Radiation of relativistic charged particles in a system with one-dimensional randomness

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Radiation of relativistic charged particles in a system of randomly spaced plates is considered in the paper. It is shown that for a large number of plates ($N\gg 1$), in the wavelength range $\lambda \ll l \ll L$ (where l is the photon mean free path and L is the system characteristic size) and for angles $|\cos\theta|\gg(\lambda/2\pi l)^{1/3}$, pseudophoton diffusion represents the major mechanism of radiation. The total intensity of radiation is investigated and its strong dependence on the particle energy and plate number is obtained. [S1063-651X(98)03702-7]

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I. INTRODUCTION

Charged-particle radiation in layered media has been considered in many papers (see, e.g., [1,2] and references therein). The interest in these systems is caused by the possibility of their use as high-energy particle detectors [2]. Detecting properties of these systems are based on the transition radiation. Transition radiation originating in such systems can be explained in the following way (see [1,2]). A charge moving in a medium creates an electromagnetic field (a pseudophoton), which is scattered by the inhomogeneities of dielectric permittivity and converted into radiation. The key problem is to account correctly for the scattering of pseudophotons on the inhomogeneities.

In earlier articles (see, for example, [2]) that addressed the problem of radiation of relativistic charged particles in a system of plates embedded in a homogeneous medium the reflection of the electromagnetic field by an individual plate is neglected. However, from experience with three-dimensional random media [3,4] we know that the multiple scattering of electromagnetic fields plays an essential role. This role is particularly important in the optical region.

In the present paper we consider multiple-scattering effects (taking into account also reflection) when a charged particle radiates passing through a one-dimensional random medium. Such media can be in particular those systems in which the plates are randomly spaced in a homogeneous medium.

It turns out that multiple scattering of the pseudophoton leading to its diffusion is dominant in the medium and this diffusion contributes to the radiation intensity. The diffusion contribution leads to a strong dependence of the radiation intensity on particle energy and plate number, a fact that is important for the detecting properties of the system. Note that the diffusion contribution is absent in an ordered stack of plates.

II. FORMULATION OF THE PROBLEM

The system considered in the paper consists of a stack of plates randomly spaced in a homogeneous medium. Let the plates fill the regions $z_i - a/2 < z < z_i + a/2$ (where a is the plate thickness and z_i are random coordinates). The dielectric permittivity of the system may be represented in the form

$$\varepsilon(z,\omega)=\varepsilon_0(\omega)+\sum_i [b(\omega)-\varepsilon_0(\omega)] \times[|\Theta(z-z_i-a/2)-\Theta(z-z_i+a/2)|], \quad (1)$$

where $\varepsilon_0(\omega)$ and $b(\omega)$ are, respectively, dielectric permittivities of the homogeneous medium and of the plates and Θ is a step function. It is convenient to represent the dielectric permittivity as a sum of average and fluctuating parts

$$\varepsilon(z,\omega)=\varepsilon+\varepsilon_r(z,\omega), \quad \langle\varepsilon_r(z,\omega)\rangle=0, \quad (2)$$

where $\varepsilon=\langle\varepsilon(z,\omega)\rangle$, $\varepsilon_r\ll\varepsilon$, and averaging over the random coordinates of plates is determined as

$$\langle f(z,\omega)\rangle=\int\prod_i \frac{dz_i}{L_z} f(z,z_i,\omega), \quad (3)$$

where L_z is the system size in the z direction. In the frequency domain, Maxwell's equations have the form

$$\vec{\nabla}\times\vec{E}(\vec{r},\omega)=\frac{i\omega}{c}\vec{B}(\vec{r},\omega),$$

$$\vec{\nabla}\times\vec{B}(\vec{r},\omega)=\frac{4\pi e}{c}\frac{\vec{v}}{v}\delta(x)\delta(y)e^{i\omega z/v}-\frac{i\omega}{c}\vec{D}(\vec{r},\omega), \quad (4)$$

$$\vec{\nabla}\cdot\vec{D}(\vec{r},\omega)=\frac{4\pi e}{v}\delta(x)\delta(y)e^{i\omega z/v},$$

$$\vec{\nabla}\cdot\vec{B}(\vec{r},\omega)=0, \quad \vec{D}(\vec{r},\omega)=\varepsilon(z,\omega)\vec{E}(\vec{r},\omega).$$

