

Anomalous scaling of fracture surfaces

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(Received 1 July 1997; revised manuscript received 30 January 1998)

We argue that fracture surfaces may exhibit *anomalous* dynamic scaling properties akin to what occurs in some models of kinetic roughening. We determine the complete scaling behavior of the local fluctuations of a brittle fracture in a granite block from experimental data. We obtain a global roughness exponent $\chi = 1.2$ which differs from the local one, $\chi_{\text{loc}} = 0.79$. Implications on fracture physics are discussed.

[S1063-651X(98)01506-2]

PACS number(s): 05.40.+j, 62.20.Mk, 61.43.-j, 81.40.Np

The study of the morphology of fracture surfaces is nowadays a very active field of research. From the early work of Ref. [1], much effort has been put into the statistical characterization of the resulting fractal surfaces in fracture processes. Scale invariance has been found in many experiments [1–9], and it is now well established that, in general, crack surfaces exhibit self-affine scaling properties in a large range of length scales (see Ref. [10] for a more detailed account of experiments).

A self-affine surface $h(\mathbf{x})$ is invariant under an anisotropic scale transformation, in the sense that $h(\mathbf{x})$ has the same statistical properties as $\lambda^{-\chi}h(\lambda\mathbf{x})$, where χ is the *roughness exponent*. Initially, the hope in studying the surface morphology was to relate geometry to mechanical properties (toughness, plasticity, etc.) in order to obtain a material characterization by means of the roughness exponent. However, experimental results in very different types of materials (from ductile aluminum alloys to brittle materials like rock) seem to support the idea of a *universal* roughness exponent which is very independent of the material properties. For three-dimensional (3D) fractures, an exponent $\chi(3\text{D}) \approx 0.8\text{--}0.9$ [1–5] has been measured, whereas in dimension two $\chi(2\text{D}) \approx 0.6\text{--}0.7$ [6–9]. It seems reasonable to expect that material properties should affect the fracture surface roughness. In particular, toughness and anisotropy should be relevant for the fracture crack morphology. However, the above mentioned experimental results seem to lead to the surprising conclusion that there is no correlation of χ with mechanical properties.

The treatment of the fracture crack as a self-affine rough surface leads in a natural way to the close field of kinetic roughening (see Ref. [11] for recent reviews in the subject). A direct mapping of the crack at the stationary state into the Kardar-Parisi-Zhang equation [12] was proposed in Ref. [13]. The crack surface has also been considered as the trace of the crack front whose propagation is modeled by other types of nonlinear Langevin equations [14].

In this paper, we will show that crack surfaces have very much in common with those obtained in growth processes exhibiting what is called an *anomalous* dynamic scaling. We argue that the scaling of the fluctuations of crack surfaces is,

in the sense of kinetic roughening, *intrinsically anomalous* rather than a simple Family-Vicsek one. The analysis of the roughness data of a crack experiment demonstrates the validity of our argument. Physical consequences of this scaling for fracture will be discussed.

FAMILY VICSEK SCALING

In practice, the self-affine character of a surface in dimension $d+1$ is shown by studying the scaling of the fluctuations of the surface height over the whole system of total size L . The invariance property under the scale transformation implies that the *global* width at time t , $W(L,t) = \langle [h(\mathbf{x},t) - \bar{h}(t)]^2 \rangle_{\mathbf{x}}^{1/2}$, where \mathbf{x} is the position, has to scale as [15]

$$W(L,t) = L^\chi f(L/t^{1/z}), \quad (1)$$

where z is the dynamical exponent and $f(u)$ is the scaling function

$$f(u) \sim \begin{cases} \text{const} & \text{if } u \ll 1 \\ u^{-\chi} & \text{if } u \gg 1. \end{cases} \quad (2)$$

χ is the roughness exponent, and gives the scaling of the surface in saturation, $W(L,t \gg L^z) \sim L^\chi$. These two exponents characterize the universality class of the particular growth model. Equivalently, the scaling behavior of the surface might be obtained by looking at the *local* width over a window of size $l \ll L$:

$$w(l,t) \sim \begin{cases} t^{\chi/z} & \text{if } t \ll l^z \\ l^\chi & \text{if } t \gg l^z. \end{cases} \quad (3)$$

This is the method actually used in real experiments, where the size of the system L remains constant and the fluctuations are calculated over scales $l \ll L$. Note that the local width saturates at time l^z as $w(l,t \gg l^z) \sim l^\chi$ independently of the system size.

