PHYSICAL REVIEW E VOLUME 57, NUMBER 5 MAY 1998

Traversal time of acoustic plate waves through a tunneling section

A. Alippi, M. Germano, A. Bettucci, F. Farrelly, and G. Muzio
Department of Energetics, University of Rome ''La Sapienza,'' Via Antonio Scarpa 14, 00161 Rome, Italy
(Received 10 February 1997; revised manuscript received 6 November 1997)

Experimental evidence is given for the unnoticeable time delay undergone by an acoustic Lamb wave pulse in penetrating a forbidden barrier, represented by a narrowed plate region in the propagation path. Measurements, which use a technique unique to Lamb waves overcoming typical objections made in wave packet experiments, have been performed in various aluminium plates for several frequencies and different barrier lengths and show a value of the traversal time close to zero that appears to be independent from the barrier length. [S1063-651X(98)50405-9]

PACS number(s): 43.20.+g

The problem of traversal time in tunneling phenomena has been tackled since 1932 [1], though only recently has the problem drawn the attention of researchers in different fields of physics, both for its intriguing connections with superluminal velocity of signals and for its implications in modern electronic tunneling devices. At first, a quantistic approach was attempted [2–4], giving rise to nonunivocal results both in theory and experiments. In this field, the definition of traversal time depends on the particular way it is determined, indeed it can also be associated with completely different physical concepts: for example, one possible definition is the interaction time between one or more physical clocks and a tunneling particle [5–7], while another derives from theoretical studies that use the path integral approach [8–10] and this results in a distribution of times rather than a single time value.

Another approach to the analysis of this problem uses a semiclassical model that simulates a particle with a wave packet [11]; in this case a unique definition of delay time is achieved and the practical problem of measuring very short times is reduced by bringing traversal time measurements from the femtosecond range typical of the Josephson junction experiments to the nanosecond range in guided microwave ones. The final limitation in the time scale range in the latter case, derives from the fact that if the barrier length were to be extended excessively, the signal would decay below the sensitivity limit of an experimental measurement. Experiments of this kind have indicated that the traversal time seems to be, under certain assumptions, independent from barrier section length, thus leading to superluminal velocity of wave packets [12–14].

In the present paper, the effect of a wave packet tunneling through a barrier is studied and experimental evidence is given of almost ''instantaneous'' traversal time of plate elastic waves (Lamb waves) through forbidden propagation sections. In the case of acoustic waves, not only is there a further scaling down of the typical traversal times from the nanosecond range of microwave radiation to the microsecond range in the acoustic case [15] but, by using a property unique to a specific Lamb mode a typical objection, that is based on the strong distortion of the wave packet in similar experiments, can be avoided, as will be shown below. In addition, elastic waves also allow the direct probing of the acoustic field within the barrier using laser probe techniques [17].

Although, in the present case, errors due to the acoustical wave packet arrival time determination greatly exceed any real possibility of ascertaining true superluminal velocity of the wave, nevertheless, on the acoustical time scale, the measured traversal times definitely demonstrate that the corresponding velocity of the wave packet through a forbidden barrier is far higher than the velocity of any acoustical mode and in particular of any bulk propagating wave. Based on the identity of the formal problem, this corresponds to a velocity of an electromagnetic perturbation within a forbidden barrier greater than its speed in vacuum.

The objection mentioned above is due to the fact that higher frequency components have a higher velocity (group velocity in the quasimonochromatic approximation [11]) with respect to lower frequency ones and so they constitute the front part of the wave packet during propagation. These components could reach the barrier and pass beyond it even before the arrival of the main part of the packet; furthermore this effect is enhanced because higher frequencies are more energetic and undergo smaller or even a negligible attenuation. The objection is that, in this case, there is no causality relation between the input and output signals, so that an outgoing peak could be observed beyond the barrier prior to the arrival of the main part of the packet at the barrier interface [4,16].

