Thomson scattering from ion acoustic waves in laser plasmas

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This work is concerned with the description of ion acoustic fluctuations in electron-ion plasmas relevant to laser-plasma interaction experiments. A nonlocal closure to the linearized moment equations for the fluctuating hydrodynamic quantities is introduced. These equations are used to construct practical expressions for the dynamical form factor and Thomson scattering cross section, which are valid in the entire region of particle collisionality in plasmas with high Z and large ZT_e/T_i. [S1063-651X(98)05403-8]

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I. INTRODUCTION

The creation of hot dense plasmas with lasers is an essential feature of x-ray lasing schemes as well as inertial fusion experiments. In such plasmas Thomson scattering is useful for both characterizations, which is necessary in order to calibrate and verify computer simulations and also in the investigation of basic plasma physics. It has recently become possible for Thomson scattering to measure ion acoustic wave features such as damping and phase velocity in laser plasmas, which allows ionization and temperature to be time resolved [1,2]. Advances in the understanding of scattering instabilities have been made possible by Thomson scattering from enhanced levels of plasma fluctuations (see, e.g., [3,4]). Furthermore, Thomson scattering has been used as a tool for understanding basic plasma physics close to thermodynamic equilibrium. For example, both branches of the ion acoustic dispersion relation have been directly observed in a plasma with two ion species [5] and the ion plasma wave dispersion relation has been verified [6]. The utility of Thomson scattering, of which the above are examples, can be further enhanced when used in conjunction with better theoretical models.

The cross section for the Thomson scattering of laser light from plasmas is determined by $S(k,\omega)$, the Fourier transform of the electron density autocorrelation function (from here on referred to as the dynamic form factor). This is well known in both the strongly collisional and collsionless limits, while the wide intermediate (weakly collisional) region of importance to laser plasmas has not yet been addressed. This paper sets out to give a self-consistent approach to the problem. We will evaluate electron density-density correlation functions by following the theory of fluctuations as described by Oberman and Williams and the results contained therein [7]. One of the key results of this theory is the demonstration that the two-point correlation function of the phase-space fluctuation $\langle \delta f^{\alpha}(\vec{x},\vec{v},t)\,\delta f^{\beta}(\vec{x}_0,\vec{v}_0,t_0)\rangle$ obeys a linearized version of the kinetic equation for the one-particle distribution function $f^{\alpha}(\vec{x},\vec{v},t)$ in the $\vec{x},\vec{v},t$ variables. This is a kinetic version of Onsager's ''regression of fluctuations'' [8] whereby fluctuations evolve from their initial values according to the equations of linearized hydrodynamics. Indeed it can be shown that this kinetic description reduces to Onsager's prescription in the hydrodynamic regime ($l_\alpha/L_H \ll 1$, $\nu_\alpha\tau_H \gg 1$) by a modification of the Chapman-Enskog method (see, e.g., [9]). Here l_α and ν_α are the collisional mean free path and collision frequency of species α respectively, and L_H,τ_H are the length and times scales for the evolution of the fluctuating hydrodynamic variables. The derivation of Onsager's method from kinetic theory can be used to justify the validity of the method not only for thermodynamic equilibrium, but also for fluctuations about some nonequilibrium background state that may, for example, support a heat flux. We will further extend the method's validity outside the usual hydrodynamic regime by making use of hydrodynamiclike models that capture kinetic effects.

Hydrodynamiclike theories that model kinetic effects have generated much interest recently. This is due to the need to describe plasmas with strong gradients that violate the usual ordering necessary for the applicability of classical transport theory [10]. These nonlocal models incorporate frequency and wave-vector-dependent transport coefficients resulting in the response of the ''fluxes'' to the thermodynamic ''forces'' becoming delocalized in both space and time. Bychenkov *et al.* [11,12] have developed nonlocal hydrodynamic models that are relevant for plasmas characterized by large $Z, ZT_e/T_i$ and slow processes that evolve on the ion acoustic time scale. In this paper these hydrodynamic models are used to construct useful expressions for the dynamic form factor $S(k,\omega)$ that are valid outside the usual domain of validity for classical transport theory. In particular, they are accurate in the weakly collisional region that is of importance to laser plasmas. In the strongly collisional limit our expressions agree with the usual two-fluid results of Braginskii [10], while in the collisionless limit we connect with the results derived from the Vlasov equation (within the approximations relevant to each case).

We will analyze in detail two cases of our general expression for the dynamic form factor $S(k,\omega)$: the ion weakly collisional case where ion viscosity (modified by finite frequency) is important together with collisionless electron Landau damping and the weakly collisional electron case in which the ions are collisional and the electron transport is nonlocal. In the ion weakly collisional case we present our analytical expression for $S(k,\omega)$ which describes the effect of ion-ion collisions on the position and width of the ion acoustic peaks in the scattered spectrum and we relate this to some experiments reported in the literature [1,2]. We will also outline the range of parameters in which ion collisional effects are important and the usual collisionless theory of $S(k,\omega)$ is inadequate [13]. Our theory of $S(k,\omega)$ also predicts the correct line shape for plasmas with weakly collisional electrons that are commonly encountered in laser-plasma interaction experiments. The height of the ion acoustic peaks are determined by the damping of ion acoustic waves. Since this damping depends on plasma transport properties, in particular electron thermal conductivity, we propose that the nonlocality of heat transport may be inferred from the scattered spectra. We assert that these descriptions are correct not only for hydrodynamic fluctuations, but also for fluctuations whose ratio of wavelength to mean free path is arbitrary. A comparison of our results with the standard collisionless cases will also be used in order to justify our method.

Our paper is organized in the following way. Section II discusses the general theory of fluctuations. Section III A outlines how the general closure problem can be addressed, Sec. III B gives the explicit form of the closure, Sec. III C gives the closed set of equations satisfied by the fluctuating hydrodynamic quantities, and Sec. III D uses these in order to calculate the dynamic form factor $S(k,\omega)$. Implications for previous experiments and proposals for the observation of nonlocal transport are discussed in Secs. IV A and IV B. Finally, Sec. V is a summary.

II. THEORY OF FLUCTUATIONS IN PLASMAS

The dynamic form factor $S(k,\omega)$ determines the cross section for the Thomson scattering of laser light from the plasma, where $\vec{k}=\vec{k}_0-\vec{k}'$ and $\omega=\omega_0-\omega'$ are the momentum and energy transfer, i.e., the difference in the wave vector and frequency between the probe $(\vec{k}_0,\omega_0)$ and scattered $(\vec{k}',\omega')$ electromagnetic waves [13]. For stable plasmas, $S(k,\omega)$ is well known in two opposite limits. These are the collisionless limit, given by $kl_\alpha \gg 1$, $\omega \gg \nu_\alpha$, and the hydrodynamic limit in which the opposite is true, $kl_\alpha \ll 1$, $\omega \ll \nu_\alpha$.

