Electron cavitation and relativistic self-focusing in underdense plasma

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An improved cavitation model shows that stable beam channeling and electron cavitation occur for relativistic laser intensities even at powers hundreds of times larger than the critical power for self-focusing. [S1063-651X(98)11906-2]

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I. INTRODUCTION

Recent advances in laser technology [1], based on the chirped pulse amplification technique, make intensities greater than 10^{18} W/cm^2 available for experiments. At such intensities, electron motion in the laser field is essentially relativistic and the physics of the laser-plasma interaction is very different from the well studied case of more moderate intensities.

In the present paper, we discuss self-focusing of intense laser beams in underdense ($n<n_c$) plasmas. It was shown many years ago [2] that in the weakly relativistic regime, steady-state self-focusing is similar to self-focusing in a Kerr medium: the focusing is described by a nonlinear Schrödinger equation (NSE). For beam power P greater than a critical power $P_{\mathrm{cr}}=16n_c/n$ GW, nonlinear self-focusing overcomes diffractive spreading and the beam is focused into a field singularity. More exactly, this behavior is followed up to the breaking of the paraxial approximation. Strong radiation scattering takes place after the focus.

If the initial beam power P is well above P_{cr}, the laser beam first breaks into filaments with power P about P_{cr}, each of which then undergoes catastrophic self-focusing.

Recent studies [3–7] demonstrate that self-focusing changes qualitatively for very intense beams. In this case, the laser field becomes so strong that the ponderomotive force evacuates electrons from macroscopic regions (electron cavitation). Stable channeling with confined power $P\gg P_{\mathrm{cr}}$ can take place inside the resultant empty cavity since further focusing cannot take place.

Understanding self-focusing is very important for realization of the ''fast ignitor'' project [8]. In this inertial confinement fusion (ICF) scheme, a powerful ultrashort pulse produces a burst of energetic electrons to ignite an already compressed ICF target. Filamentation and subsequent anomalous scattering in the underdense plasma will reduce the fast electron generation efficiency. Thus radiation channeling can be very helpful for the ''fast ignitor'' scheme. From the above arguments, one can see that whether filamentation or channeling of radiation occurs is determined not only by the beam power, but also by focusing conditions, density profile, etc. The importance of propagation modeling for understanding is indicated by recent experiments [9,10], in which similar experimental parameters resulted in radiation filamentation in one case [9] and stable channeling in the other [10].

We show below that the previously used description of cavitation leads to nonconservation of net plasma charge. Modification of the cavitation description and a comparison of our approach with previous results is the major goal of this paper. The difficulty with the standard approach is due to loss of Hamiltonian structure of the self-focusing equations upon cavitation. The regularized solution presented here maintains the Hamiltonian structure from which some general conclusions can be made. In particular, we will demonstrate the stability of the formed channel.

In the conclusion, we discuss the applicability of our results to real situations and possible consequences for the ''fast ignitor'' scheme.

II. CAVITATION AND RELATIVISTIC CHANNELING

Propagation of ultraintense radiation in plasma involves a number of highly nonlinear phenomena characterized by different spatial and temporal scales, e.g., the scale of charge separation, the laser wavelength, the transverse beam size, and the longitudinal plasma scale. Also, the self-focusing of light in Kerr media is very different in two and three dimensions, so reliable modeling should be three dimensional. As a result, even the best particle-in-cell simulation is limited, in practice, to short simulation times and small plasma volumes that do not match typical experimental conditions [11]. On the other hand, the disparate sizes of parameters allows a simplified description [12,13]. Equations for slowly varying envelopes can be derived from the fully coupled system of equations of relativistic hydrodynamics for the electron motion and from Maxwell's equations for the laser beam. When the pulse duration is longer than $1/\omega_p$, a further simplification is to disregard charge separation in the direction of propagation. While such a model does not include wake field generation, or Raman scattering (important, e.g., for laser acceleration studies), it is quite adequate for present purposes. For propagation in a plasma with density near critical, this approximation is good even for pulses with duration as small as a few tens of femtoseconds. We show below that in the process of self-focusing, small scale modulations of the pulse can arise. For pulse durations greater than 0.5 ps, pulse splitting is important.

