Inhomogeneous broadening effects in a multiharmonic wiggler based optical klystron

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Multiharmonic wiggler based optical klystrons driven by a monoenergetic electron beam are known to be capable of producing a considerably higher gain than conventional optical klystrons. In this contribution, inhomogeneous broadening effects due to electron energy spread are considered for these multiharmonic optical klystrons. A modification to a recently developed convolution technique is derived to formulate the inhomogeneously broadened interaction gain in the small signal regime, taking account of the energy spread effects on electron bunching in the drift section. Based on the new gain formulation, numerical examples are used to demonstrate that the beam quality requirement of multiharmonic optical klystrons is essentially the same as that of their conventional counterparts. Thus for the same interaction gain, the gain enhancement achieved with a multiharmonic optical klystron configuration can be used to relax requirements for both the quality and current of the electron beam. In addition, it is suggested that the new formulation may be used to improve the accuracy of gain spectrum based techniques for beam quality measurement.
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I. INTRODUCTION

Conventional free electron lasers (FEL's) based on wiggler magnets of single period are well understood in terms of their interaction mechanisms and device performance. Their operation has been demonstrated over a very large portion of the spectrum from microwave to ultraviolet with their peak output power up to gigawatts. To enable a wider range of applications in medicine and industry, however, their performance for a given accelerator system needs to be improved still further and as such there has been much interest to explore novel beam-wave interaction mechanisms based on unconventional magnet or/and interaction cavity structures [1–5]. For instance, it was suggested to use a double wiggler system of similar periods to control the FEL spectrum in the high gain Compton regime [1,2]. A quasiperiodic wiggler configuration was also conceived for an effective control of both the strength and wavelength of its harmonic radiation in the low gain Compton regime [3]. In addition, a two-sectioned wiggler structure with two constituent parts having different field strengths and periodicities was found useful for mode selection purpose in low gain waveguide FEL's [4,5].

The most commonly used unconventional wiggler configuration is an optical klystron (OK), which employs a drift section between two essentially identical wiggler magnets to enhance the electron beam bunching and thus increase the small signal gain [6]. To optimize the electron beam bunching, it was proposed recently to replace the first wiggler of usually single periodicity with an alternative modulator having a series of harmonically related periodicities [7]. With the same drift section and radiator (the second wiggler), it was shown that this alternative arrangement is capable of increasing the small signal gain up to 75%. Such an arrangement is known as a multiharmonic optical klystron (MHOK) and one of its advantages is to use the elevated gain to relax the beam quality requirement. However the electron energy spread effects were not taken into account in the gain formulation of MHOK's [7] and as a result it remains to be answered whether or not the gain enhancement in a multiharmonic optical klystron is achieved at the expense of a more stringent beam quality requirement.

One major consequence of electron energy spread is to cause the interaction gain to reduce and the gain spectrum to undergo an inhomogeneous broadening [8–10]. Based on a convolution technique [11], analytical formulation of the inhomogeneous broadening effects has recently been obtained for both free electron lasers [12,13] and optical klystrons [14,15]. Nevertheless, these analytical treatments have so far ignored the energy spread effects on electron bunching in the drift section. Since it is mainly in the electron beam bunching process that a multiharmonic optical klystron differs from its conventional counterpart, this effect needs to be taken into account. To this end, an extension of the convolution technique [11] is suggested here to formulate the inhomogeneous broadening effects in multiharmonic optical klystrons taking account of the electron bunching dependence on energy spread. Based on the gain formulation derived, it is shown through numerical examples that the beam quality requirement of multiharmonic optical klystrons is essentially the same as that of conventional optical klystrons. Therefore, the gain enhancement achieved with a MHOK configuration can be realistically realized in practice, thus permitting a freedom to relax requirements for both the quality and current of the electron beam. In addition, it is shown that the newly developed gain formulation results in a slightly different spectrum broadening from that derived with some approximations but nevertheless widely used in practice [8,16]. Discussion suggests that the new gain formulation may be used to derive a more accurate diagnostic tool for beam quality measurement.

II. THE EFFECTS OF INITIAL ELECTRON ENERGY SPREAD ON BEAM BUNCHING

The dependence of electron bunching on the beam's initial energy spread in optical klystrons has not been analytically formulated, possibly because it is considered to make a relatively small contribution to the inhomogeneous broadening effects in OK devices [8,10,14]. However, since a finite initial energy spread will alter, to some degree, the electron bunching process in the drift section of an optical klystron, it is possible that such an alteration may become significant under some operation conditions. Thus for a more complete assessment of the inhomogeneous broadening effects in optical klystrons, it is in general desirable to take account of the initial energy spread in the formulation of electron beam bunching. This is particularly important for the analysis of the inhomogeneous broadening effects in multiharmonic optical klystrons since it is predominately in the electron bunching process that they differ from conventional optical klystrons.

