Nonequilibrium phase transition in the kinetic Ising model: Is the transition point the maximum lossy point?

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The nonequilibrium dynamic phase transition, in the kinetic Ising model in presence of an oscillating magnetic field, has been studied both by Monte Carlo simulation (in two dimensions) and by solving the mean-field dynamical equation of motion for the average magnetization. The temperature variations of hysteretic loss (loop area) and the dynamic correlation have been studied near the transition point. The transition point has been identified as the minimum-correlation point. The hysteretic loss becomes maximum above the transition point. An analytical formulation has been developed to analyze the simulation results. A general relationship among hysteresis loop area, dynamic order parameter, and dynamic correlation has also been developed. [S1063-651X(98)03807-0]

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I. INTRODUCTION

The dynamics of magnetization reversal in simple ferromagnetic systems has recently attracted considerable scientific interest to study *nonequilibrium* responses. In this regard, the dynamical responses of the Ising system in the presence of an oscillating magnetic field have been studied extensively [1–6]. The dynamical hysteretic response [1–3] and the nonequilibrium dynamical phase transition [4–9] are two main subjects of interest to study the dynamic responses of the kinetic Ising model in the presence of an oscillating magnetic field.

Tome and Oliviera [4] first studied the dynamic transition by solving the mean-field (MF) dynamic equation of motion (for the average magnetization) of the kinetic Ising model in presence of a sinusoidally oscillating magnetic field. By defining the order parameter as the time-averaged magnetization over a full cycle of the oscillating magnetic field, they showed that the order parameter vanishes, depending upon the value of the temperature and the amplitude of the oscillating field. In the field amplitude and temperature plane they have drawn a phase boundary separating dynamic ordered (nonzero value of the order parameter) and disordered (order parameter vanishes) phases. They [4] have also observed and located a *tricritical point* (TCP) [separating the nature (discontinuous or continuous) of the transition] on the phase boundary line.

Since this transition exists even in the static (zero-frequency) limit, such a transition, observed [4] from the solution of a mean-field dynamical equation, cannot be dynamic in the true sense. This is because, for a field amplitude less than the coercive field (at a temperature less than the static ferro-para transition temperature), the response magnetization varies periodically but asymmetrically even in the zero-frequency limit; the system remains locked to one well of the free energy and cannot go to the other one, in the absence of noise or fluctuation. On the other hand, in the presence of thermal fluctuations, in the static limit, the system can go from one well to another via the formation of nucleating droplets. A vanishingly small field is required to push the system from one to the other well. Consequently, the dynamic phase boundary collapses, in the presence of thermal fluctuations.

To study the true dynamic phase transition (which should disappear in the static limit) one has to consider the effect of thermal fluctuations. In this regard, Lo and Pelcovits [5] first attempted to study the dynamic nature of this phase transition (incorporating the effect of fluctuation) in the kinetic Ising model by Monte Carlo (MC) simulation. However, they [5] have not reported any precise phase boundary. Acharyya and Chakrabarti [6] studied the nonequilibrium dynamic phase transition in the kinetic Ising model in the presence of an oscillating magnetic field by extensive MC simulation. They [6] have drawn the phase boundary and located a tricritical point (as observed) on the boundary. It has been also observed [6] that this dynamic phase transition is associated with the breaking of the symmetry of the dynamic hysteresis (m-h) loop. In the dynamically disordered (value of the order parameter vanishes) phase the corresponding hysteresis loop is symmetric, and loses its symmetry in the ordered phase (giving a nonzero value of the dynamic order parameter). They have [6] also studied the temperature variation of the ac susceptibility components near the dynamic transition point. It has been observed [6] that the imaginary or lossy (real) part of the ac susceptibility gives a peak (dip) near the dynamic transition point (where the dynamic order parameter vanishes). It was concluded that this is a possible indication of the thermodynamic nature of this kind of nonequilibrium dynamical phase transition.

The statistical distribution of the dynamic order parameter has been studied by Sides *et al.* [7]. The nature of the distribution changes (from bimodal to unimodal) near the dynamic transition point. They have also observed [7] that the fluctuation of the hysteresis loop area grows and becomes considerably large as one approaches the dynamic transition point.

The relaxation behavior, of the dynamic order parameter, near the transition point (in the disordered phase), has been

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studied [8] recently by MC simulation and by solving the mean-field dynamic equation. It has been observed that the relaxation is Debye type and the relaxation time diverges near the transition point. The ''specific heat'' and the ''susceptibility'' also diverge [9] near the transition point in a similar manner as that of fluctuations of the order parameter and fluctuation of the energy, respectively. These observations [9] (divergences of fluctuations) indirectly support earlier facts [7] where the distribution of the dynamic order parameter becomes wider and the fluctuation of the hysteresis loop area becomes considerably large near the transition point.