A complementary technique to determine the critical exponents of a growing surface is to study the Fourier transform of the interface height in a system of linear size L , $\hat{h}(\mathbf{k},t) = L^{-d/2} \sum_{\mathbf{x}} [h(\mathbf{x},t) - \bar{h}(t)] \exp(i\mathbf{k} \cdot \mathbf{x})$, where the spatial average of the height has been subtracted. In this representation, the properties of the surface can be investigated by

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calculating the power spectrum $S(k,t) = \langle \hat{h}(\mathbf{k},t) \hat{h}(-\mathbf{k},t) \rangle$, which contains the same information on the system as the local width. In most growth models, the power spectrum scales as

$$S(k,t) = k^{-(2\chi+d)} s(kt^{1/z}), \quad (4)$$

where s is the simple scaling function

$$s(u) \sim \begin{cases} \text{const} & \text{if } u \gg 1 \\ u^{2\chi+d} & \text{if } u \ll 1. \end{cases} \quad (5)$$

This form for the power spectrum can be easily inverted to obtain the scaling behavior of both local and global widths described above.

ANOMALOUS SCALING AND IMPLICATIONS

In very recent studies of several growth models [16,17], it has been found that the surface fluctuations may exhibit an *anomalous* scaling, in the sense that, although the global width behaves as in Eqs. (1) and (2), local surface fluctuations do not satisfy Eq. (3), but scale as

$$w(l,t) \sim \begin{cases} t^{\chi/z} & \text{if } t \ll L^z \\ t^{\beta_*} l^{\chi_{\text{loc}}} & \text{if } t \gg L^z, \end{cases} \quad (6)$$

where the exponent $\beta_* = (\chi - \chi_{\text{loc}})/z$ is an anomalous time exponent and χ_{loc} the local roughness exponent. Thus in the case of anomalous scaling, two exponents χ_{loc} and χ enter the scaling, and must be taken into account to give a complete description of the scaling behavior of the surface. An outstanding consequence is that the local width does not saturate at times L^z but when the whole system does, i.e., at times L^z , giving an unconventional dependence of the stationary local width on the system size as

$$w(l, t \gg L^z) \sim l^{\chi_{\text{loc}}} L^{\chi - \chi_{\text{loc}}}, \quad (7)$$

in such a way that the magnitude of the roughness over regions of the same size l at saturation is not just a function of the window size but also of the system size, which is distinctly different from what happens in the Family-Vicsek case.

Owing to several experimental limitations, anomalous scaling is difficult to observe (see Ref. [18] for kinetic roughening experiments in which anomalous scaling was found). Since the system size L of experiments can hardly be changed over a broad range, the dependence of the global width on the system size cannot actually be determined. Only local fluctuations over a window l can be measured. Moreover, very often the time evolution of a crack cannot be monitored, and in most experiments only the final crack surface is analyzed, i.e., Eq. (7) for a fixed system size L . This immediately leads to the conclusion that, whether anomalous scaling exists, only χ_{loc} is actually within the reach of the methods currently used in experiments.

In terms of the power spectrum, the existence of a local exponent $\chi_{\text{loc}} \neq \chi$ comes from a nonstandard form of the scaling function $s(u)$ in Eq. (4). It has recently been shown [19] that the anomalous scaling (6) is associated with either

