_{0d}}-\mathcal{Z}\frac{\delta Z_d}{Z_{0d}}\right)=-\frac{v_{Te}^2}{n_{0e}\Omega_e}\overline{\mathbf{z}}\times\nabla\phi\cdot\nabla_\perp n_{0e}+\frac{k_z^2v_{Te}^2}{\nu_{ed}}\left(\phi-\frac{\delta n_e}{n_{0e}}\right)-\frac{v_{Te}^2}{\Omega_e}\nabla_\perp\left(\frac{\delta n_e}{n_{0e}}\hat{\mathbf{z}}\times\nabla\phi\right), \tag{40}$$

where $\nabla_\perp$ stands for the gradient perpendicular to the external magnetic field [such that $\nabla=(\nabla_\perp,\nabla_z)$], and we use the perturbation expansion of Eq. (18),

$$\frac{\delta\overline{\nu}_{ed}}{\overline{\nu}_{ed}}=-\mathcal{Z}\frac{\delta Z_d}{Z_{0d}}+\frac{\delta n_d}{n_{0d}}, \tag{41}$$

which can easily be found by taking into account the quasineutrality condition (26) together with the perturbation of the charging equation (14).

For plasma ions, invoking Eqs. (26), (27), and (40), and using an equation for the perturbations of the ion capture rate [similar to Eq. (41)],

$$\frac{\delta\overline{\nu}_{id}}{\overline{\nu}_{id}}=-\frac{\mathcal{Z}}{\tau+\mathcal{Z}}\frac{\delta Z_d}{Z_{0d}}+\frac{\delta n_d}{n_{0d}}, \tag{42}$$

the continuity equation can be written as

$$\frac{P}{1+P}\partial_t\left(\frac{\delta n_d}{n_{0d}}\right)-\partial_t(1+\rho_s^2\nabla_\perp^2)\phi+\frac{v_s^4}{\Omega_i^3}\nabla\cdot\hat{\mathbf{z}}\times[(\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla)\hat{\mathbf{z}}\times\nabla\phi]$$
$$=-\frac{v_s^2}{(1+P)n_{0e}\Omega_i}\overline{\mathbf{z}}\times\nabla\phi\cdot\nabla_\perp(n_{0d}Z_d)-\frac{v_s^2}{(1+P)\Omega_i}\nabla_\perp\cdot\left(P\frac{\delta Z_d}{Z_{d0}}+P\frac{\delta n_d}{n_{d0}}\right)\overline{\mathbf{z}}\times\nabla\phi$$
$$+\frac{k_z^2v_s^2}{\nu_{id}}\left[\phi+\frac{\tau}{1+P}\frac{\delta n_e}{n_{0e}}+\frac{\tau P}{1+P}\left(\frac{\delta Z_d}{Z_{0d}}+\frac{\delta n_d}{n_{0d}}\right)\right]-\frac{k_z^2v_s^2}{(1+P)\nu_{ed}}\left(\phi-\frac{\delta n_e}{n_{0e}}\right). \tag{43}$$

Furthermore, we introduce the dimensionless variables

$$t\to\Omega_it\frac{\rho_s}{L_n},\quad \mathbf{r}\to\frac{\mathbf{r}}{\rho_s},\quad \zeta\to\frac{\delta Z_d}{Z_{0d}}\frac{L_n}{\rho_s},\quad n_{e(d)}\to\frac{\delta n_{e(d)}}{n_{0e(d)}}\frac{L_n}{\rho_s},\quad \phi\to\phi\frac{L_n}{\rho_s}. \tag{44}$$

Thus we find from Eq. (43)

$$\partial_t(1+\nabla_\perp^2)\phi-s\partial_y\phi-\frac{P}{1+P}\partial_tn_d+c_i\left[\phi+\frac{\tau}{1+P}n_e+\frac{\tau P}{1+P}(\zeta+n_d)\right]-\frac{c_e}{1+P}(\phi-n_e)$$
$$=\{\nabla_\perp^2\phi,\phi\}-\frac{P}{1+P}\{\zeta+n_d,\phi\}, \tag{45}$$

where the Poisson bracket is defined by

$$\{A,B\}=\partial_x A\partial_y B-\partial_x B\partial_y A \tag{46}$$

and the following parameters have been introduced:

$$c_e=\frac{k_z^2 v_{Te}^2 L_n}{\nu_{ed}\Omega_i\rho_s},\quad c_i=\frac{k_z^2 v_s^2 L_n}{\nu_{id}\Omega_i\rho_s},$$

$$s=\frac{P}{1+P}\frac{L_d^{-1}}{1+P-PL_d^{-1}},\quad L_d^{-1}=\frac{\partial \ln n_{0d}Z_{0d}}{\partial \ln n_{0i}}. \tag{47}$$