Here $\tilde{v}\|\hat{z}$ is the velocity of the particle. For convenience we introduce the potentials of electromagnetic field

$$\vec{E}(\vec{r},\omega)=\frac{i\omega}{c}\vec{A}(\vec{r},\omega)-\vec{\nabla}\varphi(\vec{r},\omega). \quad (5)$$

Using Eqs. (4) and (5), we obtain the equation for $\vec{A}(\vec{r},\omega)$,

$$\vec{\nabla}^2\vec{A}+\frac{\omega^2}{c^2}\vec{A}(\vec{r},\omega)\varepsilon(\vec{r},\omega)-\vec{\nabla}\left[\vec{\nabla}\cdot\vec{A}-\frac{i\omega}{c}\varepsilon(\vec{r},\omega)\varphi(\vec{r},\omega)\right]=\vec{j}(\vec{r},\omega), \quad (6)$$

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where $\vec{j}(\vec{r},\omega)$ is the Fourier transform of the current of the charged particle

$$\vec{j}(\vec{r},\omega)=\frac{4\pi e}{c}\frac{\vec{v}}{v}\,\delta(x)\delta(y)e^{i\omega z/v}. \tag{7}$$

Imposing the Lorentz gauge condition on the potentials, we finally obtain

$$\begin{gathered}\vec{\nabla}\cdot\vec{A}-\frac{i\omega}{c}\,\varepsilon(\vec{r},\omega)\varphi(\vec{r},\omega)=0,\\ \vec{\nabla}^2\vec{A}+\frac{\omega^2}{c^2}\,\varepsilon(\vec{r},\omega)\vec{A}(\vec{r},\omega)=\vec{j}(\vec{r},\omega).\end{gathered} \tag{8}$$

It follows from the symmetry of the problem that the vector potential $\vec{A}$ is directed in the z direction: $A_i=\delta_{\hat{z}i}A(\vec{r},\omega)$.

III. RADIATION TENSOR

As usual, we decompose the electric field into two parts $\vec{E}=\vec{E}_0+\vec{E}_r$. Here $\vec{E}_0$ is the electric field of the charged particle moving in the homogeneous medium with dielectric permittivity ε and E_r is the radiation field caused by fluctuations in the dielectric permittivity. We define the radiation tensor as

$$I_{ij}(\vec{R})=E_{ri}(\vec{R})E^*_{rj}(\vec{R}). \tag{9}$$

Here $\vec{R}$ is the radius vector to the observation point, which is far from the system $R\gg L$. The vector potential is decomposed in a similar way: $\vec{A}=\vec{A}_0+\vec{A}_r$, where $\vec{A}_0$ and $\vec{A}_r$, as follows from Eq. (8), satisfy the equations

$$\vec{\nabla}^2\vec{A}_0+\frac{\omega^2}{c^2}\,\varepsilon\vec{A}_0=\vec{j}(\vec{r},\omega),$$

$$\vec{\nabla}^2\vec{A}_r+\frac{\omega^2}{c^2}\,\varepsilon\vec{A}_r+\frac{\omega^2}{c^2}\,\varepsilon_r\vec{A}_r=-\frac{\omega^2}{c^2}\,\varepsilon_r\vec{A}_0. \tag{10}$$

It is convenient to express the radiation intensity in terms of the radiation potential $\vec{A}_r$,

$$\begin{aligned}\langle I_{ij}(\vec{R})\rangle=&\frac{\omega^2}{c^2}\,\delta_{\hat{z}i}\delta_{\hat{z}j}\langle A_r(\vec{R},\omega)A^*_r(\vec{R},\omega)\rangle\\ &+\frac{\delta_{\hat{z}i}}{\varepsilon}\left\langle A_r(\vec{R},\omega)\,\frac{\partial^2}{\partial R_j\partial z}\,A^*_r(\vec{R},\omega)\right\rangle\\ &+\frac{\delta_{\hat{z}j}}{\varepsilon}\left\langle A^*_r(\vec{R},\omega)\,\frac{\partial^2}{\partial R_i\partial z}\,A_r(\vec{R},\omega)\right\rangle\\ &+\frac{c^2}{\omega^2\varepsilon^2}\left\langle\frac{\partial^2}{\partial R_i\partial z}\,A_r(\vec{R},\omega)\,\frac{\partial^2}{\partial R_j\partial z}\,A^*_r(\vec{R},\omega)\right\rangle.\end{aligned} \tag{11}$$