Historically there have been many differing approaches taken in order to calculate $S(k,\omega)$. The formalism of fluctuations described by Oberman and Williams [7] is particularly suited to our needs. They have derived kinetic equations for the hierarchy of phase-space fluctuations $\langle \delta f^\alpha(\vec{x},\vec{v},t)\,\delta f^\alpha(\vec{x}_0,\vec{v}_0,t_0)\rangle$, $\langle \delta f^\alpha(\vec{x},\vec{v},t)\,\delta f^\beta(\vec{x}\,',\vec{v}\,', t')\,\delta f^\alpha(\vec{x}_0,\vec{v}_0,t_0)\rangle$, etc., where $\delta f^\alpha(\vec{x},\vec{v},t)$ is the difference between the Klimontovich microdensity

$$f^\alpha(\vec{x},\vec{v},t)=\sum_{i=1}^{N_\alpha} \delta(\vec{v}-\vec{v}_i(t))\,\delta(\vec{x}-\vec{x}_i(t)) \tag{1}$$

and its statistical average

$$\delta f^\alpha(\vec{x},\vec{v},t)=f^\alpha(\vec{x},\vec{v},t)-\langle f^\alpha(\vec{x},\vec{v},t)\rangle. \tag{2}$$

In all the following we will assume the species label α to mean both electrons and ions, $\alpha=e,i$. Starting from the two-time Liouville equation (as originally done by Rostoker [14]) and arriving at a linearized form of the Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy, they derived a key result. Namely, for stable plasmas the phase-space fluctuation obeys a kinetic equation that is the linearization of the usual equation for the single-particle distribution function. For example, in the collisionless limit δf^α obeys the linearized Vlasov equation

$$\left(\frac{\partial}{\partial t}+\vec{v}\cdot\frac{\partial}{\partial \vec{x}}\right)\delta f^\alpha-\frac{e_\alpha}{m_\alpha}\frac{\partial\langle f^\alpha\rangle}{\partial \vec{v}}\cdot\frac{\partial}{\partial \vec{x}}\delta\phi=0, \tag{3}$$

$$\delta\phi(\vec{x},\vec{v},t)=\sum_\beta e_\beta\int d\vec{x}\,'d\vec{v}\,'\,\frac{\delta f^\beta(\vec{x}\,',\vec{v}\,',t)}{|\vec{x}-\vec{x}\,'|}. \tag{4}$$

This can be solved by a Fourier space and Laplace time transform with the initial condition $\delta f^\alpha(\vec{k},\vec{v},t=0)$. Correlations are then obtained by multiplying the solution $\delta f^\alpha(\vec{k},\vec{v},\omega)$ by $\delta f^{\beta *}(\vec{k},\vec{v}\,',t=0)$ and ensemble averaging in order to obtain $C^\dagger_{\alpha\beta}(\vec{k},\vec{v},\vec{v}\,',\omega)=\langle \delta f^\alpha(\vec{k},\vec{v},\omega)\,\delta f^{\beta *}(\vec{k},\vec{v}\,',0)\rangle$ in terms of the initial correlations. $C^\dagger_{\alpha\beta}(\vec{k},\vec{v},\vec{v}\,',\omega)$ is the Laplace transform of the correlation function $C_{\alpha\beta}(\vec{k},\vec{v},\vec{v}\,',t)=\langle \delta f^\alpha(\vec{k},\vec{v},t)\,\delta f^{\beta *}(\vec{k},\vec{v}\,',0)\rangle$,

$$C^\dagger_{\alpha\beta}(\vec{k},\vec{v},\vec{v}\,',\omega)=\int_0^\infty dt\exp(i\omega t)\,C_{\alpha\beta}(\vec{k},\vec{v},\vec{v}\,',t). \tag{5}$$

The initial conditions for a weakly coupled equilibrium plasma are given by

$$\langle \delta f^\alpha(\vec{x},\vec{v},0)\,\delta f^\beta(\vec{x}\,',\vec{v}\,',0)\rangle = \delta_{\alpha\beta}\delta(\vec{v}-\vec{v}\,')\,\delta(\vec{x}-\vec{x}\,')\,f^\alpha_M(v)/n_\alpha, \tag{6}$$

where $f^\alpha_M(v)$ is a Maxwellian distribution function,

$$f^\alpha_M(v)=n_\alpha/(\sqrt{2\pi}v_{T_\alpha})^3\exp(-v^2/2v^2_{T_\alpha}) \tag{7}$$

and $v_{T_\alpha}=\sqrt{T_\alpha/m_\alpha}$ is the thermal velocity of particles of species α. Due to the time-reversal symmetry of $C_{\alpha\beta}(\vec{k},\vec{v},\vec{v}\,',t)$ and the fact it is a real quantity, its Fourier transform can be written in terms of $C^+_{\alpha\beta}$ according to

$$\begin{aligned}C_{\alpha\beta}(\vec{k},\vec{v},\vec{v}\,',\omega)&=\int_0^\infty dt\exp(i\omega t)\,C_{\alpha\beta}(\vec{k},\vec{v},\vec{v}\,',t)\\&\quad+\int_{-\infty}^0 dt\exp(i\omega t)\,C_{\alpha\beta}(\vec{k},\vec{v},\vec{v}\,',t)\\&=C^+_{\alpha\beta}(\vec{k},\vec{v},\vec{v}\,',\omega)+[C^+_{\alpha\beta}(\vec{k},\vec{v},\vec{v}\,',\omega)]^*\\&=2\ \mathrm{Re}\ C^+_{\alpha\beta}(\vec{k},\vec{v},\vec{v}\,',\omega).\end{aligned} \tag{8}$$

$C_{\alpha\beta}(\vec{k},\vec{v},\vec{v}\,',\omega)$ [Eq. (8)] may be used in order to obtain spectral functions of macroscopic quantities. Of particular importance is the dynamical form factor $S(k,\omega)$,

$$S(\vec{k},\omega)=\frac{1}{n_e}\int d\vec{v}\, d\vec{v}\,'C_{ee}(\vec{k},\vec{v},\vec{v}\,',\omega)$$
$$=2\,\mathrm{Re}\,\frac{\langle \delta n_e(\vec{k},\omega)\,\delta n_e^*(\vec{k},0)\rangle}{n_e}. \qquad (9)$$

Following Eqs. (3)–(9), one finds

$$S(k,\omega)=\frac{2\pi}{k}\,\frac{|1+\chi_i|^2F_e(\omega/k)+Z|\chi_e|^2F_i(\omega/k)}{|\epsilon(k,\omega)|^2}, \quad (10)$$

where $\epsilon=1+\Sigma_\alpha\chi_\alpha$, χ_α is the collisonless form for the partial susceptibility of species α, and $F_\alpha(\omega/k)$ are the one-dimensional distribution functions evaluated at the phase velocity and normalized to unity. Due to its simplicity, this form of $S(k,\omega)$ is the most often used in applications (even when its validity is questionable). On taking particle discreteness (collisions) into account Eq. (3) is modified by the addition of a collision term on the right-hand side,

$$\left(\frac{\partial}{\partial t}+\vec{v}\cdot\frac{\partial}{\partial \vec{x}}\right)\delta f^\alpha-\frac{e_\alpha}{m_\alpha}\frac{\partial\langle f^\alpha\rangle}{\partial \vec{v}}\cdot\frac{\partial}{\partial \vec{x}}\delta\phi$$
$$=\sum_\beta C(\delta f^\alpha,\langle f^\beta\rangle)+C(\langle f^\alpha\rangle,\delta f^\beta). \qquad (11)$$

