PHYSICAL REVIEW E VOLUME 57, NUMBER 1 JANUARY 1998

Controlling Hamiltonian chaos by medium perturbation in periodically driven systems

Zhihua Wu
Department of Neurobiology, Institute of Biophysics, Academia Sinica, Beijing 100101, China

Zhaoxuan Zhu
Department of Mechanics and Engineering Science, Peking University, Beijing 100871, China

Chengfu Zhang
Department of Physics, Peking University, Beijing 100871, China

(Received 11 February 1997; revised manuscript received 7 July 1997)

By employing another external field with intensity not larger than 60% of the original driving force, the Hamiltonian chaos in a driven Morse oscillator could be controlled in the sense that the bound state regions can be changed at will. Based on our understanding about the complex dynamics of a periodically driven Hamiltonian system with one degree of freedom by two external fields, an idea of decreasing bound regions is proposed. A formula for island width, being helpful for selecting a proper controlling field, is derived. The mechanism in increasing bound regions extracted from a large number of numerical experiments is still not so clear at present, but we have gotten a good grasp of the roles of controlling parameters with suppression of Hamiltonian chaos. Though no small perturbations with this method can have a satisfactory effect, as previous studies, some remarks about the magnitude of perturbation are presented. [S1063-651X(98)03701-5]

PACS number(s): 05.45.+b

I. INTRODUCTION

Since Ott, Grebogi, and Yorke (OGY) [1] clearly pointed out that one had the possibility of utilizing chaos in practice by controlling, a wide variety of methods have been developed for controlling chaotic systems [1–5]. However, a large number of controlling works in the literature so far have concentrated on dissipative systems. One of the reasons might come from the fact that conservative systems, unlike dissipative systems, are generally difficult to meet in practical engineering. Another important reason is that due to area conservation in Hamiltonian maps, many control approaches, e.g., the OGY method that was developed for the dissipative systems, cannot be applied directly to Hamiltonian systems [6]. In order to overcome this difficulty, Lai *et al.* [6] gave a modified OGY algorithm that is successfully used in stabilizing not only the unstable periodic orbits bounded to a finite volume in phase space in the standard map [6] but also the unbound nonhyperbolic chaotic scattering trajectories in the Gaspard-Rice scattering map [7]. This algorithm features a possible, precise tailoring of the controlling process by applying weak parameter perturbations around preselected unstable periodic orbits. As pointed out in Ref. [6], its efficiency will be greatly enhanced if a general scheme of targeting for the layered Hamiltonian phase space structures emerges in the future.

In this paper, we propose a different way to control Hamiltonian chaos, which could be generally suitable for a type of periodically driven system with one degree of freedom. A driven Hamiltonian system is no longer conservative, and only its time average may be such. The specific condition for the average Hamiltonian to be conservative is that there exist a periodic orbit along which the natural frequency of the system and the forcing frequency are commensurable. For other orbits such as quasiperiodic or chaotic ones, the time average Hamiltonian is not conservative. A driven Hamiltonian system is more appropriate as a model for the study of controlling chaos than a conservative one, since the latter usually does not exchange energy with its environment and it is not easy to find an externally controllable parameter in practice. On the other hand, by introducing a pair of canonically conjugated variables t and $-H$, any one of the time-dependent Hamiltonian systems with n degrees of freedom $H=H(\vec{q},\vec{p},t)$ can be transformed into a conservative one $H'=H(\vec{q},\vec{p},t)-H=H'(\vec{q}\,',\vec{p}\,')$ with a $2(n+1)$ dimensional phase space, where $q_i'=q_i$, $p_i'=p_i$, $q_{n+1}'=t$, $p_{n+1}'=-H$ $(i=1,2,\ldots,n)$ and the canonical equations associated with H' are $\dot{\vec{p}}\,'=-\partial H'/\partial\vec{q}\,'$ and $\dot{\vec{q}}\,'=\partial H'/\partial\vec{p}\,'$, leading to some common features between controlling processes of a driven Hamiltonian and a conservative one. Therefore, some conclusions obtained from driven Hamiltonian systems are believed to be beneficial in controlling of chaos in conservative ones.