In obtaining Eq. (11) we assumed that the fluctuations of dielectric permittivity are much smaller than its mean value $\varepsilon_r\ll\varepsilon$. To carry out averaging over the random coordinates of plates, we express the radiation potential A_r in terms of the Green's function of Eq. (10),

$$\begin{gathered}A_r(\vec{R})=-\frac{\omega^2}{c^2}\int\varepsilon_r(\vec{r})A_0(\vec{r})G(\vec{R},\vec{r})d\vec{r},\\ \left[\vec{\nabla}^2+k^2+\frac{\omega^2}{c^2}\,\varepsilon_r(z)\right]G(\vec{r},\vec{r})=\delta(\vec{r}-\vec{r}),\end{gathered} \tag{12}$$

where $k=\omega\sqrt{\varepsilon}/c$.

IV. GREEN'S FUNCTION

The bare Green's function of Eq. (12) satisfies the equation

$$[\vec{\nabla}^2+k^2+i\delta]G_0(\vec{r}-\vec{r})=\delta(\vec{r}-\vec{r}), \tag{13}$$

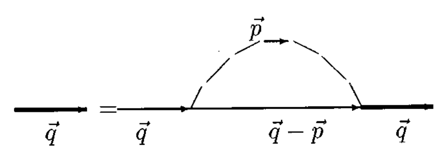
where $i\delta$, as usual, is an infinitesimal imaginary term. The solution in the momentum representation has the form

$$G_0(\vec{q})=\frac{1}{k^2-q^2+i\delta}. \tag{14}$$

In the coordinate representation, one has

$$G_0(r)=-\frac{1}{4\pi r}\,e^{ikr}. \tag{15}$$

To perform the averaging, we use the impurity-diagram method [5]. Summing the diagrams in the ladder approximation, we obtain Dyson's equation for the average Green's function



(16)

The dotted line denotes the correlation function of the one-dimensional random field

$$\begin{gathered}---=B(\vec{p})=(2\pi)^2\delta(\vec{p}_\rho)B(|p_z|),\\ B(|z-z'|)=\frac{\omega^4}{c^4}\,\langle\varepsilon_r(z)\varepsilon_r(z')\rangle,\end{gathered} \tag{17}$$

where $\vec{p}_\rho$ is the transverse component of $\vec{p}$. The solution of Eq. (16) can be represented in the form

$$G(\vec{q})=\frac{1}{G_0^{-1}(\vec{q})-\int\frac{d\vec{p}}{(2\pi)^3}\,B(\vec{p})G_0(\vec{q}-\vec{p})}. \tag{18}$$

Using expression (14), we obtain for the averaged Green's function the expression

$$G(\vec{q})=\frac{1}{k^2-q^2+i\ \mathrm{Im}\ \Sigma(\vec{q})}, \tag{19}$$

in which the imaginary part Im Σ of the self-energy is determined by Ward's identity

$$\mathrm{Im}\,\Sigma(\vec{q})=\int \frac{d\vec{p}}{(2\pi)^3}\,B(\vec{p})\mathrm{Im}\,G_0(\vec{q}-\vec{p})$$
$$=\frac{1}{4\sqrt{k^2-q_\rho^2}}\,[B(|q_z-\sqrt{k^2-q_\rho^2}|)$$
$$+B(|q_z+\sqrt{k^2-q_\rho^2}|)],\quad |\vec{q}_\rho|<k. \tag{20}$$

The decay length of the pseudophoton in the z direction is determined by the imaginary part of the Green's function as (see, e.g., [6])

$$l(\vec{q})=\frac{\sqrt{k^2-q_\rho^2}}{\mathrm{Im}\,\Sigma(\vec{q})}. \tag{21}$$

As expected, the decay length depends on the pseudophoton momentum direction. In the case where the momentum is directed in the z direction, one obtains from Eqs. (21) and (20)

$$l(\theta=0)=\frac{4k^2}{B(0)+B(2k)}. \tag{22}$$

We shall call this quantity the pseudophoton mean free path.