This is found to be the linearized Balescu-Guernsey-Lenard (BGL) collision term, which in turn can be approximated by the Landau equation and further simplified by using the Lorentz operator for electron-ion collisions. In the hydrodynamic regime this equation can be solved by a modification of the usual Chapman-Enskog [10] method, resulting in a system of linear fluid equations for the fluctuating hydrodynamic quantities $\{\delta n_\alpha,\delta\vec{u}_\alpha,\delta T_\alpha\}$. These may be obtained by linearizing the usual Braginskii fluid equations

$$\frac{\partial n_\alpha}{\partial t}+\frac{\partial}{\partial \vec{x}}(n_\alpha\vec{u}_\alpha)=0, \qquad (12)$$

$$\left(\frac{\partial}{\partial t}+\vec{u}_\alpha\cdot\frac{\partial}{\partial \vec{x}}\right)\vec{u}_\alpha=-\frac{1}{m_\alpha n_\alpha}\frac{\partial}{\partial \vec{x}}(n_\alpha T_\alpha)-\frac{1}{m_\alpha n_\alpha}\frac{\partial}{\partial \vec{x}}\cdot\hat{\sigma}_\alpha$$
$$-\frac{e_\alpha}{m_\alpha}\frac{\partial}{\partial \vec{x}}\phi+\frac{1}{m_\alpha n_\alpha}\vec{R}_\alpha, \qquad (13)$$

$$\left(\frac{\partial}{\partial t}+\vec{u}_\alpha\cdot\frac{\partial}{\partial \vec{x}}\right)T_\alpha+\frac{2T_\alpha}{3}\frac{\partial}{\partial \vec{x}}\cdot\vec{u}_\alpha$$
$$=-\frac{2}{3n_\alpha}\frac{\partial}{\partial \vec{x}}\cdot\vec{q}_\alpha-\frac{2}{3n_\alpha}\hat{\sigma}_\alpha\cdot\frac{\partial\vec{u}_\alpha}{\partial \vec{x}}+\frac{2}{3n_\alpha}Q_\alpha, \qquad (14)$$

with $n_\alpha\rightarrow n_\alpha+\delta n_\alpha$ and so on. The linearization of equations (12)–(14) together with the linearized closure relations that relate the fluctuating fluxes $\{\delta\vec{q}_\alpha,\delta\hat{\sigma}_\alpha,\delta\vec{R},\delta Q\}$ to the forces $\{-\nabla\delta T_\alpha,\delta\hat{W},\delta\vec{u}\}$ as a result of the Chapman-Enskog procedure [10] and initial conditions [obtained by taking moments of Eq. (6)] will form a complete set of equations from which one can calculate the thermal correlations of any of the hydrodynamic variables, for example, $\langle\delta n_\alpha\delta n_\beta^*\rangle/n_e$. The choice of fluxes (heat flux $\delta\vec{q}_\alpha$, stress $\delta\hat{\sigma}_\alpha$, friction $\delta\vec{R}$, and heat generation δQ) and the corresponding forces (temperature gradient $-\nabla\delta T_\alpha$, rate of strain $\delta\hat{W}$, and relative velocity $\delta\vec{u}$) are those of Braginskii. For a discussion of different choices of hydrodynamic closure relations see also Balescu [15].

The fact that fluctuations on a hydrodynamic scale in thermal equilibrium relax according to the equations of linearized hydrodynamics has been known for a long time [16,17]; however, the derivation from kinetic theory rather than from thermodynamics shows the much wider validity of the method. The linear hydrodynamic fluctuations may be linearized about a nonequilibrium flow, for example, a state with heat flux. In summary, the correctness of the reduced hydrodynamic description rests on the validity conditions for the closure which in this case is the Chapman-Enskog procedure. This requires $kl_\alpha\ll 1$, $\nu_\alpha/\omega\ll 1$. If these conditions are not met, and often they are not, then some alternative closure to the fluid moments must be sought or the linearized BGL equation (11) solved by some other means.

III. THEORY OF LOW-FREQUENCY FLUCTUATIONS

A. The closure problem

The Chapman-Enskog method of closure to the hydrodynamic moment equations fails in the weakly collisional case; however, we still wish to retain the fluidlike description. We are then presented with the problem of closure of the moment equations (12)–(14). This arises because each velocity moment of the kinetic equations introduces still higher-order velocity moments, for example, the heat flux and stress tensor $\vec{q}_\alpha$ and $\hat{\sigma}_\alpha$, which must be expressed in terms of the lower-order hydrodynamic moments. We will present two methods of closure that together cover a wide range of conditions encountered in laser plasmas.

Laser plasmas are quite often nonisothermal as a result of inverse bremsstrahlung heating that preferentially heats the electrons $T_e\gtrsim T_i$. The ionization can also be large especially for heavy elements such as gold, $Z\gg 1$. Therefore, in many experiments there exists a separation in scale between the electron and ion collisionalities expressed by the relation $l_{ei}=(ZT_e/T_i)^2 l_i/\sqrt{2}$, where l_{ei} and l_i are the electron-ion and ion-ion collisional mean free paths $l_{ei}=v_{T_e}/\nu_{ei}$ and $l_i=v_{Ti}/\nu_i$. Here we have adopted the usual definition of collision frequencies

$$\nu_{ei}=\frac{4\sqrt{2\pi}Ze^4n_e\Lambda_e}{3\sqrt{m_e}T_e^{3/2}},\qquad \nu_i=\frac{4\sqrt{\pi}Z^4e^4n_i\Lambda_i}{3\sqrt{m_i}T_i^{3/2}}, \qquad (15)$$

where Λ_α are the Coulomb logarithms. Considering an ion acoustic fluctuation in the plasma with a wave vector k and the separation $l_{ei}\gg l_i$ we consider the possibilities

$$kl_i,kl_{ei}\ll 1 \quad \text{strongly collisional case} \quad \text{(Braginskii)}, \qquad (16)$$

$$kl_i\ll 1,\; kl_{ei}\sim 1 \quad \text{weakly collisional electrons}, \qquad (17)$$

$$kl_i\sim 1,\; kl_{ei}\gg 1 \quad \text{weakly collisional ions}, \qquad (18)$$

$$kl_i, kl_{ei} \gg 1 \quad \text{collisionless case} \quad \text{(Vlasov)}. \tag{19}$$

In the first case (16), the linearized fluid equations of Braginskii [10] correctly describe the evolution of the fluctuations and ion acoustic damping is determined in terms of the classical transport coefficients of thermal conduction and viscosity. In the last case (19), the collisionless, linearized Vlasov descriptions of fluctuations is appropriate (3) and damping is then due to wave-particle resonance (Landau damping), which depends on the form of the distribution function in velocity space at the phase velocity of the wave. These two cases are well known, but as yet the two intermediate cases are not and have no self-consistent description. They are, however, very important because with typical k vectors and conditions in laser-plasma experiments one invariably finds oneself in either of the two intermediate cases, for example, the experiments of La Fontaine *et al.* [1,2].

