Nonequilibrium phase transition in the kinetic Ising model: Dynamical symmetry breaking by randomly varying magnetic field

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The nonequilibrium dynamic phase transition, in the two-dimensional kinetic Ising model in the presence of a randomly varying (in time but uniform in space) magnetic field, has been studied both by Monte Carlo simulation and by solving the mean-field dynamic equation of motion for the average magnetization. In both the cases, the time-averaged magnetization vanishes from a nonzero value depending upon the values of the width of randomly varying field and the temperature. The phase boundary lines are drawn in the plane formed by the width of the random field and the temperature. [S1063-651X(98)03207-3]

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I. INTRODUCTION

The problem of the random field Ising model having a *quenched* random field has been investigated both theoretically [1–4] and experimentally [5] in the past few years because it helps to simulate many interesting but complicated problems. The effects of a randomly *quenched* magnetic field on the critical behavior near the ferromagnetic phase transition is the special focus of the modern research. The recent developments in this field can be found in a review article [6]. However, the dynamical aspects of the Ising system in the presence of a randomly varying field has not yet been studied thoroughly. It would be interesting to know if there is any dynamical phase transition in the presence of a randomly varying magnetic field.

For completeness and continuity, it would be convenient to review briefly the previous studies on the dynamic transition in the kinetic Ising model. Tome and Oliviera [7] observed and studied the dynamic transition in the kinetic Ising model in the presence of a sinusoidally oscillating magnetic field. They solved the mean-field (MF) dynamic equation of motion (for the average magnetization) of the kinetic Ising model in the presence of a sinusoidally oscillating magnetic field. By defining the order parameter as the time-averaged magnetization over a full cycle of the oscillating magnetic field they showed that the order parameter vanishes depending upon the value of the temperature and the amplitude of the oscillating field. Precisely, in the field amplitude and temperature plane they have drawn a phase boundary separating dynamic ordered (nonzero value of the order parameter) and disordered (order parameter vanishes) phases. They [7] have also observed and located a *tricritical point* [separating the nature (discontinuous or continuous) of the transition] on the phase boundary line. However, such a transition, observed [7] from the solution of the mean-field dynamical equation, is not dynamic in a true sense. This is because, for the field amplitude less than the coercive field (at a temperature less than the transition temperature without any field), the response magnetization varies periodically but asymmetrically even in the zero-frequency limit; the system remains locked to one well of the free energy and cannot go to the other one, in the absence of noise or fluctuation.

Lo and Pelcovits [8] attempted to study the dynamic nature of this phase transition (incorporating the effect of fluctuation) in the kinetic Ising model by a Monte Carlo (MC) simulation. In this case, the transition disappears in the zero-frequency limit; due to the presence of fluctuations, the magnetization flips to the direction of the magnetic field and the dynamic order parameter (time-averaged magnetization) vanishes. However, they [8] have not reported any precise phase boundary. Acharyya and Chakrabarti [9] studied the nonequilibrium dynamic phase transition in the kinetic Ising model in presence of oscillating magnetic field by extensive MC simulation. They [9] have successfully drawn the phase boundary for the dynamic transition and observed or located a tricritical point on it. It was also noticed by them [9] that this dynamic phase transition is associated with the breaking of the symmetry of the dynamic hysteresis (m-h) loop. In the dynamically disordered (the value of the order parameter vanishes) phase the corresponding hysteresis loop is symmetric and loses its symmetry in the ordered phase (giving a nonzero value of the dynamic order parameter). They [9] have also studied the temperature variation of the ac susceptibility components near the dynamic transition point. They observed that the imaginary (real) part of the ac susceptibility gives a peak (dip) near the dynamic transition point (where the dynamic order parameter vanishes). They [9] have the following conclusions: (i) This is a distinct signal of a phase transition and (ii) this is an indication of the thermodynamic nature of the phase transition.

Recently, the relaxation behavior of the dynamic order parameter near the transition point has been studied [10] both by MC simulation and by solving the mean-field dynamic equation. It has been observed that the relaxation is Debye type and the relaxation time diverges near the transition point, showing a *critical slowing down*. The ''specific heat'' and the ''susceptibility'' also diverge [11] near the transition point in a manner similar to that of fluctuations of the dynamic order parameter and energy, respectively.

