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# Towards a simple model of compressible Alfvénic turbulence

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A simple model of collisionless, dissipative, compressible magnetohtdrodynamics (Alfvénic) turbulence in a magnetized system is investigated. In contrast to more familiar paradigms of turbulence, dissipation arises from Landau damping, enters via nonlinearity, and is distributed over all scales. The theory predicts that two different regimes or phases of turbulence are possible, depending on the ratio of steepening to damping coefficient $(m_1/m_2)$. For strong damping $(|m_1/m_2|<1)$, a regime of smooth, hydrodynamic turbulence is predicted. For $|m_1/m_2|>1$, steady state turbulence does not exist in the hydrodynamic limit. Rather, spikey, small scale structure is predicted. [S1063-651X(97)51509-1]

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The theory of compressible magnetohydrodynamic (MHD) (e.g., Alfvénic) turbulence has been a topic of interest for some time [1]. Alfvén wave turbulence presents several novel challenges, due to the fact that the $k-\omega$ selection rules preclude three Alfvén-wave resonance. Thus, in incompressible MHD, two Alfvén waves can interact only with the vortex (i.e., eddy) mode. Compressibility relaxes this constraint by allowing interaction with accoustic and ion-ballistic modes (i.e., Landau damping), along with waveform steepening. This naturally leads to the formation of Alfvénic shocklets. Thus, one approach, which is analogous to the noisy Burgers model in hydrodynamics, is based on the study of nonlinear wave evolution equations with external noise drive [e.g., the noisy derivative nonlinear Schrödinger (DNLS) equation, in space physics]. Such theories describe turbulence as an ensemble of nonlinear structures, e.g., shocks, discontinuities, and high-amplitude waves, which are typically observed in compressible (e.g., interplanetary [2]) plasmas. This course of investigation was pursued computationally to study the noisy DNLS equation [3]. Stationarity was achieved by inserting *ad hoc* viscous damping (later linked to finite plasma conductivity [4]) into the otherwise conservative DNLS equation. The DNLS model fails, however, for the important case of $\beta\sim1$ ($\beta=4\pi\alpha p/B_0^2$ is the ratio of plasma pressure to magnetic pressure, $B_0^2$ is an external magnetic field) and the electron-to-ion temperature ratio $T_e/T_i\sim1$ (for instance, in the solar wind plasma), when Alfvén waves couple to strongly damped ion acoustic modes. As a consequence, the kinetically modified DNLS [5,6], referred to as the kinetic nonlinear Schrödinger equation (KNLS), which exhibits intrinsically dissipative nonlinear coupling, emerges as the superior basic model. Numerical solution of the KNLS reveals a new class of dissipative structures, which appear through the balance of nonlinear steepening with collisionless nonlinear damping. These structures include arc-polarized and *S*-polarized rotational discontinuities [7], observed in the solar wind plasma and not predicted by other models. The resulting quasistationary structures typically have narrow spectra.

Here, we present the first analytical study of the noisy-KNLS equation as a generic model of collisionless, large-amplitude Alfvénic shocklet turbulence. Indeed, this is, to our knowledge, the first structure-based theory of compressible MHD turbulence in a collisionless system. Stationarity is maintained via the balance of noise and dissipative nonlinearity. Dissipation here results from ion Landau damping, which balances the parallel ponderomotive force produced by modulations of the compressible Alfvén wave train. A one-loop renormalization group (RG) calculation (equivalent [8] to a direct interaction approximation [9] closure) is utilized. Although the KNLS describes both quasiparallel and oblique waves [5], we consider here the simpler case of quasiparallel propagation. The general case will be addressed in a future publication. The noisy KNLS is, thus, a generic model of strong, compressible Alfvénic turbulence and may be relevant to the solar wind, interstellar medium, shock acceleration, as well as to compressible MHD theory, as a whole. Note that this perspective is analogous to that of the noisy Burgers equation model of compressible fluid turbulence [10]. Several features which are not common in standard MHD turbulence theories appear in this model. It is shown that the dissipative *integral* coupling renormalizes the *wave train velocity*, in addition to inducing nonlinear damping and dispersion. Moreover, consideration of the resulting solvability condition for a stationary state in the hydrodynamic limit $(\omega,k\rightarrow0)$ suggests that KNLS turbulence can exist in one of *two different states* or *phases*. In the hydrodynamic regime, turbulence consists of large-scale, smooth $(\omega,k\rightarrow0)$ wave forms and dissipative structures. In the regime when the hydrodynamic limit does not exist, one may expect a small-scale, spikey, intermittent $(\omega,k\nrightarrow0)$ shocklet turbulence. This hypothesis, however, needs further (e.g., numerical) study.

The "noisy KNLS" equation is

$$\frac{\partial\phi}{\partial t}+v_0\frac{\partial\phi}{\partial z}+\lambda\frac{\partial}{\partial z}(\phi U_2)-i\mu_0\frac{\partial^2\phi}{\partial z^2}=\tilde{f}, \tag{1}$$

where $\phi=(b_x+ib_y)/B_0$ is the wave magnetic field, $\tilde{f}$ is the random noise, $\mu_0=v_A^2/2\Omega_i$ is the dispersion coefficient, $v_0$ is