The electron continuity equation (40) can thus be rewritten as

$$\partial_t n_e+\alpha(n_e+n_d-\mathcal{Z}\zeta)+\partial_y\phi-c_e(\phi-n_e)=\{n_e,\phi\}. \tag{48}$$

The charging equation (14) is now given by

$$\partial_t\zeta+\beta\zeta=\alpha(1+P)(n_e-n_d), \tag{49}$$

where

$$\beta=\frac{L_n(\overline{\nu_{id}}+\nu_d^{ch})}{\Omega_i\rho_s}=\alpha(1+P)\left(1+\frac{\nu_d^{ch}}{\overline{\nu_{id}}}\right). \tag{50}$$

Note the term containing n_d on the right hand side of Eq. (49) that is appearing due to the perturbation of the dust motion.

The combination of the dust continuity equation together with Eq. (37) gives us

$$\partial_t^2(1-\gamma^{-1}\partial_t)n_d=-\nabla\cdot\mathbf{f}-\kappa\nabla^2\phi, \tag{51}$$

where κ can be expressed via the dust sound velocity $v_{ds}=(T_e/m_d)^{1/2}$:

$$\kappa=\frac{v_{sd}^2}{v_s^2}\frac{L_n^2}{\rho_s^2}, \tag{52}$$

and γ describes the rate of damping of the dust motion due to the dust–neutral-particle collisions

$$\gamma=\frac{\nu_{dn}}{\Omega_i}\frac{L_n}{\rho_s}\frac{\mu+1}{\mu}. \tag{53}$$

Furthermore, in Eq. (51) the divergence of the force **f** can be written in the form

$$\nabla\cdot\mathbf{f}=\frac{\hat{\omega}_{pd}^2}{\Omega_i^2}\frac{L_n^2}{\rho_s^2}\hat{K}n_d, \tag{54}$$

where the Fourier transform of the operator $\hat{K}$ is given by

$$K=\frac{\pi}{4}\frac{\eta_r ka}{\rho_s}-\eta_a\frac{a^2}{\lambda_D^2}, \tag{55}$$

and we have $\eta_a=\eta_b+\eta_c$,

$$\hat{\omega}_{pd}^2=\frac{4\pi n_{0d}Z_{0d}^2e^2}{\hat{m}_d},\quad \hat{m}_d=m_d\left(1+\frac{\mu\nu_{dn}}{\partial_t+\nu_{dn}}\right). \tag{56}$$

Note that the coefficient c_e describes the nonadiabaticity introduced by dust (namely, because of collisions of plasma electrons, moving along the magnetic field lines, with dust grains). The smaller the c_e the larger the nonadiabaticity. The coefficient α is connected with the charging process and describes the influence of the charging collisions. The nonadiabaticity introduced by ions and described by c_i is usually small, and below we neglect it in our stability analysis.

The set of nonlinear equations for the drift-wave potential (45), the electron density perturbation (48), the perturbation of the charging process (49), and the dynamics of the dust density perturbations (51) [connected also with the perturbation in the motion of neutral particles via Eq. (36)] are the general result of the consideration. Below, we present the stability analysis of the linearized equations found on the basis of the general nonlinear set.

VI. LINEAR INSTABILITIES OF DRIFT WAVES

In the linear approximation, the dust density perturbation is given by

$$n_d=-\frac{\kappa k^2}{\Omega_1^2}\phi, \tag{57}$$

where

$$\Omega_1^2=\omega^2\left(1+\frac{i\omega}{\gamma}\right)-\hat{\omega}_{pd}^2K. \tag{58}$$

For the dimensionless electron perturbations, we find

$$n_e=\frac{\phi}{-i\Omega_2}\left[c_e-ik_y+\frac{\alpha\kappa k^2}{\Omega_1^2}\left(1+\alpha\mathcal{Z}\frac{1+P}{-i\omega+\beta}\right)\right], \tag{59}$$

where

$$-i\Omega_2=-i\omega+\alpha+c_e-\alpha^2\mathcal{Z}\frac{1+P}{-i\omega+\beta}. \tag{60}$$