In order to describe the electron weakly collisonal regime $kl_{ei} \sim 1$ [Eq. (17)] we will make use of a nonlocal theory of electron transport that has been developed by Bychenkov *et al.* [12]. This theory is based upon the solution to the linearized electron Fokker-Planck equation by a Legendre polynomial expansion $\delta f^e(k,\vec{v},\omega) = \Sigma_l \delta f_l(v) P_l(\cos\theta)$. In this work the authors have been able to express the first Legendre coefficient δf_1 in terms of the hydrodynamic variables $\vec{E}^*$, the effective electric field; T_e, the electron temperature; and $\vec{u}_i$, the ion velocity in a way reminicent of the Chapman-Enskog development, but without the restrictions of strong collisions. This has been achieved by the introduction of a renormalized collision frequency that includes the effects of all higher Legendre modes that are negligible in the strongly collisional limit, but necessary in order to describe properly the collisionless limit. This solution for δf_1 is sufficient to achieve the necessary closure as δf_1 is responsible for transport. For example, δf_1 can be substituted into the expressions for heat flux $\vec{q}_e$ and current $\vec{j}$. Since the phase-space fluctuation δf^α [Eq. (2)] obeys the same equations as the perturbation of the distribution function in the work of Bychenkov *et al.* [12] we may here interpret the δf^α to be the phase-space fluctuation. We emphasize that this theory has a domain of validity beyond that of classical transport theory.

In describing the ion weakly collisional case (18) the usual classical transport for ions is not sufficient. To address this problem the analytic method of expansion of the ion kinetic equation in tensor Hermite polynomials is used. The full set of moment equations can be thought of as a representation of the kinetic equation with closure being achieved by truncation of the hierarchy. A truncation at the 21 moment level (Grad 21M) [15] retaining explicitly the frequency dependence in the ion tensor moment equation results in an ion viscosity that is frequency dependent and hence nonlocal in time. This method has been shown by [11] to correctly describe ion acoustic wave properties in the limit $\omega \gg kv_{T_i}$. The damping of ion waves is in agreement with Braginskii in the collisional limit $\omega \ll \nu_i$ and also agrees with Fokker-Planck solutions in the intermediate regime of collisionality $\omega \gtrsim \nu_i$ for large ZT_e/T_i [11]. We will use the above closures together with the linearization of Eqs. (12)–(14) in order to cover both cases (17) and (18) of weakly collisional plasmas that are often encountered experimentally [1,2,5,6].

B. Nonlocal closure

We start by writing the system of linearized moment equations for the fluctuating hydrodynamical quantities $\delta n_\alpha, \delta\vec{u}_\alpha, \delta T_\alpha$, obtained from the kinetic equation for the phase-space particle density fluctuation $\delta f^\alpha(\vec{x},\vec{v},t)$ $(\alpha = e,i)$ as prescribed by Eqs. (12)–(14). Since ions are predominantly responsible for momentum transport, we write the ion momentum equation with the viscous term but neglect the ion thermal transport effect and the electron-ion energy exchange $\delta Q \to 0$ in Eq. (14) as these terms are small in comparison to momentum transport described by the viscosity tensor $\delta\hat{\sigma}^i$, particularly for plasmas with $ZT_e/T_i \gg 1$. While it is the ions that carry the momentum, it is the electrons that are responsible for the heat transport. We also make approximations pertinent to low-frequency fluctuations. We assume the quasineutral limit $\delta n_e \approx Z\delta n_i$, so that we restrict ourselves to long-wavelength perturbations $k\lambda_{De} \ll 1$, where λ_{De} is the electron Debye length:

$$\frac{\partial \delta n_\alpha}{\partial t} + n_\alpha \frac{\partial}{\partial \vec{x}} \cdot \delta\vec{u}_\alpha = 0, \tag{20}$$

$$\frac{\partial \delta\vec{u}_i}{\partial t} = -\frac{Ze}{m_i} i\vec{k}\delta\phi - \frac{i\vec{k}}{m_i n_i}(\delta n_i T_i + \delta T_i n_i) + \frac{1}{m_i n_i} i\vec{k}\cdot\delta\hat{\sigma}_i + \frac{1}{m_i n_i}\delta R_{ie}, \tag{21}$$

$$\frac{\partial \delta T_i}{\partial t} + \frac{2}{3} T_i i\vec{k}\cdot\delta\vec{u}_i = 0, \tag{22}$$

$$\frac{\partial \delta T_e}{\partial t} + \frac{2}{3n_e}\frac{\partial}{\partial \vec{x}}\cdot\delta\vec{q}_e + \frac{2}{3}T_e \frac{\partial}{\partial \vec{x}}\cdot\delta\vec{u}_e = 0, \tag{23}$$

where $\delta\vec{u}_e = \delta\vec{u}_i - \delta\vec{j}/en_e$. The phase-space fluctuation δf^e is solved for in terms of the hydrodynamic moments $\delta\vec{u}_i, \delta n_e, \delta T_e$ and the potential $\delta\phi$ as described in [12]. On substituting the solution for $\delta f_e(\vec{k},\vec{v},\omega)$ that is dependent on $\delta\vec{u}_i, \delta n_e, \delta T_e, \delta\phi$ into the expressions for current, heat flux, and friction, one finds the closure relations

$$\delta\vec{j} = \sigma\delta\vec{E}^* + \alpha i\vec{k}\delta T_e + \beta_j e n_e \delta\vec{u}_i, \tag{24}$$

$$\delta\vec{q}_e = -\alpha T_e \delta\vec{E}^* - \chi i\vec{k}\delta T_e - \beta_q n_e T_e \delta\vec{u}_i, \tag{25}$$

$$\delta\vec{R}_{ie} = -(1-\beta_j) n_e e\,\delta\vec{E}^* + \beta_q n_e i\vec{k}\delta T_e - \beta_r m_e n_e \nu_{ei}\delta\vec{u}_i, \tag{26}$$

where $\delta\vec{E}^* = -i\vec{k}\delta\phi + i\vec{k}/en_e(\delta n_e T_e + n_e\delta T_e)$ is the effective electric field usually introduced in classical transport theory [15]. These closure relations are written in Fourier space as the transport coefficients are all k and ω dependent. In real space the closure relations will become convolution operators. Since we are concerned with quasineutral fluctuations the relation $\delta\vec{j} = \vec{0}$ gives the expression for the heat flux

$$\delta\vec{q}_e = -\kappa i\vec{k}\delta T_e - \beta n_e T_e \delta\vec{u}_i, \tag{27}$$

where $\kappa = \chi - \alpha^2 T_e/\sigma$ and $\beta = \beta_q - e\alpha\beta_j/\sigma$. The transport coefficients in this theory are α, the thermocurrent coefficient; χ, the thermal conductivity; σ, the electrical conductivity; and the new transport coefficients β_q, β_j, and β_r that are related to the ion flow. All the coefficients are dependent on the ionization, Z, k, and ω. Rather than tabulate numerical values for the coefficients, one of us (J.M.) has made available upon request a Fortran code that calculates all the necessary transport coefficients.