To demonstrate the initial energy spread effects on electron bunching in multiharmonic optical klystrons, we consider a simple case in which the modulator contains the fundamental and the second harmonics only. The methodology developed below should, however, apply to any multiharmonic wiggler based optical klystron, albeit with a more complicated algebra. Suppose such a modulator of length L has an on-axis magnetic field of

$$\mathbf{B}_w=\hat{\mathbf{y}}(B_{w1}\cos k_w z+B_{w2}\cos 2k_w z), \tag{1}$$

with its dimensionless field strength parameters $a_{wn}=eB_{wn}/mc(nk_w)$ $(n=1,2)$, and an on-axis laser field of

$$\mathbf{E}_s=-\hat{\mathbf{x}}(E_1\cos\Phi_1+E_2\cos\Phi_2), \tag{2a}$$

$$\mathbf{B}_s=-\hat{\mathbf{y}}(B_1\cos\Phi_1+B_2\cos\Phi_2), \tag{2b}$$

with $E_n=(n\omega/nk)B_n$, $\Phi_n=n\omega t-nkz+\phi_n$, and the dimensionless field strength parameter $a_{sn}=eE_n/mc(n\omega)$ $(n=1,2)$. In addition, the drift section is assumed to be a straightforward free space of length D. Furthermore, we assume that the radiator consists of a conventional wiggler of period $\lambda_w=2\pi/k_w$ and length $L=N_w\lambda_w$, and a radiation field at $f=\omega/2\pi$ with its dimensionless field strength parameter $a_s=eE_0/mc\omega$.

In the small signal regime, the energy exchange between an electron beam and its amplifying laser field in an optical klystron is usually considered to be significant only at the second, or higher, order of the laser field [8–10]. However, with a sizeable density modulation formed in the drift section of the OK, the beam-wave interaction in its radiator can be significant at the first order of the laser field [7,17]. For a monoenergetic electron beam in a MHOK, its energy loss to the first order of the radiation field may be expressed as [7]

$$\langle\Delta\gamma\rangle=-\frac{a_w a_s}{\gamma\beta_z}\frac{\omega L}{2c}\frac{\sin(\Delta kL/2)}{(\Delta kL/2)}\sin\theta_0 g(\Delta kL), \tag{3}$$

where $\Delta kL=(\omega/v_z-k-k_w)L$ is the FEL detuning parameter in one wiggler section, $\theta_0=\omega D/c\beta_z$ is the electron transit angle through the drift section, and

$$g(\Delta kL)=\frac{1}{2\pi}\int_0^{2\pi}\cos\left[x+\sum_{n=1}^{2}M_n\theta_0\sin nx\right]dx, \tag{4a}$$

$$M_n=\frac{na_{wn}a_{sn}L}{2\gamma^2\beta_z}\left[(k_w+k)-\frac{\omega}{c}\beta_z\right]\mathrm{sinc}\left[\frac{n\Delta kL}{2}\right], \tag{4b}$$

where $\mathrm{sinc}(x)=\sin(x)/x$. It is worth mentioning that $g(\Delta kL)$ represents the strength of the electron bunching and as such it is referred to as the bunching strength function.

To maximize the interaction gain, an optical klystron is usually operated with $\Delta kL=0$ (at the resonant electron energy γ_r) and $\sin\theta_0=1$ (the optimum phase at the radiator entrance) satisfied simultaneously [7]. Note that

$$\frac{\omega D}{c\beta_z}=[\Delta k+(k+k_w)]D=(\Delta kL)\frac{D}{L}+(k+k_w)D, \tag{5}$$

thus $\sin\theta_0=\sin(k+k_w)D=1$ needs to be satisfied at $\Delta kL=0$. So for $\gamma\neq\gamma_r$, $\sin\theta_0$ may be expressed as

$$\sin\theta_0=\sin\left[(\Delta kL)\frac{D}{L}+\frac{\pi}{2}\right]=\cos\left[(\Delta kL)\frac{D}{L}\right]. \tag{6}$$

Similarly $M_n\theta_0$ of Eq. (4) may be expressed as

$$M_n\theta_0=\chi_n^r\xi(\gamma)\ \mathrm{sinc}(\Delta kL/2), \tag{7}$$

where the superscript r denotes resonance and

$$\chi_n^r=\frac{na_{wn}a_{sn}}{2\gamma_r^4\beta_{zr}^3}\left(\frac{\omega L}{c}\right)^2\frac{D}{L}, \tag{8a}$$

$$\xi(\gamma)=\gamma_r^4\beta_{zr}^2(1-\beta_{zr}\beta_z)/\gamma^2\beta_z^2. \tag{8b}$$

Consequently Eq. (3) becomes

$$\langle\Delta\gamma\rangle=-\frac{a_w a_s}{\gamma\beta_z}\frac{\omega L}{2c}\,\mathrm{sinc}\,\frac{\Delta kL}{2}\cos\frac{m\Delta kL}{2}\,g(\Delta kL) \tag{9}$$

with $m=2D/L$ and

$$g(\Delta kL)=\frac{1}{2\pi}\int_0^{2\pi}\cos\left(x+\sum_{n=1}^{2}\chi_n^r\xi(\gamma)\times\mathrm{sinc}(\Delta kL/2)\sin nx\right)dx. \tag{10}$$

It should be mentioned that if $\chi_2^r=0$ is specified in Eq. (10), Eq. (9) may be used to calculate the first-order interaction gain of a conventional optical klystron having a sizeable density modulation in the small signal limit.

Equation (9) is derived for a monoenergetic electron beam. If the electron beam has an initial energy spread, however, the gain will be reduced through terms dependent upon the electron energy, which, in the case of Eq. (9), are namely $(\gamma\beta_z)^{-1}$, $\mathrm{sinc}(\Delta kL/2)$, $\cos(m\Delta kL)$, and $g(\Delta kL)$. Of these four terms, $(\gamma\beta_z)^{-1}$ affects the gain magnitude only and for relativistic electron beams its dependence on the electron energy spread may be considered negligible [8–15]. $\mathrm{sinc}(\Delta kL/2)$ represents the electron beam's spontaneous emission in one wiggler section, whereas $\cos(m\Delta kL/2)$ represents the interference between radiations from the two wiggler sections and it depends crucially on the electron phase at the entrance of the second wiggler section. These two terms are affected by the electron energy spread considerably and they have been considered [8,10,14]. $g(\Delta kL)$, on the other hand, contains information about the electron bunching in the drift section and its dependence on the initial energy spread has been ignored in previous analytical studies [8,10,14]. This dependence will be considered here to understand possible differences in the inhomogeneous broadening effects between multiharmonic and conventional optical klystrons.