In this approximation, steady state relativistic self-focusing in the comoving system of reference is described by the paraxial wave equation [3–7]:

$$2ia_z+\Delta_\perp a+\left(1-\frac{n}{\gamma}\right)a=0. \tag{1}$$

Here a is the envelope of the vector potential normalized by the electron rest energy $a=eA/mc^2$, n is the electron density in units of the critical density $n=n_e/n_c$, $\gamma=\sqrt{1+|a|^2/2}$ is the usual relativistic factor, the transverse coordinates are normalized by c/ω_p, with ω_p being the plasma frequency, and z is normalized by the Fresnel length corresponding to the plasma wavelength $\omega c/\omega_p^2 \gg c/\omega$. The electron density n in Eq. (1) is related to the light intensity by

$$n=1+\Delta_\perp\gamma. \tag{2}$$

A problem is that at high intensity, the ponderomotive force dominates the electric field produced by charge separation in a macroscopic region and the density given by Eq. (2) can take nonphysical, negative values. It was suggested in Ref. [3] that negative densities be avoided by setting the electron density to zero inside the cavitation zone, i.e.,

$$n=0 \quad \text{if} \quad 1+\Delta_\perp\gamma<0. \tag{3}$$

This type of cavitation model has been used in several published investigations [3–7]. Unfortunately, this model has a difficulty that makes quantitative results unreliable. To demonstrate the problem, consider a powerful beam propagating in plasma, with ponderomotive forces evacuating electrons up to a radius R. At $r=R$ we must have $1+\Delta_\perp\gamma=0$. The solution of Eq. (1) is completely determined by this specification, the requirement that the field should decay at large radius, and the fixed beam power P [6]. Now charge conservation requires that

$$\int (n-1)dV=0 \quad \text{or} \quad -\nabla_\perp\gamma=R/2. \tag{4}$$

The equation is thus overdetermined and solutions with cavitation obtained in Refs. [3–6] have, generally speaking, nonzero net charge. In Fig. 1, we plot the net charge of the steady-state solution of Eq. (1) versus the power of the channeling beam. We see that charge nonconservation starts just after the appearance of cavitation and increases with increasing channeling power. The net charge is negative.

The above inconsistency could be removed by the introduction of surface charge on the cavitation boundary, which would make Eq. (1) consistent with both the boundary conditions and the additional condition (4). A more natural way to solve this problem, however, is to schematically take into account the finite plasma temperature T. In this case, instead of Eq. (2) one can derive the equation

$$n=1+\Delta_\perp(\gamma+\alpha\ \ln n),$$
$$\alpha=\frac{T}{mc^2}\ll 1. \tag{5}$$

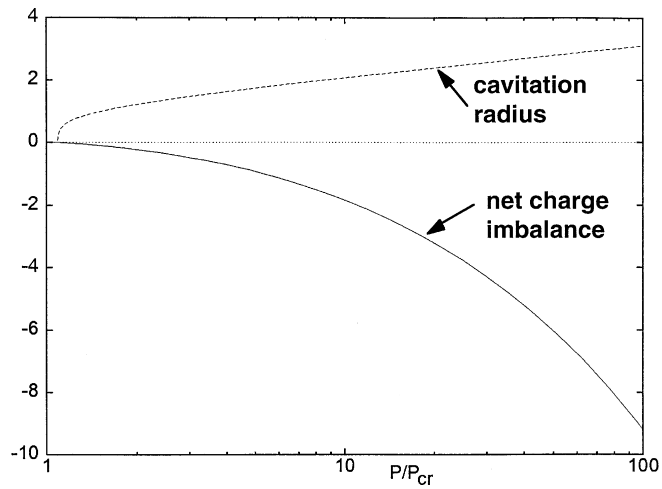

FIG. 1. Total charge imbalance $Q=\int(1-n)dV$ of the steady-state solution Eq. (1) with standard cavitation description Eq. (3) vs trapped power. Charge nonconservation starts just after the appearance of cavitation and increases with increasing channeled power. Total charge is negative. The dotted line is the cavitation radius.