In order to avoid this effect a characteristic that is unique to Lamb modes has been exploited, that is the contradirected phase and group velocity behavior of one particular mode, the first symmetrical S_1 mode in isotropic plates, within a specific range of frequencies [18]. The dispersion law of mode S_1 in aluminium plates is given in Fig. 1, where the dependence of parameter $\omega b/\pi V_s$ versus βb is shown, with $\beta=2\pi/\lambda_f$, ω being the angular frequency, λ_f the spatial periodicity along the propagation direction in the plate, b the thickness of the plate, and V_s the velocity of the shear waves. From A to B, the dispersion curve refers to waves whose energy flow is contradirected with respect to the direction of the phase progressive wave, whereas from B towards increasing values of the abscissa, it refers to waves whose energy flow is in the same direction. In our experimental case, within the frequency range from $f_B=1.42$ MHz to $f_A=1.52$ MHz and for $\beta b<1.52$, mode S_1 propagates with its

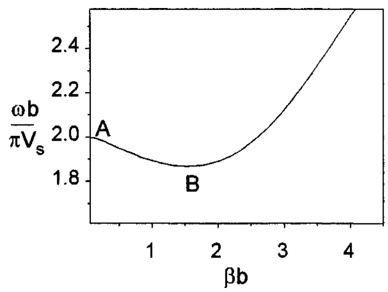

FIG. 1. Dispersion curve for the S_1 Lamb mode: the mode generated with a value for the adimensional parameter $\beta b < 1.52$ and within the proper frequency range, has the unique property that the group velocity is contradirected with respect to the phase velocity.

wave vector towards the positive axis, while its group velocity is directed backwards. Indeed, a wave packet constituted by the S_1 mode has been generated in the AB zone of the dispersion curve. In this case the situation is such to overcome the objection we mentioned above deriving from distortion effects: here the components with a frequency $f > 1.52$ MHz have a velocity opposite to that of the packet's carrier frequency (e.g., $f_{carrier} = 1.47$ MHz); furthermore, the components with a frequency higher than the carrier's but slightly lower than 1.52 MHz have a smaller velocity than that of the carrier, thus overcoming the problem that the higher frequency components could constitute the front part of the packet before it reaches the barrier, and produce a strong alteration of the outgoing packet shape.

Elastic Lamb waves in the indicated frequency range and with proper value of the propagating wave vector component ($\beta < 760$ m^{-1}) are generated and received in an aluminium plate of thickness $b = 2$ mm (40 cm long) through mode conversion of bulk longitudinal waves impinging at an appropriate incidence angle to the plate from a lower velocity medium, by mean of wedge transducers. These were placed at a distance of more than a dozen wave lengths from the narrowed zone so as to ensure the generation of Lamb waves.

Wave packets, few tens of microseconds long, of a selected Lamb mode are generated in the plate and successively detected after traveling through a forbidden section of variable length d and thickness $b' = 1$ mm, where the cut-off frequency value $f'_{cut\text{-}off} = 2.84$ MHz is well above their frequency (see Fig. 2). The time delay undergone by the wave packets along the path from the transmitting to the receiving transducer is measured and corresponds to the overall transit time of the acoustic pulse through the coupling media, the free propagation regions of the plate and the forbidden region; this enables a relative determination of the barrier traversal time keeping all other propagation parameters unaltered.

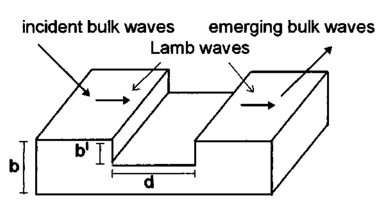

FIG. 2. Geometry of the experiment: the incident and the emerging waves are generated and detected with wedge transducers.

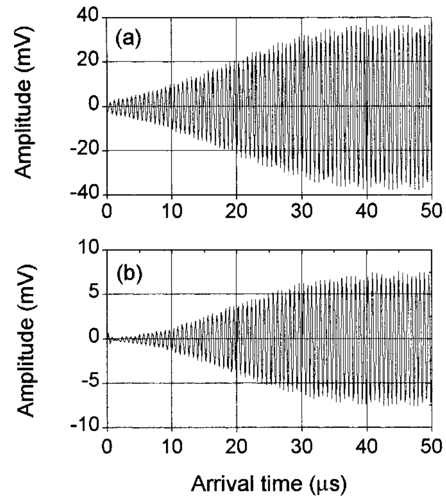

FIG. 3. Oscillograms of the detected signals for $f = 1.465$ MHz in two cases: (a) free propagation, (b) 5 mm long barrier.

Measurements of the pulse arrival time were performed by determining the arrival time of the half value of the regime signal amplitude, for three different cases of the barrier length $d = 0,5,10$ mm, and three different values of the acoustic central frequency $f = 1.460,1.465,1.470$ MHz. Figure 3 shows the oscillograms of the detected signals, at $f = 1.465$ Mhz, in the case of (a) free propagation through the plate and (b) propagation with an additional path of 5 mm through the forbidden zone. From this it is possible to see that the signal detected after tunneling is only negligibly distorted and slightly attenuated (from 36 to 7 mV) thus validating the current procedure of taking the half amplitude arrival time of the packet as the packet arrival time [19]. The figure shows that the arrival times of the two packets are the same within 0.5 μs. The traversal time, that is the difference between the arrival times in the two cases of free propagation and tunneling, is so short that it falls within the error interval of 0.5 μs.