Suppose the initial energy spread of the electron beam has a Gaussian distribution

$$f(\epsilon)=\frac{1}{\sqrt{2\pi}\sigma_\epsilon}\exp\left(-\frac{\epsilon^2}{2\sigma_\epsilon^2}\right), \tag{11}$$

where $\epsilon=(\gamma-\gamma_0)/\gamma_0$ is the relative deviation from the nominal electron energy, γ_0, and σ_ϵ is the rms energy spread. The gain for a nonmonoenergetic electron beam is the convolution of the gain for a monoenergetic beam, Eq. (9), on the energy distribution of Eq. (11). Mathematically this convolution is an integral from $\epsilon=-\infty$ to $\epsilon=+\infty$. But if the gain for a monoenergetic electron beam can be expressed by a parametric integral of

$$\langle(\Delta\gamma)\rangle=\int_0^1 \mathcal{E}(\Delta kLt)dt, \tag{12}$$

the required convolution of an infinite integral may alternatively be expressed by a finite integral [11], permitting a much more efficient numerical estimate of the convolution and hence the interaction gain for nonmonoenergetic electron beams. It has been shown that many sinusoidal functions can be expressed by a finite parametric integral [11,13,14] and this technique has been applied to both FEL's [11–13] and conventional optical klystrons [14,15].

For multiharmonic optical klystrons, however, $g(\Delta kL)$ of Eq. (10) is not a sinusoidal function and so it is found not possible to express the exact bunching strength function in the form of Eq. (12). To overcome this, we first consider $g(\Delta kL)$ as a function of two variables, $S=\mathrm{sinc}(\Delta kL/2)$ and $T=\mathrm{sinc}(2\Delta kL/2)$, and Taylor expand it,

$$\begin{aligned} g(\Delta kL)\approx\; & g(\Delta k_0L)+g_1'(\Delta k_0L)(S-S_0)\\ & +g_2'(\Delta k_0L)(T-T_0), \end{aligned} \tag{13}$$

around its nominal value at $S=S_0=\mathrm{sinc}(\Delta k_0L/2)$ and $T=T_0=\mathrm{sinc}(\Delta k_0L/2)$ with Δk_0L calculated at γ_0 and

$$g_l'(\Delta k_0L)=-\frac{\chi_l^r\xi(\gamma_0)}{2\pi}\int_0^{2\pi}\sin lx\,\sin\left(x+\sum_{n=1}^{2}\chi_n^r\xi(\gamma_0)\mathrm{sinc}\left[\frac{n\Delta k_0L}{2}\right]\sin(nx)\right)dx \tag{14}$$

with $l=1,2$. If we denote

$$A(\Delta k_0L)=g(S_0,T_0)-S_0g_1'(S_0,T_0)-T_0g_2'(S_0,T_0), \tag{15}$$

then the bunching strength function becomes

$$\begin{aligned} g(\Delta kL)=\; & A(\Delta k_0L)+g_1'(\Delta k_0L)\mathrm{sinc}(\Delta kL/2)\\ & +g_2'(\Delta k_0L)\mathrm{sinc}(\Delta kL). \end{aligned} \tag{16}$$

Note that in the above equation, $A(\Delta k_0L)$, $g_1'(\Delta k_0L)$, and $g_2'(\Delta k_0L)$ are all calculated at the nominal electron energy. Thus the electron energy spread affects the bunching strength function only through the $\mathrm{sinc}(\Delta kL/2)$ and $\mathrm{sinc}(\Delta kL)$ terms. When Eq. (16) is substituted into Eq. (9), the gain becomes a function of sinusoidal terms only and this allows the gain to be expressed in the form of Eq. (12), as will be shown in the next section.

It should be emphasized, however, that although the Taylor expansion technique allows the gain to be expressed by a finite parametric integral, this is essentially an approximation and therefore its validity needs to be examined first. To this end, we note that the change of ΔkL due to an electron energy change, $\delta\gamma$, is given by

$$\delta(\Delta kL)=\delta\left[\frac{\omega L}{c\beta_z}\right]=-\frac{4N_w\pi}{\beta_{zr}^2}\left[1+\frac{\Delta k_0L}{2N_w\pi}\right]\frac{\delta\gamma}{\gamma} \tag{17}$$

and thus the actual FEL detuning parameter can be calculated using $\Delta kL=\Delta k_0L+\delta(\Delta kL)$. This may be used to compare the bunching strength function of Eq. (10) and its approximation of Eq. (16). In Fig. 1, the bunching strength function and its Taylor expansion are plotted against the nominal FEL detuning parameter, Δk_0L, for $\gamma_0=100$, $N_w=10$, $\delta\gamma/\gamma=5\times10^{-3}$, $\chi_1^r=1.92$, and $\chi_2^r=0.81$. It is clear from Fig. 1 that Eq. (16) represents an excellent approximation of $g(\Delta kL)$ for $\delta\gamma/\gamma\leqslant5\times10^{-3}$. It should be noted, however, that an identical energy shift of 0.5% is assumed for all electrons in Fig. 1. For an electron beam with a rms