Using Eqs. (1), (2), and (17) one finds for correlation function

$$B(q_z)=\frac{4(b-\varepsilon)^2 n\,\sin^2 q_z a/2}{q_z^2}\,\frac{\omega^4}{c^4}. \tag{23}$$

Here $n=N/L_z$ is concentration of plates in the system. From Eq. (23) it follows that $B(0)=\omega^4/c^4(b-\varepsilon)^2na^2$. On the other hand, when $ka\gg1$, $B(2k)/B(0)\sim 1/(ka)^2\ll1$. Therefore, the photon mean free path is

$$l\equiv l(\theta=0)\approx\begin{cases}4k^2/B(0), & ka\gg1\\ 2k^2/B(0), & ka\ll1.\end{cases} \tag{24}$$

The calculation carried out above is correct only in the weak-scattering regime, when $\mathrm{Im}\,\Sigma(\vec{q})/(k^2-q_\rho^2)\ll1$. Using Eq. (20) we obtain

$$\frac{B(0)+B(2k|\cos\theta|)}{4k^3|\cos\theta|^3}\ll1. \tag{25}$$

From Eq. (25) it follows that at $\theta\approx\pi/2$ the condition of weak scattering is not satisfied. This is natural because in this case the pseudophoton moves parallel to the plates. Taking $\theta=\pi/2-\delta$ and using Eqs. (22) and (25), one has $\delta\gg(1/kl)^{1/3}$.

V. RADIATION INTENSITY IN THE SINGLE SCATTERING APPROXIMATION

Substitution of Eq. (12) into Eq. (11) gives the following expression for the radiation tensor:

$$I_{ij}(\vec{R})=\delta_{\hat{z}i}\delta_{\hat{z}j}\,\frac{\omega^6}{c^6}\int d\vec{r}\,d\vec{r}\,'A_0(\vec{r})A_0^*(\vec{r}\,')$$
$$\times\langle\varepsilon_r(\vec{r})\varepsilon_r(\vec{r}\,')G(\vec{R},\vec{r})G^*(\vec{r}\,',\vec{R})\rangle$$
$$+\frac{\omega^2}{c^2}\frac{1}{\varepsilon^2}\int d\vec{r}\,d\vec{r}\,'A_0(\vec{r})A_0^*(\vec{r}\,')$$
$$\times\left\langle\varepsilon_r(\vec{r})\varepsilon_r(\vec{r}\,')\,\frac{\partial^2}{\partial R_i\partial z}\,G(\vec{R},\vec{r})\,\frac{\partial^2}{\partial R_j\partial z}\,G^*(\vec{r}\,',\vec{R})\right\rangle$$
$$+\delta_{\hat{z}j}\,\frac{\omega^4}{c^4\varepsilon}\int d\vec{r}\,d\vec{r}\,'A_0(\vec{r})A_0^*(\vec{r}\,')$$
$$\times\left\langle\varepsilon_r(\vec{r})\varepsilon_r(\vec{r}\,')G^*(\vec{r},\vec{R})\,\frac{\partial^2}{\partial R_i\partial z}\,G(R,r)\right\rangle$$
$$+\delta_{\hat{z}i}\,\frac{\omega^4}{c^4\varepsilon}\int d\vec{r}\,d\vec{r}\,'A_0(\vec{r})A_0^*(\vec{r}\,')$$
$$\times\left\langle\varepsilon_r(\vec{r})\varepsilon_r(\vec{r}\,')G(\vec{R},\vec{r})\,\frac{\partial^2}{\partial R_j\partial z}\,G'^*(\vec{r},\vec{R})\right\rangle. \tag{26}$$

In the single-scattering approximation, we substitute the Green's functions appearing in Eq. (26) by bare functions (15). Since the observation point $\vec{R}$ is far from radiating system, one finds, using Eq. (15), the useful relations

$$G_0(\vec{R},\vec{r})\approx-\frac{1}{4\pi R}\,e^{ik(R-\vec{n}\cdot\vec{r})},$$
$$\frac{\partial^2G_0(\vec{R},\vec{r})}{\partial R_i\partial z}\approx\frac{k^2n_in_z}{4\pi R}\,e^{ik(R-\vec{n}\cdot\vec{r})},\quad R\gg r. \tag{27}$$

Here $\vec{n}$ is the unit vector in the direction of observation point $\vec{R}$. Inserting Eq. (27) into Eq. (26) and using Eq. (17), for the radiation tensor we find

$$I_{ij}^0(\vec{R})=\frac{\omega^2}{c^2}\,\frac{1}{16\pi^2R^2}\int d\vec{r}\,d\vec{r}\,'e^{ik\vec{n}\cdot(\vec{r}-\vec{r}\,')}$$
$$\times B(|z-z'|)A_0(\vec{r})A_0^*(\vec{r}\,')$$
$$\times[\delta_{\hat{z}i}\delta_{\hat{z}j}-\delta_{\hat{z}i}n_jn_z-\delta_{\hat{z}j}n_in_z+n_in_jn_z^2]. \tag{28}$$