The statistical distribution of the dynamic order parameter, near the dynamic transition point, has been studied by Sides *et al.* [12]. They observed that the distribution widens (to a double hump from a single hump type) as one crosses the dynamic transition phase boundary. Since the fluctuation

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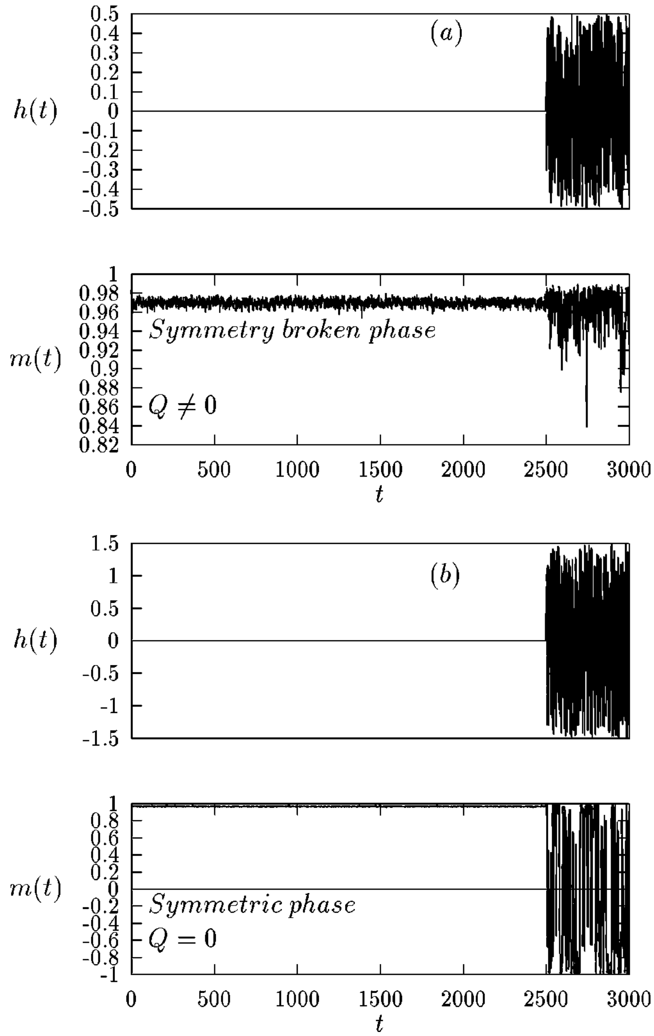

FIG. 1. Monte Carlo results of the time t variations of randomly varying field $h(t)$ and the response magnetization $m(t)$ for $T=1.7$ and (a) $h_0=1.0$ and (b) $h_0=3.0$. The symmetry (about the zero line) breaking is clear from the figure.

increases as the width of a distribution increases, this observation is consistent with the independent observation [11] of critical fluctuations of a dynamic order parameter. They [12] have also observed that the fluctuation of the hysteresis loop area becomes considerably large near the dynamic transition point.

In this paper the dynamic phase transition has been studied in the two-dimensional kinetic Ising model in the presence of a randomly varying (in time but uniform in space) magnetic field, both by MC simulation and by solving the mean-field dynamical equation. This paper is organized as follows. In Sec. II the model and the MC simulation scheme with the results are given. In Sec. III the mean-field dynamical equation and its solution with the numerical results are given. The paper ends with a summary of the work in Sec. IV.

II. MONTE CARLO STUDY

A. Model and simulation scheme

The Hamiltonian of an Ising model (with ferromagnetic nearest neighbor interaction) in the presence of a time-varying magnetic field can be written as

$$H=-\sum_{\langle ij\rangle} J_{ij}s_i^z s_j^z - h(t)\sum_i s_i^z. \tag{2.1}$$

Here s_i^z $(=\pm 1)$ is Ising spin variable, J_{ij} is the interaction strength, and $h(t)$ is the randomly varying (in time but uniform in space) magnetic field. The time variation of $h(t)$ can be expressed as

$$h(t)=\begin{cases} h_0 r(t) & \text{for} \quad t_0<t<t_0+\tau \\ 0 & \text{otherwise,}\end{cases} \tag{2.2}$$

where $r(t)$ is a random variable distributed uniformly between $-1/2$ and $+1/2$. The field $h(t)$ varies randomly from $-h_0/2$ to $h_0/2$ and

$$\frac{1}{\tau}\int_{t_0}^{t_0+\tau} h(t)dt=0. \tag{2.3}$$

The system is in contact with an isothermal heat bath at temperature T. For simplicity the values of all J_{ij} are taken to be equal to unity. The periodic boundary condition is used here.