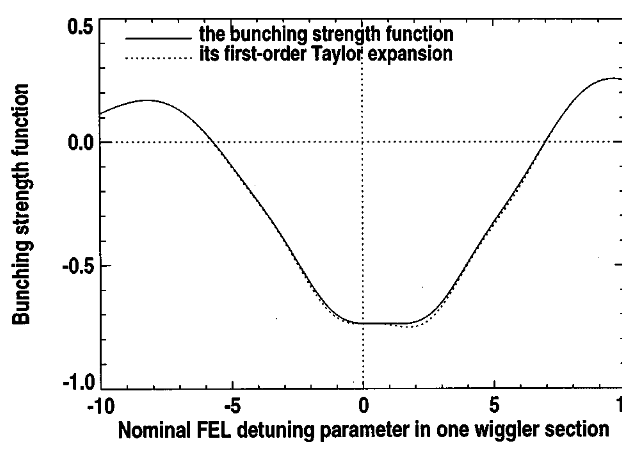

FIG. 1. Validation of the Taylor expansion approximation of $g(\Delta kL)$ for $\gamma_0=100$, $N_w=10$, and $\delta\gamma/\gamma=5\times10^{-3}$.

energy spread of $\sigma_\epsilon = 5\times10^{-3}$, a large percentage of electrons has an energy deviation of less than 0.5% from their nominal value. Thus Fig. 1 overestimates the deviation of Eq. (16) from the actual bunching strength function and so Eq. (16) should in practice be an even better approximation than suggested in Fig. 1.

III. OPTICAL KLYSTRON GAIN FOR NONMONOENERGETIC ELECTRON BEAMS

With the bunching strength function expressed by the two (ΔkL) dependent sinusoidal functions in Eq. (16), the averaged electron energy loss of Eq. (9) has now three ΔkL dependent sinusoidal functions, namely,

$$\mathcal{K}_1(\Delta kL) = \mathrm{sinc}(\Delta kL/2)\cos(m\Delta kL/2), \tag{18a}$$

$$\mathcal{K}_2(\Delta kL) = \mathrm{sinc}^2(\Delta kL/2)\cos(m\Delta kL/2), \tag{18b}$$

$$\mathcal{K}_3(\Delta kL) = \mathrm{sinc}(\Delta kL)\,\mathrm{sinc}\left[\frac{\Delta kL}{2}\right]\cos\left[\frac{m\Delta kL}{2}\right]. \tag{18c}$$

These three sinusoidal functions may be expressed in the form of Eq. (12). If we introduce

$$U_l(x) = \frac{l}{2}\int_0^1 \cos(lxt)\,dt, \tag{19a}$$

$$V_l(x) = \frac{l^2}{4}\int_0^1 (1-t)\cos(lxt)\,dt \quad (l=1,2,3), \tag{19b}$$

it can be shown that

$$\mathcal{K}_1(x) = U_{m+1}(x) - U_{m-1}(x), \tag{20a}$$

$$\mathcal{K}_2(x) = V_{m+2}(x) + V_{m-2}(x) - V_{2m}(x), \tag{20b}$$

$$\mathcal{K}_3(x) = [V_{m+3}(x) + V_{m-3}(x) - V_{m+1}(x) - V_{m-1}(x)]/2. \tag{20c}$$

For the simplicity of mathematical expression, we further introduce

$$\mathcal{P}_l = \frac{l}{2}\int_0^1 \cos\left[\frac{l(\Delta k_0 L)t}{2}\right]\exp\left[-\frac{l^2\mu_\epsilon^2 t^2}{8}\right]dt, \tag{21a}$$

$$\mathcal{R}_l = \frac{l^2}{4}\int_0^1 (1-t)\cos\left[\frac{l(\Delta k_0 L)t}{2}\right]\exp\left[-\frac{l^2\mu_\epsilon^2 t^2}{8}\right]dt, \tag{21b}$$

where $l=1,2,3$ and $\mu_\epsilon = \alpha_0\sigma_\epsilon$. By noting that

$$\Delta kL \approx \alpha_0\left(\frac{\gamma-\gamma_0}{\gamma_0} + \frac{\gamma_0-\gamma_r}{\gamma_0}\right) = \alpha_0(\epsilon+\epsilon_r), \tag{22a}$$

$$\alpha_0 = (\omega/c)L/(\gamma_0^2\beta_z^3), \tag{22b}$$

the convolution of the three functions in Eq. (20) on the distribution of Eq. (11),

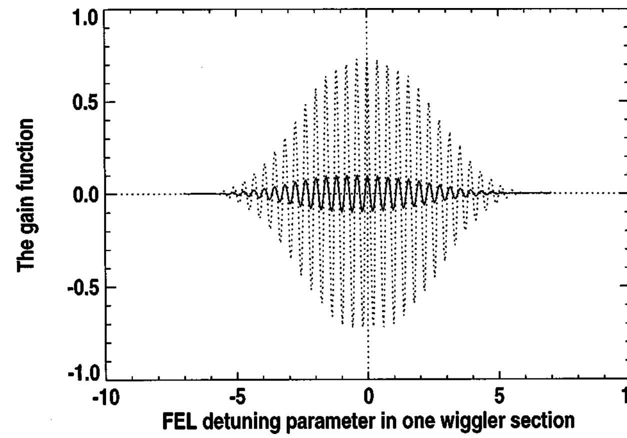

FIG. 2. The gain function with $\sigma_\epsilon = 10^{-3}$ (solid curve) and $\sigma_\epsilon = 0$ (dotted curve) for $\gamma_0 = 100$, $N_w = 10$, and $D/L = 16$.