Recently experimental evidence [10] for the dynamic transition has been found. Dynamical symmetry breaking (associated with the dynamic transition) across the transition point of the hysteresis loop has been observed, in highly anisotropic (Ising-like) and ultrathin Co/Cu(001) ferromagnetic films by the surface magneto-optic Kerr effect, as one passes through the transition point. Dynamical symmetry breaking in the hysteresis loops has also been observed [11] in ultrathin Fe/W(110) films. However, the detailed natures of the dynamic transition and the phase boundary have not yet been studied experimentally.

In this paper, the dynamic phase transition has been studied in the kinetic Ising model in the presence of a sinusoidally oscillating magnetic field by MC simulation and by solving the mean-field dynamical equation of motion for the average magnetization. The temperature variations of the hysteresis loss (or loop area), the dynamic correlation, and the phase lag are studied near the dynamic transition point. This paper has been organized as follows: In Sec. II simple analytic forms are given for the loop area, dynamic correlation, and dynamic order parameter. In Sec. III a general relationship has been developed among the various dynamical quantities. In Sec. IV the models are introduced and in Sec. V the numerical results are given. The paper ends with a summary of the work in Sec. VI.

II. ANALYTIC FORMS OF THE LOOP AREA AND THE DYNAMIC CORRELATION NEAR THE TRANSITION POINT

The form of the oscillating magnetic field is

$$h(t)=h_0\cos(\omega t). \tag{2.1}$$

The dynamic order parameter is defined as

$$Q=\frac{\omega}{2\pi}\oint m(t)dt, \tag{2.2}$$

which is nothing but the time-averaged magnetization over a full cycle of the oscillating magnetic field. The hysteresis loop area is

$$A=-\oint m\,dh=h_0\omega\oint m(t)\sin(\omega t)dt, \tag{2.3}$$

which corresponds the energy loss due to the hysteresis. The dynamic correlation is defined as

$$C=\langle m(t)h(t)\rangle-\langle m(t)\rangle\langle h(t)\rangle,$$

where $\langle\ \rangle$ denotes the time average over the full cycle of the oscillating magnetic field. Since $\langle h(t)\rangle=0$, one can write

$$C=\frac{\omega}{2\pi}\oint m(t)h(t)dt=\frac{\omega h_0}{2\pi}\oint m(t)\cos(\omega t)dt. \tag{2.4}$$

The dynamic correlation has another physical interpretation. For the cooperatively interacting spin system, this is the negative of the time-averaged spin-field interaction energy (per spin) $[\langle E_f\rangle=-(\omega/2\pi L^2)\oint\Sigma_i\sigma_i\ h(t)dt]$ over a complete cycle of the oscillating field.

In the dynamically disordered $(Q=0)$ phase and near the transition point, the time series of the magnetization $[m(t)]$ can be approximated as a square wave with a phase lag δ with the applied sinusoidal magnetic field:

$$m(t)=\begin{cases}1 & \text{for}\quad 0<t<\tau/4+\delta/\omega,\\ -1 & \text{for}\quad \tau/4+\delta/\omega<t<3\tau/4+\delta/\omega,\\ 1 & \text{for}\quad 3\tau/4+\delta/\omega<t<2\pi/\omega,\end{cases} \tag{2.5}$$

where τ is the time period of the oscillating field and δ is the phase lag between magnetization $m(t)$ and the magnetic field $h(t)=h_0\cos(\omega t)$. The value of the hysteresis loop area can easily be calculated as

$$A=4h_0\sin(\delta). \tag{2.6}$$

This form of the loop area was also obtained [6] from the assumption that it is approximately equal to 4 times the product of coercive field and remanent magnetization (here the remanent magnetization is equal to unity), where the coercive field is identified as $h_0\sin(\delta)$ (the change in field during the phase lag). Considering the same form of the magnetization the dynamic correlation C can also be calculated exactly as

$$C=\frac{2h_0}{\pi}\cos(\delta). \tag{2.7}$$

From the above forms of A and C it can be written as

$$\frac{A^2}{(4h_0)^2}+\frac{C^2}{(2h_0/\pi)^2}=1. \tag{2.8}$$

The above relation tells us that the loop area A and the dynamic correlation C are elliptically related to each other. It may be noted here that the previously studied ac susceptibility components [6] obey a circular relationship $[\chi'^2+\chi''^2=(m_0/h_0)^2]$, where m_0 is the amplitude of the magnetization.

The ordered region $(Q\neq 0)$ can be approximated by considering the following form of the magnetization:

$$m(t)=\begin{cases}1 & \text{for}\quad 0<t<\tau/4+\delta/\omega,\\ 1-m_r & \text{for}\quad \tau/4+\delta/\omega<t<3\tau/4+\delta/\omega,\\ 1 & \text{for}\quad 3\tau/4+\delta/\omega<t<2\pi/\omega.\end{cases} \tag{2.9}$$

