Dielectric function of a two-component plasma including collisions

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A multiple-moment approach to the dielectric function of a dense nonideal plasma is treated beyond the random phase approximation including collisions in the Born approximation. Sum rules are checked, and the relation to the dc electrical conductivity is pointed out. Enlarging the number of moments used, converging results are obtained. The proposed approach is compared to a perturbation expansion of the Kubo formula. [S1063-651X(98)02606-3]

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I. INTRODUCTION

The dielectric function $\epsilon(\vec{k},\omega)$ depending on the wave vector $\vec{k}$ and frequency ω is a physical quantity providing us with various information about the plasma. In homogeneous, isotropic systems, the dielectric function is related to the electrical conductivity $\sigma(k,\omega)$ and the polarization function $\Pi(k,\omega)$ according to

$$\epsilon(k,\omega)=1+\frac{i}{\epsilon_0\omega}\sigma(k,\omega)=1-\frac{1}{\epsilon_0 k^2}\Pi(k,\omega). \qquad (1)$$

A quantum statistical approach to these quantities can be found from kinetic theory as well as from linear response theory. As an example of the latter approach, the Kubo formula relates the dielectric function to equilibrium correlation functions, which can be treated by perturbation theory. In the lowest order of perturbation theory, a well-established expression is the random phase approximation (RPA), valid for collisionless plasmas. The inclusion of collisions, however, is connected with difficulties. Partial summations have to be performed which are sometimes in conflict with sum rules.

Different approximations are known to go beyond the RPA. In the static limit, local field corrections [1] have been studied extensively. Making use of sum rules to parametrize the dielectric function, this treatment was extended to finite frequencies in [2]. A study of the dynamical corrections within a time-dependent mean-field theory was reported in [3]. However, damping effects are neglected in this treatment.

A particular problem is the appropriate treatment of the long-wavelength limit $k\to 0$ at small frequencies where the dc conductivity should be obtained. In a previous paper [4] an approach has been given where this limiting case coincides with the Chapman-Enskog approach [5] to the dc conductivity.

In particular, within a generalized linear response approach the polarization function was found as

$$\Pi(k,\omega)=i\frac{k^2}{\omega}\epsilon(k,\omega)\beta\Omega_0 \times\begin{vmatrix} 0 & M_{0n}(k,\omega)\\ M_{m0}(k,\omega) & M_{mn}(k,\omega)\end{vmatrix}\Big/\,|M_{mn}(k,\omega)|. \qquad (2)$$

As pointed out in [4], this expression is a generalization of the well-known Kubo formula. The matrix elements $M_{mn}(k,\omega)$ are equilibrium correlation functions which are explicitly given in the following section. They contain operators B_n which specify the nonequilibrium state of the system under the influence of an external field.

For the evaluation of the dielectric function, we have to deal with two problems: (i) the choice of the operators B_n to describe the relevant fluctuations in the linear response regime and (ii) the evaluation of the equilibrium correlation functions such as $M_{mn}(k,\omega)$.

The equilibrium correlation functions in a nonideal plasma can be evaluated using different methods. Computer simulations such as quantum molecular dynamics are able to obtain results for an arbitrary density and coupling strength. This promising approach will not be discussed here in detail. On the other hand, analytical results are found using the method of thermodynamic Green functions, which can be treated by perturbation theory. We shall restrict ourselves to only the lowest-order terms corresponding to the Born approximation of the collision integral as described in [4]. Higher-order terms of the perturbation theory can be taken into account in a systematic way; see [6].

With respect to the choice of the operators B_n, only the current density operator J has been considered in [4]. In the spirit of the Chapman-Enskog approach we will include here higher moments of the single-particle distribution function. In the limiting case of the dc conductivity [7] it has been shown that the improvement by using higher moments converges to the Spitzer result.

Note that different approaches based on different sets of relevant observables B_n are formally equivalent as long as no approximations in evaluating the correlation functions are performed. However, within a finite-order perturbation theory, the results for the conductivity are improved if the set of relevant observables is extended.

Results for the dielectric function for two-component plasmas are shown within a one-moment approach and compared with a two-moment approach in Sec. II. To study the internal consistency of the approach, some exact relations such as sum rules are discussed in Sec. III. Of particular interest is the relation to the Kubo formula which under certain circumstances may be treated by perturbation theory as discussed in Sec. IV.

II. MOMENT APPROACH FOR THE POLARIZATION FUNCTION OF A TWO-COMPONENT PLASMA

To evaluate the dielectric function we use the expression (2) for the polarization function, where the matrix elements are given by [9]

$$M_{0n}(k,\omega)=(J_k;B_n),\quad M_{m0}(k,\omega)=(B_m;\epsilon(k,\omega)J_k),$$

$$M_{mn}(k,\omega)=-i\omega(B_m;B_n)-(\dot{B}_m;B_n)+\langle\dot{B}_m;\dot{B}_n\rangle_{\omega+i\eta}+\begin{vmatrix} 0 & \langle\dot{B}_m;B_j\rangle_{\omega+i\eta} \\ \langle B_i;\dot{B}_n\rangle_{\omega+i\eta} & \langle B_i;B_j\rangle_{\omega+i\eta}\end{vmatrix}\Big/\,|\langle B_i;B_j\rangle_{\omega+i\eta}|. \tag{3}$$

The equilibrium correlation functions are defined as

$$(A;B)=(B^+;A^+)=\frac{1}{\beta}\int_0^\beta d\tau\ \mathrm{Tr}[A(-i\hbar\tau)B^+\rho_0],$$

$$\langle A;B\rangle_z=\int_0^\infty dt\ e^{izt}(A(t);B), \tag{4}$$

with $A(t)=\exp(iHt/\hbar)A\exp(-iHt/\hbar)$ and $\dot{A}=(i/\hbar)[H,A]$. The averages are performed with the equilibrium statistical operator $\rho_0=\exp(-\beta H+\beta\Sigma_c\mu_c N_c)/\mathrm{Tr}\exp(-\beta H+\beta\Sigma_c\mu_c N_c)$.

We will consider a two-component plasma consisting of electrons ($c=e$) and ions ($c=i$). In particular, results are given below for a hydrogen plasma. We introduce the single-particle operators

$$n^c_{p,k}=(n^c_{p,-k})^\dagger=a^\dagger_{c,p-k/2}\,a_{c,p+k/2}, \tag{5}$$

where $a^\dagger$ and a are creation and annihilation operators in momentum representation. Choosing the z axis parallel to $\vec{k}$, the current density operator is given by $\vec{J}_k=J_k\vec{e}_z$ with

$$J_k=\frac{1}{\Omega_0}\sum_{c,p}\frac{e_c}{m_c}\hbar p_z n^c_{p,k}. \tag{6}$$

To select the relevant operators B_n, we restrict ourselves to only single-particle observables,

$$B_n=\sum b^c_n(p)n^c_{p,k}. \tag{7}$$

This corresponds to the ordinary kinetic approach, where only the single-particle distribution is considered. The inclusion of higher-order correlations is also possible; see [6].

Within the kinetic approach, the nonequilibrium state of the plasma is described by the mean values of the single-particle operators (5) corresponding to an induced single-particle distribution function with wave number k. Instead of treating an infinite number of operators depending on the momentum p, we can restrict ourselves to a finite number of moments of the distribution function. This procedure is familiar from the theory of dc conductivity [6]. Whereas in that case only moments with respect to p have to be selected, in the general case of arbitrary k to be considered here moments of p as well as $\vec{p}\cdot\vec{k}$ have to be taken into account.