Equation (5) has a simple physical meaning. The displacement of electron density produces an electrostatic potential Φ, according to the relation $\Delta_\perp\Phi=1-n$. Hence, Eq. (5) describes a Boltzmannian electron distribution in joint electrostatic and ponderomotive potentials. It is clear that in this case the electron density in the region of laser field localization can be extremely small, but, nevertheless, nonzero. There will be no singularities in the solution. This phenomenological finite temperature correction is important only in a very thin boundary layer. The actual value of T is determined by the process by which the plasma was formed. Its self-consistent evaluation is beyond our description. In typical experiments [9,10], it is on the order of 1 keV. The additional term regularizes and selects the physically correct solution of Eq. (1). Also, our calculations demonstrate that the solution is not sensitive to the exact ''temperature'' value chosen.

Figures 2 and 3 compare the evolution of initially Gaussian beams propagated in plasma as described by the cavitation model and by the system Eqs. (1) and (5). Figure 2 compares the laser field amplitude, and Fig. 3 the electron density distribution. One can see that while being qualitatively similar, the solutions are very different quantitatively. Our charge conserving solution is smoother, and the channel is wider. One sees that the regular solution does not exhibit the high spikes of laser intensity typical of the cavitation model simulation [5]. Even for power as low as 100 times P_{cr}, the cavitation model overestimates peak intensity by a factor of 4. This is an important factor in interpreting real experiments.

We note that Eqs. (1) and (5) can be written in the Hamiltonian form,

$$2ia_z=\frac{\delta H}{\delta a^*} \tag{6}$$

with the Hamiltonian being

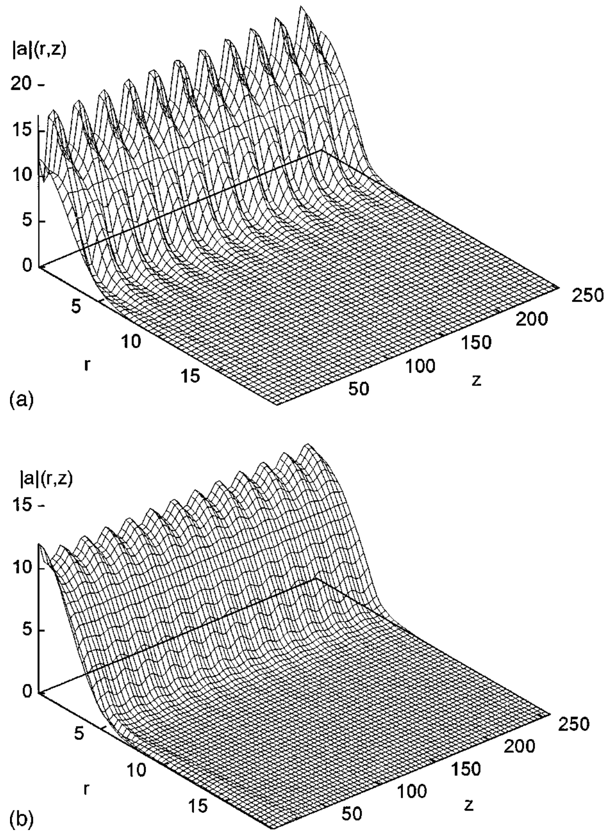

FIG. 2. Evolution of the dimensionless amplitude $a(r,z)$ of the vector potential for a pulse with power $P/P_{\mathrm{cr}}=118.5$ and initial distribution $a(z=0,r)=12\exp[-(r/3.5)^2]$. (a) Cavitation model, (b) regular description Eq. (5).

$$H=\int[|\nabla_{\perp}a|^2-|a|^2+2(\gamma n-1)-(n-1)\Delta_{\perp}^{-1}(n-1)+2\alpha(n\ln n-n+1)]dV. \tag{7}$$

Together with the additional constraint $\delta H/\delta n=0$, Eq. (6) is equivalent to Eq. (5). The Hamiltonian structure implies that Eqs. (1) and (5) conserve the Hamiltonian H in addition to the power $P=\int|a|^2dV$. In the usual cavitation model, the equations have Hamiltonian structure outside the cavitation zone: H is given by Eq. (7) with $\alpha=0$. The equations are also Hamiltonian inside the cavity, but due to the moving surface effect, the overall Hamiltonian is not conserved within the cavitation model. The evolution of the Hamiltonian in the cavitation model is presented in Fig. 4. Jumps in the value of H are directly correlated with the appearance of cavitation.