In the above simplified approximation, it was considered that the magnetization cannot jump to the other well; however,

the value of the initial magnetization is reduced by the amount m_r. In a real situation it has been observed that this well is not fully square [as assumed above in the form of $m(t)$]; it has a cusplike (or parabolic) shape. For $m_r=2$, the above functional form of $m(t)$ will take the form of (2.5) and one can get the disordered ($Q=0$) phase. Taking the above form of magnetization the dynamic order parameter Q can be calculated as $Q=(2-m_r)/2$. It may be noted that, in this simplified approximation, the dynamic order parameter Q is independent of the phase lag δ, which is not observed in the real situation (the phase lag shows a peak at the transition point). However, this simple picture can anticipate the convex (looking from the origin) nature [6] of the dynamic phase boundary. As the temperature increases m_r increases and it also increases as the field amplitude increases. In the simplest asumption, one can consider that m_r is proportional to the product of h_0 and T. Demanding $m_r=2$ for the dynamic transition ($Q=0$), one can readily obtain $(h_0)_d T_d = \text{const}$. This equation tells us that the dynamic phase boundary will be convex. The convex nature of the phase boundary remains invariant even if one assumes that m_r is any increasing function of both T and h_0 [for example, the power law $m_r \sim T^x h_0^y$; in this particular case the equation of the dynamic phase boundary becomes $T_d^x (h_0)_d^y = \text{const}$, and it is easy to see that this gives the convex shape of the dynamic phase boundary]. However, this very simple asumption cannot describe the entire form of the phase boundary accurately, particularly near the end points [$(h_0)_d=0$ and $T_d=0$].

III. GENERAL RELATION AMONG THE DYNAMIC ORDER PARAMETER, HYSTERESIS LOOP AREA, AND DYNAMIC CORRELATION

From the usual definitions (given in Sec. II) of C and A, one can write

$$\frac{1}{\sqrt{2\pi}}\left(\frac{2\pi C}{\omega h_0}-i\frac{A}{\omega h_0}\right)=\frac{1}{\sqrt{2\pi}}\oint m(t)\exp(-i\omega t)dt,$$

where $m(\omega)=(1/\sqrt{2\pi})\oint m(t)\exp(-i\omega t)dt$. So

$$C=\frac{h_0\omega}{\sqrt{2\pi}}\mathrm{Re}[m(\omega)]$$

and

$$A=-h_0\omega\sqrt{2\pi}\,\mathrm{Im}[m(\omega)].$$

The general (complex) form of $m(\omega')$ will be

$$m(\omega')=|m(\omega')|\exp(i\phi),$$

$$m(\omega')=\frac{1}{\sqrt{2\pi}}\left(\frac{4\pi^2C^2}{h_0^2}+\frac{A^2}{h_0^2\omega'^2}\right)^{1/2}\exp i\left[-\tan^{-1}\frac{A}{2\pi C}\right].$$

So Q is related to A and C as follows:

$$Q=\frac{1}{\tau}\oint m(t)dt=\frac{1}{\sqrt{2\pi}\tau}\int d\omega'\oint m(\omega')\exp(i\omega' t)dt$$

$$=\frac{1}{2\pi\tau}\int d\omega'\oint\sqrt{\left(\frac{4\pi^2C^2}{h_0^2\omega'^2}+\frac{A^2}{h_0^2\omega'^2}\right)}\times e^{i(\omega' t-\tan^{-1}A/2\pi C)}dt, \tag{3.1}$$

where

$$\frac{1}{\tau}\oint\cdots dt\equiv\lim_{n\to\infty}\frac{1}{2n\tau}\int_{-\infty}^{\infty}\cdots dt.$$

The above equation gives the general relationship among Q, A, and C.

It has been observed that the steady response $m(t)$, to a sinusoidally oscillating magnetic field [$h(t)=h_0\cos(\omega t)$], is periodic (with phase lag δ) and has the same periodicity ($\tau=2\pi/\omega$) of the field. So one can write $m(t)$ in a Fourier series as

$$m(t)=a_0+\sum_{n=1}^{\infty}a_n\cos(n\omega t)+\sum_{n=1}^{\infty}b_n\sin(n\omega t). \tag{3.2}$$

From the usual definitions of Q, A, and C, it is easy to see that

$$a_0=Q,\quad a_1=2C/h_0,\quad \text{and}\quad b_1=A/(\pi h_0).$$

So one can write

$$m(t)=Q+\frac{2C}{h_0}\cos(\omega t)+\cdots+\frac{A}{\pi h_0}\sin(\omega t)+\cdots. \tag{3.3}$$

Keeping only the first harmonic terms (ignoring higher harmonics) one can easily express the instantaneous magnetization as

$$m(t)=Q+m_0\cos(\omega t-\delta), \tag{3.4}$$

where the amplitude of the magnetization is $m_0=\{(2C/h_0)^2+[A/(\pi h_0)]^2\}^{1/2}$ and the phase lag is $\delta=\tan^{-1}[A/(2\pi C)]$.

IV. MODEL AND THE SIMULATION SCHEME

A. Monte Carlo study

The local field (at time t) at any site i of a nearest neighbor ferromagnetic Ising model in the presence of a time-varying external magnetic field $h(t)$ with homogeneous and unit interaction energy can be written as

$$h_i(t)=\sum_j\sigma_j(t)+h(t), \tag{4.1}$$

where $\sigma_i(t)=\pm1$ and j runs over the nearest neighbor of site i. The local field (at site i) $h_i(t)$ has an external field part $h(t)$, which is oscillating sinusoidally in time:

$$h(t)=h_0\sin(2\pi ft), \tag{4.2}$$

where h_0 and f are the amplitude and frequency of the oscillating field.