In this paper we investigate how the lowest-moment approach in the Born approximation is modified if further moments are included. From the theory of dc conductivity we know that important modifications are obtained by including the energy current density in addition to the particle current density, i.e., if we include also the moment related to the average of $\vec{p}^2p_z$. Then, the electrical conductivity is not only described by the electron-ion interaction, but includes also the effects of the electron-electron interaction which are not effective in the lowest-moment approximation due to the conservation of total momentum.

The two-moment approach for a two-component plasma to be considered in this paper is given by the following moments of the electron ($c=e$) or ion ($c=i$) distribution function, respectively:

$$b^c_1(p)=\frac{\hbar}{\sqrt{2m_ck_BT}}p_z,$$

$$b^c_2(p)=\left(\frac{\hbar}{\sqrt{2m_ck_BT}}\right)^{3/2}\vec{p}^2p_z. \tag{8}$$

The evaluation of the corresponding correlation functions in the Born approximation is given in Appendixes A and B for the nondegenerate case. As a trivial result, in the lowest approximation with respect to the interaction the RPA result is recovered. In general the matrix elements are given in terms of integrals of expressions containing the Dawson integral.

It is interesting to study the difference between the RPA and the approximation detailed above for a dense plasma. As an example for a dense plasma we consider a plasma under the conditions found in the center of the sun, i.e., with a temperature of $T=98$ Ry and a density of $n_e=8.9a_B^{-3}$ [10]. To simplify the analysis, the plasma considered here shall consist of electrons and protons only, neglecting, e.g., α particles. We will see that the results are comparable to the results obtained in [4] for parameter values corresponding to laser produced high-density plasmas [8].

Results for the imaginary and real parts of the dielectric function in the one-moment approximation given by $b^c_1(p)$ are shown in Figs. 1 and 2, respectively. The dielectric function is shown as a function of the frequency for two different wave numbers, $k=0.1a_B^{-1}$ and $k=10^{-5}a_B^{-1}$. For the sake of comparison the RPA is displayed as well. As can be seen from Fig. 1, the differences between the improved dielectric function and the RPA are small at higher wave numbers ($k=0.1a_B^{-1}$), while significant changes occur at smaller wave numbers ($k=10^{-5}a_B^{-1}$). In both situations, the high-frequency behavior of the improved dielectric function is inverse proportional to the cube of the frequency, while the

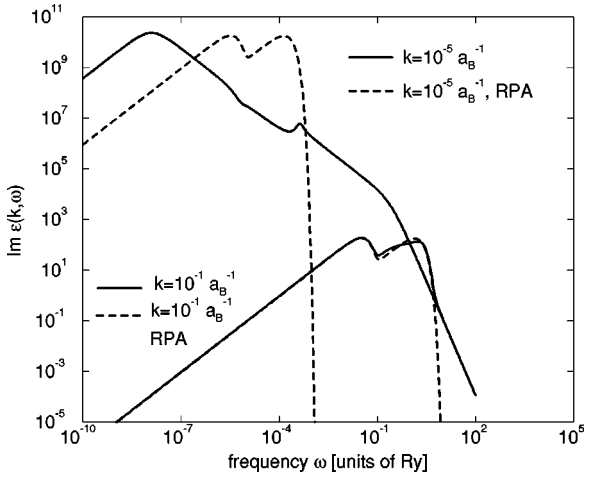

FIG. 1. Imaginary part of the dielectric function for a hydrogen plasma (parameter values: temperature $T=98$ Ry, density $n=8.9a_B^{-1}$) as a function of the frequency at two wave numbers $k=0.1a_B^{-1}$ (lower left) and $k=10^{-5}a_B^{-1}$ (upper left). The dielectric function including collisions (solid lines) is compared with the RPA (dashed lines).

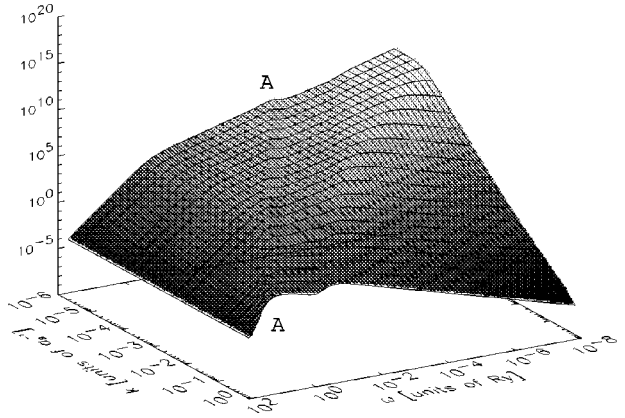

FIG. 3. The imaginary part of the dielectric function for a hydrogen plasma (parameter values: temperature $T=98$ Ry, density $n=8.9a_B^{-1}$) as a function of frequency ω and wave number k. The letter A indicates the line of largest single-particle damping.

RPA decreases exponentially. Such a $1/\omega^3$ dependence is also known from the simple Drude relation. Inspecting further the situation at the small wave number ($k=10^{-5}a_B^{-1}$), several regions can be distinguished. From $\omega=10^{-1}$ Ry down to $\omega=10^{-4}$ Ry there is a $1/\omega$ proportionality, as is anticipated from Eq. (1). This region ends where single-particle damping comes into play. At very small frequencies, a second $1/\omega$ region shows up. At even smaller frequencies, the imaginary part increases linearly with the frequency as is known from RPA.

Figure 2 shows that the static limit is given by the Debye law, which is also the limit for the RPA. In analogy to the imaginary part, the improved dielectric function resembles the RPA at high wave numbers, while a drastic difference emerges at small wave numbers.

In Fig. 3, the imaginary part of the dielectric function for the solar core is presented as a surface plot, showing the dependence on the wave number and the frequency. The results shown in Fig. 1 can be recognized as cuts along the frequency axis. As was already mentioned above, a conductivitylike behavior shows up at small wave numbers, while at high wave numbers a RPA-like behavior exists. This conductivity region enlarges the smaller the wave number. The lower bound to the conductivity region can be traced back to the electronic maximum in the RPA dielectric function, which is situated at $\omega=k\sqrt{k_BT/m_e}$. The line connecting the letters A indicates this position. The upper bound is approximatively given by the relaxation time.

The dc conductivity can easily be obtained by inspection of $\omega\ \mathrm{Im}\epsilon(k,\omega)$, as given in Fig. 4. The conductivity region discussed before appears as a plateau. As expected from Fig. 3 the plateau gets more pronounced the smaller the wave number. In the long-wavelength limit, the Drude result is reproduced. Within a one-moment approximation the dc conductivity is $\sigma(0,0)/\epsilon_0=1703$ Ry. This value coincides with the dc conductivity yielded in the Chapman-Enskog approach using only the moment b_1^c. Note that a second plateau appears at small frequencies.