In the presence of an external field, the Poincaré surface of section of a driven Hamiltonian system is divided into bound and unbound state regions. These regions are separated by a bounding torus, i.e., the most robust Kolmogorov-Arnold-Moser (KAM) torus that breaks up eventually as the amplitude of the external field is gradually increased. Our aim is to control the structures of phase space, i.e., to increase the area of bound regions (or the area of unbound regions depending on one's requirements). This is realized by employing a second external field whose frequency and amplitude are appropriately chosen. In order to distinguish these two fields, we call the second one a controlling field. Usually by controlling chaos one means a process achieved by small perturbations less than a few percent. We also expect here that the controlling field has an intensity as small as possible. However, the results show that our method features a medium perturbation by which we mean that the amplitude of the controlling field ranges from 40% to 60% of the original driving field. The possible reasons will be analyzed in Sec. IV.

Furthermore, the Morse oscillator is used to illustrate our controlling method. At this point it is worth comparing our approach with another one that controls the chaotic scattering between two atoms interacting via a Morse potential in the presence of a laser field [8]. The latter is focusing on the unbound motion above the field-free dissociation energy in an impulsively driven model, while our interest is concentrated on the phase space below the field-free dissociation energy in a sinusoidal driven model. Additionally, their control mechanism makes full use of a newly created stable resonance island to trap the chaotic scattering trajectories by switching on the driving force during the collision, while ours is based on the analysis of the interaction between two sets of resonances induced by the driving force and the controlling field, respectively. Another important difference lies in the strength of perturbations. The required field in Ref. [8] must be very strong, while in the present paper the amplitude of a controlling field is smaller than that of the original driving force.

This paper is organized as follows. In Sec. II we shall describe the model and present the island width formula [9]. Chirikov's overlap criterion shall be used to analyze the width of islands in a periodically driven system, giving rise to ''the island width formula,'' which provides preliminaries for the controlling method. In Sec. III we shall develop the control method. Numerical results and discussions and concluding remarks are given in Secs. IV and V, respectively.

II. MODEL AND THE ISLAND WIDTH FORMULA

A periodically driven Morse oscillator is extensively used in investigating the dynamical behaviors of a molecular system under intensive laser radiation [10–14]. It is described by a Hamiltonian

$$H=\frac{p^2}{2\mu}+D(1-e^{-\alpha\gamma})^2-\varepsilon\gamma\cos(ft), \tag{1}$$

where the symbols carry their usual meanings, as in Ref. [13]. Considering our purpose in the following we shall first briefly review the resonance structures in system (1) analyzed by Gu *et al.* [13], and then present our method.

We define dimensionless variables as

$$x=1-e^{-\alpha\gamma},$$

$$y=\frac{p}{\sqrt{2D\mu}},$$

$$\tau=\alpha\left(\frac{2D}{\mu}\right)^{1/2}t,$$

$$A=\frac{\varepsilon}{2D\alpha},$$

$$\Omega=\frac{f}{\alpha}\left(\frac{\mu}{2D}\right)^{1/2}.$$

Hamilton's equations associated with Eq. (1) can be written as

$$\begin{aligned}\frac{dx}{d\tau}&=(1-x)y,\\ \frac{dy}{d\tau}&=-(1-x)x+A\cos(\Omega\tau).\end{aligned} \tag{2}$$

