Heliconic band structure of one-dimensional periodic metallic composites

Chul-Sik Kee, Jae-Eun Kim, and Hae Yong Park
Department of Physics, Korea Advanced Institute of Science and Technology, Taejon 305-701, Korea
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We study the propagation modes of right circularly polarized electromagnetic waves traveling parallel to an external static magnetic field $\mathbf{H}_{\rm ex}$ in one-dimensional (1D) periodic metallic composites, i.e., *the heliconic band structure*. Although the dielectric function of metal in this case depends on frequency ω, the characteristics of the heliconic band structures are very similar to those of the photonic band structures of 1D periodic dielectric composites. The heliconic band structure can be easily controlled by $\mathbf{H}_{\rm ex}$. Thus it is possible to control the defect modes of helicons and the coupling modes of the helicon-transverse sound wave in 1D periodic metallic composites using $\mathbf{H}_{\rm ex}$. We also discuss the localization of helicons in 1D disordered metallic composites. [S1063-651X(98)00802-2]

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I. INTRODUCTION

It is well known that periodic dielectric structures (photonic crystals) can give rise to photonic band gaps (PBG's), frequency regions where electromagnetic (EM) waves cannot propagate [1,2]. Such PBG's can affect radiation-matter interactions in photonic crystals. That is, the dipole-dipole interaction between two atoms can be suppressed and a quantum electrodynamic photon-atom bound state can exist [3,4]. Photonic crystals can be used in high-efficiency semiconductor lasers, optical diodes, solar cells, optical switches, and high-Q resonant cavities. Recently, metallic photonic crystals have attracted much attention among photonic crystals because they have new PBG's that extend from zero frequency to the cutoff frequency ν_c [5–7].

The photonic band structure depends on the dielectric constant, the shape, and the volume fraction of an artificial dielectric atom with which dielectric photonic crystals are made. Since they are almost unchanged by such external parameters as temperatures, pressure, electric fields, and magnetic fields, it is nearly impossible to control the size and position of PBG's of dielectric photonic crystals using external parameters. However, it may be possible to control the PBG's of metallic photonic crystals using external parameters. For instance, the presence of a magnetic field can greatly change the dielectric response of a free electron gas to the low frequency EM wave of $\omega<\omega_c$, where ω_c is the cyclotron frequency. In the absence of a static magnetic field, the dielectric function of a metal can be assumed to be the free electron gas form in the long wavelength limit,

$$\epsilon(\omega)=1-\omega_p^2/\omega^2, \tag{1}$$

where ω_p is the plasma frequency of the conduction electrons. When $\omega<\omega_p$, $\epsilon(\omega)$ is negative, so that EM waves cannot propagate in this kind of media, but are totally reflected. This explains the high reflectivity of most metals in the visible, infrared, and microwave ranges. In the presence of a large static magnetic field, the effective long wavelength dielectric response of an electron gas for circularly polarized EM waves traveling parallel to the external static magnetic field $\mathbf{H}_{\rm ex}$ is

$$\epsilon(\omega)=1-\omega_p^2/\omega(\omega\mp\omega_c). \tag{2}$$

The sign in front of ω_c refers to the sense of circularly polarized EM waves. We only consider right circularly polarized EM waves, the negative sign. In the frequency region of $\omega<\omega_c$, $\epsilon(\omega)$ is positive. For a magnetic field of the order of 1 kG and $m^*\simeq m$, ω_c lies in the microwave region, i.e., $\omega_c\simeq10^{10}$ Hz, while $\omega_p\simeq10^{15}$ Hz. We thus anticipate a dramatic phenomenon that opens a window for the propagation of right circularly polarized EM waves in the frequency region where EM waves are normally forbidden to propagate. These propagating EM waves are well known as *helicons* in solid state plasma physics [8]. In the limit of $\omega\ll\omega_c$, the dielectric function of metals for right circulary polarized EM waves traveling parallel to $\mathbf{H}_{\rm ex}$ can be approximated as

$$\epsilon(\omega)=\omega_p^2/\omega_c\omega=4\pi N_e ec/H_{\rm ex}\omega, \tag{3}$$

where N_e is the electron density, e the charge of an electron, c the vacuum velocity of EM waves, and $H_{\rm ex}$ the magnitude of an external static magnetic field in cgs units. This indicates that the dispersion relation of ω to $\mathbf{k}$ for helicons is parabolic. Since $\epsilon(\omega)$ depends on the external parameter $H_{\rm ex}$, it is possible to control the propagation modes of the right circularly polarized EM waves traveling parallel to $\mathbf{H}_{\rm ex}$.