In order to demonstrate the convergence, we have enlarged the set of relevant observables. The effect on the dielectric function is shown in Fig. 5. In addition to b_1^c also the moments b_2^c were considered. In this figure, the product

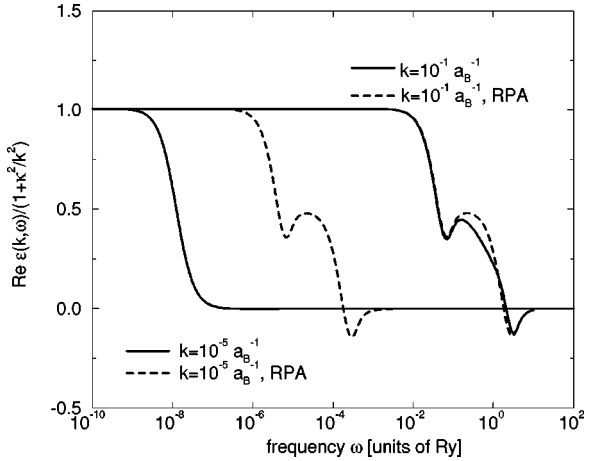

FIG. 2. The same as Fig. 1 for the real part. The real part is normalized to the Debye law.

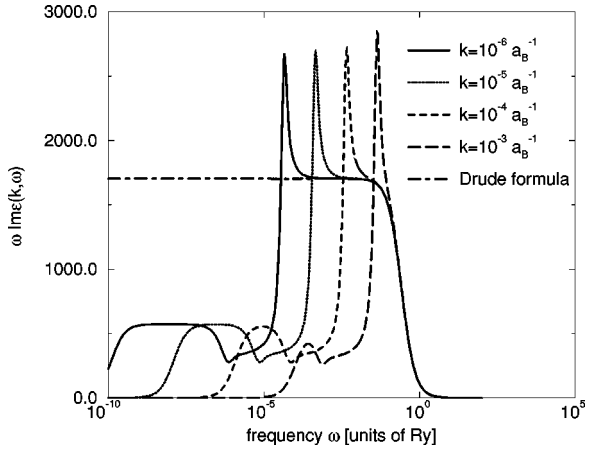

FIG. 4. $\omega\ \mathrm{Im}\epsilon(k,\omega)$ as a function of the frequency ω for different wave numbers k.

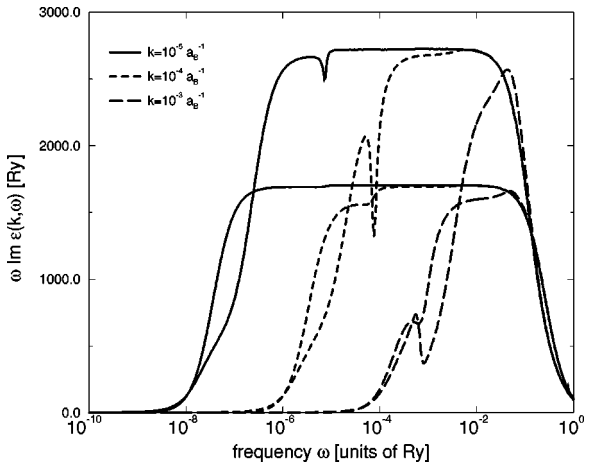

FIG. 5. Long wavelength limit of $\omega\ \mathrm{Im}\epsilon(k,\omega)$ for a two-moment approach (upper plateau) compared to the one-moment approach (lower plateau). Note that the two-moment collision integrals are calculated for $k\to 0$.

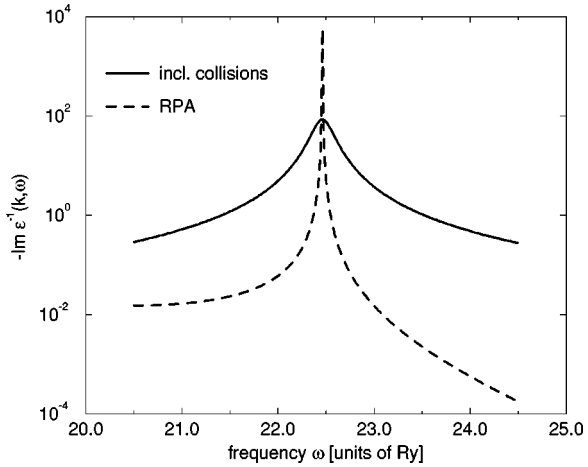

FIG. 6. The imaginary part of the inverse dielectric function compared to the RPA at wave number $k=0.3a_B^{-1}$.

$\omega\ \mathrm{Im}\epsilon(k,\omega)$ is shown for different wave numbers as a function of the frequency setting against a calculation with b_1^c only. Since it is tedious to calculate the correlation functions $M_{mn}(k,\omega)$, the long-wavelength limit $k\to 0$ of these functions has been used in determining the dielectric function. The details of the calculation can be found in Appendix B. The position of the plateau changes significantly, leading to a dc conductivity of $\sigma(0,0)/\epsilon_0=2720$ Ry. This is in agreement with the improvement one gets in the Chapman-Enskog method by considering b_1^c and b_2^c . The reason for this significant change is that electron-electron and proton-proton collisions come into play via b_2^c only.

In Fig. 5, large differences between the one-moment and the two-moment calculations occur in the lower-frequency region only. However, we notice that the dc conductivity within our approach coincides with the corresponding approximation in the Chapman-Enskog method. On the other hand, we know that the Chapman-Enskog approach converges to the Spitzer result if higher moments are included; cf. [7]. Therefore we conclude that also our approach for the dielectric function is converging, if the number of moments is enlarged.

Results for the inverse dielectric function, which describes the response to the external potential, are shown in Fig. 6 for a wave number of $k=0.3a_B^{-1}$ and compared with the RPA. The imaginary part of the dielectric function including collisions is considerably broader than the RPA one. While the imaginary part of the inverse dielectric function in the RPA approximation becomes δ-like in the long-wavelength limit, the improved dielectric function converges to a broad profile (see Fig. 7), reflecting the broadening of the plasmons, by taking into account collisions. Qualitatively, this kind of broadening is also found in molecular dynamic simulations as reported in [11] and [12].

III. EXACT RELATIONS FOR THE DIELECTRIC FUNCTION AND LIMITING CASES

Approximations are to be made to calculate the dielectric function for a system of interacting particles. A possibility to assess the quality and internal consistency of an approximation is the check of exact known properties for the dielectric function. Several exact properties independent of the model discussed are known [13] such as sum rules and the high-frequency as well as long-wavelength limit. Since the sum rules are closely related to conservation laws, it is considered virtuous for an approximation to obey these sum rules. Sum rules can be formulated for a product of the imaginary part of the dielectric function with odd moments of the frequency. In the following, only the lowest-order sum rules are considered, since the sum rules are directly connected to the density. Closely connected to collective excitations in the system is the f-sum rule

$$-\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\,\omega\ \mathrm{Im}\epsilon^{-1}(k,\omega)=\omega_{\mathrm{pl}}^2, \tag{9}$$

whereas a similar sum rule

$$\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\,\omega\ \mathrm{Im}\epsilon(k,\omega)=\omega_{\mathrm{pl}}^2 \tag{10}$$