As it is not convenient to apply Chirikov's nonlinear resonance theory in such a form, Eq. (2) is rewritten in terms of action-angle variables (I and θ) of an isolated Morse oscillator by using the transformation relations [13]:

$$\begin{aligned}x&=\frac{E+\sqrt{E}\cos\theta}{1+\sqrt{E}\cos\theta},\\ y&=\frac{-\omega\sqrt{E}\sin\theta}{1+\sqrt{E}\cos\theta},\end{aligned} \tag{3}$$

where $E=I-I^2/4$ is the dimensionless energy of the unperturbed Morse oscillator and $\omega=1-I/2$ is the natural frequency of the oscillator. Obviously, E takes a value between 0 and 1 for a bound state. In terms of new variables the Hamiltonian H becomes [13] $K=H/D=E-2A\sum_{n=0}^{\infty}f_n(E)\cos(n\theta)\cos(\Omega\tau)$. The Chirikov's standard theory [15] gives the following formula for the primary resonance width:

$$\Delta E^{1/n}=8\left(\frac{A(1-E^{1/n})}{n}\right)^{1/2}\left(\frac{\sqrt{E^{1/n}}}{1+\sqrt{1-E^{1/n}}}\right)^{n/2}, \tag{4}$$

where $E^{1/n}$ denotes the energy determined by conditions for occurrences of primary resonances

$$n\omega(I)\pm\Omega=0,\quad n=1,2,\ldots\ . \tag{5}$$

Now let us discuss the islands in a chaotic sea. According to Chirikov's theory, when two primary resonances overlap each other, all the KAM tori between them break up, and almost all the nearby phase points fall within the stochastic layer except those initiated within small islands. We think that the remaining islands deserve to be noticed, because the existence of them implies that a driven molecular system with higher energy can be kept in a bound state by employing a specially designed initial condition. In order to illustrate how to estimate the width of an island, we take the winding number $\omega/\Omega=1/1$ island as an example that denotes the remaining regular region around the 1/1 primary resonance center after two primary resonances with $\omega/\Omega=1/1$ and 1/2 overlap, as shown in Fig. 1.

When a touch between separatrices of the 1/1 and 1/2 primary resonances occurs at the critical field strength, the global instability arises. What happens to the motion of the system when the separatrices overlap further? To answer this question, we first restrict ourselves to the vicinity of the 1/1 resonance center. The dynamics with a driving strength beyond a critical value can be understood in such a way that a phase point initiated within the intersected part AB or its symmetric line segment $B'A'$ about the center of the 1/1

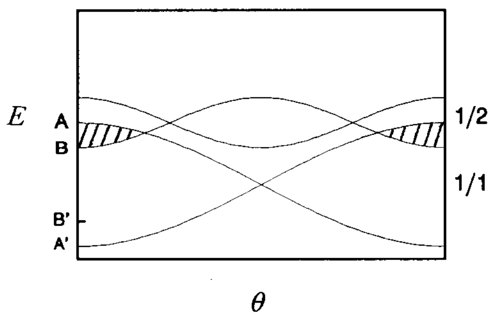

FIG. 1. Overlap of two primary resonances with the winding numbers 1/1 and 1/2 in the Poincaré surface of section. Line segments AB and $B'A'$ along the E axis are symmetric about the center of the 1/1 resonance.

resonance along the E axis will wander from one resonance region to another, and finally escape out. A phase point initiated within the BB' segment will be bounded around the center of the 1/1 resonance. Thus the width of the 1/1 island can be estimated by

$$\Delta E_{i(\text{theo})}^{1/1} = 2(E^{1/2} - E^{1/1}) - \Delta E^{1/2}. \tag{6}$$