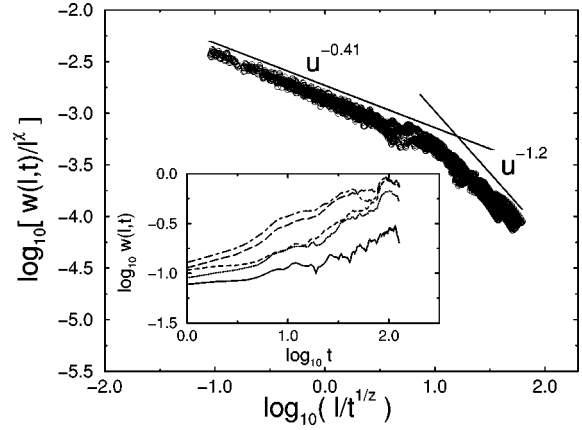


FIG. 1. Inset shows the local width vs time for the higher resolution data of the crack experiment calculated over windows of sizes $l = 100$ (solid line), 300 (dotted line), 500 (dashed line), 700 (long-dashed line), and 900 (dot-dashed line) (all in units of the grid step Δx). Data are collapsed for $\chi = 1.2$ and $z = 1.2$ in the main panel for windows sizes ranging from $l = 10$ to 1200. The nonconstant behavior at small values of the argument (main panel) reflects the anomalous character of the scaling, which agrees with Eq. (9).

super-roughening [20] or a power spectrum that satisfies the dynamic scaling behavior stated in Eq. (4), but with a distinct scaling function

$$s(u) \sim \begin{cases} u^{2(\chi - \chi_{\text{loc}})} & \text{if } u \gg 1 \\ u^{2\chi+d} & \text{if } u \ll 1. \end{cases} \quad (8)$$

So, in the stationary regime (at times $t \gg L^z$), the power spectrum scales as $S(k,t) \sim k^{-(2\chi_{\text{loc}}+d)} L^{2(\chi - \chi_{\text{loc}})}$, and not simply as $k^{-(2\chi+d)}$, as corresponds to a standard scaling. This means that experimental determinations of the roughness exponent of fracture crack surfaces from the decay of the power spectrum with k also give a measure of χ_{loc} and not χ .

Most of the experimental studies are unable to follow the crack in time, and much important information about the complete scaling is lost. In the majority of the experiments one has to deal with a static fracture surface, and fluctuations of its height are evaluated over windows of different sizes l . So, *neither scaling form (3) nor (6) is actually tested in fracture experiments.*

EXPERIMENT

In the following we present an analysis of data describing growth of the crack roughness. In this experiment a fracture was initiated from a straight notch in a granite sample ($25 \times 25 \times 12 \text{ cm}^3$) [5]. It is a mode I unstable crack. The crack roughness increases from two hundredths of a millimeter to several millimeters. Topographies of two areas of $5 \times 4 \text{ cm}^2$ were recorded with a first mechanical profiler along a regular grid (100 parallel profiles). The x direction, which is parallel to the initial notch, was sampled with 1050 points ($\Delta x = 50 \mu\text{m}$). The grid step along the perpendicular direction (i.e., the crack propagation direction) was $\Delta y = 350 \mu\text{m}$. A third map ($5 \times 5 \text{ cm}^2$) was obtained from a second and independent mechanical profiler with a higher resolution. Two hundred parallel profiles were recorded with 2050 points per profile (in which $\Delta x = 32.5 \mu\text{m}$ and $\Delta y = 250 \mu\text{m}$). We as-

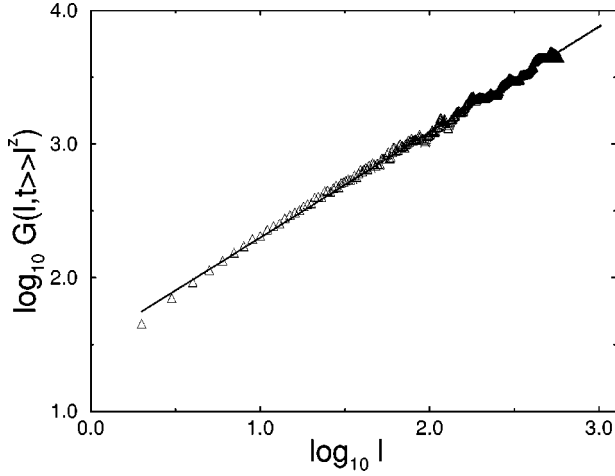


FIG. 2. Height difference correlation function for the experimental data displayed in Fig. 1. The straight line is a fit of the data, and its slope 0.79 gives a determination of the local roughness exponent.

sumed that the crack speed was constant, which translates into a linear relationship between position y and time t . As a consequence, we consider the one-dimensional profiles as descriptions of the advancing crack $h(x, t)$. The complete spatiotemporal behavior of the surface can thus be obtained.