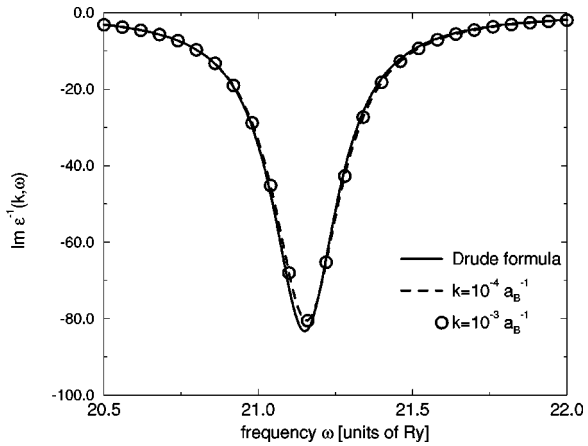

FIG. 7. The imaginary part of the inverse dielectric function at wave numbers $k=10^{-3}a_B^{-1}$ and $k=10^{-4}a_B^{-1}$. The Drude formula is shown as well.

can be related to the conductivity. The perfect screening behavior reads

$$\lim_{k\to 0}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\frac{1}{\omega}\,\mathrm{Im}\epsilon^{-1}(k,\omega)=-1. \tag{11}$$

Here $\omega_{\mathrm{pl}}^2=\Sigma_{c=e,i}(e^2n_c)/(\epsilon_0 m_c)$ denotes the plasma frequency.

Furthermore, the Kramers-Kronig relation holds which connects the real and imaginary parts of the dielectric function:

$$\mathrm{Re}\epsilon(k,\omega)=1+\mathrm{P}\int\frac{d\omega'}{\pi}\frac{\mathrm{Im}\epsilon(\vec{k},\omega')}{\omega-\omega'}. \tag{12}$$

Here, P denotes the Cauchy principal value integration. Usually, this relation follows from the causality principle using the assumption that the dielectric function vanishes sufficiently fast at large frequencies. The inverse dielectric function obeys a corresponding relation. Combining the Kramers-Kronig relation with the sum rules results in rigorous statements about the asymptotic behavior at high frequencies:

$$\lim_{\omega\to\infty}\mathrm{Re}\epsilon(k,\omega)=1-\frac{\omega_{\mathrm{pl}}^2}{\omega^2}+O\left(\frac{1}{\omega^4}\right). \tag{13}$$

Further, important relation can be derived which relate the dielectric function to thermodynamical properties. In particular, the static limit of the real part of the dielectric function obeys the compressibility sum rule

$$\lim_{k\to 0}\mathrm{Re}\epsilon(k,0)=1+V(k)n^2K. \tag{14}$$

Here K denotes the isothermal compressibility. This sum rule follows from the well-known relation between the dynamical structure factor

$$S(k,\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dt\langle\rho_k^{\dagger}(t)\rho_k\rangle e^{i\omega t} \tag{15}$$

and the dielectric function which can be established via the fluctuation-dissipation theorem,

$$S(k,\omega)=-\frac{1}{\pi}\frac{1}{e^{\beta\omega}-1}\mathrm{Im}\epsilon^{-1}(k,\omega-i0). \tag{16}$$

Further extensions for a many-component system can be found in [14].

The RPA, which describes the collisionless plasma, satisfies the relations (9)–(16). While this can be shown analytically, only numerical verifications of the sum rules for the improved dielectric function are possible.

The results for a numerical check for the sum rules (10) and (9) are presented in Table I. It is found that the sum rules are fulfilled within the numerical accuracy ($\approx 0.1\%$). The numerical evaluation shows that the main contribution to the f-sum rule at small wave numbers is provided at frequencies close to the plasma frequency, as is also known from the RPA. The low-frequency region, where the conductivity behavior is found, contributes most significantly to the integral in Eq. (10).

TABLE I. Check of sum rules for different wave numbers. The integrals in Eq. (9) and Eq. (10) are normalized by division with ω_{pl}^2.

Sum rule	Wave number k (units of a_B^{-1})	Normalized integral
Eq. (10)	10^{-3}	0.9990
Eq. (10)	10^{-4}	0.9989
Eq. (10)	10^{-5}	0.9990
Eq. (9)	0.3	0.9990
Eq. (9)	10^{-1}	0.9989
Eq. (9)	10^{-2}	0.9989
Eq. (9)	10^{-3}	0.9992
Eq. (9)	10^{-4}	0.9992
Eq. (9)	10^{-5}	0.9991

The improved dielectric function exhibits perfect screening behavior. It can be shown analytically from the expression for $M_{mn}(k,\omega)$ given in Appendix A that the static limit of the dielectric function is given by the Debye law

$$\lim_{\omega\to 0}\mathrm{Re}\epsilon(k,\omega)=1+\frac{\kappa^2}{k^2}, \tag{17}$$

$$\lim_{\omega\to 0}\mathrm{Im}\epsilon(k,\omega)=0, \tag{18}$$

implying Eq. (11). Here, κ denotes the inverse Debye screening length, $\kappa^2=\Sigma_c n_c e_c^2/(\epsilon_0 k_B T)$.

It is found numerically that the Kramers-Kronig relation holds as well. As discussed above, the imaginary part of the dielectric function falls off as ω^{-3}. Keeping in mind the Kramers-Kronig relation, the real part declines in concordance with Eq. (13). The compressibility sum rule will be studied in future work. In order to check this sum rule, one has to determine the isothermal compressibility within the scope of the approximation given here. This has not been carried out yet.

Thus, the lowest-order sum rules are satisfied by the improved dielectric function reported here in contrast to several other microscopically based approaches beyond the RPA. This problem, which is closely related to so-called conserving approximations, will be discussed in the next section. In many phenomenological approaches [15], the sum rules are incorporated to parametrize the dielectric function. A further possibility to test the dielectric function is to calculate the dynamical structure factor via Eq. (16), since this quantity is determined in molecular dynamics simulations; cf. [11,12].

IV. COMPARISON WITH THE KUBO FORMULA

An alternative approach to improve the dielectric function beyond the RPA is a perturbative treatment starting from the Kubo formula [9] as given by

$$\Pi(k,\omega)=-\frac{ik^2\beta\Omega_0}{\omega}\epsilon(k,\omega)\langle J_k;J_k\rangle_{\omega+i\eta}. \tag{19}$$

As shown in [4], this result follows as a special case within the generalized linear response theory. As also shown there, the different expressions are identical in the limit $\eta \to 0$ if no further approximations are performed. However, the different expressions might be more or less suited for a perturbation expansion. Indeed, several problems arise in carrying out a perturbation expansion based on the Kubo formula. Two of these problems, which are connected to the dc conductivity and the sum rules, shall be discussed here.

The correlation function can be calculated by perturbation theory which is most effectively formulated within the concept of thermodynamic Green functions [16]. The prefactor $\epsilon(k,\omega)$ can be omitted if only irreducible diagrams are taken into account in evaluating the correlation function $\langle J_k ; J_k \rangle_{\omega + i\eta}$.