According to heat-bath dynamics, the probability $p_i(t)$ that the spin $\sigma_i(t)$ will be up at time t is given as

$$p_i(t)=\frac{e^{h_i(t)/K_BT}}{e^{h_i(t)/K_BT}+e^{-h_i(t)/K_BT}}, \tag{4.3}$$

where K_B is the Boltzmann constant which has been taken equal to unity for simplicity. It may be noted here that the spin-spin interaction strength J has been taken equal to unity. The temperature T is measured in units of J/K_B. The field is measured in units of J. The spin $\sigma_i(t)$ is oriented (at time t) as

$$\sigma_i(t+1)=\mathrm{sgn}[p_i(t)-r_i(t)], \tag{4.4}$$

where $r_i(t)$ are independent random fractions drawn from the uniform distribution between 0 and 1.

In the simulation, a square lattice ($L\times L$) is considered under periodic boundary conditions. The initial condition is that all spins are up [i.e., $\sigma_i(t=0)=1$, for all i]. The multispin coding technique is employed here to store ten spins in a computer word consisting of 32 bits. Ten spins are updated simultaneously (or parallel) by a single command. All words (containing ten spins) are updated sequentially and one full scan over the entire lattice consists of one Monte Carlo step per spin (MCSS). This is the unit of time in the simulation. The instantaneous magnetization $[m(t)=(1/L^2)\Sigma_i\sigma_i(t)]$ is calculated easily. Some transient loops were discarded to have a stable loop and all the dynamical quantities were calculated from the stable loop.

This simulation is performed in a SUN workstation cluster and the computational speed recorded is 7.14×10^6 updates of spins per second.

B. Mean-field study

The mean-field dynamical equation of an Ising ferromagnet in the presence of a time-varying magnetic field is [4]

$$\frac{dm}{dt}=-m+\tanh\left(\frac{m(t)+h(t)}{T}\right), \tag{4.5}$$

where the external time-varying field $h(t)$ has the previously described sinusoidal form. T is the temperature measured in units of zJ/K_B (z is the coordination number and K_B is the Boltzmann constant). This equation has been solved for $m(t)$ by the fourth-order Runge-Kutta method by taking the initial condition $m(t=0)=1.0$. The value of the time differential (dt) was taken to be 10^{-3}, so that the error is $O(dt^5)$ $\sim10^{-15}$. The frequency ω of the oscillating field is kept fixed ($\omega=2\pi$) throughout the study. Some transient loops were discarded and all the values of the response are calculated from a stable loop.

V. RESULTS

A. Monte Carlo results

In the MC simulation, a square lattice of linear size $L=1000$ is considered. The frequency ω of the oscillating field has been kept fixed ($\omega=2\pi\times0.01$) throughout the study. From the Monte Carlo simulation technique described above, m-h or hysteresis loops were obtained. Some (~600) initial transient loops were discarded so as to have a the stable loop. From this one can easily estimate the length of the simulation. For the above choice of frequency, 100 MCSS are required to form a complete loop (or cycle), and 600 such loops were discarded. It has been checked carefully that the loop gets stabilized (within a reasonably useful error bars) for this choice. The dynamic order parameter $Q=(\omega/2\pi)\oint m(t)dt$ is readily calculated. The loop area A and the dynamic correlation C have been calculated from the usual definitions. The phase lag δ (between field and magnetization) has been calculated by taking the difference between the positions of the minimum of magnetization and the magnetic field [6]. All values of Q, A, δ, and C for a particular temperature were obtained by averaging over ten different random samples to obtain a smooth variation. Figure 1 demonstrates the dynamic transition (with dynamic symmetry breaking) and the related phemomena (e.g., temperature variations of A, δ, etc.) at a glance. For a fixed field amplitude $h_0=0.7$ the time variations of $h(t)$ and $m(t)$ are plotted for various temperatures in the pictures in the left column and the corresponding m-h loops are shown in the right column. For a very low temperature (topmost pictures of Fig. 1), since no spin flip occurs (within the time period), the magnetization $m(t)$ remains constant (unity) and consequently the m-h loop is a straight line having zero loop area. The dynamic order parameter is unity. The concept of phase lag [between $m(t)$ and $h(t)$] is not applicable here. Obviously the dynamic correlation is zero. After a slight increase of temperature (pictures in the second row) some small number of spin flips occurs (within the time period). For some time, $m(t)$ decreases from unity and again it becomes equal to unity. The phase lag is the frequency (ω) times the time difference between the positions of the minimum of $m(t)$ and $h(t)$. The m-h loop encloses a finite but small area, giving a Q less than unity. The dynamic correlation starts to grow. As the temperature increase further the phase lag (δ) and the loop area A increases (pictures in the third row) and the dynamic order parameter Q decreases. In all three cases, described so far, the asymmetric shapes of the m-h loops are observed due to asymmetric time variation of the response magnetization $m(t)$. The dynamic correlation decreases. The temperature is very close to the dynamic transition temperature (fourth row), where the time variation of the response magnetization is almost symmetric, giving maximum values of δ. The m-h loop is symmetric and the dynamic order parameter Q is almost zero. The dynamic correlation C becomes negative and minimal. As one increases the temperaure further (last row), the phase lag decreases, and the loop area decreases. The dynamic correlation starts to grow further. It may be noted here that the conventional hysteresis or m-h loop is observed in this region of temperature. As the