Similar to the overlap criterion [15], only the 1/1 and 1/2 primary resonances are considered, and all other primary resonances and the sequences of secondary or higher resonances are neglected. So Eq. (6) can be used to estimate an order of magnitude of island width which usually gives too large values. For instance, by applying a similar expression to the overlap between 1/2 and 1/3 primary resonances in Eq. (2) with $A=0.025$ and $\Omega=0.9$, the 1/2 island has a width $\Delta E_{i(\text{theo})}^{1/2} = 2(E^{1/3} - E^{1/2}) - \Delta E^{1/3} = 0.087$ along the E axis, while numerical simulation yields $\Delta E_{i(\exp)}^{1/2} = 0.031$. However, in this formula it is highly important for us to devise a controlling method for Hamiltonian chaos, because it discloses qualitatively the relation between the width of the island and the driving frequency, as shown in Fig. 2. In this plot we present both the $\Delta E_{i(\text{theo})}^{1/1}$ and $\Delta E_{i(\exp)}^{1/1}$ curves versus Ω (C_1 and C_2) for Eq. (2) with the driving amplitude $A=0.025$. It can be seen that the theoretical results describe, to a good approximate extent, the relation between the width of the 1/1 island and Ω. The discrepancy between C_1 and C_2 is due to the fact that other resonances, especially the infinite number of $(n-1)/n$ secondary resonances generated via the interaction of these 1/1 and 1/2 primary resonances, have effects on the size of the 1/1 island.

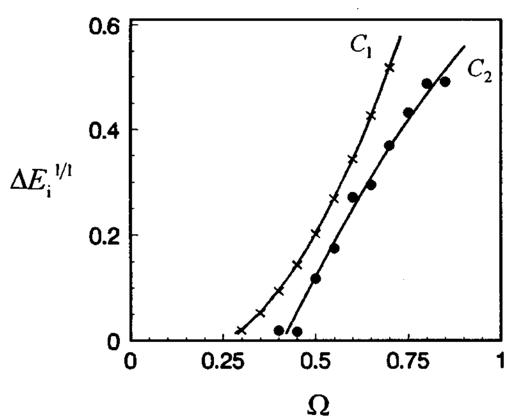

FIG. 2. The width of the 1/1 island with response to the driving frequency Ω in Eq. (2) with $A=0.025$. C_1 is obtained from formula (6) while C_2 corresponds to numerical experiment results.

III. THE CONTROLLING METHOD

Using the driven Morse oscillator (1) as a model we can investigate in this section the controlling of Hamiltonian chaos. As mentioned in Sec. I, our aim is to change the ratio of bound state regions to unbound state regions by employing a weak controlling field with amplitude A_c and frequency Ω_c, where the bound regions denote the region below the bounding torus and the tiny islands in the chaotic sea in the Poincaré surface of section. This definition of the bound regions comes from the fact that, although there is a stochastic layer below the bounding torus, the trajectories initiated below it will be made to remain below it forever.

In the presence of both $A\cos(\Omega\tau)$ and $A_c\cos(\Omega_c\tau)$, Hamilton's equations corresponding to a driven Morse oscillator can be written as

$$\begin{aligned} \frac{dx}{d\tau} &= (1-x)y, \\ \frac{dy}{d\tau} &= -(1-x)x + A\cos(\Omega\tau) + A_c\cos(\Omega_c\tau). \end{aligned} \tag{7}$$

Although the dynamics of a driven Morse oscillator by two external fields with different frequencies simultaneously is so complex that it is very difficult to analyze by using existing theories, the problem regarding the size of bound regions can be simplified by means of the following presumption. Each external field induces a set of resonances as if the other field did not exist, and each primary resonance with the winding number $1/n$ or $1/n_c$ has an equal status in the sense that the overlap criterion [15] and the width of island formula (6) can be applied to any two adjacent primary resonances no matter to which set they belong, where $n_c=1,2,\ldots$ and $1/n_c$ denotes the winding number of the primary resonances induced by $A_c\cos(\Omega_c\tau)$. Therefore, we can choose A_c and Ω_c deliberately so that the largest one among the $1/n_c$ resonances is embedded just between two resonances with the winding number $1/n$ and $1/(n+1)$, where there are yet undestroyed KAM tori. This procedure results in an emergence of stochastic layers according to Chirikov's nonlinear resonance theory. In order to decrease the bound regions as much as possible, the area between 1/1 and 1/2 resonances is usually chosen as the embedded region so that the bounding torus breaks down and the global instability arises. The width of the remaining bound regions of the controlled system can be estimated by means of formula (6). Obviously, the above presumption describes the structures of phase space only in the zeroth-order approximation, but numerical experiments given in the next section indicate that this method is simple and practical.