The time so measured, corresponds to what is known as phase time. An analytical estimate for the phase time cannot really be done through any modeling with classical or quantum waves, since it would require to define univocal imaginary wave number, that is not the case of Lamb waves, because of their composite nature resulting from superposition of shear vertical (SV) and longitudinal (L) modes. However, an order of magnitude of the traversal time can be drawn by computing the (extreme) two values obtainable through modelling Lamb waves either with pure longitudinal or pure shear waves. The traversal time is obtained from the phase displacement $\Delta\phi$ undergone by the complex transmission coefficient. The dispersion relation for pure modes in a waveguide of thickness b and bulk waves velocity V is

$$k^2 = \left(\frac{\omega}{V}\right)^2 - \left(\frac{n\pi}{b}\right)^2 \quad (n = 0,1,2,\ldots). \tag{1}$$

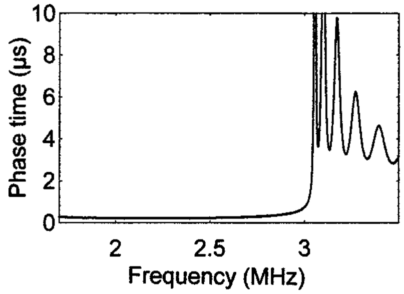

FIG. 4. Theoretical estimate for the phase time of a SH mode ($n=2$); the oscillating part of the curve is above the cut-off frequency.

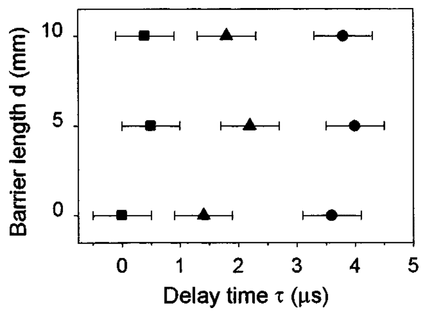

FIG. 5. Relative arrival times vs barrier length for three frequencies: 1.470 MHz (squares), 1.465 MHz (triangles), and 1.460 MHz (circles).

In a narrower section of length d and thickness b', the wave number becomes:

$$k'^2=\left(\frac{\omega}{V}\right)^2-\left(\frac{n\pi}{b'}\right)^2 \quad (n=0,1,2,\dots). \tag{2}$$

and by defining quantities $q=\imath k'$ and

$$k_0=\sqrt{\left(\frac{n\pi}{b'}\right)^2-\left(\frac{n\pi}{b}\right)^2}, \tag{3}$$

the expression for the phase time can be computed through the complex transmission coefficient of a forbidden potential wall in quantum mechanics, by simple substitution of the particle momentum with the wave momentum [15,20]; one then obtains

$$\tau_\phi=\frac{1}{v_g}\frac{\partial(\Delta\phi)}{\partial k}=\frac{1}{v_g q}\frac{2qdk^2(q^2-k^2)+k_0^4\sinh(2qd)}{4k^2q^2+k_0^4\sinh^2(qd)}, \tag{4}$$

where $v_g=V^2k/\omega$ is the bulk wave group velocity.

Values of τ_ϕ would, then, result to be equal to 0.296 and 0.304 μs for L and SV second mode ($n=2$, corresponding to Lamb S_1 mode), respectively, if they both would have the same group velocity of the Lamb wave used in our experiment. Of course, pure L or SV waves would not propagate as separate modes in a plate waveguide with free boundary and could only exist in case of complex boundary conditions. Pure shear modes, however, are realistic in case of a free plate if their displacement is parallel to the free surface (shear horizontal waves SH). In the approximation, then, that a SH wave substitutes our Lamb mode, the behavior can be computed of the phase time vs the barrier length. Figure 4 shows the phase time τ_ϕ vs frequency for a SH mode ($n=2$) in the same conditions of the experimental case for a barrier section 5 mm long. For a shear wave, then, that would have the same group velocity of our Lamb mode, the theoretical estimate of the phase time is 0.304 μs, which is within the experimental error; identical values are also obtained in the case of a 10-mm-long barrier so that the traversal time is unaffected by the barrier length [15].