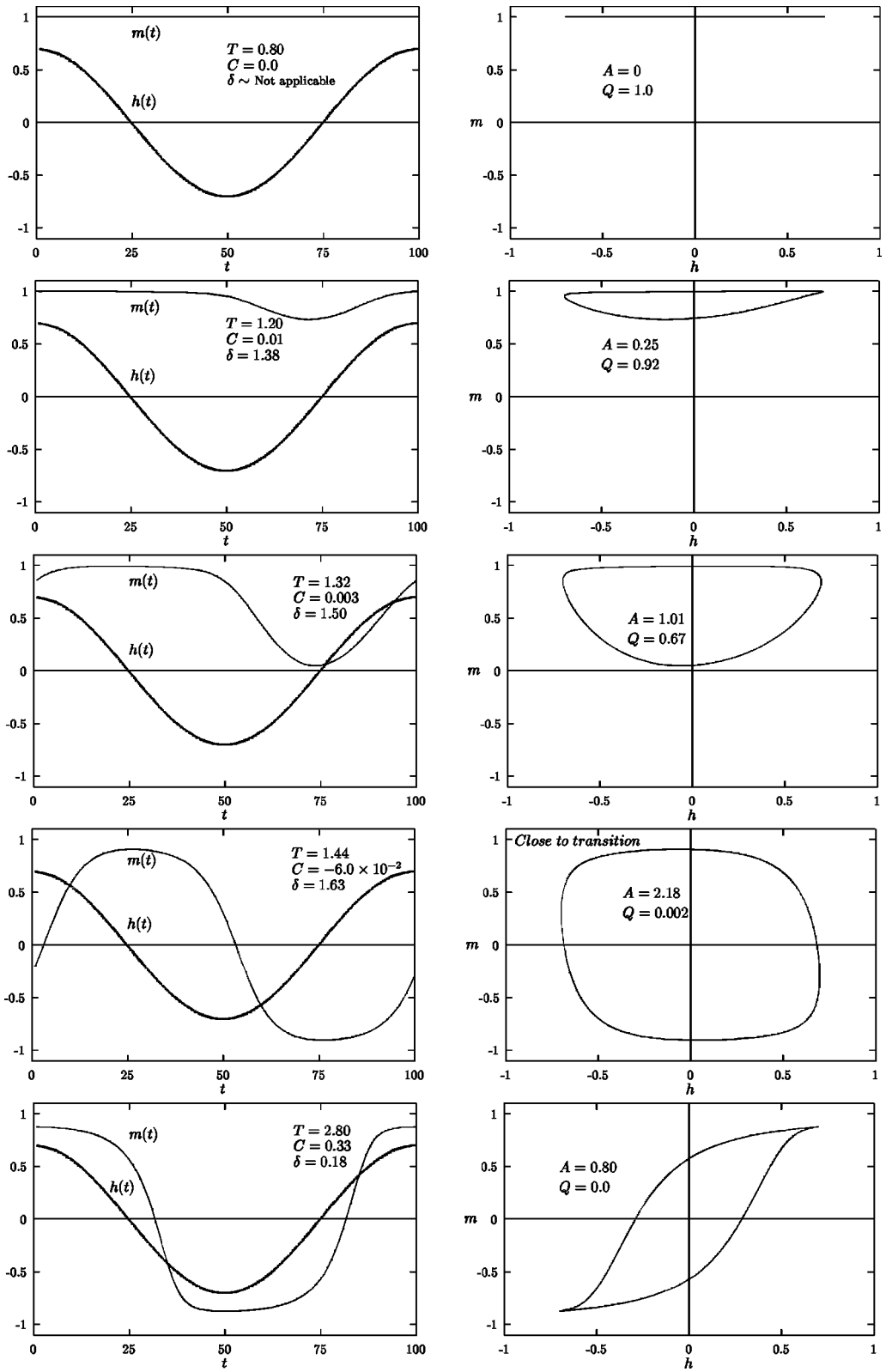

FIG. 1. A pictorial demonstration of the dynamic transition and associated phemomena. The figures in the left column represent the time variation of $h(t)$ and $m(t)$ for different temperatures and the corresponding m-h loops are shown in the right column. Temperature increases from top to bottom. Monte Carlo results for $L=1000$, $\omega=2\pi\times0.01$, and $h_0=0.7$.

temperature increases further the dynamic correlation grows, shows a maxima or peak, and then decreases. The loop area monotonically decreases.