On the other hand, in a large number of experiments we find that the bound regions can also be increased by employing a method similar to the above except that the controlling

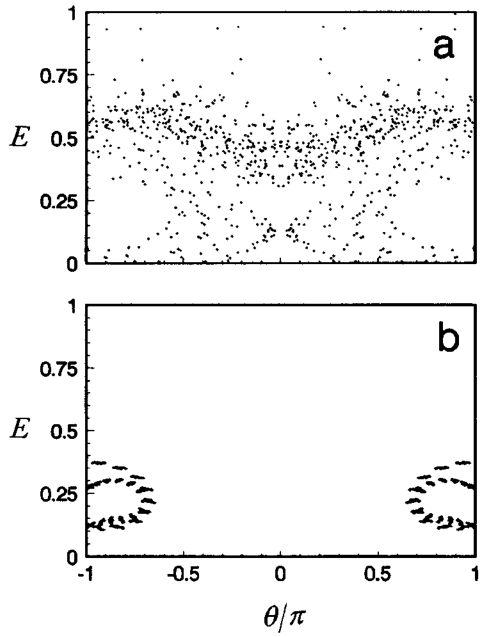

FIG. 3. Numerical results of decreasing bound regions in Eq. (7) with $A=0.025$, $\Omega=0.9$, $A_c=0.010$, and $\Omega_c=0.7$. (a) An unbound trajectory initiated at $(E_0,\theta_0)=(0.330,-\pi)$. (b) A bound trajectory initiated at $(E_0,\theta_0)=(0.280,-\pi)$.

field must satisfy $\Omega_c=2\Omega$ and $\Delta\Phi=\pi$, where $\Delta\Phi$ is the phase difference between the driving field $A\cos(\Omega\tau)$ and the controlling field. The mechanism in such a suppression of Hamiltonian chaos is still not so clear, but through numerical experiments we have gotten a good grasp of the roles of controlling parameters Ω_c, $\Delta\Phi$, and A_c in increasing bound regions, as will be seen in the next section.

IV. NUMERICAL RESULTS

For a driven Morse oscillator (2) with $A=0.025$ and $\Omega=0.9$, about a 50% phase space area in the Poincaré surface of section is dominated by bound states. From Eqs. (4) and (5) we obtain $(E^{1/1},\theta)=(0.1900,-\pi)$, $(E^{1/2},\theta)=(0.7975,-\pi)$, $\Delta E^{1/1}=0.5453$, and $\Delta E^{1/2}=0.2479$, where $(E^{1/1},\theta)$ and $(E^{1/2},\theta)$ are centers of 1/1 and 1/2 primary resonances, respectively. Suppose that the strength of a controlling field is fixed at $A_c=0.010$. The proper frequency that will be used in decreasing bound regions is selected as $\Omega_c=0.7$ by applying the method proposed in Sec. III. By substituting A_c and Ω_c in Eqs. (4) and (5), we can obtain the largest primary resonance $E^{1/1}_{\Omega_c}=0.5100$ and $\Delta E^{1/1}_{\Omega_c}=0.3630$ induced by the controlling field. Thus, when the controlling field is turned on, the global instability sets in. The bound state is restricted mainly to the 1/1 island with a width $\Delta E^{1/1}_{i(\text{theo})}=2(E^{1/1}_{\Omega_c}-E^{1/1})-\Delta E^{1/1}_{\Omega_c}=0.277$.