By solving Eq. (10), we easily obtain

$$A_0(\vec{q})=-\frac{8\pi^2e}{c}\,\frac{\delta(q_z-\omega/v)}{k^2-q^2}. \tag{29}$$

Using Eq. (29) in Eq. (28) and integrating, we find the radiation intensity $I(\vec{n})=(c/2)R^2I_{ii}(\vec{R})$ in the single-scattering approximation

$$I^0(\vec{n})=\frac{\pi e^2}{c}\,\delta(0)\,\frac{B(|k_0-kn_z|)n_\rho^2}{[k^2n_z^2-k_0^2]^2}\,\frac{\omega^2}{c^2}. \tag{30}$$

Here $k_0=\omega/v$, while the δ-type singularity of Eq. (30) is caused by the infinite path of the charged particle in the medium. If one takes into account the finite size of the system, $\delta(0)$ must be replaced by $L_z/2\pi$. To analyze the angular dependence of Eq. (30), it is convenient to represent it in the form

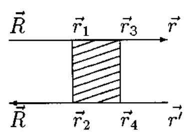
$$I^0(\theta)=\frac{e^2}{2c}\,\frac{L_z B(|k_0-k\cos\theta|)\sin^2\theta}{[\gamma^{-2}+\sin^2\theta k^2/k_0^2]^2}\,\frac{\omega^2}{k_0^4c^2}. \qquad (31)$$

Here $\gamma=(1-\varepsilon v^2/c^2)^{-1/2}$ is the Lorentz factor of the particle. Note some features of expression (31): For relativistic energies ($\gamma\gg 1$, $k_0\to k$), the radiation intensity in the forward direction, for short waves $ka\gg 1$, is significantly higher than in the backward direction. The maximum lies in the range of angles $\theta\sim\gamma^{-1}$. This result is consistent with the results of [1,2]. Since $B\sim n$, the radiation intensity in this approximation, as expected, is proportional to the total number N of plates in the system.

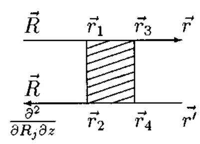
VI. DIFFUSION CONTRIBUTION TO THE RADIATION INTENSITY

In the diffusion approximation, the averages appearing in Eq. (26) are determined by the diagrams

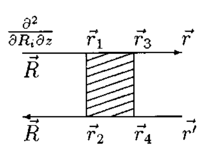
$$< G(\vec R,\vec r)G^*(\vec r\,',\vec R) >^D =$$



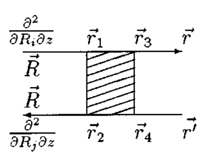
$$< G(\vec R,\vec r)\frac{\partial^2}{\partial R_j\partial z}G^*(\vec r\,',\vec R) >^D =$$



$$< \frac{\partial^2}{\partial R_i\partial z}G(\vec R,\vec r)G^*(\vec r\,',\vec R) >^D =$$



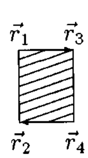
$$< \frac{\partial^2}{\partial R_i\partial z}G(\vec R,\vec r)\frac{\partial^2}{\partial R_j\partial z}G^*(\vec r\,',\vec R) >^D =$$



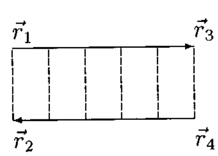
. (32)

Here the filled rectangle corresponds to the diffusion propagator

$$P(\vec r_1,\vec r_2,\vec r_3,\vec r_4)=$$



$$=\sum$$



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Using Eqs. (26), (32), and (33), we obtain the expression for the diffusion contribution

$$I_{ij}^D(\vec R)=\frac{k^2}{16\pi^2R^2\varepsilon}\int d\vec r\,d\vec r\,'B(r-r')A_0(\vec r)A_0^*(\vec r\,')$$
$$\times\int d\vec r_1 d\vec r_2 d\vec r_3 d\vec r_4 e^{-ik\vec n\cdot(\vec r_1-\vec r_2)}P(\vec r_1,\vec r_2,\vec r_3,\vec r_4)$$
$$\times G(\vec r_3,\vec r)G^*(\vec r\,',\vec r_4)[\delta_{\hat z i}\delta_{\hat z j}+n_in_jn_z^2-\delta_{\hat z i}n_jn_z$$
$$-\delta_{\hat z j}n_in_z]. \qquad (34)$$