A square lattice of linear size $L(=100)$ has been considered. Initially all spins are taken to be directed upward and $h(t)=0$. At any finite temperature T, the dynamics of this system has been studied here by Monte Carlo simulation using Metropolis single spin-flip dynamics. The transition rate $(s_i^z \rightarrow -s_i^z)$ is specified as

$$W(s_i^z \rightarrow -s_i^z)=\min[1,\exp(-\Delta H/k_BT)], \tag{2.4}$$

where ΔH is the change in energy due to the spin flip and k_B is the Boltzmann constant, which has been taken to be unity here for simplicity. Each lattice site is updated here sequentially and one such full scan over the lattice is defined as the time unit [Monte Carlo step per spin (MCSS)] here. The magnitude of the field $h(t)$ changes after every MCSS obeying Eq. (2.2). The instantaneous magnetization (per site) $m(t)=(1/L^2)\Sigma_i s_i^z$ has been calculated. After bringing the system to a steady state [$m(t)$ becomes stablized with some fluctuations], the switch of the randomly varying magnetic field $h(t)$ has been turned on (at time t_0 MCSS) and the instanteneous magnetization has been calculated. t_0 has been taken to be equal to 2×10^6 and even more (3.15×10^6) near the static ferro-para transition temperature ($\sim 2.269\ldots$). By inspecting the data and the time variation, it is observed that $m(t)$ becomes stabilized for this choice of the value of t_0. The time-averaged (over the active period of the magnetic field) magnetization $Q=(1/\tau)\int_{t_0}^{t_0+\tau} m(t)dt$ has been calculated over a sufficiently large time $\tau(1.75\times 10^6)$. By changing the values of τ the stabilization of Q has been checked quite carefully. It is observed that Q does not change much (apart from the small fluctuations) and this choice of the value of τ is considered to be good enough to believe that the value of Q becomes stable. Here Q plays the role of the dynamic order parameter. It may be noted that one measures the value of the order parameter, i.e., spontaneous magnetization, in the same way as in the case of a normal ferro-para transition. However, there is a remarkable difference. In the latter (or normal ferro-para) case, the system reaches a steady

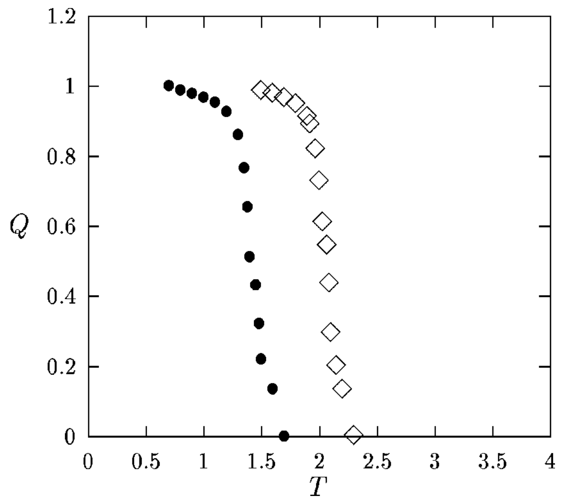

FIG. 2. Monte Carlo results of the temperature T variations of the dynamic order parameter Q for two different values of h_0: $h_0=2.4$ (●) and $h_0=0.8$ (◇).