Finally, the dust charge perturbation is

$$\zeta=\frac{\alpha(1+P)\phi}{-i\omega+\beta}\left\{\frac{\kappa k^2}{\Omega_1^2}+\frac{1}{-i\Omega_2}\left[c_e-k_y+\frac{\alpha\kappa k^2}{\Omega_1^2}\left(1+\alpha\mathcal{Z}\frac{1+P}{-i\omega+\beta}\right)\right]\right\}. \tag{61}$$

Note that the coefficient c_e describes the nonadiabaticity introduced by dust (in particular, because of collisions of plasma electrons, moving along the magnetic field lines, with dust grains), such that smaller c_e corresponds to larger nonadiabaticity. The coefficient α is connected with the charging process and describes the influence of the charging collisions, while the parameter β appears due to the Coulomb collisions. The nonadiabaticity introduced by ions and described by c_i is usually small, and we neglect it in our linear stability analysis.

Thus, ignoring the ion nonadiabaticity (i.e., putting $c_i = 0$), we obtain the following dispersion relation for the frequency and the instability rate of the drift waves in a weakly ionized dusty plasma:

$$\omega(1+k_\perp^2)=k_y s-\frac{c_e}{1+P}-\frac{\kappa k^2}{\Omega_1^2}\frac{P\omega}{1+P}+\frac{1}{-i\Omega_2}\frac{1}{1+P}\left[c_e+P\tau\frac{\alpha(1+P)}{-i\omega+\beta}\right]\times\left[c_e-ik_y+\frac{\alpha\kappa k^2}{\Omega_1^2}\left(1+\alpha\mathcal{Z}\frac{1+P}{-i\omega+\beta}\right)\right]. \tag{62}$$

Note that the third term on the right hand side of this equation (which contains Ω_1) is different compared with the previous study [9]; it appears due to the motion of dust (factor $\hat{\omega}_{pd}$, connected with dust particle oscillations, and factor γ, describing the friction of dust with neutral particles) and also includes the dust-dust interaction [factor K given by Eq. (55)]. The presence of this term strongly affects the dissipative drift-wave instability in a partially ionized dusty plasma. Indeed, if we move this term to the left hand side, and take into account that Ω_1^2 can change the sign of its real part, depending not only on the relation between the frequency ω and the plasma dust frequency ω_{pd}, but also on the balance of the repulsive and attractive forces (factor K), we can qualitatively conclude, that because of this factor Ω_1 completely different regimes of the instability are possible. In effect, this means that the instability appears for those regimes that are stable in the absence of the dust motion and the dust-dust interaction.

Equation (62) is solved numerically and some dependences of the instability growth rate vs the wave number of the dissipative drift wave are presented in Figs. 1–3. Note that in these figures the dimensionless wave frequency is measured in units of $\omega = \Omega_i\rho_s/L_n$, and the dimensionless wave vector is measured in units of $k \equiv k_y\rho_s$ (for simplicity, we assume $k_y \gg k_x$). It is important that the instability presented on these figures correspond to those values of plasma parameters, in particular $P=10$, which are related to a *stable* situation in the absence of the dust motion and the dust-dust interaction.

The analysis demonstrates that for the case considered there are two important controlling parameters, which are different compared with the case [9] of no motion and interaction of dust grains: (1) γ [given by Eq. (53)], which is the damping of the dust density perturbations due to their friction with neutral particles; and (2) $\Omega = \omega_{pi}^2/\hat{\omega}_{pd}^2$, which describes the ratio of the ion plasma and dust plasma frequencies [note that the latter takes into account dust mass renormalization because of the interactions with neutral particles; see Eq. (56)]. Thus, an increase of the friction (see Fig. 1) dumps the instability, as well as the increased effective density of the dust component (Fig. 3). Note that the