The numerical experiment from Eq. (7) shows that $\Delta E^{1/1}_{i(\text{exp})}=0.140$, and all the phase points with initial conditions $E_0\geqslant 0.290$ ($\theta_0=\pm\pi$) or $E_0\leqslant 0.130$ ($\theta_0=\pm\pi$) dissociate within a few optical cycles, as shown in Fig. 3. We present separately the Poincaré surface of section of a trajectory initiated at $(E_0,\theta_0)=(0.330,-\pi)$ in Fig. 3(a) and that at $(0.280,-\pi)$ in Fig. 3(b). The former corresponds to a chaotic unbound motion, while the latter corresponds to a complicated bound motion whose frequency cannot be determined easily. These results show that the numerical experiment agrees qualitatively with our theory.

Further experiments show that all attempts to control such a system by small perturbations have failed, and the most suitable intensity ranges from 40% to 50% of the original driving force. Although such a result can be easily explained by applying the theoretical formula in our controlling method, a radical reason is needed. In a dissipative chaotic system, there exists a strange attractor, and due to the ergodicity the system can reach an arbitrarily small vicinity of any phase point included in the attractor. So the small perturbations that are deliberately chosen can stabilize a chaotic trajectory if the system enters a proper small controlling region (the neighborhood of desired periodic orbit). However, for a Hamiltonian system, the phase space is divided into layered components that are separated from each other, and the trajectories initiated at different energies are constrained within the distinct regions in phase space. For example, according to the resonance structures in a driven Morse oscillator, the stable 1/1 island does not get in touch with the stable 1/2 island, and a phase point initiated in a stochastic layer under the bounding torus will not enter the unbound stochastic region forever, etc. The layered structure is so universal in Hamiltonian systems that the phase space of the same stochastic layer is separated by cantori, remains of KAM tori, and particles initialized in one layer of the chaotic region wander in that layer for a long period of time before they cross the cantori and wander in the next layer. Furthermore, the layered structure repeats itself in each smaller space scale, such as the scale of secondary or higher resonances. Different layers correspond to different initial conditions. If an initial condition and the intended target are in the same layer, the needed perturbations may be as small as possible after the system enters the desired controlling region (the problem of long chaotic transients is not discussed here); e.g., the maximum range of parameter variation in Ref. [6] is 1% of the unperturbed parameter. When the initial condition and the target are in different layers, as the situation met in most cases, the controlling force must be large in order to bring the system from one layer to another. We guess the amount of perturbation would be proportional to the initial energy difference of these two layers. In our controlling method, $A_c\cos(\Omega_c\tau)$ is used to induce a sufficiently wide resonance that can touch the two adjacent separatrices of resonances in two different layers, so a medium A_c (about 40% of A) can be reasonably understood.

On the other hand, for the same driven oscillator (2) with $A=0.025$ and $\Omega=0.9$ as above, it is expected that the bound regions will be increased by adding a controlling field $A_c\cos(\Omega_c\tau+\Delta\Phi)$ with $A_c=0.010$. As pointed out in Sec. III, we must set $\Omega_c=1.8$ and $\Delta\Phi=\pi$. The numerical results are displayed in Fig. 4(a). In this figure, we present seven typical trajectories whose initial conditions are $(0.410,-\pi)$, $(0.448,-\pi)$, $(0.520,-\pi)$, $(0.580,-\pi)$, $(0.617,-\pi)$, $(0.670,-\pi)$, and $(0.745,-\pi)$, respectively. It can be seen that not only is the bounding torus of the system raised from $E_0=0.548$ ($\theta_0=-\pi$) to $E_0=0.670$ ($\theta_0=-\pi$), but also the width of the 1/2 island is increased from $\Delta E^{1/2}_{i(\text{exp})}=0.031$ to