In our regular model, the Hamiltonian H of Eq. (7) was conserved with high accuracy during numerical evaluation of Eqs. (1) and (5).

Now consider steady-state solutions of Eq. (1). Similar to steady state solutions for the NSE, these solutions realize extremums of H for fixed values of power P (see, e.g., the review Ref. [14]). It was shown in Ref. [15] that for $\alpha=0$, H is bounded from below for a fixed value of P. Small thermal corrections do not change this result. The boundedness of H means that the solution corresponding to minimum H for a fixed P is stable, according to the Lyapunov theorem [14].

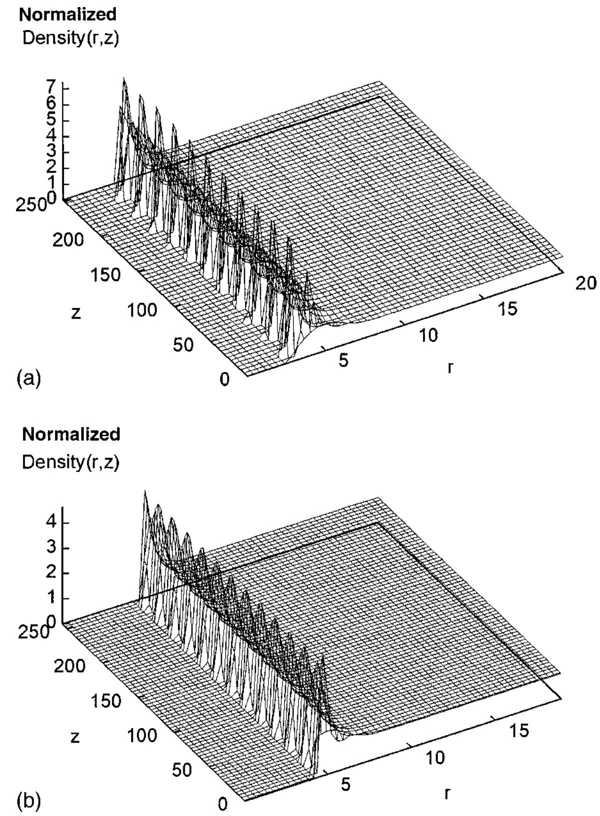

FIG. 3. Normalized electron density evolution for the same case as Fig. 2. Part (a) corresponds to cavitation model and part (b) to the regular description.

It may appear, at first, impossible to arrive at this stable solution from general initial conditions. The initial value of the Hamiltonian is different from the H value for the steady-state solution, and H is a conserved quantity. However, this discrepancy can be removed by radiation out of the channel. Thus, the efficiency of trapping in the channeling regime can be sensitive to focusing conditions, plasma density, etc. Residual oscillations of the trapped power can occur after channel formation as is seen in Fig. 2.

The last two figures illustrate the sensitivity of propagation to the beam focusing conditions. We treated the evolu-

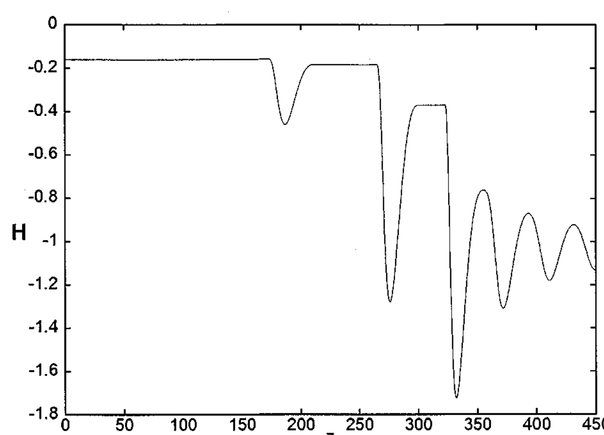

FIG. 4. Evolution of the Hamiltonian H [Eq. (7)] in the usual cavitation model. The jumps in Hamiltonian value coincides with appearance of cavitation.