Similar effects can appear in two-dimensional metallic photonic crystals. With $\mathbf{H}_{\rm ex}$ applied to the direction of the axes of metallic rods, the dielectric function of the metallic rods is not changed when the electric field of the incident EM waves is parallel to the rods (TE modes). However, when the magnetic field of the incident EM waves is parallel to the rods (TM modes), external field $H_{\rm ex}$ affects the value of the dielectric function of the metallic rods. Therefore, it is also possible to control the propagation of TM modes using $H_{\rm ex}$, when $\mathbf{H}_{\rm ex}$ is parallel to the axes of metallic rods.

In this paper, we only investigate the propagation modes of right circularly polarized EM waves propagating parallel to $\mathbf{H}_{\rm ex}$ in one-dimensional (1D) periodic metallic composites (PMC's), i.e., *the heliconic band structures*. We also discuss the defect modes of helicons and the helicon-transverse sound wave interaction in 1D PMC's and the localization of helicons in 1D disordered metallic composites.

II. MODEL AND METHOD

We consider right circularly polarized EM waves of frequency ω propagating in the z direction, which is normal to 1D PMC's. In this situation, a vector equation for the electric field of the EM wave is

$$\nabla^2\mathbf{E}+\epsilon(\omega)\frac{\omega^2}{c^2}\mathbf{E}=\mathbf{0}. \tag{4}$$

When $\mathbf{H}_{\rm ex}=H_{\rm ex}\hat{\mathbf{z}}$, the dielectric function can be assumed to be $\omega_p^2/\omega_c\omega$. We define a dimensionless function, $\Theta(z)$,

$$\Theta(z)=\begin{cases}1 & \text{if } \frac{d}{2}<|z|\leq\frac{a}{2}\\ \left(\frac{\omega_{\rm pb}}{\omega_{\rm ph}}\right)^2 & \text{if } |z|\leq\frac{d}{2},\end{cases} \tag{5}$$

where $\omega_{\rm pb}$ is the plasma frequency of the background metal, $\omega_{\rm ph}$ the plasma frequency of the host metal, d the width of the host metal, and a the lattice constant. Using $\Theta(z)$, the vector equation becomes

$$\Theta(z)\nabla^2\mathbf{E}+\frac{\omega_{\rm pb}^2}{\omega_c c^2}\omega\mathbf{E}=\mathbf{0}. \tag{6}$$

Because of the periodicity of $\Theta(z)$, we can use the Bloch theorem to expand $\mathbf{E}$ in plane waves, i.e., $\mathbf{E}=\hat{\mathbf{c}}\Sigma_{\mathbf{K}'}E_{\mathbf{K}'}e^{-i(\mathbf{K}'\cdot\mathbf{r})}$, where $\mathbf{K}'=\mathbf{k}+\mathbf{G}'$; $\mathbf{k}$ is a wave vector in the first Brillouin zone of the 1D lattice, $\mathbf{G}'$ a 1D reciprocal lattice vector, and $\hat{\mathbf{c}}$ a polarization vector of a right circularly polarized EM wave. Substituting this into Eq. (6), we obtain the matrix equation

$$-\sum_{\mathbf{K}'}\Theta(\mathbf{K}-\mathbf{K}')|\mathbf{K}||\mathbf{K}'|F_{\mathbf{K}'}+\frac{\Lambda F_{\mathbf{K}}}{c^2}=\mathbf{0}, \tag{7}$$

where $F_{\mathbf{K}}=E_{\mathbf{K}}|\mathbf{K}|$, $\Lambda=(\omega_{\rm pb}^2/\omega_c)\omega$, and $\Theta(\mathbf{K}-\mathbf{K}')$ the Fourier transform of $\Theta(z)$.

We solved Eq. (7) using the standard matrix diagonalization method. The number of plane waves used in obtaining our results was 501. When 1001 plane waves were used, the difference was less than 0.3%. Thus, we believe that the results are well converged within at least 1% of their true values.