In order to close the set (20)–(23) all that remains is the closure for the ion stress tensor $\delta\sigma_i$ that is valid for the case (18). This has been previously derived by Bychenkov *et al.* [11] using the frequency-dependent Grad 21M closure. The Grad 21M closure for the longitudinal part of the viscosity tensor

$$\delta\sigma_i = \frac{\vec{k}\cdot\delta\hat{\sigma}_i\cdot\vec{k}}{k^2} = \frac{4}{3}\frac{n_i T_i}{\nu_i}\tilde{\eta}_i(\omega) i\vec{k}\cdot\delta\vec{u}_i \tag{28}$$

results in a frequency-dependent ion viscosity that has both a real and an imaginary part

$$\tilde{\eta} = \frac{i\nu_i(\omega + 1.46i\nu_i)}{(\omega + 1.20i\nu_i)(\omega + 1.46i\nu_i) + 0.23\nu_i^2}. \tag{29}$$

In previous work [11] that was concerned with ion acoustic damping it has been demonstrated that the real part of Eq. (29) produces the correct damping of ion acoustic waves with a smooth transition from the strongly collisional Braginskii limit $\gamma_i \approx 0.64 k^2 v_{T_i}^2/\nu_i$ to the saturated Rukhadze limit [18], where $\gamma_i \approx 0.8\nu_i T_i/ZT_e$. It also compares well to the Fokker-Planck simulations of [19,20] in the intermediate region of collisionality. The imaginary part effects the transition from the adiabatic to the isothermal phase speed as ω exceeds the ion-ion collision frequency ν_i. We now set out our generalized version of Onsager's ''regression of fluctuations'' that was outlined in Sec. II using the closures $\delta\vec{j} = 0$ [Eqs. (24), (26), and (27)] to the linear hydrodynamic moment equations (20)–(23).

C. Correlations of the fluctuating hydrodynamic variables

In order to be able to calculate hydrodynamic correlations we take the Laplace transform in time of the set (20)–(23) and the Fourier transform in space,

$$-i\omega\delta n_e + n_e i\vec{k}\cdot\delta\vec{u} = \delta n_e(0), \tag{30}$$

$$-i\omega\delta\vec{u} = -\frac{Ze}{m_i}i\vec{k}\delta\phi - \frac{i\vec{k}}{m_i n_i}(n_i\delta T_i + T_i\delta n_i) + \frac{1}{m_i n_i}i\vec{k}\cdot\delta\hat{\sigma}_i + \frac{1}{m_i n_i}\delta\vec{R}_{ie} + \delta\vec{u}(0), \tag{31}$$

$$-i\omega\delta T_e + \frac{2}{3n_e}i\vec{k}\cdot\delta\vec{q}_e + \frac{2}{3}T_e i\vec{k}\cdot\delta\vec{u} = \delta T_e(0), \tag{32}$$

$$-i\omega\delta T_i + \frac{2}{3}T_i i\vec{k}\cdot\delta\vec{u} = \delta T_i(0). \tag{33}$$

Here δu is the hydrodynamic velocity perturbation which is the same for ions and electrons since we conside quasineutral perturbations $\delta j = 0$. Equations (30)–(33) describe the evolution of the fluctuating hydrodynamic variables from their initial values at time $t = 0$. This is sufficient for the calculation of the correlations of any of the hydrodynamic variables by following the prescription outlined in Sec. II. For example, $\langle\delta T_e \delta T_e^*(0)\rangle$ may be formed by solving the set (30)–(33) (with the appropriate closure) for the transformed δT_e in terms of the initial fluctuations, multiplying by $\delta T_e^*(0)$, and then ensemble averaging. The solution is then given in terms of the initial correlations that are known (6). The initial correlations are simplified as the different hydrodynamic variables are independent of each other by virtue of the initial condition (6). The Fourier transform of the correlation function $\langle\delta T_e \delta T_e^*\rangle$ is then related to the Laplace transform by $\langle\delta T_e\delta T_e^*\rangle = 2\ \mathrm{Re}\ \langle\delta T_e \delta T_e^*(0)\rangle$, as explained in Sec. II, Eq. (8). We now specialize this to the calculation of $S(k,\omega) = \langle\delta n_e \delta n_e^*\rangle/n_e$ because of its usefulness in determining the cross section for Thomson scattering.

D. Calculation of the dynamic form factor

In solving Eqs. (30)–(33), we will ignore the time derivative in the electron heat equation (32) as it is consistent with our desire to describe isothermal ion acoustic fluctuations ($\omega \sim kc_s$ and $kl_{ei} \gg c_s/v_{T_e}$), where c_s is the cold ion sound speed $c_s = \sqrt{ZT_e/m_i}$. Also, in calculating $S(k,\omega)$ we can neglect all initial conditions except $\delta n_e(0)$, since all others are uncorrelated with the choice of initial conditions (6). The condition of zero current $\delta\vec{j} = 0$ [Eq. (24)] gives an expression for the fluctuating potential

$$i\vec{k}\delta\phi = \frac{i\vec{k}}{en_e}(\delta n_e T_e + \delta T_e n_e) + \frac{\alpha}{\sigma}i\vec{k}\delta T_e + \frac{\beta_j}{\sigma}en_e\delta\vec{u}. \tag{34}$$

This can be used to eliminate the potential term in the ion momentum equation (31) and also in the expression for the friction $\delta\vec{R}_{ie}$ [Eq. (26)],

$$\delta\vec{R}_{ie} = n_e\left(\beta + \frac{e\alpha}{\sigma}\right)i\vec{k}\delta T_e + (1-\beta_j)\beta_j\frac{e^2 n_e^2}{\sigma}\delta\vec{u} - \beta_r m_e n_e \nu_{ei}\delta\vec{u}. \tag{35}$$

With the closure (27) for the electron heat flux, the electron temperature equation (32) can be solved for δT_e,

$$\delta T_e = -\frac{n_e T_e}{k^2\kappa}(1-\beta)i\vec{k}\cdot\delta\vec{u}. \tag{36}$$

On substituting Eqs. (34)–(36) together with the expression (29) for the ion viscosity and ion temperature into the ion momentum equation (31) and after using the continuity equation (31) in order to express the velocity in terms of density, the density perturbation is expressed in terms of the initial perturbation

$$\delta n_e(k,\omega)=\frac{i\,\delta n_e(0)}{\omega D(k,\omega)}\quad \text{where}\quad D=1-\frac{k^2(c_s^2+v_{T_i}^2)}{\omega(\omega-\Delta+2i\gamma_a)}. \tag{37}$$

Here D is the dispersion equation for ion acoustic waves,

$$\gamma_a=\frac{n_e c_s^2(1-\beta)^2}{2\kappa}+\frac{n_e e^2 c_s^2}{2\sigma T_e}\beta_j^2+\beta_r \nu_{ei}\frac{c_s^2}{2v_{T_e}^2}+\gamma_i \tag{38}$$

is the damping rate with the ion viscous contribution,

$$\gamma_i=\frac{2}{3}\frac{k^2 v_{T_i}^2}{\nu_i}\mathrm{Re}\ \tilde{\eta}=k^2 v_{T_i}^2\frac{\nu_i(1.49\nu_i^2+0.80\omega^2)}{\omega^4+4.05\nu_i^2\omega^2+2.33\nu_i^4}, \tag{39}$$

and

$$\Delta=\frac{2}{3}\frac{k^2 v_{T_i}^2}{\omega}+\frac{4}{3}\frac{k^2 v_{T_i}^2}{\nu_i}\mathrm{Im}\ \tilde{\eta} \tag{40}$$

accounts for ion contribution to the acoustic wave dispersion due to ion viscosity and heating. Using Eq. (37) we can express $\langle \delta n_e \delta n_e^*(0)\rangle/n_e$ in terms of the initial correlations as given by Eq. (6), $\langle \delta n_e(0)\delta n_e^*(0)\rangle/n_e=1$. From the relation $S(k,\omega)=2\ \mathrm{Re}\langle \delta n_e \delta n_e^*(0)\rangle/n_e$ the dynamic form factor is determined

$$S(k,\omega)=\frac{4k^2(c_s^2+v_{T_i}^2)\gamma_a}{(\omega^2-k^2v_s^2)^2+4\omega^2\gamma_a^2}, \tag{41}$$

where we have introduced the definitions

$$v_s=\sqrt{c_s^2+\Gamma_i v_{T_i}^2},\quad \Gamma_i=\frac{5}{3}+\frac{4}{3}\frac{\omega}{\nu_i}\mathrm{Im}\ \tilde{\eta}$$
$$=\frac{9\omega^4+29.7\omega^2\nu_i^2+11.7\nu_i^4}{3(\omega^4+4.05\omega^2\nu_i^2+2.33\nu_i^4)} \tag{42}$$

for the ion acoustic group velocity and ion specific heat ratio.