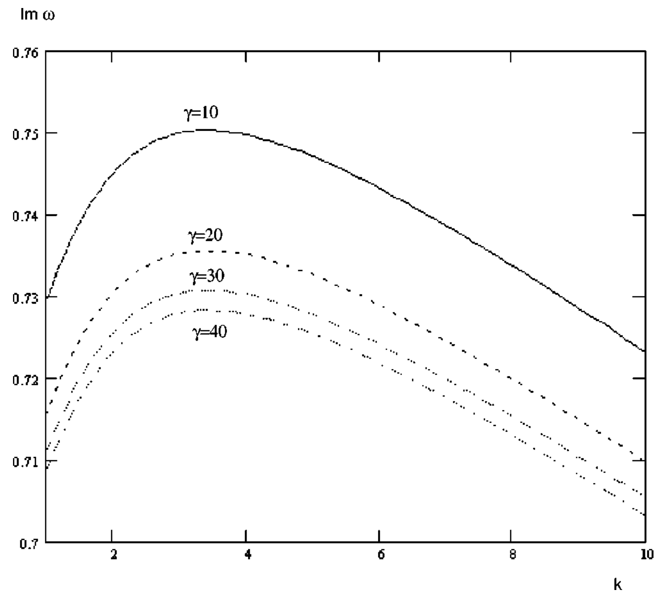

FIG. 1. The growth rate of the dissipative drift instability for four different values of the damping of dust density perturbations γ. Other parameters are $P=10$, $\Omega \equiv \omega_{pi}^2/\hat{\omega}_{pd}^2 = 10$, $c_e=10$, $\alpha = 0.1$, and $\tau = 0.01$.

latter occurs because the studied instability regime corresponds to negative K, causing an increase of the real part of Eq. (58) with larger $\hat{\omega}_{pd}$. We also stress that the dissipative instability can develop for relatively large values of the parameter P [given by (12)], which is the ratio of the total charge on the dust to the total charge of plasma electrons. This feature is completely different, compared with the results in [9], in a fully ionized dusty plasma, when the drift wave instability was suppressed with increasing of P, such that there was no instability for $P>1$.

VII. DISCUSSION

Previously, it was shown in [9] that in a dusty plasma the drift-wave instability can be substantially enhanced due to the increase of the dissipation rate along the magnetic field lines as compared to the dust-free case. In this paper, we have demonstrated that because of the dust–neutral-particle friction and dust-dust interaction, the instability in a partially ionized dusty plasma can also develop in a completely different parameter range, which corresponds to the stable drift waves in a fully ionized dusty plasma. The developed instability can lead to formation of nonlinear combined structures, including the drift-wave vortices coupled with the vortices of the neutral gas component via dust grains.

The long-lived dust-plasma structures have time to redistribute the dust particles in the structure. One can expect that dust is concentrated in the structure and the structure becomes negatively charged, attracting the fluxes of electrons and ions. Presently, the full program of investigation of such neutral-particle–dust–plasma structures can be only formulated. The first step, which is done in the present paper, is the generalization of the nonlinear equations, taking into account the dust-dust interactions (including the dust-dust attraction and repulsion). The next step is to solve these nonlinear equations; this requires a lot of numerical calculation and

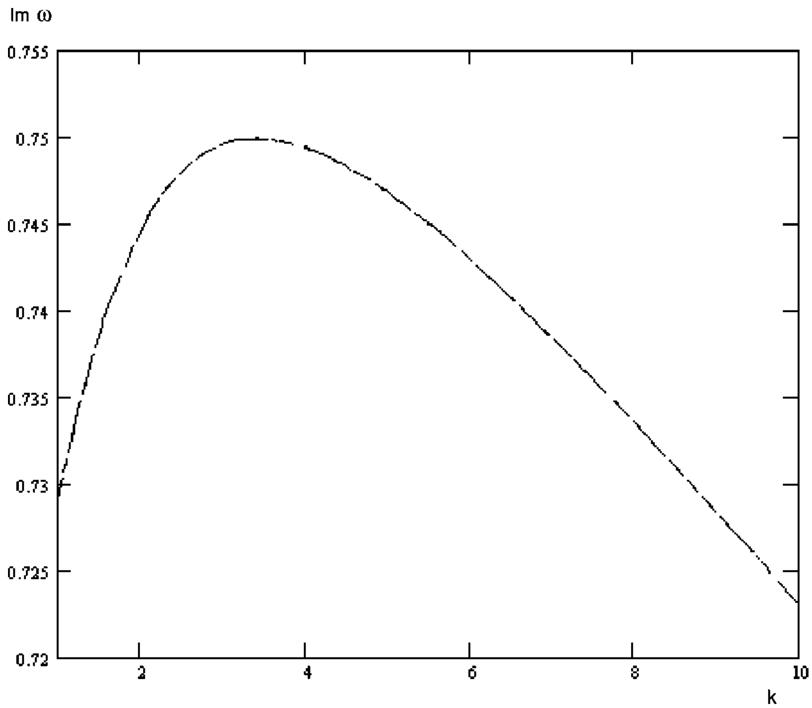

FIG. 2. The growth rate of the dissipative drift instability for four different values of the electron adiabaticity parameter $c_e = 10,100,10^3,10^4$ and $\gamma = 10$, $P = 10$, $\Omega = 10$, $\alpha = 0.1$, and $\tau = 0.01$. There is very weak dependence of the growth rate on c_e: the four curves are almost exactly superimposed on each other.

computer time, and is a subject for further investigation.