III. RESULTS AND DISCUSSION

Since $\Theta(\mathbf{K}-\mathbf{K}')$ depends on the filling fraction f and the ratio $(\omega_{\rm pb}/\omega_{\rm ph})^2$, the eigenvalues of Λ in Eq. (7) can be solved as a function of $\mathbf{k}$ for each value of f and $(\omega_{\rm pb}/\omega_{\rm ph})^2$. Then, we can obtain the dispersion relation of ω to $\mathbf{k}$ for the parameters of f, $(\omega_{\rm pb}/\omega_{\rm ph})^2$ and $H_{\rm ex}$ from the definition of Λ. Figure 1 shows the heliconic band structure of 1D PMC's, when the filling fraction f is 0.8 and the ratio $(\omega_{\rm pb}/\omega_{\rm ph})^2$ 16. The normalized frequency Ω is given by $\Lambda a^2/(2\pi c)^2=\alpha(\omega a/2\pi c)$, where $\alpha=(\omega_{\rm pb}a/2\pi c)^2/(\omega_c a/2\pi c)$. The shaded areas represent the heliconic band gaps. Although $\epsilon(\omega)$ depends on ω as Eq. (3), the heliconic band structure of 1D PMC's in Fig. 1 is similar to the photonic band structure of 1D periodic dielectric composites (PDC's). It has been suggested that flat bands are a common feature in systems with frequency-dependent dielectric functions [9]. However, flatbands do not appear in the helicon band structure, as shown in Fig. 1. There is no cutoff frequency ν_c, either, below which no EM waves can propagate when $H_{\rm ex}=0$. This can be easily understood from the eigenvalue equation for helicons, Eq. (7). This is very similar to that for 1D PDC's; the dispersion relation of Ω to $\mathbf{k}$ for helicons is identical to that of $\epsilon_b(\omega a/2\pi c)^2$ to $\mathbf{k}$ for the EM waves in 1D PDC's when $\epsilon_b/\epsilon_h=(\omega_{\rm pb}/\omega_{\rm ph})^2$. Here ϵ_b and ϵ_h are the dielectric constants of the background material and the host material in 1D PDC's, respectively. This means that all the interesting phenomena of EM waves in 1D PDC's can be applicable to helicons in 1D PMC's. Furthermore, the localization of EM waves in 1D disordered dielectric composites should appear as the localization of helicons in 1D PMC's [10].

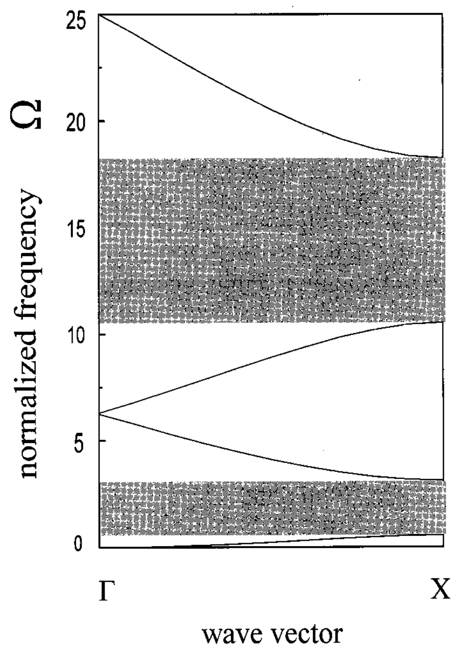

FIG. 1. Heliconic band structure of right circularly polarized EM waves in 1D periodic metallic composites for $f=0.8$ and $(\omega_{\rm pb}/\omega_{\rm ph})^2=16$. The normalized frequency Ω is $\alpha(\omega a/2\pi c)$, where $\alpha=(\omega_{\rm pb}a/2\pi c)^2/(\omega_c a/2\pi c)$. The heliconic band gaps are shaded.

The midgap frequency of the heliconic band gap is given by $\omega_{\rm mid}=\pi cH_{\rm ex}\Omega_{\rm mid}/N_b e a^2$, where N_b and $\Omega_{\rm mid}$ are the electron density of the background metal and the normalized midgap frequency, respectively. Since $\Omega_{\rm mid}$ is independent of $H_{\rm ex}$, $\omega_{\rm mid}$ depends linearly on $H_{\rm ex}$. Furthermore, the width of the gap normalized to $\omega_{\rm mid}$, $\Delta\omega/\omega_{\rm mid}=\Delta\Omega/\Omega_{\rm mid}$, does not depend on $H_{\rm ex}$, but the absolute size of the gap $\Delta\omega$ depends linearly on $H_{\rm ex}$. This means that we can control both the position and the size of the heliconic band gap using the external static magnetic field. It is also