IV. APPLICATIONS

A. Application of the nonlocal theory in the limit of collisional electrons

There are two main issues that can be addressed concerning the application of our theory for the ion acoustic feature in the Thomson scattered spectrum in this regime (17). The first is ion acoustic damping, which determines the height of the ion acoustic peaks. In the intermediate regime of collisonality $kl_{ei}\sim 1$ the electron contribution to ion acoustic damping has been investigated both theoretically [21] and numerically [22], as it is important for stimulated scattering processes. The damping may be calculated from the theory comprising Eqs. (20)–(22) and (24)–(27) for the wavelengths $kl_{ei}\gg c_s/v_{T_e}$, $kl_i\ll 1$,

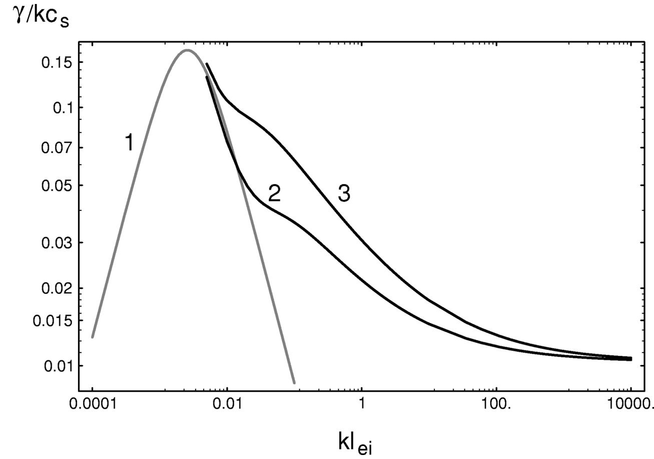

FIG. 1. Electron part of ion acoustic damping γ/kc_s as a function of electron collisionality kl_{ei}. The gray curve 1 shows the prediction of fluid theory with classical thermal conductivity. The black lines show the damping from the analytic theory of [12] for $Z=8$ (curve 2) and $Z=64$ (curve 3).

$$\gamma_a=\frac{n_e c_s^2}{2}\left[\frac{(1-\beta)^2}{\kappa}+\frac{e^2\beta_j^2}{T_e\sigma}+\frac{\beta_r}{n_e v_{T_e} l_{ei}}\right]+0.64\frac{k^2 v_{T_i}^2}{\nu_i}, \tag{43}$$

and this compares well with the numerical solution to the Fokker-Planck kinetic equation [22] and the analytic theory [21]. It has the proper hydrodynamic form in the long-wavelength limit $kl_{ei}\ll 1$ and takes the form of collisionless electron Landau damping in the short-wavelength region $kl_{ei}\gg 1$. Figure 1 shows the damping as a function of electron collisonality kl_{ei} as predicted by Eq. (43). It is interesting to note that the deviation from classical Braginskii theory occurs early, while the wavelength is still hundreds of times larger than a mean free path. This will be reflected in $S(k,\omega)$ [Eq. (41)], whose form may be interpreted with the aid of Fig. 1.

The other issue is concerned with transport. The parameters of many laser-plasma experiments fall in the regime of nonlocal transport as is demonstrated in Fig. 2 for the case of a high-Z plasma. Since the line shape or height of the ion acoustic peaks described by Eq. (41) is expressed in terms of transport coefficients, Thomson scattering may be used as a probe for this nonlocality. The probed k vector in the plasma is determined by $k=2k_0\sin(\theta/2)$, where k_0 is the wave vector of the incident probe beam and θ is the scattering angle chosen by the experimentalist. We propose a comparison between the spectrum for two (or more) different scattering angles. In this way the k dependence of the transport coefficients may be inferred. In choosing experimental parameters, Z should be sizable for the validity of the nonlocal transport theory [12]. In Fig. 2 there are three lines that identify $\alpha=1/k\lambda_{De}$ and the contours show electron-ion collisonality. Figures 3 and 4 show a comparison between the spectrum predicted by Eq. (41) and collisionless theory (10) for different scattering angles. In particular Fig. 4 shows how the effect of collisions alters the k dependence of the peak height from that expected from collisonless theory where fluctuations are only Landau damped. These parameters have been chosen to be close to those encountered experimentally, for

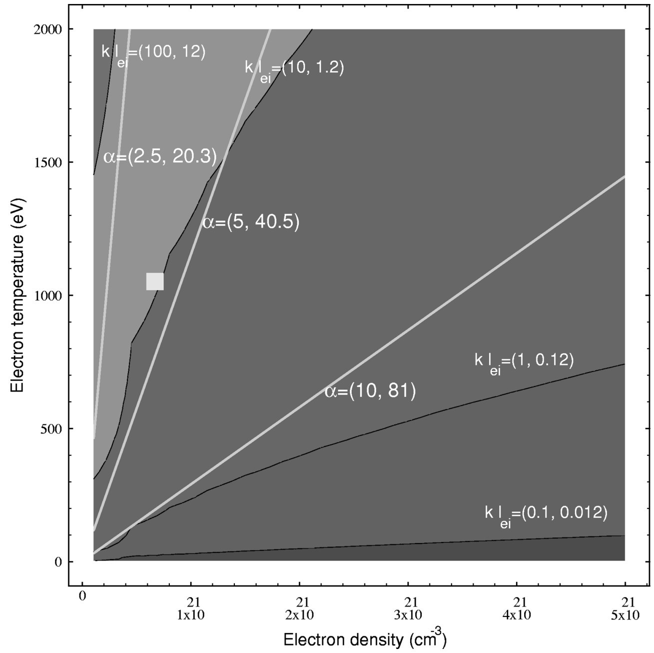

FIG. 2. Parameter regime for gold plasma. The contour plot shows electron-ion collisionality kl_{ei} for scattering angles of 90° and 10°. The first number in the parentheses corresponds to 90° and the second to 10° for a 0.35-μm probe beam. Also shown is $\alpha = 1/k\lambda_{De}$ again for 90° and 10° scattering. The box shows the plasma parameters of Figs. 3 and 4.

example, a gold plasma with the conditions $n_e = 0.5\times10^{21}$ cm^{-3}, $T_e = 1$ keV, $T_i = 500$ eV, and $Z = 50$ and a 0.35-μm probe. Figure 5 shows a more collisional regime due to the use of a longer-wavelength probe, which is compared to Braginskii theory. In this case the effect of changing the angle from 10° to 180° changes the collisionality of the probed ion acoustic fluctuation from $kl_{ei}\sim0.01$ (where classical transport just starts to break down) to $kl_{ei}\sim0.1$ (classical transport inadequate). This is an interesting regime as the main contribution in Eq. (41) to the scattering then comes from κ, the electron thermal conductivity. Investigation of the spectra in this regime could be used to test models of nonlocal thermal conductivity.