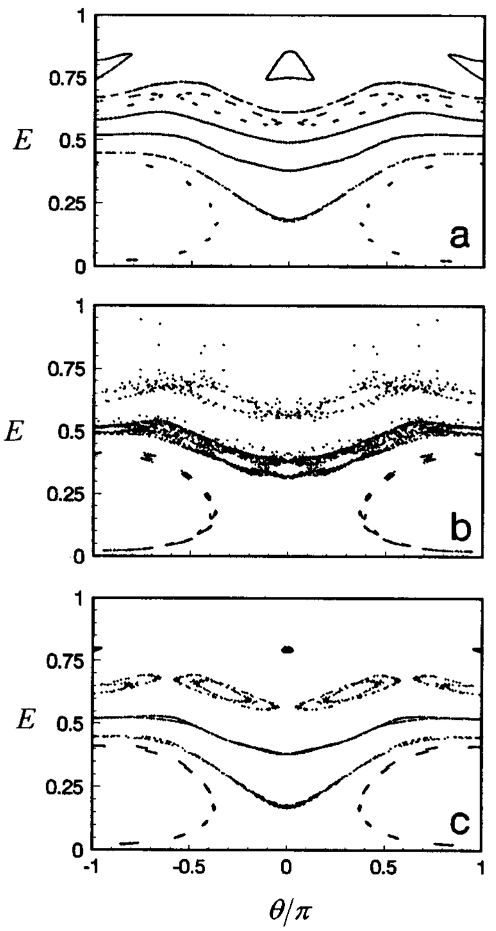

FIG. 4. Influence of a small perturbation to the frequency Ω_c and the phase difference $\Delta\Phi$ separately on the effect in increasing the bound regions in Eq. (7) with $A=0.025$, $\Omega=0.9$, and $A_c=0.010$. (a) Seven trajectories with $\Omega_c=1.8$ and $\Delta\Phi=\pi$. (b) Five trajectories with $\Omega_c=1.7995$ and $\Delta\Phi=\pi$. (c) Three trajectories with $\Omega_c=1.8$ and $\Delta\Phi=\frac{9}{10}\pi$. The initial phase point (E_0,θ_0) of each trajectory is given in the text.

0.075 along the E axis. Furthermore, the stochastic layer under the bounding torus is much thinner than before controlling.

In order to investigate the effect of Ω_c, we cut down Ω_c by 3/10 000 and repeat the above simulation. Three trajectories are given in Fig. 4(b). The lower trajectory initiated from $(0.410,-\pi)$ represents a bound motion, the middle one from $(0.520,-\pi)$ dissociates from the bound state very slowly, and the upper one from $(0.617,-\pi)$ escapes quickly. This fact demonstrates that a slight perturbation to Ω_c can result in a disastrous or qualitative change in the phase space structures, which is quite different from the situation in decreasing bound regions where the resonance structure is not so sensitive to the perturbation of Ω_c.

It is interesting that the effect of $\Delta\Phi$ is not so important as Ω_c in suppressing Hamiltonian chaos of Eq. (2). Reducing $\Delta\Phi$ by 1/10, we present the Poincaré surface of section of five trajectories initiated from $(0.410,-\pi)$, $(0.448,-\pi)$, $(0.520,-\pi)$, $(0.617,-\pi)$, and $(0.800,-\pi)$ in Fig. 4(c), where the area of bound regions is clearly greater than that in Fig. 4(b). We can conclude that the dynamical behavior of a driven Morse oscillator controlled by this method changes with $\Delta\Phi$ quantitatively or continuously.

As an addition we also explore the effect of A_c. In numerical experiments it is found that the controlling field with a too large or a too small strength cannot reach a good suppressing effect; e.g., for the situation with $A=0.025$, $\Omega=0.9$, $\Omega_c=1.8$, and $\Delta\Phi=\pi$, a suitable range of A_c is {0.010,0.015}. A great number of experimental results show that the most suitable A_c used in increasing the bound regions is about 50% of A. Whether this is due to the layered structure of the Hamiltonian system as discussed above or other reasons remains a open question, which needs further investigation.