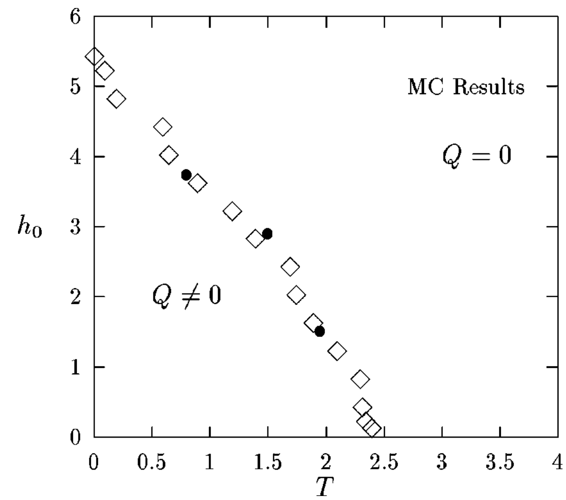

FIG. 3. Phase boundary for the dynamic transition obtained from MC simulations. The data points represented by ● are obtained by using random updating and those represented by ◇ are obtained by sequential updating.

equilibrium state; however, in the former (or present) case the state lies in a *nonequilibrium* (time-dependent) state. This dynamic order parameter Q is observed to be a function of width h_0 of the randomly varying magnetic field and the temperature T of the system, i.e., $Q=Q(h_0,T)$. Each value of Q has been calculated by averaging over at least 25 different random samples.

B. Results

Taking all spins to be up ($s_i^z=+1$) as the initial condition, the simulation was performed using the above form of the time-varying field $h(t)$. It has been observed numerically that, for a fixed values of h_0, if T increases, Q decreases continuously and ultimately vanishes at a fixed value of T. Similarly, for a fixed temperature, if h_0 increases, the value of Q decreases and finally vanishes at a particular value of h_0. Figure 1 shows the time variation of magnetization $m(t)$ at a particular temperature T and for two different values of field width h_0. For a small value of h_0, the system remains in a dynamically symmetry broken phase [Fig. 1(a)] where the magnetization oscillates asymmetrically about the zero line. As a result, the dynamical order parameter Q (the time-averaged magnetization) is nonzero. As the field increases, the system gets sufficient energy to flip dynamically in such a way that the magnetization oscillates symmetrically [Fig. 1(b)] about the zero line and as a result Q vanishes.

It is possible to let Q vanish by either increasing the temperature T for a fixed field width h_0 or vice versa. It has been observed that for any fixed value of h_0 the transition is continuous. Two such transitions (for two different values of h_0) are depicted in Fig. 2. So, in the plane formed by the temperature T and the field width h_0, one can think of a boundary line, below which Q is nonzero and above which it vanishes. Figure 3 displays such a phase boundary in the h_0-T plane obtained numerically by Monte Carlo simulation. The transition observed here is continuous irrespective of the values of h_0 and T, i.e., unlike the earlier cases [9,7], no *tricritical point* has been observed. It may be noted here that the transition temperature, for field width going to zero, reduces to the equilibrium (zero field) ferro-para transition point. In Fig. 3, the overestimated (from the Onsager value $T_c=2.269\ldots$) value of the transition temperature in the $h_0\to 0$ limit may be due to the small size of the system. It should be mentioned here that all the results are obtained by using sequential updating scheme. Although there is some study [13] regarding the updating scheme to simulate the dynamic processes, in the present study no significant deviation was found for these two different (sequential and random) updating techniques. Few data points of the phase boundary (marked by bullets in Fig. 3) are obtained by using the random updating scheme and no significant deviation was observed.

III. MEAN-FIELD STUDY

A. Mean-field dynamical equation and solution

The mean-field dynamical equation of motion for the average magnetization m [7] is

$$\frac{dm}{dt}=-m+\tanh\left(\frac{m(t)+h(t)}{T}\right), \tag{3.1}$$

where $h(t)$ is the randomly varying external magnetic field satisfying the condition

$$\frac{1}{\tau}\int_{t_0}^{t_0+\tau}h(t)dt=0, \tag{3.2}$$

with the same distribution $P(h)$ discussed earlier. Eq. (3.1) has been solved by employing the fourth-order Runge-Kutta method subjected to the above condition. The initial magnetization is set equal to unity, which serves as a boundary condition. First, the magnetization $m(t)$ has been calculated for $h(t)=0$, bringing the system into an equilibrium state. After that the switch of the randomly varying magnetic field has been turned on. The dynamic order parameter Q