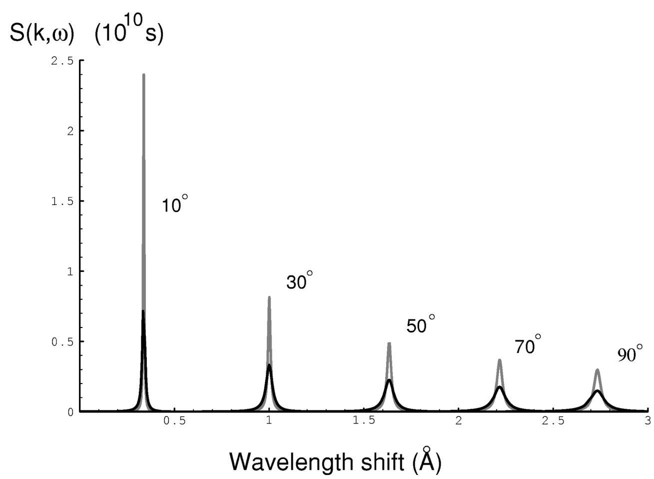

FIG. 3. Dynamic form factor $S(k,\omega)$ for a weakly collisional gold plasma, $n_e = 0.5\times10^{21}$ cm^{-3}, $T_e = 1$ keV, $T_i = 0.5$ keV, and $Z = 55$ for a 0.35-μm probe and different scattering angles. Gray is collisionless theory, black is nonlocal theory.

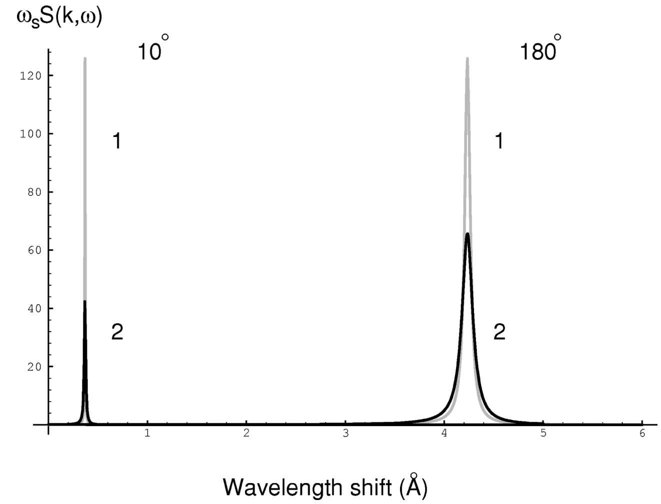

FIG. 4. Dynamic factor $\omega_s S(k,\omega)$ normalized by the ion acoustic frequency ω_s for a gold plasma, $n_e = 0.5\times10^{21}$ cm^{-3}, $T_e = 1$ keV, $T_i = 0.5$ keV, and $Z = 55$ for a 0.35-μm probe. This figure illustrates the difference between the Vlasov theory (curve 1) and the nonlocal theory (curve 2) for the scattering angles of 10° and 180°. Gray is collisionless theory, black is nonlocal theory.

B. Application of the theory in the limit of collisonless electrons

In this regime of collsionless electrons $kl_{ei}\gg1$ and semicollisional ions $kl_i\sim1$ the damping γ_a [Eq. (38)] takes the form

$$\gamma_a = \sqrt{\frac{\pi}{8}}\frac{c_s}{v_{T_e}}kv_s + \frac{2}{3}\frac{k^2 v_{T_i}^2}{\nu_i}\mathrm{Re}\ \tilde{\eta}, \tag{44}$$

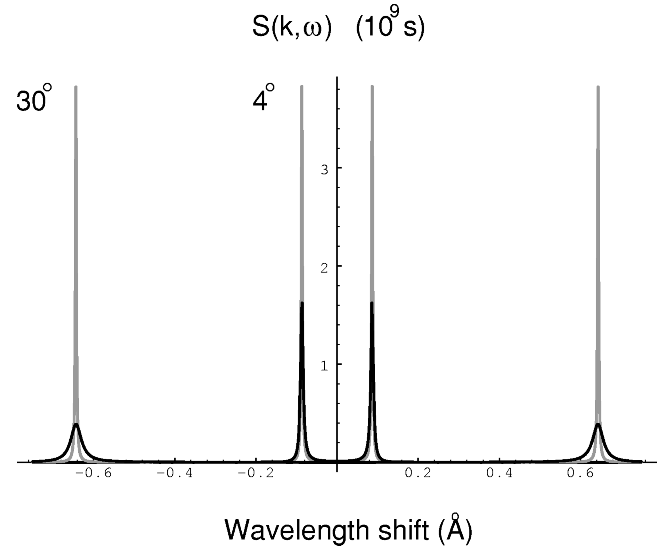

FIG. 5. Dynamic form factor $S(k,\omega)$ for a more collisonal gold plasma, $n_e = 2\times10^{21}$ cm^{-3}, $T_e = 1$ keV, $T_i = 0.5$ keV, and $Z = 55$ for a 10.6-μm probe. The figure shows $S(k,\omega)$ at angles of 10° (left) and 180° (right) and demonstrates the departure from classical hydrodynamics. Gray is Braginskii fluid equations with classical heat conductivity, black is nonlocal theory.

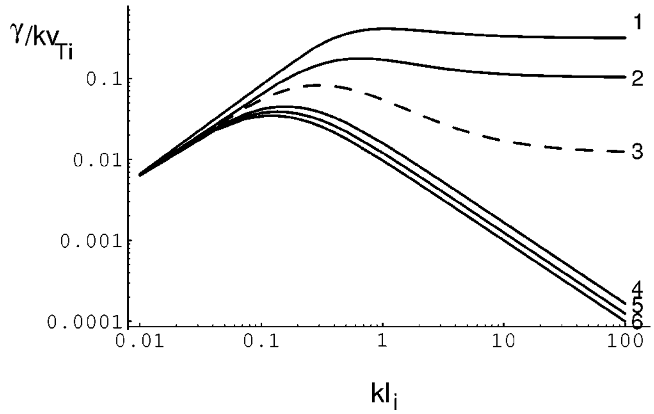

FIG. 6. Dependence of γ_i/kv_{T_i}, the normalized ion part of ion acoustic damping, on ion collisionality kl_i. This damping includes an ion Landau damping contribution in addition to collisions. There are six curves (1–6) plotted for the temperature ratios ZT_e/T_i of 4, 8, 16, 48, 64, and 80, respectively. The dashed curve shows the electron Landau damping contribution for the case $ZT_e/T_i=16$ and shows how the importance of the ions depends strongly on the ion collisionality.

which will be appropriate for discussing the experiments [1,2]. The fluctuation spectrum (41) does not account for the entropy mode since we have neglected the ion thermal conductivity ($\omega \gg kv_{T_i}$). To assess this formula (41) we will compare the predictions to those of the collisionless theory for plasma parameters similar to those of the experiment due to La Fontaine *et al.* [2]. We define the range of plasma parameters for which ion-ion collisions can be important in determining the fluctuation spectra. Figure 6 shows the ion damping of ion acoustic waves as a function of ion-ion collisionality from Ref. [11]. Note that the effect of ion Landau damping, which is missing in Eqs. (39) and (44), has been added phenomenologically in Fig. 6 according to [11]. For plasmas with $ZT_e/T_i>40$ we have the situation where although the ion damping differs from the collisionless limit, the ion contribution is much less than that due to the electrons (electron Landau damping). We therefore identify the interesting range of parameters to be given by $8 \lesssim ZT_e/T_i \lesssim 40$. As an example, for $ZT_e/T_i=16$ the ion damping is a few times smaller than the electron contribution in the collisionless limit, but with the addition of ion-ion collisions it becomes (for $kl_i \sim 0.2$) a few times larger than the electron (Landau damping) contribution; see Fig. 6. Ion acoustic waves will be more strongly damped in this regime than the collisionless theory would predict. This range of parameters has relevance to several recent experiments [1,2,23,6].