The diffusion propagator P that appears in Eq. (34) is found similarly to the three-dimensional case [4]. It follows from Eq. (33) that $P(\vec r_1,\vec r_2,\vec r_3,\vec r_4)$ can be represented in form

$$P(\vec r_1,\vec r_2,\vec r_3,\vec r_4)=B(\vec r_1-\vec r_2)B(\vec r_3-\vec r_4)$$
$$\times P(\vec R\,',\vec r_1-\vec r_2,\vec r_3-\vec r_4), \qquad (35)$$

where $\vec R\,'=\frac{1}{2}(\vec r_3+\vec r_4-\vec r_1-\vec r_2)$ and P satisfies the equation

$$\int\frac{d\vec p}{(2\pi)^3}\left[1-\int\frac{d\vec q}{(2\pi)^3}f(\vec q,\vec K)B(\vec p-\vec q)\right]P(\vec K,\vec p,\vec q\,')$$
$$=f(\vec q\,',\vec K). \qquad (36)$$

Here

$$f(\vec q,\vec K)=G(\vec q+\vec K/2)G^*(\vec q-\vec K/2). \qquad (37)$$

As it will be seen further, one has to know P when $\vec K\to 0$. In this limit, the diffusion propagator has the form [4]

$$P(\vec K\to 0,\vec p,\vec q)=\frac{\mathrm{Im}\,G(\vec p)\mathrm{Im}\,G(\vec q)}{\mathrm{Im}\,\Sigma(\vec q)}A(\vec K), \qquad (38)$$

where

$$A(\vec K)=\left[3\int\frac{(\vec q\vec K)^2\,\mathrm{Im}\,G(\vec q)}{\mathrm{Im}^2\,\Sigma(\vec q)}\frac{d\vec q}{(2\pi)^3}\right]^{-1}. \qquad (39)$$

Choosing $\vec K\|\hat z$ and using Eq. (20), we obtain

$$A(K)=\left[\frac{6K^2k^5}{\pi}\int_{-1}^{1}\frac{dx\,x^4}{[B(0)+B(2k|x|)]^2}\right]^{-1}. \qquad (40)$$

Here we have changed variables while integrating over the angles. It follows from the form of the correlation function (23) that the main contribution into the integral (40) is given by the values of x close to unity (the corresponding angles are close to zero). Taking into account this fact, for $A(K)$, we have approximately

$$A(K)=\frac{1}{k}\,\frac{20\pi}{3K^2l^2}, \qquad (41)$$

where $l=4k^2/B(0)$ is the pseudophoton's mean free path. In the expression for radiation intensity it is convenient to turn to new variables of integration

$$\vec{R}'=\frac{1}{2}(\vec{r}_3+\vec{r}_4-\vec{r}_1-\vec{r}_2), \quad \vec{x}_1=\vec{r}_1-\vec{r}_2, \qquad (42)$$

$$\vec{x}_2=\vec{r}_3-\vec{r}_4, \quad \vec{r}_4\equiv\vec{r}_4,$$

which gives

$$I_{ij}^D(\vec{R})=\frac{k^2}{16\pi^2R^2\varepsilon}(\delta_{\hat{z}i}\delta_{\hat{z}j}+n_in_jn_z^2-\delta_{\hat{z}i}n_jn_z-\delta_{\hat{z}j}n_in_z)D, \qquad (43)$$

where D is given by the expression

$$D=\int d\vec{r}\, d\vec{r}'d\vec{R}'d\vec{x}_1d\vec{x}_2d\vec{r}_4A_0(\vec{r}) \times B(\vec{r}-\vec{r}')A_0^*(\vec{r}')e^{-ik\vec{n}\cdot\vec{x}_1}B(\vec{x}_1)B(\vec{x}_2)P(\vec{R}',\vec{x}_1,\vec{x}_2) \times G(\vec{x}_2+\vec{r}_4-\vec{r})G^*(\vec{r}'-\vec{r}_4). \qquad (44)$$