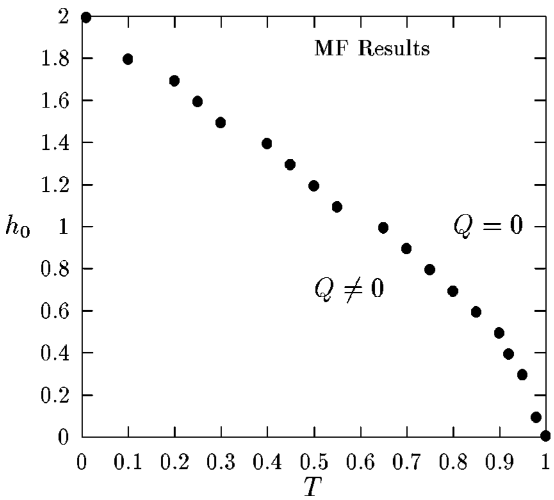

FIG. 4. Phase boundary for the dynamic transition obtained from the solution of the MF dynamic equation (3.1).

$=(1/\tau)\int_0^\tau m(t')dt'$ has been calculated from the solution $m(t)$ of Eq. (3.1). The Q is averaged over 25 different random samples.

B. Results

Observations similar to the MC case are made in this case. For quite small values of h_0 and T the systems remains in a dynamically asymmetric phase ($Q\neq 0$) and get into a dynamically symmetric ($Q=0$) phase for higher values of h_0 and T. It is observed that one can force Q to vanish continuously by tuning h_0 and T. A phase boundary line for the dynamic transition is shown in Fig. 4. Here also the limiting ($h_0\to 0$) transition temperature reduces to the MF equilibrium (zero-field) transition point ($T_c^{MF}=1$). It may be noted here that the coordination number z (= 4 in two dimension) has been absorbed in the interaction strength J in calculating the $T_c^{MF}(=1)$ and the temperature (shown here in Fig. 4) is measured in units of Jz.

IV. SUMMARY

The nonequilibrium dynamic phase transition in the two-dimensional kinetic Ising model in the presence of a randomly varying (in time but uniform over the space) magnetic field is studied both by Monte Carlo simulation and by solving the mean-field dynamical equation of motion. In both the cases, it is observed that the system remains in a dynamically symmetric phase ($Q=0$) for large values of the width of the randomly varying magnetic field and the temperature. By reducing the value of field width and temperature one can bring the system in a dynamically symmetry broken phase ($Q\neq 0$). The time-averaged magnetization, i.e., the dynamic order parameter, vanishes continuously depending upon the value of width of the randomly varying field and the temperature. The nature of the transition observed here is always continuous and the phase boundaries are drawn. It may be mentioned here that the dynamic responses of the Glauber kinetic Ising model are studied [9] for a quasiperiodic time variation of the magnetic field.

There is some experimental evidence of the dynamic transition. Recently, Jiang *et al.* [14] observed indications of the dynamic transition, associated with the dynamical symmetry breaking, in the ultrathin Co/Cu(001) sample put in a sinusoidally oscillating magnetic field by the magneto-optic Kerr effect. For small values of the amplitude of the oscillating field the m-h loop lies asymmetrically (dynamic ordered phase) in the upper half plane and becomes symmetric (dynamic disordered phase) for higher values of the field amplitude. Very recently, the dynamical symmetry breaking has been observed experimentally in highly anisotropic (Ising-like) ultrathin ferromagnetic Fe/W(110) films and is nicely depicted in Fig. 1 of Ref. [15]. However, the detailed quantitative study to draw the phase boundary has not yet been done experimentally.

Very recently, the stationary properties of the Ising ferromagnet in the presence of a randomly varying (having a bimodal distribution) magnetic field have been studied [16] in the mean-field approximation. The transition observed from the distribution of stationary magnetization is discontinuous. However, the present study deals with the *dynamical* properties (dynamical symmetry breaking) of the system and the nonequilibrium transitions in the Ising ferromagnet in the presence of a randomly varying (uniformly distributed) magnetic field. In the former case the transition was observed from the distribution of stationary magnetization. However, in the present study, this was observed from the temperature variation of the dynamic order parameter associated with a dynamical symmetry breaking and the transition observed is continuous. An extensive numerical effort is required to characterize and to know the details of this dynamical phase transition. It would also be important to see whether the different kinds of distributions of the randomly varying field would give different results.

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