In lowest order with respect to the interaction, from Eq. (19) the RPA result is obtained immediately, in coincidence with all other approaches including J_k within the set of relevant operators. Evaluating the polarization function up to second order with respect to the screened interaction reads [17]

$$\Pi(k,\omega_\lambda) = \sum_p \frac{f(E_p+\Delta_p) - f(E_{p-k}+\Delta_{p-k})}{E_p+\Delta_p-\omega_\lambda-E_{p-k}-\Delta_{p-k}} - n_{\rm ion} \sum_{pq} V_q^2 f_p \left(\frac{kq}{m}\right)^2 \left(\frac{1}{E_p - E_{p-q}}\right)^2 \times \frac{1}{E_p-\omega_\lambda-E_{p-k}} \frac{1}{E_p-\omega_\lambda-E_{p-k-q}} \times \frac{1}{E_{p-k}-E_{p-k-q}} + (\omega_\lambda, k \leftrightarrow -\omega_\lambda, -k), \tag{20}$$

where the self-energy shift is introduced via

$$\Delta_p = n_{\rm ion} \sum_q V_q^2 \frac{1}{E_p - E_{p-q}}. \tag{21}$$

The first term corresponds to a quasiparticle RPA originating from the self-energy corrections. The second term is the vertex correction. For the sake of simplicity, we have taken the adiabatic limit where $m_i/m_e \to \infty$ (Lorentz plasma). In particular, we find, for $k \to 0$,

$$\mathrm{Im}\Pi(k,\omega) = n \sum_{pq} V_q^2 \left(\frac{kq}{m}\right)^2 \pi \delta(E_p - \omega - E_{p-q}) \times e^{-\beta(E_p-\mu)} \frac{1-e^{\beta\omega}}{\omega^4}, \tag{22}$$

which gives the frequency-dependent conductivity.

However, this perturbation expansion diverges at $\omega \to 0$. Thus, this expression cannot be used to derive a dc conductivity. To tackle this problem, partial summations have to be performed. For instance, a simple approximation for the polarization function including interactions with further particles would be a polarization function given by the product of two full propagators. This way, the polarization function contains the shift and damping of the single-particle states due to the interaction with the medium. However, this approximation for the polarization function does not fulfill rigorous relations such as sum rules. This can be traced to the omission of vertex corrections, which are of the same order in the density as the self-energy corrections considered here. A rigorous relation between the self-energy and the vertex corrections is given by the Ward identities [18], which must be fulfilled in order to satisfy the sum rules.

As early as 1961, Baym and Kadanoff proposed a scheme to consider self-energy and vertex corrections on the same footing. They started from an approximation of the ground-state–energy functional in a way that the conservation laws for energy, momentum, and particle number are obeyed. Following Baym and Kadanoff [19], a consistent vertex can be constructed to a given self-energy. As a consequence, the Ward identities are fulfilled. A detailed description of the connection between sum rules and conservation laws can be found in [20]. A straightforward way to construct conserving vertices within the language of parquet theory can be found in [21].

Although a lot of work has been done in deriving conserving vertex equations, the explicit solution of this equation is still in its infancy. The solution of the vertex equation cannot be given in a simple algebraic form and detailed numerical studies have to be carried out to solve this equation because of the complicated multifrequency dependence of the kernel. Up to now, no solutions beyond the Born approximation are known to the authors. Although some further approximations can be made as shown in [17], leading to an expression for $\Pi(k,\omega_\lambda)$, in that paper no expression for the physical relevant retarded polarization function could be given.

Summarizing, a finite-order perturbative treatment of the Kubo formula is not feasible in contrast to the more general approach given in Sec. II, which allows for a perturbative treatment even near $(k=0,\omega=0)$. In the Kubo theory, one is forced to perform partial summations which sometimes leads to contradictions with exactly known properties.

V. CONCLUSIONS

An approach to the dielectric function has been investigated which includes the effects of collisions and can be used in the entire k,ω space. Within a one-moment approach to a two-component plasma, the Born approximation has been evaluated, and important rigorous properties for the dielectric function such as sum rules are found to be fulfilled. As a main result, an approximation for the dc conductivity is found.

The method proposed here permits a systematic improvement of the dielectric function by enlarging the number of moments. In particular, a two-moment approach was studied. An improvement of the dc conductivity is found due to the inclusion of electron-electron and ion-ion collisions. On each level of the approximation, the approximative value for the dc conductivity coincides with the one calculated in the Chapman-Enskog approach using corresponding moments.

Within the quantum statistical approach described here, a systematic way to account for interaction effects is possible by evaluating the correlation functions beyond the Born approximation. This can be done most effectively using thermodynamic Green's functions. E.g., the inclusion of strong

collisions and bound states can be done by treating T-matrix-like contributions. The treatment of the dynamic screening of the interaction is possible by partial summation of particle-hole excitations. Degeneracy effects can be incorporated by inclusion of exchange diagrams. An interesting point would be the comparison of the quantum version of our formalism with the well-known expressions for the local field corrections.

Alternative approaches to evaluate the dielectric function beyond the RPA are given by a perturbative treatment of the Kubo formula and by computer simulation methods. In contrast to our approach, the Kubo formula requires a partial summation in order to get a finite dc conductivity. Thus, our approach seems to be more appropriate for a perturbation expansion. The comparison of our approach with computer simulations is also a promising perspective. Work in this direction is in progress.

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APPENDIX A: EVALUATION OF THE MATRIX ELEMENTS OF Π

We start from the general expressions (1) and (2) for the dielectric function and specify the relevant observables B_1, B_2, B_3, and B_4 by the moments $b_1^e(p)$, $b_2^e(p)$, $b_1^i(p)$, and $b_2^i(p)$, respectively, according to Eq. (7). We introduce a doubled index $(m\alpha)$ with $m=1,2$, $\alpha=e,i$, so that the four moments $B_{m\alpha}$ are given by B_{1e}, B_{2e}, B_{1i}, and B_{2i}, respectively. We extract some factors so that

$$\epsilon(k,\omega) = 1-\frac{\beta n e^2 \epsilon(k,\omega)}{\epsilon_0 k\omega} \times \begin{vmatrix} 0 & \tilde{M}_{0,n\beta}(k,\omega) \\ \tilde{M}_{m\alpha,0}(k,\omega) & \tilde{M}_{m\alpha,n\beta}(k,\omega) \end{vmatrix} \Big/ |\tilde{M}_{m\alpha,n\beta}(k,\omega)|, \tag{A1}$$

with the matrix elements

$$\tilde{M}_{0,n\beta}(k,\omega) = \frac{z_\beta}{ne}\frac{k}{\omega}M_{0,n\beta}(k,\omega) = \frac{z_\beta}{ne}\frac{k}{\omega}\frac{1}{\Omega_0}\sum_{\gamma,p,p'}\frac{e_\gamma}{m_\gamma}\hbar p_z b_n^\beta(p')(n_{p,k}^\gamma;n_{p',k}^\beta), \tag{A2}$$

$$\tilde{M}_{m\alpha,0}(k,\omega) = \frac{z_\alpha}{ne}\frac{k}{\omega}M_{m\alpha,0}(k,\omega) = \frac{z_\alpha}{ne}\frac{k}{\omega}\frac{1}{\Omega_0}\sum_{\delta,p,p'}\frac{e_\delta}{m_\delta}\hbar p_z' b_m^\alpha(p)(n_{p,k}^\alpha;n_{p',k}^\delta), \tag{A3}$$