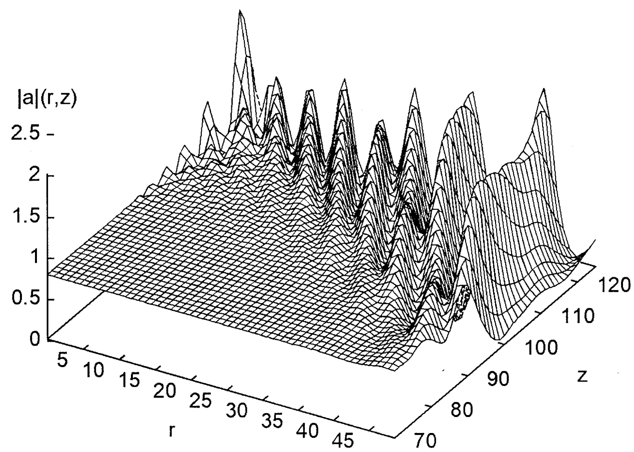

FIG. 5. Evolution of the amplitude $a(r,z)$ of the vector potential for a flat top pulse with $a=0.8$, normalized beam radius R of 70, and power $P=686P_{\mathrm{cr}}$. Only the central part of the beam is plotted.

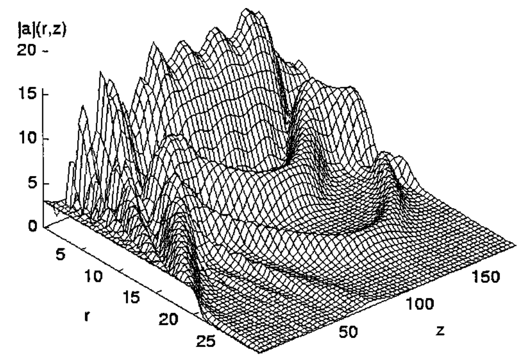

FIG. 6. Evolution of the amplitude $a(r,z)$ of the vector potential for a flat top pulse with $a=3$, $R=20$, and $P=515\ P_{\mathrm{cr}}$.

tion of an initially flat top beam to make the filamentation process more visible.

In Fig. 5, one can see the development of filamentation in a very powerful beam. The initial beam with $a=0.8$ breaks into multiple rings. These rings are unstable and would break into filaments in a full 3D description (see, e.g., [5]).

When the beam is tightly focused, however, and the intensity is high enough to produce cavitation, the initial discrete filaments coalesce as a result of nonlinear interactions (see Fig. 6). The channel formed transports many times the critical power P_{cr}. Qualitatively, such behavior can be explained as follows. With well developed cavitation, the main determinant of H is the electrostatic energy of separated charges. Coalescence of ''empty'' channels decreases the total electrostatic energy and makes the formation of one channel to transport most of the beam energetically preferable.

III. CONCLUSION

In conclusion, we demonstrated that an adequate description of electron cavitation does not qualitatively change the results of previous studies. Stable channeling of intense laser radiation through underdense plasma is possible if the laser beam is powerful enough and tightly focused. But our results for channel width, axial intensity, and distribution smoothness differ quantitatively from those of previous studies.

We found above that for pulses with power much greater than P_{cr}, stable channeling of radiation through the underdense plasma can be realized. But beam power is not the only parameter that characterizes propagation. If the beam is not focused tightly enough, or is defocused during propagation through very underdense plasma, beam filamentation can take place even at high power. Thus, only detailed numerical modeling is capable of deciding whether filamentation will occur in a specific experimental setting.

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