In the experiments of La Fontaine *et al.* [1,2], a difficulty is expressed in fitting the width of the observed spectra to the collisionless theory (10) (see also [24]). They note that this is possibly due to the effects of ion-ion collisions and point out the need for further investigation. We address this situation for the plasma conditions of their experiment. Two cases considered are for carbon plasmas, in the first ZT_e/T_i is ~ 12 and in the latter ~ 8.6. The authors obtain T_i from the width of the peaks, as in the collisionless limit this is due to ion Landau damping. However, in this experiment the ions are not collisionless $kl_i \sim 1$ and our ion acoustic peaks are twice as broad for the same T_i. A comparison of our spectra and

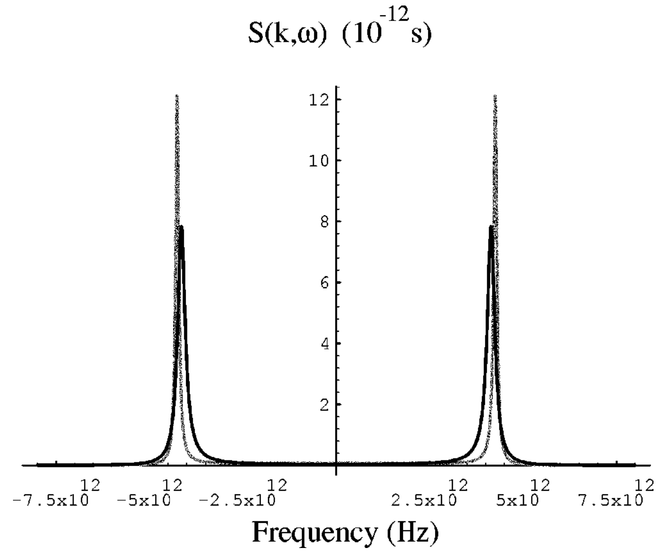

FIG. 7. Ion acoustic peaks as predicted from Eq. (41) (black lines) and from collisionless theory (gray lines) for a carbon plasma, $ZT_e/T_i=12$.

the collisionless spectra appears in Figs. 7 and 8. The authors correctly point out that ion collisions can broaden the ion acoustic peaks. In addition, however, ion collisions modify the specific-heat ratio and alter the phase speed of the ion acoustic mode. The phase speed $v_s \approx c_s\sqrt{1+3T_i/ZT_e}$ in the collisionless limit, where the coefficient 3 corresponds to the isothermal specific-heat ratio for ions. The effect of collisions is to reduce this coefficient towards 5/3 [23]. This effect is not large (a few percent), but it adds more error to the inferred electron temperatures (cf. Fig. 8).

V. SUMMARY

The importance and range of applicability of Thomson scattering as a plasma diagnostic technique depends on the accuracy of the theoretical model of fluctuations and scatter-

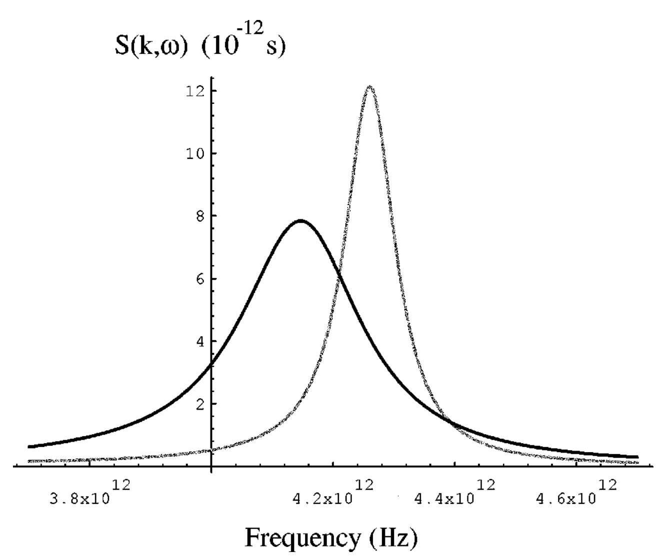

FIG. 8. Closeup of the ion acoustic peaks for the parameters of Fig. 7. The gray line corresponds to the prediction of collisionless theory and the black line to Eq. (41).

ing cross section. We have described a theory for the dynamical form factor $S(k,\omega)$, which is valid for arbitrary particle collisionality in plasmas with large Z and ZT_e/T_i. Our theory properly describes the ion acoustic resonance in the entire region of parameters between collision dominated hydrodynamics and the collisionless formulation based on the Vlasov description. This has been achieved using generalized nonlocal hydrodynamics [11,12,21] for the fluctuating variables.

The starting point has been an exact result of fluctuation theory [7] that demonstrates that the two-point correlation function of the phase-space fluctuation satisfies the usual linearized kinetic equation with the Landau collision operator. We have solved this equation and reduced the problem of finding fluctuations of the phase-space densities to the solution of the linear generalized hydrodynamical equations for the fluctuating hydrodynamical variables. The closure leading to the hydrodynamical model has been achieved with the help of frequency-dependent ion transport coefficients [11] and the full set of nonlocal electron transport coefficients [12]. This derivation involves the frequency-dependent Grad 21-moment approximation for the ion fluctuations and a generalized Laguerre expansion of the electron fluctuation density. Calculations of the dynamical form factor $S(k,\omega)$ are completed assuming an equilibrium electron density correlation function at the initial moment in time.

Starting from our general theory of the dynamic form factor, we have analyzed in detail two different regimes of ion acoustic fluctuations with weakly collisional electrons and cold ions $kl_i \ll 1$, $kl_{ei} \sim 1$ and with weakly collisional ions and collisionless electrons $kl_i \sim 1$, $kl_{ei} \gg 1$. Equation (41) provides an expression for the dynamical form factor in the first limit of weakly collisional electrons. The k-dependent transport coefficients are calculated by a Fortran code that is available from us. The ion acoustic resonance line shape calculated from Eq. (41) has been used to demonstrate the effect of nonlocal inhibited electron thermal transport. The possibility of directly inferring electron thermal transport properties from Thomson scattering measurements is proposed for realistic experimental parameters. Equations (41) and (44) give an expression for $S(k,\omega)$ in the regime of weak ion collisionality and for collisionless electrons. This is the regime of parameters often encountered in x-ray lasers plasmas [1,2], where our theory predicts variations of the Thomson scattering cross section that are consistent with experimental observations.

ACKNOWLEDGMENTS

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