V. SUMMARY

In this paper we suggest an idea that the complex dynamics of a periodically driven Hamiltonian system with one degree of freedom by two external fields can be understood qualitatively based on the interaction between two sets of resonances induced by every single driving force, respectively, and all primary resonances can be regarded as being on an equal status in the sense that Chirikov's theory can be used in any two adjacent resonances. Another contribution of the present work is the island width formula, which provides a necessary preparation for this controlling method. The area of bound regions in a driven Hamiltonian system can be changed at will by adding another medium driving field, and specifically a controlling field used to decrease the bound regions is selected by means of the island width formula while that used to increase the bound regions must satisfy $\Omega_c=2\Omega$ and $\Delta\Phi=\pi$.

Numerical results for a periodically driven Morse oscillator show that our method is quite effective. The discrepancy between theory and numerical experiments in decreasing the bound regions is due to the zeroth-order approximation of our method, i.e., we only consider the primary resonances and neglect all secondary and other higher resonances. In fact, the resonance structures of a driven Morse oscillator are actually much more complicated than those that appear explicitly in the Hamiltonian [15].

With the development of high-power infrared lasers, there has been considerable interest in investigating the dynamical behavior of a molecular system under intensive laser radiation [10–14,16]. Our method not only gives a useful way for controlling chaos in such a system but also provides some intuitive examples for study of the complex dynamics of a driven molecular system under two external fields. For example, since increasing the bound region in the Poincaré surface of section of a Hamiltonian system is related to the molecular association rate in a chemical reaction while decreasing the bound region is related to the dissociation rate, by employing a second external field whose intensity is smaller than that of the original driving field these processes would find applications in controlling chemical reactions by using a laser field. In particular, it is well known that the laser intensity required to dissociate a diatomic molecular is far too high to be practical. We hope that the approach for decreasing the bound region in this paper may give a challenging idea for a universal solution to this problem. Moreover, in order to get a better effect on lowering the strength of a driving force required in all molecular photodissociation processes, is it more practical to apply two or more weak controlling fields to a driven diatomic molecular than one? This question needs further investigation on the dynamics of a driven Hamiltonian system by three or more external fields both theoretically and practically.

ACKNOWLEDGMENTS

The first author wishes to express her gratitude to Professor Y. Gu for direction on Sec. II and for helpful discussions. This work was supported by the National Science Foundation of China.

[1] E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **64**, 1196 (1990).
[2] K. Pyragas, Phys. Lett. A **170**, 421 (1992).
[3] E. R. Hunt, Phys. Rev. Lett. **67**, 1953 (1991).
[4] G. Hu, Z. L. Qu, and K. F. He, Int. J. Bifurcation Chaos Appl. Sci. Eng. **5**, 901 (1995).
[5] G. Chen and X. Dong, Int. J. Bifurcation Chaos Appl. Sci. Eng. **3**, 1363 (1993).
[6] Y. C. Lai, M. Ding, and C. Grebogi, Phys. Rev. E **47**, 86 (1993).
[7] Y. C. Lai, T. Tel, and C. Grebogi, Phys. Rev. E **48**, 709 (1993).
[8] Z. M. Lu, M. Vallieres, J. M. Yuan, and J. F. Heagy, Phys. Rev. A **45**, 5512 (1992).
[9] Z. H. Wu, Master, thesis, Lan Zhou University, 1989 (unpublished).
[10] S. K. Gray, Chem. Phys. **75**, 67 (1983).
[11] P. S. Dardi and S. K. Gray, J. Chem. Phys. **80**, 4738 (1984).
[12] R. M. O. Galvano, L. C. M. Miranda, and J. T. Mendonca, J. Phys. B **17**, L577 (1984).
[13] Y. Gu and J. M. Yuan, Phys. Rev. A **36**, 3788 (1987).
[14] J. F. Heagy and J. M. Yuan, Phys. Rev. A **41**, 571 (1990).
[15] B. V. Chirikov, Phys. Rep. **52**, 263 (1979).
[16] R. C. Brown and R. E. Wyatt, Phys. Rev. Lett. **57**, 1 (1986).