$$\tilde{M}_{m\alpha,n\beta}(k,\omega) = -i\frac{\sqrt{m_\alpha m_\beta}}{2k_BTnk}\frac{1}{\Omega_0}M_{m\alpha,n\beta}(k,\omega) = -i\frac{\sqrt{m_\alpha m_\beta}}{2k_BTnk}\frac{1}{\Omega_0}\sum_{p,p'}b_m^\alpha(p)b_n^\beta(p') \times\{([\dot{n}_{p,k}^\alpha - i\omega n_{p,k}^\alpha];n_{p',k}^\beta) + \langle\dot{n}_{p,k}^\alpha;\dot{n}_{p',k}^\beta\rangle_{\omega+i\eta}\} - \begin{vmatrix} 0 & C_{m\alpha,j\delta} \\ \overline{C}_{i\gamma,n\beta} & \tilde{B}_{i\gamma,j\delta} \end{vmatrix} \Big/ |\tilde{B}_{i\gamma,j\delta}|, \tag{A4}$$

where $z_\alpha=[m_\alpha/(2k_BT)]^{1/2}\omega/k$ and the following abbreviations were used:

$$\tilde{B}_{i\gamma,j\delta} = -i\frac{\sqrt{m_\gamma m_\delta}}{2k_BTnk}\frac{1}{\Omega_0}\langle B_{i\gamma};B_{j\delta}\rangle_{\omega+i\eta} = -i\frac{\sqrt{m_\gamma m_\delta}}{2k_BTnk}\frac{1}{\Omega_0}\sum_{p,p'}b_i^\gamma(p)b_j^\delta(p') \times\langle n_{p,k}^\gamma;n_{p',k}^\delta\rangle_{\omega+i\eta}, \tag{A5}$$

$$C_{m\alpha,j\delta} = -i\frac{\sqrt{m_\alpha m_\delta}}{2k_BTnk}\frac{1}{\Omega_0}\langle\dot{B}_{m\alpha};B_{j\delta}\rangle_{\omega+i\eta} = -i\frac{\sqrt{m_\alpha m_\delta}}{2k_BTnk}\frac{1}{\Omega_0}\sum_{p,p'}b_m^\alpha(p)b_j^\delta(p') \times\langle\dot{n}_{p,k}^\alpha;n_{p',k}^\delta\rangle_{\omega+i\eta}, \tag{A6}$$

$$\overline{C}_{i\gamma,n\beta} = i\frac{\sqrt{m_\gamma m_\beta}}{2k_BTnk}\frac{1}{\Omega_0}\langle B_{i\gamma};\dot{B}_{n\beta}\rangle_{\omega+i\eta} = -i\frac{\sqrt{m_\gamma m_\beta}}{2k_BTnk}\frac{1}{\Omega_0}\sum_{p,p'}b_i^\gamma(p)b_n^\beta(p') \times\langle n_{p,k}^\gamma;\dot{n}_{p',k}^\beta\rangle_{\omega+i\eta}. \tag{A7}$$

For $n_e=n_i=n$, $e_e=-e_i=e$ we have in Born approximation (see [6])

$$\tilde{M}_{0,1e}=\tilde{M}_{1e,0}=\frac{1}{2},\quad \tilde{M}_{0,2e}=\tilde{M}_{2e,0}=\frac{5}{4},$$
$$\tilde{M}_{0,1i}=\tilde{M}_{1i,0}=-\frac{1}{2},\quad \tilde{M}_{0,2i}=\tilde{M}_{2i,0}=-\frac{5}{4}, \tag{A8}$$

$$\tilde{M}_{m\alpha,n\beta}(k,\omega) = -2\frac{k}{\omega}z_\alpha^2 N_{m\alpha,n\beta} - \frac{k}{\omega}z_\alpha^4 B_{m\alpha,n\beta} - i\frac{\sqrt{m_\alpha m_\beta}}{2k_BTnk\Omega_0}\langle\dot{B}_{m\alpha};\dot{B}_{n\beta}\rangle_{\omega+i\eta}^{(1)} - \begin{vmatrix} 0 & C_{m\alpha,j\delta} \\ \overline{C}_{i\gamma,n\beta} & \tilde{B}_{i\gamma,j\delta} \end{vmatrix} \Big/ |\tilde{B}_{i\gamma,j\delta}|, \tag{A9}$$

with $\tilde{B}_{i\gamma,j\delta}=-z_\gamma^4 B_{i\gamma,j\delta}k/\omega^3$,

$$C_{m\alpha,j\delta}=i\frac{k}{\omega^2}z_\alpha^2 N_{m\alpha,j\delta}+i\frac{k}{\omega^2}z_\alpha^4 B_{m\alpha,j\delta}$$
$$-i\frac{\sqrt{m_\alpha m_\delta}}{2k_BTnk\Omega_0}\langle\dot{B}_{m\alpha};B_{j\delta}\rangle^{(1)}_{\omega+i\eta}, \tag{A10}$$

and

$$\overline{C}_{i\gamma,n\beta}=i\frac{k}{\omega^2}z_\gamma^2 N_{i\gamma,n\beta}+i\frac{k}{\omega^2}z_\gamma^4 B_{i\gamma,n\beta}$$
$$+i\frac{\sqrt{m_\gamma m_\beta}}{2k_BTnk\Omega_0}\langle B_{i\gamma};\dot{B}_{n\beta}\rangle^{(1)}_{\omega+i\eta}. \tag{A11}$$

Introducing the Dawson integral

$$D(z)=\lim_{\delta\to+0}\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}dx e^{-x^2}\frac{1}{x-z-i\delta}, \tag{A12}$$

we have

$$N_{1\alpha,1\beta}=\frac{1}{2}\delta_{\alpha\beta},\quad N_{1\alpha,2\beta}=N_{2\alpha,1\beta}=\frac{5}{4}\delta_{\alpha\beta},$$
$$N_{2\alpha,2\beta}=\frac{35}{8}\delta_{\alpha\beta} \tag{A13}$$

and

$$B_{1\alpha,1\beta}=[1+z_\alpha D(z_\alpha)]\delta_{\alpha\beta},$$
$$B_{1\alpha,2\beta}=B_{2\alpha,1\beta}=\left[\frac{3}{2}+z_\alpha^2+(z_\alpha+z_\alpha^3)D(z_\alpha)\right]\delta_{\alpha\beta},$$
$$B_{2\alpha,2\beta}=\left[\frac{15}{4}+\frac{5}{2}z_\alpha^2+z_\alpha^4+(2z_\alpha+2z_\alpha^3+z_\alpha^5)D(z_\alpha)\right]\delta_{\alpha\beta}. \tag{A14}$$