In Ref. [5], the scaling form (3) was checked for the two first data sets. However, a careful inspection of the data collapse reported, Fig. 1 in Ref. [5], reveals that the scaling function goes like a power law for large abscissa values rather than be a constant. Also, the slope for small values of the argument does not match well with the correct value. As we will see, much better and more accurate results are obtained if, instead of assuming a Family-Vicsek behavior, we analyze the data on the basis of an anomalous scaling. In this case, from Eq. (6) it is easy to see that the corresponding scaling function would be

$$g_A(u) \sim \begin{cases} u^{-(\chi - \chi_{\text{loc}})} & \text{if } u \ll 1 \\ u^{-\chi} & \text{if } u \gg 1. \end{cases} \quad (9)$$

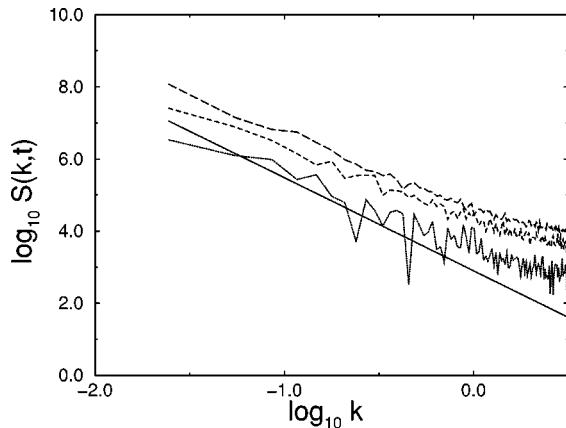


FIG. 3. Power spectrum at times $t=50, 75$, and 100 calculated from the experimental data of higher resolution. The shift of the curves for different times is apparent and characteristic of an *intrinsic* anomalous scaling. The straight line has a slope -2.58 and is in agreement with the power decay $k^{-(2\chi_{\text{loc}}+1)}$ with $\chi_{\text{loc}}=0.79$.

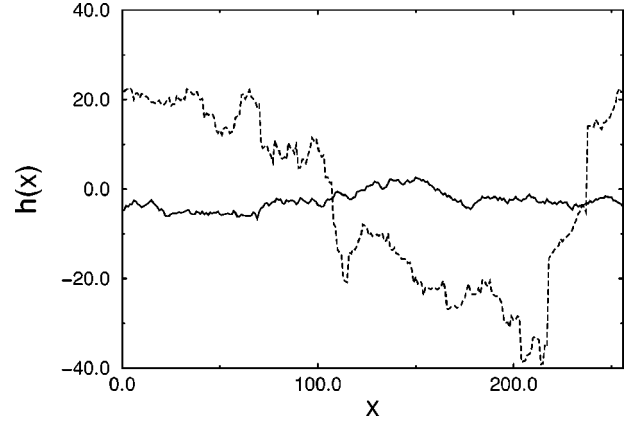


FIG. 4. Example of two fractal curves with the same local roughness exponent but different global one. One interface (solid) has $\chi = \chi_{\text{loc}} = \frac{1}{2}$ and is a realization of the simple $\partial_t h = \partial_x^2 h + \xi$ equation, where ξ is a Gaussian white noise, and it is thus a true self-affine interface. The other curve (dashed) has $\chi = \frac{3}{4}$ and $\chi_{\text{loc}} = \frac{1}{2}$, and is a typical front of the random diffusion growth process [17,19]), which is well known to exhibit anomalous roughening.

in such a way that $w(l, t)/l^X = g_A(l/t^{1/z})$, where the label A denotes the anomalous scaling form.

In Fig. 1, we present the data collapse of $w(l, t)/l^X$ vs $l/t^{1/z}$ for the high resolution map of the crack surface obtained in the experiment. The best data collapse occurs for a global roughness exponent $\chi = 1.2 \pm 0.1$ and $z = 1.2 \pm 0.15$. The same results were also found when the other two lower resolution data sets were used. The nonconstant behavior for $u \ll 1$ in Fig. 1 is the main fingerprint of the anomalous character of the scaling. Figure 1 is in excellent agreement with a scaling function like Eq. (9).