The main problem related to the redistribution of dust in the dissipative structures in a partially ionized plasma is connected with the dust–neutral-particle interaction. The amount of the neutral nonionized component is not small for many plasmas; e.g., in the lower ionosphere (where the drift structures and dust are observed), in which the ionization degree is low and the dust–neutral-particle collisions are important in transferring momentum to the dust grains. Since the neutral particles have no charges, the cross section of their interaction with dust is just the geometrical cross section of the dust particles. This allows us to write the frequency of the neutral-particle–dust collisions in a simple manner. In addition to the neutral-particle–dust collisions, the friction of neutral particles with plasma ions can be important in the energy and momentum exchange between neutral particles and ions. Comparison of the ion–neutral-particle collision rate with that of the dust–neutral-particle collisions shows that there is a broad range of parameters where the dust–neutral-particle collisions can dominate.

The interest in nonlinear dust–neutral-particle drift-wave structures is also connected with the possibility of merging in one structure the properties of two types of vortices. If neutral particles have no interaction with dust, their motion (as

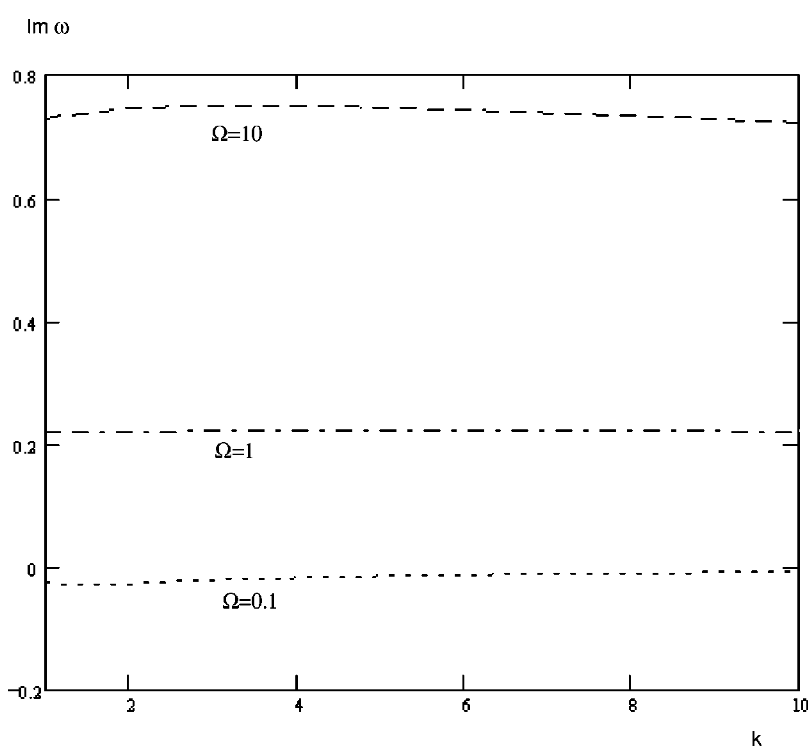

FIG. 3. The growth rate of the dissipative drift instability for three different values of the parameter $\Omega \equiv \omega_{pi}^2/\hat{\omega}_{pd}^2 = 0.1,1,10$ and $\gamma = 10$, $P = 10$, $c_e = 10$, $\alpha = 0.1$, and $\tau = 0.01$. Note the strong dependence of the growth rate on Ω: the wave is stable for $\Omega = 0.1$ and becomes unstable as it increases.

any subsonic motion in neutral gas) is described by motion of a vortex or superposition of vortices. On the other hand, in a dusty plasma, where interactions of dust particles with neutral particles are not taken into account, the elementary nonlinear structures will be the drift vortices, modified by dust. In the presence of the dust–neutral-particle interaction, these vortices cannot be separated, and the resulting nonlinear structure will appear to have the properties of the neutral gas vortices and the drift vortices simultaneously. In the lower ionosphere and the upper atmosphere such structures can probably be created once conditions for them appear. These combined drift structures are strongly influenced by the degree of plasma ionization as well as solar activity, and can create specific motions in the upper atmosphere that depend on the amount of dust present and therefore on the degree of existing pollution.

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