The dynamical phase transition, via the dynamical symmetry breaking of the hysteresis loops, has been observed in highly anisotropic and ultrathin [two-dimensional (2D) Ising-like] ferromagnetic films [Co/Cu(001) and Fe/W(110)] [10,11] by using a surface magneto-optic Kerr effect study at room temperature. In a recent experimental study [11] in ultrathin Fe/W(110), the dynamical symmetry breaking of

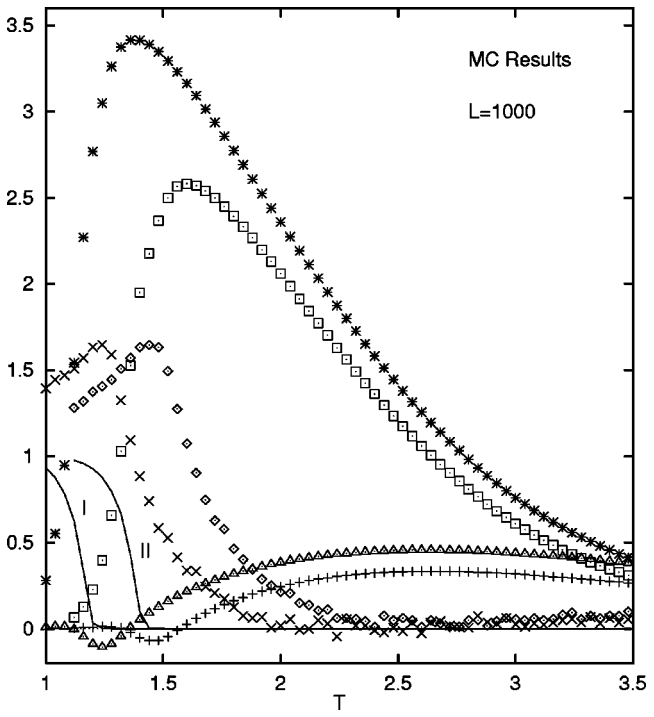

FIG. 2. The Monte Carlo results for the temperature variations of Q, δ, C, and A for two different values of field amplitudes. Q [solid lines (I) for $h_0=0.9$ and (II) for $h_0=0.7$], δ ($\times$ for $h_0=0.9$ and $\diamond$ for $h_0=0.7$), C ($\triangle$ for $h_0=0.9$ and $+$ for $h_0=0.7$), and A ($\star$ for $h_0=0.9$ and $\square$ for $h_0=0.7$).

the hysteresis loop was nicely depicted in Fig. 1 of Ref. [11].

The temperature variations of Q, δ, C, and A for two different values of field amplitudes h_0 are shown in Fig. 2. In sboth cases, it has been observed that, near the dynamic tranition point ($Q=0$), the phase lag gives a peak and the dynamic correlation C gives a shallow dip. The dynamic correlation C gives a smeared peak much above (around $T=2.6$) the static (ferro-para) transition point ($T_c=2.269\ldots$) (see Fig. 2). The hysteresis loop area A shows a peak above the dynamic transition point.

It is possible to explain these observations from the very simple analytical results described above (Sec. II). The phase lag δ becomes maximum near the dynamic transition point. So according to the analytical formulation (for C and A) for a fixed value of the field amplitude as the temperature increases the loop area $A(=4h_0\sin\delta)$ starts to increases as the dynamic order parameter Q starts to decrease and above the dynamic transition point (complete spin reversal) the loop area will be maximum and after that A will start to decrease. Similarly, the dynamic correlation C will remain approximately equal to zero until a considerable amount of spin flip occurs and Q changes appreciably and then starts to increase. Above and near the transition point, where the phase lag δ decreases as temperature increases, $C=(2h_0/\pi)\cos(\delta)$ should start to increase, which has been observed indeed. However, near the transition point it gives a shallow dip, where the value of the dynamic correlation C is minimum and negative. The phase lag δ should be less than or at most equal to $\pi/2$. The field $[h(t)=h_0\cos(\omega t)]$ crosses zero first at the phase value $\pi/2$ and it becomes minimum (maximum negative) at the value of phase (ωt) equal to π. The response magnetization should change its sign (cross zero) within this period. This is true for the $\omega\rightarrow 0$ limit; however, for finite but sufficiently high frequency, this will not happen. A phase difference of more than $\pi/2$ would be observed, yielding the unconventional shapes of m-h or hysteresis loops. In practice, it was observed that some asymmetric shape of the $(m-h)$ loop gives a value of the phase lag δ slightly higher than $\pi/2$. In this region, $\cos(\delta=\pi/2+\epsilon)=-\sin(\epsilon)$, which is negative and will show a shallow dip (cusplike shape) at the point where δ is maximum. According to the analytical prediction, the loop area A $[=4h_0\sin(\delta)]$ should show a maximum at the transition point. However, strictly speaking and in practice it has been observed that the loop area A becomes peaked above the transition temperature, since the loop area is much more strongly dependent on the actual shape of the magnetization (which is not a perfect square wave in the temperature range concerned here). As the field amplitude increases the transition points shift towards the lower temperature. The maximum of δ also increases and consequently the dip of C becomes deeper and it remains negative over a wider range of temperature (since δ remains larger than $\pi/2$ over a wider range). It may be noted that the dynamic correlation C becomes zero (in the disordered or $Q=0$ region) where the phase lag $\delta=\pi/2=1.570\,80\ldots$. The dynamic correlation C shows a smeared peak at quite higher temperature (above the Onsager value), which was misinterpreted [12] as a signature of the *stochastic resonance*. In the MF study (next section), it was shown that this is also present in the absence of fluctuations (or stochasticity).