$$F_l = \int_{-\infty}^{+\infty} \mathcal{K}_l(\Delta kL) f(\epsilon)\,d\epsilon \quad (l=1,2,3), \tag{23}$$

may be shown to be

$$F_1 = \mathcal{P}_{m+1} - \mathcal{P}_{m-1}, \tag{24a}$$

$$F_2 = \mathcal{R}_{m+2} - \mathcal{R}_{m-2} - 2\mathcal{R}_m, \tag{24b}$$

$$F_3 = [\mathcal{R}_{m+3} + \mathcal{R}_{m-3} - \mathcal{R}_{m+1} - \mathcal{R}_{m-1}]/2. \tag{24c}$$

Consequently the interaction gain for nonmonoenergetic electron beams is obtained as

$$\begin{aligned}\langle(\Delta\gamma)\rangle = -\frac{a_w a_s}{\gamma_r\beta_{zr}}\frac{\omega L}{2c}\,\zeta(\Delta k_0 L)\{&A(\Delta k_0 L)F_1(\Delta k_0 L,\mu_\epsilon)\\ &+ g_1'(\Delta k_0 L)F_2(\Delta k_0 L,\mu_\epsilon)\\ &+ g_2'(\Delta k_0 L)F_3(\Delta k_0 L,\mu_\epsilon)\},\end{aligned} \tag{25}$$

where $\zeta(\Delta k_0 L) = 1 - \beta_{zr}^2(\Delta k_0 L)/4N_w\pi$. It should be mentioned that although Eq. (25) is derived for multiharmonic wiggler optical klystrons, it is also applicable to conventional optical klystrons when $\chi_2^r = 0$ is specified.

IV. APPLICATIONS

To illustrate the inhomogeneous broadening effects on both the spectrum and the magnitude of the interaction gain in Eq. (25), we introduce a gain function defined as

$$\mathcal{G}(\Delta k_0 L,\mu_\epsilon) = \frac{-\langle(\Delta\gamma)\rangle}{(a_w a_s/\gamma_r\beta_{zr})(\omega L/2c)}. \tag{26}$$

We first consider a multiharmonic wiggler based optical klystron driven by an electron beam of $\gamma_0 = 100$ with $N_w = 10$ and $D/L = 16$. With the electron beam bunching optimized at $\chi_1^r = 1.92$ and $\chi_2^r = 0.81$ [7], the gain function for this MHOK is calculated from Eq. (26) and plotted as a function of $\Delta k_0 L$ for both $\sigma_\epsilon = 0$ and $\sigma_\epsilon = 10^{-3}$ in Fig. 2. It is clear that the interaction gain is reduced considerably in the presence of a finite energy spread. At $\sigma_\epsilon = 10^{-3}$, the peak value of the gain function at $\Delta k_0 L = 0$ is reduced to 0.01 from 0.74 at σ_ϵ

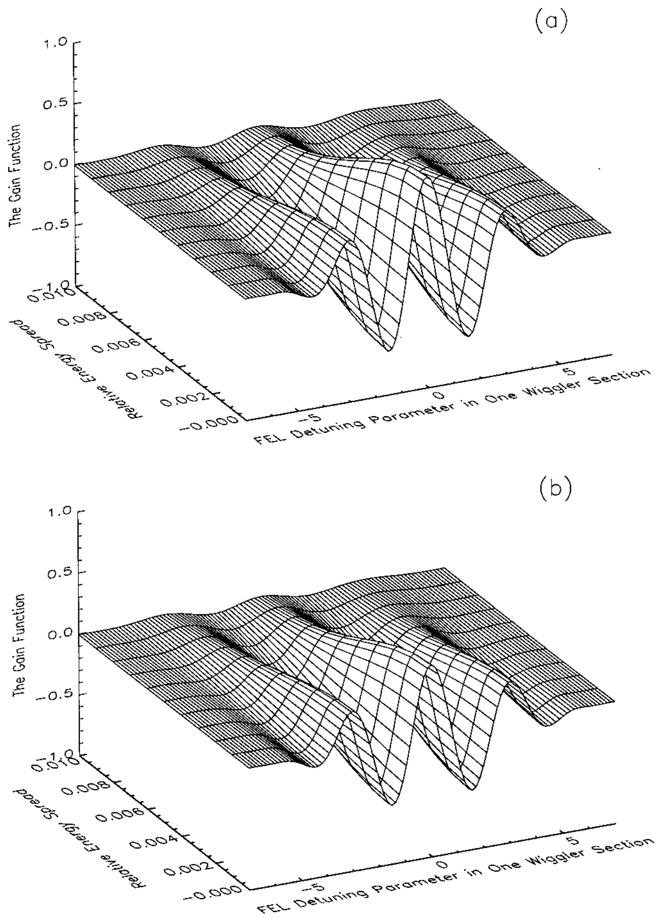

FIG. 3. The optimized gain function for (a) a MHOK and (b) its conventional counterpart with $D/L=2$.