Expanding the matrices in Eq. (A7) we find, up to the first Born approximation,

$$\tilde{M}_{m\alpha,n\beta}(k,\omega)=-\begin{vmatrix}0 & i\frac{k}{\omega^2}z_\alpha^2 N_{m\alpha,j\delta}\\ i\frac{k}{\omega^2}z_\beta^2 N_{i\gamma,n\beta} & \tilde{B}_{i\gamma,j\delta}\end{vmatrix}\Big/|\tilde{B}_{i\gamma,j\delta}|-i\frac{\sqrt{m_\alpha m_\beta}}{2k_BTnk\Omega_0}\langle\dot{B}_{m\alpha};\dot{B}_{n\beta}\rangle^{(1)}_{\omega+i\eta}$$
$$-\omega\frac{\sqrt{m_\alpha m_\beta}}{2k_BTnk\Omega_0}\langle\dot{B}_{m\alpha};B_{n\beta}\rangle^{(1)}_{\omega+i\eta}+\omega\frac{\sqrt{m_\alpha m_\beta}}{2k_BTnk\Omega_0}\langle B_{m\alpha};\dot{B}_{n\beta}\rangle^{(1)}_{\omega+i\eta}$$
$$-\begin{vmatrix}0 & -i\frac{\sqrt{m_\alpha m_\delta}}{2k_BTnk\Omega_0}\langle\dot{B}_{m\alpha};B_{j\delta}\rangle^{(1)}_{\omega+i\eta}\\ i\frac{k}{\omega^2}z_\beta^2 N_{i\gamma,n\beta} & \tilde{B}_{i\gamma,j\delta}\end{vmatrix}\Big/|\tilde{B}_{i\gamma,j\delta}|$$
$$-\begin{vmatrix}0 & i\frac{k}{\omega^2}z_\alpha^2 N_{m\alpha,j\delta}\\ i\frac{\sqrt{m_\gamma m_\beta}}{2k_BTnk\Omega_0}\langle B_{i\gamma};\dot{B}_{n\beta}\rangle^{(1)}_{\omega+i\eta} & \tilde{B}_{i\gamma,j\delta}\end{vmatrix}\Big/|\tilde{B}_{i\gamma,j\delta}|. \tag{A15}$$

The first contribution in Eq. (A15) is of zeroth order with respect to the interaction and reproduces the RPA result. The remaining terms contain the electron-ion interaction and the electron-electron or ion-ion interaction, respectively, in the Born approximation. We use a screened interaction with the Debye screening factor $\exp(-\kappa r)$, $\kappa^2=\Sigma_c n_c e_c^2/(\epsilon_0 k_B T)$. Evaluating the Born terms we find

$$\frac{\sqrt{m_\alpha m_\beta}}{2k_BTnk\Omega_0}(i\langle\dot{B}_{m\alpha};\dot{B}_{n\beta}\rangle^{(1)}_{\omega+i\eta}+\omega\langle\dot{B}_{m\alpha};B_{n\beta}\rangle^{(1)}_{\omega+i\eta}-\omega\langle B_{m\alpha};\dot{B}_{n\beta}\rangle^{(1)}_{\omega+i\eta})$$
$$=\frac{i}{8(2\pi)^{3/2}}\frac{1}{k}n\frac{e^4}{\epsilon_0^2}\left(\frac{1}{k_BT}\right)^{5/2}\int_0^\infty dp e^{-p^2}\left[\left(\frac{m_e m_i}{M_{ei}}\right)^{1/2}g_0(m\alpha,n\beta;p)+\delta_{\alpha\beta}\left(\frac{m_\alpha}{2}\right)^{1/2}h_0(m\alpha,n\alpha;p)\right], \tag{A16}$$

$$-\omega\frac{\sqrt{m_\alpha m_\beta}}{2k_BTnk\Omega_0}\langle\dot{B}_{m\alpha};B_{n\beta}\rangle^{(1)}_{\omega+i\eta}=-\frac{i}{8(2\pi)^{3/2}}\frac{1}{k}n\frac{e^4}{\epsilon_0^2}\left(\frac{1}{k_BT}\right)^{5/2}\int_0^\infty dp\ e^{-p^2}\left[\left(\frac{m_e m_i}{M_{ei}}\right)^{1/2}g_1(m\alpha,n\beta;p)\right.$$
$$\left.+\delta_{\alpha\beta}\left(\frac{m_\alpha}{2}\right)^{1/2}h_1(m\alpha,n\alpha;p)\right], \tag{A17}$$

$$\omega\frac{\sqrt{m_\alpha m_\beta}}{2k_BTnk\Omega_0}\langle B_{m\alpha};\dot{B}_{n\beta}\rangle^{(1)}_{\omega+i\eta}=-\frac{i}{8(2\pi)^{3/2}}\frac{1}{k}n\frac{e^4}{\epsilon_0^2}\left(\frac{1}{k_BT}\right)^{5/2}\int_0^\infty dp\ e^{-p^2}\left[\left(\frac{m_em_i}{M_{ei}}\right)^{1/2}g_2(m\alpha,n\beta;p)\right.$$
$$\left.+\delta_{\alpha\beta}\left(\frac{m_\alpha}{2}\right)^{1/2}h_2(m\alpha,n\alpha;p)\right], \tag{A18}$$

where the g_i refer to the electron-ion collisions and the h_i to the electron-electron or ion-ion collisions, respectively. While the evaluation of terms involving b_1^c is simple [4], the determination of the terms related to b_2^c is quite tedious. The reader is referred to the next appendix for details concerning the explicit form of the correlation functions.

APPENDIX B: EXPLICIT FORM OF THE CORRELATION FUNCTIONS

Here, the functions g_i and h_i defined in Appendix A are given for the moments b_1^c and b_2^c defined in Eq. (8). It is convenient to introduce the following abbreviations:

$$z_{ei}=\frac{\omega}{k}\sqrt{\frac{M_{ei}}{2k_BT}},\quad z_{ei,e}=z_{ei}-\sqrt{\frac{m_i}{m_e}}cp,\quad z_{ei,i}=z_{ei}-\sqrt{\frac{m_e}{m_i}}cp,\quad z_{ee,e}=\sqrt{2}z_e-cp \tag{B1}$$

and $\lambda^{ei}=1+\hbar^2\kappa^2M_{ei}/(4m_em_ik_BTp^2)$, $\lambda^{ee}=1+\hbar^2\kappa^2/(2m_ek_BTp^2)$,

$$\Lambda_1=\ln\left(\frac{\lambda^{ei}-1}{\lambda^{ei}+1}\right)+\frac{2}{\lambda^{ei}+1},\quad \Lambda_3=\frac{2}{(\lambda^{ei})^2-1}, \tag{B2}$$

$$\Lambda_2=\lambda^{ei}\ln\left(\frac{\lambda^{ei}-1}{\lambda^{ei}+1}\right)+2,\quad \Lambda_2^{ee}=\lambda^{ee}\ln\left(\frac{\lambda^{ee}-1}{\lambda^{ee}+1}\right)+2. \tag{B3}$$

For the first expression (A16) we find

$$g_0(1e,1e;p)=-g_0(1e,1i;p)=\Lambda_1\frac{2}{3}p, \tag{B4}$$

$$g_0(1e,2e;p)=g_0(2e,1e;p)=-g_0(2i,1e;p)=-g_0(2e,1i;p)=\frac{1}{M_{ei}}\Lambda_1\left[\frac{5}{3}m_ep+\frac{2}{3}m_ip^3\right], \tag{B5}$$

$$g_0(2e,2e;p)=\frac{1}{M_{ei}^2}\Lambda_1\left[\frac{47}{6}m_e^2p+\frac{10}{3}m_em_ip^3+\frac{2}{3}m_i^2p^5\right]+\frac{1}{M_{ei}^2}\Lambda_2\left[\frac{8}{15}m_e^2p+\frac{8}{3}m_em_ip^3\right]-\frac{1}{M_{ei}^2}\Lambda_3\frac{4}{15}m_e^2p, \tag{B6}$$

$$g_0(2e,2i;p)=\frac{1}{M_{ei}^2}\Lambda_1\left[-\frac{47}{6}m_em_ip-\frac{5}{3}m_e^2p^3-\frac{5}{3}m_i^2p^3-\frac{2}{3}m_em_ip^5\right]+\frac{1}{M_{ei}^2}\Lambda_2\left[-\frac{8}{15}m_em_ip+\frac{8}{3}m_em_ip^3\right]$$
$$+\frac{1}{M_{ei}^2}\Lambda_3\frac{4}{15}m_em_ip, \tag{B7}$$

$$h_0(1e,1e;p)=h_0(1e,2e;p)=h_0(2e,1e;p)=0, \tag{B8}$$

$$h_0(2e,2e;p)=\Lambda_2^{ee}\frac{4}{3}p^3. \tag{B9}$$

The remaining terms are obtained by interchanging e and i.