A similar previous study [6] showed that the ac susceptibility components would give a peak (or dip) near the transition point. In that case, the susceptibility components were calculated from the phase lag δ. The phase lag δ would show a peak at the transition point. As a consequence the susceptibility components would show a peak (or dip) reflecting the behavior of the phase lag δ. However, in this case, the three measurements of the phase lag δ, loop area A, and dynamic correlation C are completely independent, and indicate the transition point separately.

B. Mean-field results

By solving the above mean-field equation the m-h or hysteresis loops were obtained. The dynamic order parameter $Q=(\omega/2\pi)\oint m(t)dt$ is readily calculated. The loop area A and the dynamic correlation C have been calculated by using the above definitions. The phase lag (between field and magnetization) has been calculated by taking the difference between the minima positions of the magnetization and the magnetic field [6]. Figure 3 shows the temperature variations of Q, δ, C, and A for two different values of field amplitudes h_0. In both cases, it has been observed that, near the dynamic transition point ($Q=0$), the phase lag gives a peak and the dynamic correlation C gives a shallow dip. The hysteretic loss A gives a peak above the transition (dynamic) point. The dynamic correlation C gives a smeared peak much above (around $T=1.3$) the static (ferro-para) transition point ($T_c=1.0$) (for a closer view see Fig. 4).

This high-temperature peak of the dynamic correlation was misinterpreted as a signature of *stochastic resonance*

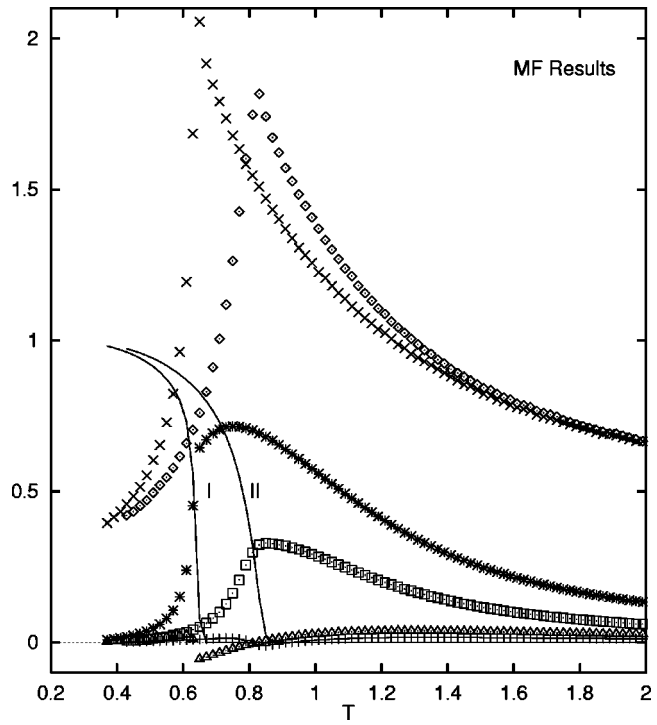

FIG. 3. The mean-field results for the temperature variations of Q, δ, C, and A for two different values of field amplitudes. Q [solid lines (I) for $h_0=0.3$ and (II) for $h_0=0.2$], δ ($\times$ for $h_0=0.3$ and $\diamond$ for $h_0=0.2$), C ($\triangle$ for $h_0=0.3$ and $+$ for $h_0=0.2$), and A ($\star$ for $h_0=0.3$ and $\square$ for $h_0=0.2$).

[12]. This peak is indeed present in the case where the fluctuation is absent (MF case). The appearance of this peak at higher temperature can be explained as follows: for a much higher temperature the time variation of instantaneous magnetization is no longer square wave like and becomes almost sinusoidal with a phase lag δ. In a very simple view, it can be approximated as $m(t)=m_0\cos(\omega t-\delta)$ [from Eq. (3.4); $Q=0$ at very high temperature). The dynamic correlation becomes $C=(m_0h_0/2)\cos(\delta)$, where m_0 is the amplitude of the magnetization which monotonically decreases as the temperature increases. The phase lag δ also monotonically decreases at higher temperature. Consequently $\cos(\delta)$ increases and m_0 decreases as the temperature increases. So one would obviously expect a peak at a finite temperature (high enough) where the competition, between the fall of m_0 and rise of $\cos(\delta)$ with respect to the temperature T, becomes comparable. Amazingly, no stochasticity is involved in it. The loop area A also gives a peak above the transition point. For a similar reason, given in Sec. V A, the dynamic correlation C gives a shallow dip near the transition point.