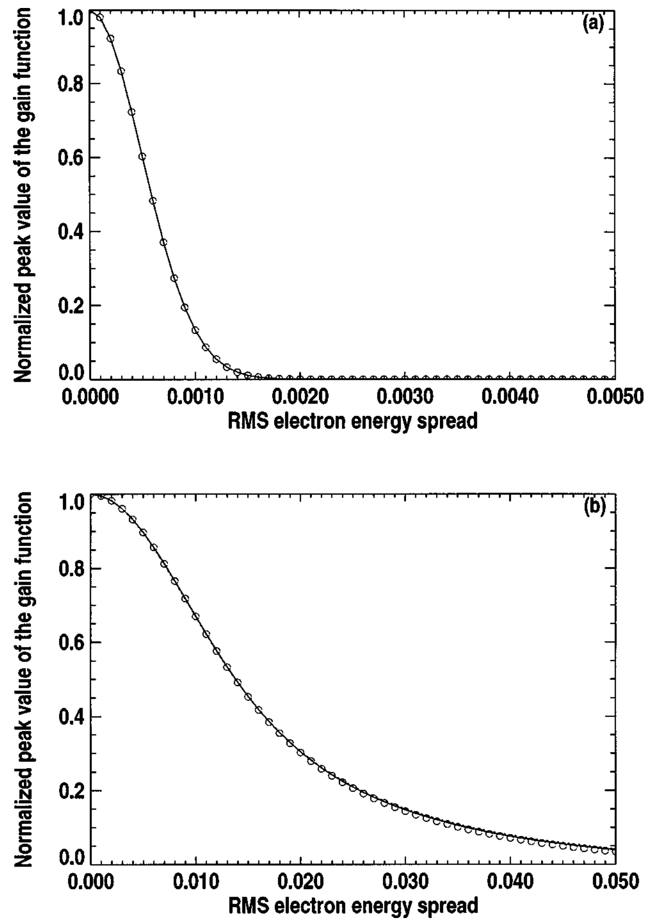

FIG. 4. Peak gain function at $\chi_1^r=1.92$ against σ_ϵ for (a) $D/L=16$ and (b) $D/L=0.7$. The solid curve is for a MHOK ($\chi_2^r=0.81$), whereas the circles and the dashed curve are for a conventional OK ($\chi_2^r=0$) with its $g(\Delta kL)$ calculated using the nominal value and Eq. (16), respectively.

$=0$, a reduction of 87%. For the corresponding conventional OK optimized at $\chi_1^r=1.84$ ($\chi_2^r=0$), calculation using Eq. (26) indicates the same peak gain reduction of 87%.

If the optimized bunching (at $\chi_1^r=1.92$ and $\chi_2^r=0.81$) is achieved over a shorter drift length, the inhomogeneous broadening effects become less severe. In Fig. 3, the optimized gain of a MHOK is plotted for $D/L=2$ and compared to the case of its conventional counterpart. For an energy spread of $\sigma_\epsilon=10^{-3}$, the gain function of the MHOK at $\Delta k_0 L=0$ is reduced to 0.71 from 0.74 for the monoenergetic case, representing a reduction of only 4.2%, whereas for the conventional OK this reduction is slightly less at 3.4% with the gain function at $\Delta k_0 L=0$ decreased to 0.56 from 0.58 at $\sigma_\epsilon=0$.

Also shown in Fig. 3 is a very similar trend of gain degradation for the two different types of optical klystrons. To illustrate this comparison more clearly, the peak value of the gain function under the optimized bunching condition of $\chi_1^r=1.92$ and $\chi_2^r=0.81$ [7] is calculated in the unit of its value for a monoenergetic electron beam and plotted in Fig. 4 as a function of σ_ϵ for two drift lengths. For the case of $D/L=16$ in Fig. 4(a), the gain of the MHOK is seen to have an identical energy spread dependence to that of its conventional counterpart. Further calculation suggests that this agreement is also true for longer drift lengths. When a shorter drift section of $D/L=0.7$ is used, the two optical klystrons become slightly different in their energy spread dependences for $\sigma_\epsilon \geqslant 0.04$ as indicated in Fig. 4(b). However, since it is unlikely to operate an optical klystron with the electron beam quality worse than $\sigma_\epsilon=0.04$, the peak interaction gain for both types of optical klystrons may be considered to experience the same degradation in practical devices.

It is of interest to note that in Fig. 4 the peak gain calculated with the nominal $g(\Delta kL)$ is in an excellent agreement with that calculated with the Taylor expansion technique for $\sigma_\epsilon \leqslant 0.04$. This implies that at $\Delta k_0 L=0$ the energy spread effects on the bunching strength function are negligible. The above observation about the peak gain agrees with the findings of our previous study reached with a phenomenological argument [7]. The implication is that around $\Delta k_0 L=0$ the presence of an electron energy spread affects the beam-wave interaction predominately through the $\cos(m\Delta kL/2)$ and $\mathrm{sinc}(\Delta kL/2)$ terms in Eq. (9), whereas its effects on $g(\Delta kL)$, or the electron beam bunching, are much less significant.

The inhomogeneous broadening effects in optical klystrons used for storage ring free electron lasers are often considered in terms of the following dependence of the peak gain on the electron energy spread [8,16,18]:

$$G \propto \exp[-8\pi^2 N_w^2 (1+D/L)^2 \sigma_\epsilon^2]. \tag{27}$$

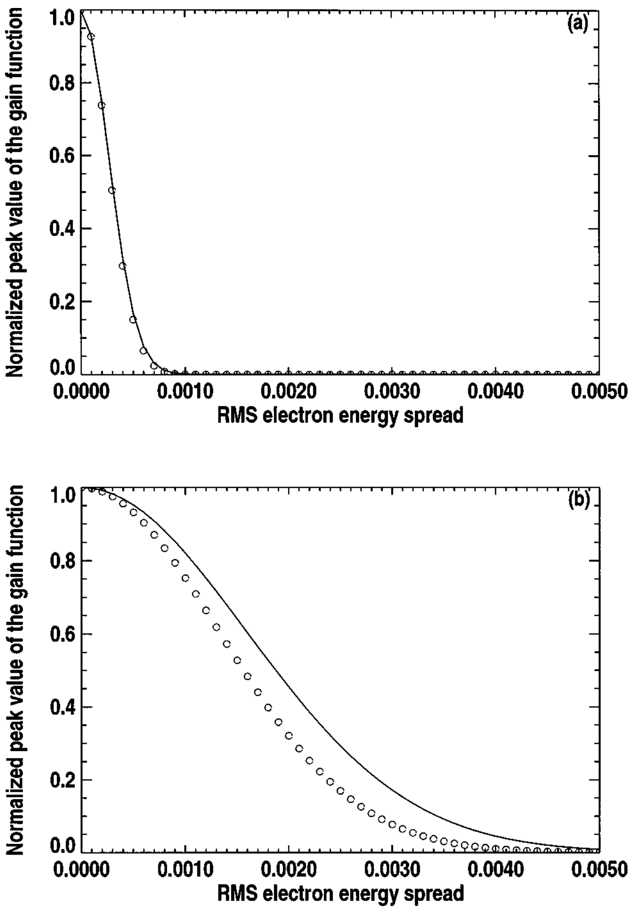

FIG. 5. Peak gain function in an MHOK against σ_ϵ for (a) $D/L=30$ and (b) $D/L=5$. The solid curve and circles are calculated from Eqs. (26) and (27), respectively.

It is therefore of interest to compare this energy spread dependence with that in Eq. (25). Since most storage ring FEL's are driven by a high-quality electron beam and the optical klystron arrangement used usually employs a very large effective D/L, we first consider a MHOK and its conventional counterpart both with a drift section of $D/L=30$. As shown in Fig. 5(a), the exponential dependence of Eq. (27) agrees very well with our calculation. For a shorter drift section, however, Fig. 5(b) shows an appreciable disagreement at $D/L=5$ at large values of the energy spread especially when $\sigma_\epsilon \geqslant 10^{-3}$. In view of the fact that Eq. (27) is derived under the assumption of $\sigma_\epsilon \ll 1$ [8], its application to cases with a sizeable initial energy spread appears to overestimate the actual gain degradation as indicated in Fig. 5(b). In other words, the newly developed gain formulation of Eq. (25) represents an improved account, from Eq. (27), of the gain degradation in optical klystron devices. One benefit of this improved formulation is that it may be used to develop a more accurate diagnostic tool for measuring the electron energy spread in storage rings for a wider range of energy spread.

Having now established that the degradation of the peak gain in a MHOK is no worse than that in its conventional counterparts, one can take advantage of the gain enhancement achieved with a MHOK arrangement to relax requirements on either electron beam quality or electron beam current. For instance, an electron beam of modest quality at a given current may be employed to provide the same interaction gain in a MHOK as what is needed in a conventional optical klystron arrangement. As suggested in Fig. 6, a gain function with its peak value at 0.57 requires a beam quality better than $\sigma_\epsilon = 4.4\times 10^{-5}$ with a conventional optical klystron, whereas a MHOK configuration allows the required beam quality to be relaxed to $\sigma_\epsilon = 2.8\times 10^{-4}$. Such a relaxation on the beam quality requirement should permit the same FEL performance to be achieved with a less expensive accelerator system.

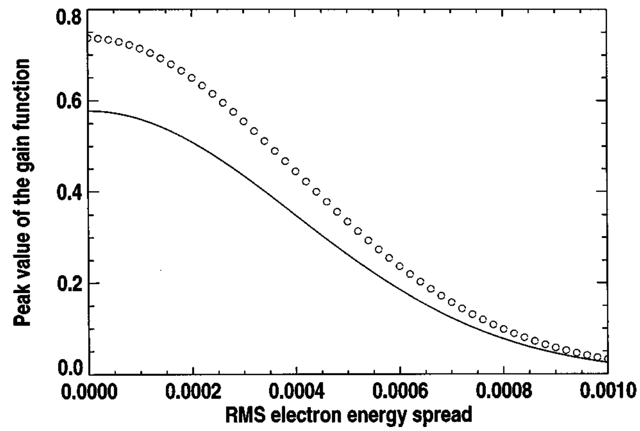

FIG. 6. Peak value of the gain function for an MHOK (dashed curve) and its conventional counterpart (solid curve), both with $D/L=20$, $\chi_1^r=1.92$, and $\chi_2^r=0.81$.

Our discussion thus far has mainly been concerned with the degradation of the peak interaction gain (at $\Delta k_0 L=0$), for which it is established that the energy spread effects on the bunching strength function may be ignored. Such an approximation, however, is less accurate for the interaction gain under nonresonance conditions (when $\Delta k_0 L\neq 0$). Figure 7 illustrates the gain function of a MHOK at $\Delta k_0 L =4.63$ calculated with and without the energy spread effects on the electron beam bunching. It is shown in Fig. 7 that the gain degradation for $\Delta k_0 L\neq 0$ is underestimated markedly if the energy spread effects on the electron beam bunching are ignored. Calculation for conventional optical klystrons suggests a similar underestimate if the bunching strength func-