In the Fourier representation Eq. (44) has the form

$$D=\int\frac{d\vec{q}_1d\vec{q}_2d\vec{q}_3d\vec{q}_4}{(2\pi)^{12}}|A_0(\vec{q}_1)|^2B(\vec{q}_2)B(\vec{q}_3)B(\vec{q}_4) \times P(K'\to 0,-\vec{q}_3-k\vec{n},\vec{q}_1+\vec{q}_2+\vec{q}_4)|G(\vec{q}_1+\vec{q}_2)|^2. \qquad (45)$$

Substituting Eq. (38) into Eq. (45) and integrating [using the Ward identity (20)], we obtain

$$D=A(K)\mathrm{Im}\,\Sigma(k\vec{n}) \times\int\frac{d\vec{q}}{(2\pi)^3}\frac{B(|\sqrt{k^2-q_\rho^2}-q_z|)+B(|\sqrt{k^2-q_\rho^2}+q_z|)}{B(0)+B(2\sqrt{k^2-q_\rho^2})} \times|A_0(\vec{q})|^2. \qquad (46)$$

Finally, we evaluate the integral over the momentum remaining in Eq. (46). Using Eqs. (20) and (29) in Eq. (46), we have

$$D=A(\vec{K})\mathrm{Im}\,\Sigma(k\vec{n})\frac{16\pi^2e^2}{c^2}L_z\int\frac{d\vec{q}_\rho}{(2\pi)^2}\frac{1}{(k^2-k_0^2-q_\rho^2)^2} \times\frac{B(|k_0+\sqrt{k^2-q_\rho^2}|)+B(|k_0-\sqrt{k^2-q_\rho^2}|)}{B(0)+B(2\sqrt{k^2-q_\rho^2})}; \qquad (47)$$

It follows from Eq. (47) that for relativistic energies $k_0\to k$, the main contribution to the integral (47) is given by the values $q_\rho\to 0$. Taking into account this fact and the fact that when $\gamma^2\gg ak$ the function B, as well as Eq. (41), varies slowly, we find

$$D\approx\frac{e^2}{c^2}\frac{20\pi^2}{3K^2l^2}L_z\frac{B(0)+B(2k|n_z|)}{k^2}\frac{1}{|n_z|}\frac{\gamma^2}{k_0^2}. \qquad (48)$$

Substituting Eq. (48) into Eq. (43) for the diffusion contribution into the radiation intensity, we obtain finally

$$I^D(n_z)=\frac{5}{6}\frac{e^2\gamma^2}{\varepsilon c}\left(\frac{L_z}{l(\omega)}\right)^3\frac{1-n_z^2}{|n_z|}. \qquad (49)$$

In deriving Eq. (49) we substitute $1/K^2$ at $K\to 0$ by L_z^2 as usual (and also assume that $L_z\ll L_x,L_y$). Note some peculiarities of the diffusion contribution. It is easy to verify that $I^D/I^0\sim L_z^2/l^2\gg 1$. This means that for $k|\cos\theta|^3l\gg 1$ and $l\ll L_z$ the diffusion contribution is the major one. As one should expect, the backward and forward intensities are equal to each other. Note that with an accuracy of unimportant numerical coefficients formula (49) is correct both for short $ka\gg 1$ and for long $ka\ll 1$ waves. All information on randomness is contained in the mean free path $l(\omega)$. In the next section we shall specify the form of $l(\omega)$ in particular cases.

VII. PSEUDOPHOTON MEAN FREE PATH

The pseudophoton mean free path in our theory is described by expression (24). In the impurity diagram method [5], as usual, we do not take into account the diagrams that correspond to the situation of three or more plates at the same point. This is valid provided that $|\sqrt{b/\varepsilon}-1|ka\ll 1$, which means that for scattering of a photon on a plate, the Born approximation is fulfilled. However, it is well known [5] that the formulas are also correct in the general case provided one employs the exact scattering amplitude instead of the Born scattering amplitude. In our case this means that formula (49) is correct in the general case provided a suitable expression is used for pseudophoton mean free path $l(\omega)$.