For the second expression (A17) we find

$$g_1(1e,1e;p)=-g_1(1i,1e;p)=\int_{-1}^1 dcD(z_{ei,e})z_{ei}^2\sqrt{\frac{m_e}{m_i}}\Lambda_1[-c], \tag{B10}$$

$$g_1(1e,2e;p)=\frac{1}{M_{ei}}z_{ei}^2\Lambda_1[2m_ep]+\int_{-1}^1 dcD(z_{ei,e})z_{ei}^2\sqrt{\frac{m_e}{m_i}}\frac{1}{M_{ei}}\{\Lambda_1[-c(1+3z_{ei,e}^2)m_e-cm_ip^2]$$
$$+\Lambda_2[2(1-3c^2)z_{ei,e}\sqrt{m_em_i}p]\}, \tag{B11}$$

$$g_1(2e,1e;p)=-g_1(1i,2e;p)=\frac{1}{M_{ei}}z_{ei}^2\Lambda_1\left[-\frac{2}{3}m_e p\right]+\int_{-1}^{1}dcD(z_{ei,e})z_{ei}^2 \times\sqrt{\frac{m_e}{m_i}}\frac{1}{M_{ei}}\Lambda_1[-c(1+z_{ei,e}^2)m_e-2c^2z_{ei,e}\sqrt{m_e m_i}p-cm_i p^2], \tag{B12}$$

$$\begin{aligned}g_1(2e,2e;p)=&\frac{1}{M_{ei}^2}z_{ei}^2\left\{\Lambda_1\left[\left(\frac{1}{3}+2z_{ei}^2\right)m_e^2p+\frac{2}{15}m_e m_i p^3\right]+\Lambda_2\left[-\frac{16}{15}m_e^2p+\frac{16}{15}m_e m_i p^3\right]+\Lambda_3\frac{8}{15}m_e^2p\right\}\\&+\int_{-1}^{1}dcD(z_{ei,e})z_{ei}^2\sqrt{\frac{m_e}{m_i}}\frac{1}{M_{ei}^2}\{\Lambda_1[-c(2+4z_{ei,e}^2+3z_{ei,e}^4)m_e^2-2c^2(z_{ei,e}+3z_{ei,e}^3)\sqrt{m_e m_i}m_e p\\&-2c(1+2z_{ei,e}^2)m_e m_i p^2-2c^2z_{ei,e}\sqrt{m_e m_i}m_i p^3-cm_i^2p^4]+\Lambda_2[2(1-3c^2)(c^2z_{ei,e}+z_{ei,e}^3)\sqrt{m_e m_i}m_e p\\&-2c(3-3c^2-2z_{ei,e}^2+6c^2z_{ei,e}^2)m_e m_i p^2+2(1-3c^2)z_{ei,e}\sqrt{m_e m_i}m_i p^3]+\Lambda_3[2c^2(1-c^2)z_{ei,e}\sqrt{m_e m_i}m_e p]\},\end{aligned} \tag{B13}$$

$$\begin{aligned}g_1(2i,1e;p)=&\frac{1}{M_{ei}}z_{ei}^2\Lambda_1[-2m_i p]+\int_{-1}^{1}dcD(z_{ei,e})z_{ei}^2\sqrt{\frac{m_e}{m_i}}\frac{1}{M_{ei}}\{\Lambda_1[c(1+3z_{ei,e}^2)m_i+cm_e p^2]\\&+\Lambda_2[2(1-3c^2)z_{ei,e}\sqrt{m_e m_i}p]\},\end{aligned} \tag{B14}$$

$$\begin{aligned}g_1(2i,2e;p)=&\frac{1}{M_{ei}^2}z_{ei}^2\left\{\Lambda_1\left[\left(-\frac{1}{3}-2z_{ei}^2\right)m_e m_i p-\frac{4}{5}m_i^2p^3+\frac{2}{3}m_e^2p^3\right]+\Lambda_2\left[\frac{16}{15}m_e m_i p+\frac{16}{15}m_e m_i p^3\right]-\Lambda_3\frac{8}{15}m_e m_i p\right\}\\&+\int_{-1}^{1}dcD(z_{ei,e})z_{ei}^2\sqrt{\frac{m_e}{m_i}}\frac{1}{M_{ei}^2}\{\Lambda_1[c(2+4z_{ei,e}^2+3z_{ei,e}^4)m_e m_i+2c^2(z_{ei,e}+3z_{ei,e}^3)\sqrt{m_e m_i}m_i p\\&+c(1+3z_{ei,e}^2)m_i^2p^2+c(1+z_{ei,e}^2)m_e^2p^2+2c^2z_{ei,e}\sqrt{m_e m_i}m_e p^3-cm_e m_i p^4]\\&+\Lambda_2[2(1-4c^2+3c^4)z_{ei,e}\sqrt{m_e m_i}m_i p+2(1-3c^2)(z_{ei,e}+z_{ei,e}^3)\sqrt{m_e m_i}m_e p-2c(3-3c^2-2z_{ei,e}^2\\&+6c^2z_{ei,e}^2)m_e m_i p^2\\&+2(1-3c^2)z_{ei,e}\sqrt{m_e m_i}m_i p^3]+\Lambda_3[-2c^2(1-c^2)z_{ei,e}\sqrt{m_e m_i}m_i p]\},\end{aligned} \tag{B15}$$

$$h_1(1e,1e;p)=h_1(1e,2e;p)=0, \tag{B16}$$

$$h_1(2e,1e;p)=\int_{-1}^{1}dcD(z_{ee,e})z_{ee}^2\frac{1}{M_{ee}}\Lambda_2^{ee}[4(1-3c^2)z_{ee,e}m_e p, \tag{B17}$$

$$\begin{aligned}h_1(2e,2e;p)=&\frac{1}{M_{ee}^2}z_{ee}^2\Lambda_2^{ee}\left[\frac{32}{15}m_e^2p^3\right]+\int_{-1}^{1}dcD(z_{ee,e})z_{ee}^2\frac{1}{M_{ee}^2}\Lambda_2^{ee}[4(1-3c^2)(z_{ee,e}+z_{ee,e}^3)m_e^2p-4c(3-3c^2-2z_{ee,e}^2\\&+6c^2z_{ee,e}^2)m_e^2p^2+4(1-3c^2)z_{ee,e}m_e^2p^3].\end{aligned} \tag{B18}$$

As above the remaining terms are obtained by interchanging e and i. Furthermore, the terms connected with the third expression (A18) are obtained as $g_2(m\alpha,n\beta;p)=g_1(n\beta,m\alpha;p)$ and $h_2(m\alpha,n\beta;p)=h_1(n\beta,m\alpha;p)$.

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