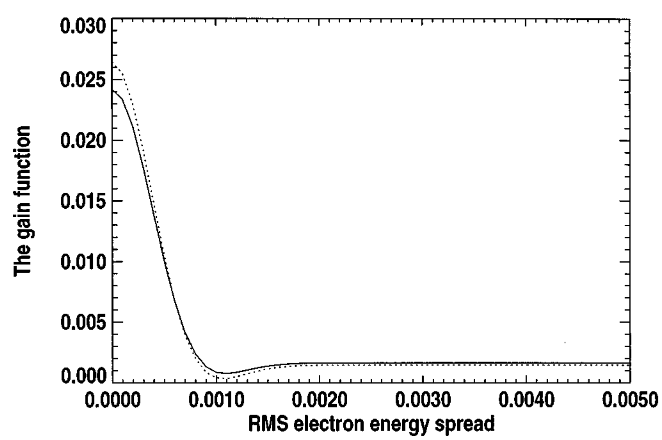

FIG. 7. The gain function optimized at $\chi_1^r=1.92$ and $\chi_2^r=0.81$ against σ_ϵ for $D/L=16$ and $\Delta k_0 L=4.63$. The solid and dashed curves are obtained with $g(\Delta kL)$ calculated from Eq. (16) and its nominal value, respectively.

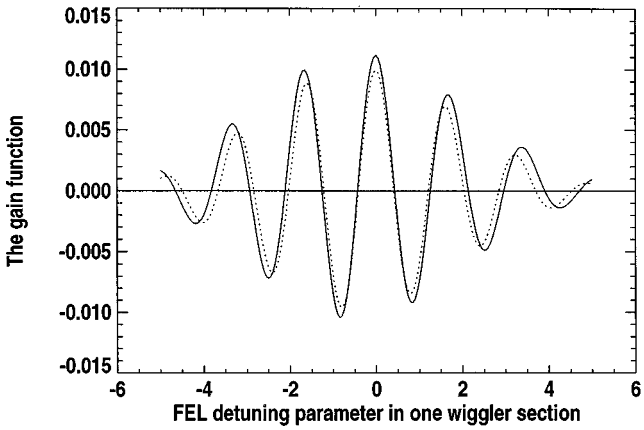

FIG. 8. The gain spectrum for $D/L=4$, $\sigma_\epsilon=5\times10^{-3}$, $\chi_1^r=0.4$, and $\chi_2^r=0.2$. The solid and dashed curves are obtained with $g(\Delta kL)$ calculated from Eq. (16) and its nominal value, respectively.

tion is calculated using the nominal electron energy. Therefore when considering the gain degradation for $\Delta k_0L\neq0$, the energy spread effects on the bunching strength function do need to be taken into account using the gain formulation in Eqs. (16), (24), and (25). This is important for an accurate picture of the FEL gain spectrum covering a wide range of Δk_0L as illustrated in Fig. 8, and as a result it bears an interesting implication to gain spectrum based diagnostic techniques. For instance, the spectrum of either the spontaneous emission or the interaction gain in optical klystron devices is often used to measure beam quality in storage rings [16,18]. It is of interest to note that there are experimental conditions under which the existing beam quality diagnostic technique based on Eq. (27) is not very accurate [19]. The formulation of the energy spread effects on beam bunching developed in this study should permit a more accurate measurement of beam quality. In fact, one can now measure the beam quality over a wider range (for an rms energy spread as large as 5×10^{-3} in the case of Fig. 8) even with a compact optical klystron (shorter drift section and hence smaller χ_1^r).

V. CONCLUSION

The inhomogeneous broadening effects in multiharmonic optical klystron devices have been analyzed. With a Taylor expansion technique, it has been demonstrated that one can express the bunching strength function in terms of sinusoidal functions and as a result the energy spread effects on the electron beam bunching have been formulated into the interaction gain analytically. Based on the newly developed gain formulation, numerical examples have been used to demonstrate that the degradation of the peak interaction gain in a MHOK is very similar to that in its conventional counterpart, suggesting that MHOK's have essentially the same beam quality requirement as that of conventional OK's. Therefore, for the same interaction gain, the gain enhancement achieved with the multiharmonic wiggler arrangement may be used to relax the requirements on either the quality or the current of the electron beam used, permitting the possibility of achieving the same FEL performance with a less expensive accelerator system.

The multiharmonic wiggler configuration was originally conceived for optical klystron applications [7] and thus its feasibility has been discussed for optical free electron lasers employing high-energy electron beams. However the basic concept may be easily extended to FEL devices driven by lower electron energies. One example is its possible implementation in the waveguide optical klystron configuration typically driven by electron beams of less than 1 MV ($\gamma<3$) [15]. It is worth noting that compact FEL's driven by lower energy electron beams are in general operated in the frequency range of 1–300 GHz where other radiation sources are available. Therefore, if such electromagnetic fields are chosen to be multiharmonically related, they may be used as the modulation signals to achieve the function of the multiharmonic wiggler configuration for lower beam energy devices [20].

The small signal gain formulated is applicable to both multiharmonic and conventional optical klystrons. The employment of the Taylor expansion technique has allowed the inclusion of the energy spread effects on beam bunching in our analytical formulation of the interaction gain, thus avoiding the otherwise time-consuming and less explicitly informative approach of large scale numerical simulation. This extension of the convolution technique should be applicable to other two-sectioned wiggler systems for a similar analytical formulation of their interaction gain [1,3,5]. Furthermore, it has been shown that under some operation conditions the gain spectrum obtained with such an extended convolution technique can be appreciably different from that predicted previously [8]. Since the latter was derived under some restrictive approximation, the newly developed gain formulation should give a more accurate assessment of the inhomogeneous broadening effects in optical klystrons. This may be used to improve the accuracy of gain spectrum based techniques for beam quality measurement.

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