The photon mean free path in the medium is related to the photon transmission coefficient through a plate

$$l(\omega)=\frac{[1-\mathrm{Re}\;t(\omega)]^{-1}}{n}, \qquad (50)$$

where $t(\omega)$ is the photon transmission coefficient through a plate with photon momentum normal to the plate [6]

$$t(\omega)=\frac{2i\sqrt{\dfrac{b(\omega)}{\varepsilon(\omega)}}\exp(-ika)}{\left[\dfrac{b(\omega)}{\varepsilon(\omega)}+1\right]\sin\sqrt{\dfrac{b(\omega)}{\varepsilon(\omega)}}ka+2i\sqrt{\dfrac{b(\omega)}{\varepsilon(\omega)}}\cos\sqrt{\dfrac{b(\omega)}{\varepsilon(\omega)}}ka}. \qquad (51)$$

It follows from Eqs. (49) and (50) that the maximum of spectral radiation intensity lies in the frequency region where the transmission coefficient is minimal. It follows from Eq. (50) that the minimal value of $l(\omega)$ is $1/n$. Now we shall clarify the conditions under which this value is achieved. In the Born approximation $|\sqrt{b/\varepsilon}-1|ka\ll 1$, using Eqs. (51) and (50), we obtain

$$l(\omega)=\frac{2}{n\left(\sqrt{\frac{b}{\varepsilon}}-1\right)^2 k^2a^2}, \tag{52}$$

which agrees with Eq. (24). More interesting for us is the geometrical optics region $|\sqrt{b/\varepsilon}-1|ka\gg 1$. Substituting Eq. (51) into Eq. (50) and neglecting the strongly oscillating terms, we have $l(\omega)\sim 1/n$. Thus, in the geometrical optics region the photon mean free path does not depend on the frequency and radiation intensity is maximal. Integrating the spectral intensity over angles and frequencies in this region, we find that the total intensity depends on the particle energy as $I^t\sim\gamma^2$. In contrast, the energy dependence of the radiation intensity in typical transition radiation from a single interface in the optical region is logarithmic (see, for example, [2]). In order to find the dependence of the radiation intensity on the number of plates, note that $L_z=N/n$ and from Eq. (49) one has $I^t\sim N^3$. One of the important conditions for the applicability of the theory is the condition $l\ll L_z$. Substituting $L_z=N/n$ and $l=1/n$ into this condition, we find a condition for the plate number $N\gg 1$.

Note that we did not take into account the absorption of photons. This is valid provided $l\ll l_{\rm in}$ (where $l_{\rm in}$ is the photon inelastic mean free path in the medium). In the theory of diffusive propagation the weak absorption ($l\ll l_{\rm in}$) is taken into account in the following way (see, for example, [7]). If the absorption is so weak that $L_z<(ll_{\rm in})^{1/2}$, then expression (49) remains unchanged. When $L_z>(ll_{\rm in})^{1/2}$ one must substitute L_z^2 by $ll_{\rm in}$ in Eq. (49),

$$I^D(n_z,\omega)=\frac{5}{6}\frac{e^2\gamma^2}{\varepsilon c}\frac{L_z l_{\rm in}(\omega)}{l^2(\omega)}\frac{1-n_z^2}{|n_z|}. \tag{53}$$

It follows from Eq. (53) that in this case the dependence of radiation intensity on the plate number is weaker $I\sim N$.

VIII. CONCLUSIONS

We have considered the diffusion contribution for radiation intensity of a relativistic particle passing through a stack of randomly spaced plates. It was shown that for a large number of plates $N\gg 1$, in the wavelength region $\lambda\ll l$ and for the angles $|\cos\theta|\gg(1/kl)^{1/3}$, the diffusion contribution is the dominant one. Note that the backward and forward intensities of relativistic charged-particle radiation intensity are equal, whereas in the regular stack case relativistic particle radiates mainly in the forward direction.

Now let us discuss the possible experimental realizations of our theory. For applicability of the theory the fulfillment of the inequalities $\lambda\ll l(\lambda)\ll l_{\rm in},L_z$ is necessary.

The transition radiation of relativistic charged particles in a stack of plates has been investigated experimentally in many papers (see, for example, [8]). Unfortunately, in these papers only the x-ray region was studied. In the x-ray region the above-mentioned inequalities are not satisfied. Optical transition radiation of relativistic particles has been investigated in experimental work [9]. However, in this experiment only one or two parallel plates were used. Samples in [9] were prepared by vacuum deposition of various metallic coatings (Al, Ag, Au, and Cu) on Mylar foils 3.5 μm thick. Note that these samples are optimal for our goals. They ensure minimal transmission due to metallic coatings and weakness of absorption due to Mylar foils. So it will be interesting to investigate experimentally the optical transition radiation of relativistic electrons passing through a stack of such samples randomly spaced in the vacuum.

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