According to Eq. (9), the power law $u^{-0.41}$ for $u \ll 1$ in Fig. 1 corresponds to a local roughness exponent $\chi_{\text{loc}} = 0.79$. This value can be compared with the value obtained from the long time behavior of the height difference correlation function, $G(l, t) = \langle [h(x+l, t) - h(x, t)]^2 \rangle_x^{1/2}$, that we plot in Fig. 2 for the long times limit, $t \gg l^z$, and the highest resolution data. At long times, $G(l)$ displays a power law behavior $l^{\chi_{\text{loc}}}$ that gives an independent determination of the *local* roughness exponent. As shown in Fig. 2, data fit to $\chi_{\text{loc}} = 0.79$, in agreement with the value obtained from the slope -0.41 in Fig. 1. This estimate of χ_{loc} was confirmed by several other independent techniques: variable bandwidth, return probability, and wavelet analysis [21,22].

We have also calculated the power spectrum, and in Fig. 3 we plot $S(k, t)$ vs k in a log-log plot for times $t=50, 75$, and 100 for the high resolution data. The curves are clearly shifted in time, as corresponds to a power spectrum scaling function like Eq. (8) and not the Family-Vicsek one in Eq. (4). $S(k, t)$ decays with a power law $k^{-2.58}$ which is consistent with $k^{-(2\chi_{\text{loc}}+1)}$ and $\chi_{\text{loc}} = 0.79$. We were unable to obtain a collapse of good quality from these data.

CONCLUSIONS

In this study, we have shown that an anomalous dynamic scaling, Eq. (6) or (7), captures much better the features of the crack geometry than the standard Family-Vicsek one. We obtained for an unstable brittle fracture of a granite block a

global exponent $\chi=1.2$, which is different from the local exponent $\chi_{\text{loc}}=0.79$. The very robust value of the local exponent has been demonstrated in the past.

The existence of two different and independent roughness exponents that characterize the complete scaling of a surface, as it follows from Eq. (7), has important implications in the appearance (and geometry) of anomalously roughened surfaces. To illustrate this point, in Fig. 4 we plot two surfaces with exactly the same local exponent, $\chi_{\text{loc}}=\frac{1}{2}$, but different global exponents, $\chi=\frac{3}{4}$ and $\frac{1}{2}$. The two interfaces plotted in Fig. 4 are quite different to the naked eye, despite the fact that they have the same local roughness exponent. From Fig. 4, it is clear that χ_{loc} necessarily gives just a part of the information about the scaling when anomalous roughening exists, and indeed the global exponent χ does give an account of the large peaks taken by the surface in the anomalous case (i.e., when $\chi \neq \chi_{\text{loc}}$). The similarity with what happens to the patterns found in experimental cracks (see for instance Fig. 1. in Ref. [9]) suggests that they may exhibit the same type of double scaling. More precisely, Fig. 1 in Ref. [9] shows that the resulting surfaces in wood for tangential and radial fractures display strongly different morphologies with the same local exponent $\chi_{\text{loc}} \approx 0.68$. These authors found *puzzling* the large peaks taken by tangential fracture surfaces, in clear contrast to the quite flat look of radial fractures, very much as occurs in the example we plot in Fig. 4. Also Zhang *et al.* [23], in a numerical model of fracture in anisotropic materials, found a local exponent

roughly constant $\chi_{\text{loc}} \approx 0.7$, *surprisingly* independent of the orientation of the fracture, although visible differences in the surfaces were noted (see Fig. 2 in Ref. [23]). Experiments in Ref. [2] with materials showing different rupture modes gave similar results. We believe that a determination of the global roughness exponent in all these experiments could allow a better characterization of the fracture surface morphology and its relationship with material properties.

A second physical consequence of the anomalous scaling is that the saturation roughness is not only a function of window size but also of the system size. Implications for the crack process are important. Information about the system size exists along the crack front during the propagation even at Rayleigh speed. This may illustrate the role of interactions between elastic waves and front propagation for the geometry of the front. More studies should be initiated to check whether or not the global roughness exponent is in general a valid index to characterize fracture surfaces.

ACKNOWLEDGMENTS

J.M.L. wishes to thank J.V. Andersen for discussions and encouragement, and R. Cuerno for a careful reading of the manuscript. J.S. thanks K.J. Málóy for very fruitful discussions. This work was supported by the MEC of the Spanish Government, the European Commission, the Ecodev-CNRS program, and the GDR "Géomécanique des Roches Profondes."

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