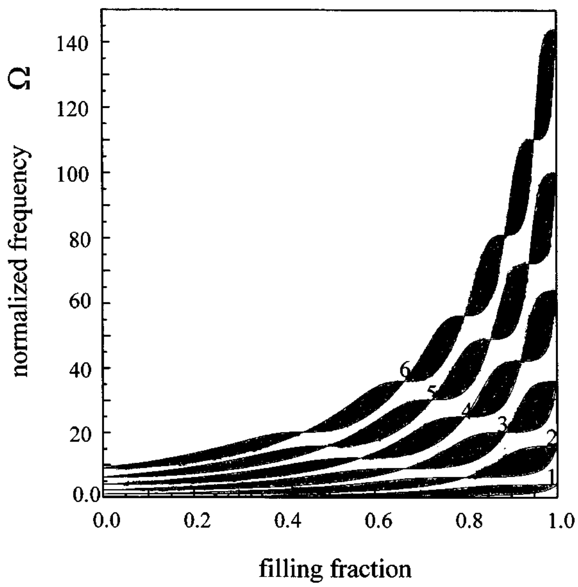

FIG. 2. Gap map as a function of the filling fraction for $(\omega_{pb}/\omega_{ph})^2=16$. The heliconic band gaps are shaded. The ith gap implies the gap between the ith and the $(i+1)$th bands ($i=1,2,\ldots,6$). Ω_n and f_n of the nodes are $(0.25k+0.25ln_b/n_h)^2$ and $l/(l+kn_h/n_b)$, respectively, where l and k are positive *even* integers, and $n_b/n_h=\omega_{pb}/\omega_{ph}=4$. Ω_n and f_n of the ith gap satisfy $l+k=2i$.

possible to control the defect modes of helicons in 1D PMC's, and the localization of helicons in 1D disordered metallic composites using this external magnetic field.

In typical metals or heavily doped semiconductors, the electron density is $10^{18}/\mathrm{cm}^3$–$10^{23}/\mathrm{cm}^3$. For example, when the values of the parameters are $N_b=10^{20}/\mathrm{cm}^3$, $a=0.05$ mm, and $H_{ex}=1$ kG in Fig. 1, the first gap position is 7.85×10^7–4.47×10^8 Hz and $\omega_c=1.7\times10^{10}$ Hz. This frequency region satisfies the condition $\omega\ll\omega_c$. The frequency of a defect mode can be positioned at the center of the forbidden frequency range by designing the width or the ω_p of the defect layer [11]. If we increase H_{ex} from 1 to 10 kG, the first gap position is 7.85×10^8–4.47×10^9 Hz and $\omega_d=1.84\times10^9$ Hz, where ω_d is the frequency of the defect mode. Since ω_c is also ten times the previous value, the condition $\omega\ll\omega_c$ is still satisfied.

Figure 2 shows the gap map of helicons as a function of the filling fraction f for $(\omega_{pb}/\omega_{ph})^2=16$. The heliconic band gaps are shaded. The ith gap implies the gap that occurs between the ith and the $(i+1)$th bands for helicons. It is interesting that $(i-1)$ nodes exist in the ith gap. We can easily understand this by the theory of dielectric multilayer optics. When the optical path difference of layers is a multiple of $\pi c/\omega$, EM waves of ω give nodes in the gap map. In our case,

$$n_h d=l\pi c/2\omega, \tag{8}$$

$$n_b(a-d)=k\pi c/2\omega, \tag{9}$$

where l and k are positive *even* integers, and n_h and n_b, the refraction index of the host metal and the background metal, respectively. The frequency of nodes, ω_n, can be derived from Eqs. (8) and (9),

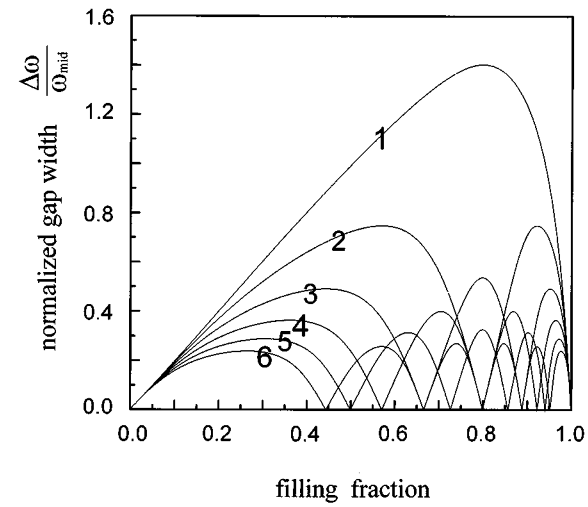

FIG. 3. $\Delta\omega/\omega_{mid}$ as a function of the filling fraction for $(\omega_{pb}/\omega_{ph})^2=16$. The numbers ($i=1,2,\ldots,6$) denote the ith gap. The normalized midgap frequency and the filling fraction at the maximum of $\Delta\omega/\omega_{mid}$ are $\Omega_{mid,m}=(0.25q+0.25pn_b/n_h)^2$ and $f_m=p(p+qn_h/n_b)$, respectively, where p and q are *odd* positive integers. $\Omega_{mid,m}$ and f_m of the ith gap satisfy $p+q=2i$.

$$\omega_n a/2\pi c=(kn_h+ln_b)/4n_hn_b. \tag{10}$$

The normalized frequencies and the filling fractions of nodes are

$$\Omega_n=(0.25k+0.25ln_b/n_h)^2 \tag{11}$$

and

$$f_n=l\pi c/2n_h\omega_n a=l/(l+kn_h/n_b), \tag{12}$$

respectively. n_h/n_b is independent of ω, and equal to ω_{pb}/ω_{ph}. When $l+k=2i$, f_n and Ω_n give the filling fractions and the normalized frequencies of nodes of the ith gap.