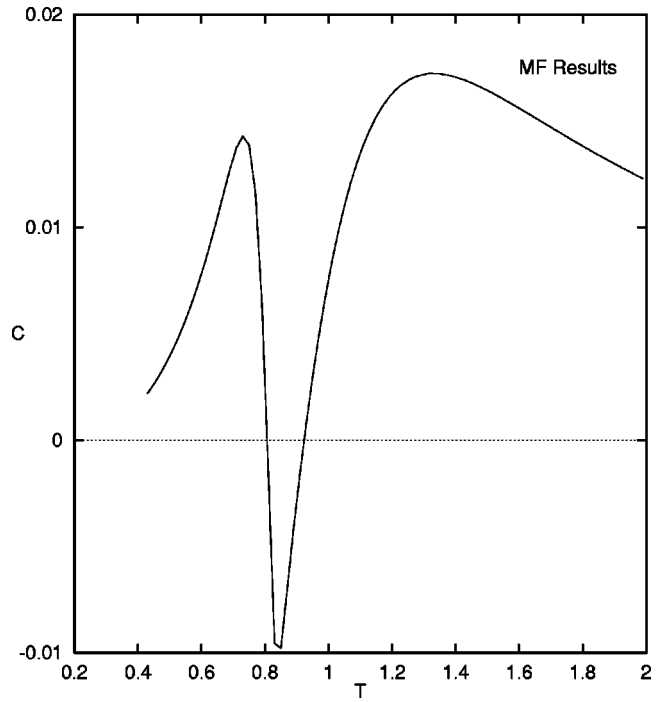

FIG. 4. A closer view of Fig. 3 for the dynamic correlation plotted against the temperature (for fixed field amplitude $h_0=0.2$).

VI. SUMMARY

The dynamical response of the kinetic Ising model in the presence of a sinusoidally oscillating magnetic field has been studied both by Monte Carlo simulation (in two dimensions) and by solving the mean-field dynamical equation of motion for the average magnetization.

A general relationship among the hysteresis loop area A, dynamic order parameter Q, and dynamic correlation C has been developed [Eq. (3.1)]. The time series of the magnetization can be decomposed in a Fourior series and the constant term is identifed as the dynamic order parameter Q; the amplitudes of the first harmonic terms are found to be related to the hysteretic loss (for the sine term) and the dynamic correlation (for the cosine term) [Eq. (3.3)].

The dynamic order parameter, the loop area, and the dynamic correlation have been calculated *separately* (both from MC and MF studies) and studied as a function of temperature. It was observed (in both cases) that the dynamic correlation shows a shallow (negative) dip near the transition point. The dynamic transition point has been identified as the minimum-correlation point. The hysteretic loss A becomes maximum above the dynamic transition point. In this sense, the dynamic transition point is not the maximum lossy point. It may be noted that an earlier study [6] of the ac susceptibility suggests that the dynamic transition point would be the maximum-lossy point, since the imaginary part (or lossy part) of the ac or complex susceptibility also shows a peak near the dynamic transition point. However, there is a remarkable distinction from the present study. In the earlier study [6], the phase lag was calculated from simulations and the ac susceptibility components were calculated from the phase lag. So it is expected that the temperature variations of the phase lag will be reflected directly in the temperature variations of the ac susceptibility components. But in the present study the measurements of the phase lag, dynamic correlation, and loop area are completely independent. This behavior of the dynamic correlation is explained from a simple square-wave-like time variation of the instantaneous response magnetization. This oversimplified assumption is incapable of explaining the peak position (above the transition point) of the hysteretic loss A. However, this simple picture can qualitatively describe the nonmonotonic temperature variations of A and C.

The high-temperature (above the static critical point T_c)

peak of the dynamic correlation was misinterpreted [12] as a signature of *stochastic resonance*. This was also discussed and an analytical form of the dynamic correlation was proposed to show that the high-temperature peak of the dynamic correlation is present even in the absence of fluctuations (or *stochasticity*).

Along with the dynamic correlation, the dynamic transition can be identified by various thermodynamic quantities like ac susceptibility [6], relaxation time [8], specific heat [8], susceptibility [9], and the fluctuations of dynamic order parameter and energy [9]. All these quantities indicate the thermodynamic natures of this kind of nonequilibrium dynamic phase transition by showing the peak, dip, or divergence near the transition point.

Related phenomena of this kind of nonequilibrium dynamic phase transition in the kinetic Ising model are mostly based on observations and not yet analyzed by using the rigorous theoretical foundations of equilibrium statistical mechanics available so far. Experimental evidence [10,11] is still in the primitive stage. Experimentally, only dynamic symmetry breaking of the hysteresis loops is observed near the transition point. However, a detailed study of the nature of the transition, the phase boundary, and the associated phenomena (described above) has not yet been done experimentally.

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