Figure 5 reports the experimental arrival times vs barrier length for three different values of the central frequency f. The shortest time has been taken as the reference one ($\tau=0$), and it corresponds to the case $f=1.470$ MHz with no barrier present ($d=0$). It is evident that no appreciable difference is obtained by changing the barrier length, which is consistent with the interpretation of an almost ''instantaneous'' velocity penetration of the pulse through the barrier. Particularly, it is worthwhile to note that the changing of frequency sensibly affects the delay time, at any barrier length, in a way that is bound to the dispersion effects, while the changing of the barrier length is almost uneffective on the delay time, at any used frequency. This fact is remarkable, considering that, at these frequencies, we measured times of 4 μs and 8 μs for a wave packet propagating distances of 5 and 10 mm, respectively, through a uniform 2-mm-thick plate; conversely, in case of tunneling, the delay is less than 0.5 μs in both cases. Our experiment does not allow a precise value for traversal time to be derived, but surely it sets an upper limit $\tau_\phi=0.5$ μs with respect to a measured delay time for normal propagation of several microseconds.

In conclusion, experimental evidence has been given showing that a classical wave, penetrating through a region where no real propagation vector is permitted, crosses this region in a time which is much shorter than the expected one, were it to travel as a propagating wave. The case that has been investigated is that of an elastic wave in a thin plate, where in a selected region the thickness is made low enough for a specific mode to be under the cut-off frequency. The result is a confirmation of several experiments done with electromagnetic wave packets [12–14,20], but using a Lamb mode with contradirected phase and group velocity avoids the effect of strong distortion of the tunneling wave packet and the consequent lack of causal relation between ingoing and outgoing signals. This gives meaning to the procedure of measuring the delay time between the transmitted and received pulses on the basis of the half maximum amplitude of the rising edge of the pulse shape. The complete analogy of the acoustic case with the electromagnetic one favors the interpretation of superluminal propagation of light pulses, giving a contribution to a question that is still under discussion: the possible violation of Einstein's causality principle [14] by finite duration pulses in a waveguide.

The present work has been partially supported by the Istituto Nazionale di Fisica della Materia, Genova, Italy.

[1] L. A. MacColl, Phys. Rev. **40**, 621 (1932).
[2] M. Büttiker and R. Landauer, Phys. Rev. Lett. **49**, 1739 (1982).
[3] E. H. Hauge and J. A. Støvneng, Rev. Mod. Phys. **61**, 917 (1989).
[4] R. Landauer and Th. Martin, Rev. Mod. Phys. **66**, 217 (1994).
[5] V. F. Rybachenko, Yad. Fiz. **5**, 895 (1967); Sov. J. Nucl. Phys. **5**, 635 (1967).
[6] Th. Martin and R. Landauer, Phys. Rev. A **47**, 2023 (1993).
[7] C. R. Leavens and W. R. McKinnon, Phys. Lett. A **194**, 12 (1994).
[8] H. W. Lee and M. O. Scully, Found. Phys. **13**, 61 (1983).
[9] D. Sokolovski and L. M. Baskin, Phys. Rev. A **36**, 4604 (1987).
[10] W. R. McKinnon and C. R. Leavens, Phys. Rev. A **51**, 2748 (1995).
[11] Th. Martin and R. Landauer, Phys. Rev. A **45**, 2611 (1992).
[12] A. Enders and G. Nimtz, J. Phys. (Paris) **3**, 1089 (1993).
[13] A. M. Steinberg, P. G. Kwiat, and R. Y. Chiao, Phys. Rev. Lett. **71**, 708 (1993).
[14] W. Heitmann and G. Nimtz, Phys. Lett. A **196**, 154 (1994).
[15] M. Germano, A. Alippi, and A. Bettucci, Riv. Italiano Acustica **20**, 3 (1996).
[16] R. Landauer and Th. Martin, Solid State Commun. **84**, 115 (1992).
[17] A. Alippi, A. Bettucci, and F. Craciun, and E. Molinari, J. Appl. Phys. **70**, 4004 (1991).
[18] B. A. Auld, *Acoustic Fields and Waves in Solids* (Wiley & Sons, New York, 1973), Vol. 2, p. 73.
[19] M. Elices and F. Garcia-Moliner, *Physical Acoustics*, edited by W. P. Mason (Academic Press, New York, 1968), Vol. 5, p. 163.
[20] A. Ranfagni, D. Mugnai, P. Fabeni, and G. P. Pazzi, Appl. Phys. Lett. **58**, 774 (1991).