Figure 3 shows the normalized gap width $\Delta\omega/\omega_{mid}$ as a function of the filling fraction. Since $\Delta\omega/\omega_{mid}$ is independent of H_{ex}, Fig. 3 holds for all values of H_{ex} that satisfy the condition $\omega\ll\omega_c$. The maximum values of $\Delta\omega/\omega_{mid}$ of the ith gap are larger than those of the $(i+1)$th gap. $\Delta\omega/\omega_{mid}$ of the first gap reaches the maximum value 140.3% at $f=0.8$, and $\Omega_{mid}=(1.25)^2$. The normalized gap width vanishes at f_n. There are i maxima of $\Delta\omega/\omega_{mid}$ in the ith gap. Using the theory of dielectric multilayer optics, $\omega_{mid,m}$ and f_m, where the normalized gap width reaches a maximum, can be obtained from ω and d/a, satisfying

$$n_h d=p\pi c/2\omega, \tag{13}$$

$$n_b(a-d)=q\pi c/2\omega, \tag{14}$$

where p and q are positive *odd* integers. The results are

$$\Omega_{mid,m}=(0.25q+0.25pn_b/n_h)^2 \tag{15}$$

and

$$f_m=p/(p+qn_h/n_b). \tag{16}$$

When $p+q=2i$, $\Omega_{\mathrm{mid},m}$ and f_m give the normalized midgap frequencies and the filling fractions, respectively, where the ith gap reaches the maximum values.

$\Delta\omega/\omega_{\mathrm{mid}}$ of helicons is much larger than that of EM waves in 1D PDC's. From the similarity of the eigenvalue equation for helicons to that for EM waves in 1D PDC's, we can derive the relation between $\Delta\omega/\omega_{\mathrm{mid}}$ for helicons and that for EM waves in 1D PDC's for $\epsilon_b/\epsilon_h=(\omega_{\mathrm{pb}}/\omega_{\mathrm{ph}})^2$;

$$(\Delta\omega/\omega_{\mathrm{mid}})_h=8(\Delta\omega/\omega_{\mathrm{mid}})_d/[4+(\Delta\omega/\omega_{\mathrm{mid}})_d^2], \quad (17)$$

where $(\Delta\omega/\omega_{\mathrm{mid}})_h$ is the normalized gap width of helicons and $(\Delta\omega/\omega_{\mathrm{mid}})_d$ that of EM waves in 1D PDC's. $(\Delta\omega/\omega_{\mathrm{mid}})_h$ is always larger than $(\Delta\omega/\omega_{\mathrm{mid}})_d$ when $(\Delta\omega/\omega_{\mathrm{mid}})_d<200\%$. In most cases, $(\Delta\omega/\omega_{\mathrm{mid}})_d$ is saturated to a value below 200% as ϵ_b/ϵ_h is increased.

Since the helicons are extremely slow EM waves, they can interact with transverse sound waves in metals [12]. This is analogous to the interaction of transverse EM waves with transverse optical phonons in ionic crystals. It has been known that the propagation modes of transverse sound waves in two-dimensional periodic elastic composites have forbidden frequency ranges, *the phononic band gap* [13]. Therefore, it may be interesting to study the coupling modes between helicon and transverse sound waves in 1D PMC's. The coupling modes of the helicon-transverse sound wave can be controlled by an external static magnetic field, as well.

IV. CONCLUSION

In conclusion, we investigated the heliconic band structure of 1D PMC's. Although the dielectric function of the metal for helicons depends on frequency ω, the characteristics of the heliconic band structure are very similar to those of the photonic band structure of 1D PDC's. The heliconic band structure can easily be controlled by an external static magnetic field. It is also possible to control the defect modes of helicons, the coupling modes of helicon-transverse sound wave in 1D PMC's, and the localization of helicons in 1D disordered metallic composites uisng this external magnetic field. The normalized gap width of helicons is larger than that of EM waves in 1D PDC's, in most cases.

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