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the reference frame velocity, $\lambda=1$ is the perturbation parameter, $v_A$ is the Alfvén speed, and $\Omega_i$ is the ion gyrofrequency. Unlike the Burgers equation, the KNLS (and DNLS) equation is not Galilean invariant, hence the $v_0$ term is explicit. The packet velocity $v_0$ is renormalized due the *broken symmetry* between $+k$ and $-k$ harmonics induced by Landau damping. This precludes the conventional practice of transforming to the frame comoving at $v_A$ to eliminate $v_0$. The macroscopic ponderomotive plasma velocity perturbation for a high amplitude Alfvén wave is

$$U_2=m_1|\phi|^2+m_2\hat{\mathcal{H}}[|\phi|^2], \quad \hat{\mathcal{H}}=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathcal{P}}{z'-z}\,dz' , \tag{2}$$

where $\hat{\mathcal{H}}$ is the (nonlocal) Hilbert operator that represents collisionless (Landau) dissipation. The coefficients $m_1$ and $m_2$ are functions of $\beta$ and $T_e/T_i$, only, i.e.,

$$m_1=\frac{1}{4}\frac{(1-\beta^*)+\chi_\parallel^2(1-\beta^*/\gamma)}{(1-\beta^*)^2+\chi_\parallel^2(1-\beta^*/\gamma)^2}, \tag{3a}$$

$$m_2=-\frac{1}{4}\frac{\chi_\parallel\beta^*(\gamma-1)/\gamma}{(1-\beta^*)^2+\chi_\parallel^2(1-\beta^*/\gamma)^2}, \tag{3b}$$

where $\beta^*=(T_e/T_i)\beta$, $\gamma=3$ is the polytropic constant, and $\chi_\parallel=\sqrt{8\beta/\pi\gamma}[(T_e/T_i)^{3/2}\exp\{(T_i-T_e)/2T_i\}]$ is the parallel heat conduction coefficient that models kinetic collisionless dissipation in fluid models. The term $m_1$ represents nonlinear steepening of a wave via coupling to the self-generated density perturbation (associated with an acoustic mode). The term $m_2$ corresponds to *kinetic* damping of a wave by resonant particles, which rapidly sinks energy from all harmonics, unlike viscous dissipation. We emphasize that the KNLS is intrinsically a *nonlinearly dissipative* equation, i.e., there is no *linear* damping retained here. In this regard, we comment that there appear to be two meaningful paradigm problems to be explored. One is to retain *both* linear growth *and* damping in the KNLS. In this case the results will necessarily be quite model dependent. The other, which we pursue here, is to study the purely nonlinear problem with noisy drive. This case allows us to isolate and focus on the intrinsically nonlinear dynamics of the KNLS equation.

In Fourier space $\hat{\mathcal{H}}=ik/|k|$, so the transformed KNLS is

$$(-i\omega+iv_0k+i\mu_0k^2)\phi_{k,\omega}+i\lambda k\sum_{k',k'',\omega',\omega''}\Big(\phi_{k',\omega'}\phi_{k'',\omega''}\phi_{k-k'-k'',\omega-\omega'-\omega''}[m_1+im_2\mathrm{sgn}(k-k')]\Big)=f_{k,\omega}, \tag{4}$$

where the function $\mathrm{sgn}(x)=x/|x|$. The stochastic noise $f_{k,\omega}$ is assumed to be zero mean, and $\delta$ correlated in space and time. To extract information from Eq. (4), we utilize the direct interaction approximation (DIA) closure [9,11,12]. We follow the approach used in [12] which is different from [11], where the correlation functions of cubic Schrödinger turbulence were found. We emphasize the statistical nature of our analysis. Indeed, turbulence renormalizes the coefficients of the evolution equation, leaving its functional form unchanged. The renormalized phase velocity $v$ and dispersion $\mu$ (see discussion below) appear in the second order in $\lambda$. Since the KNLS (and DNLS) is not Galilean invariant, the vertex $\lambda$ is also renormalized. This is a third order effect. Because of mathematical difficulties, we exactly calculate $v$ and $\mu$, only, and for $\lambda$, we provide a simple heuristic argument. In general, the noise finction $\tilde{f}$ is also renormalized at higher orders in $\lambda$. Usually, in one-loop RG analyses, such turbulent corrections are assumed to be small. Note that knowledge of turbulent $\lambda$ and $\tilde{f}$ is *not* required for predicting the existence of the hydrodynamic limit (see below).

We expand $\phi_{k,\omega}$ in a power series with respect to the perturbation parameter $\lambda$: $\phi_{k,\omega}=\phi_{k,\omega}^{(0)}+\lambda\phi_{k,\omega}^{(0)}+\lambda^2\phi_{k,\omega}^{(2)}+\cdots$ and equate terms, order by order, in $\lambda$. To second order, we have

$$\begin{aligned}(\omega-v_0k-\mu_0k^2)\phi_{k,\omega}^{(2)}&=\lambda k\sum_{k',k'',\omega',\omega''}\phi^{(0)}_{-k',-\omega'}\phi^{(0)}_{-k'',-\omega''}\phi^{(1)}_{k+k'+k'',\omega+\omega'+\omega''}[m_1+im_2\mathrm{sgn}(k+k')]\\&=-i\lambda^2\sum_{k',k'',\omega',\omega''}k(k+k'+k'')\mathrm{G}_0(k+k'+k'',\end{aligned} \tag{5}$$

$$\omega+\omega'+\omega'')(\phi^{(0)}_{-k',-\omega'})\phi^{(0)}_{k',\omega'}(\phi^{(0)}_{-k'',-\omega''}\phi^{(0)}_{k'',\omega''})\phi^{(0)}_{k,\omega}\times[m_1+im_2\mathrm{sgn}(k+k')][m_1+im_2\mathrm{sgn}(k+k'')] ,$$

where the bare propagator $G_0(\omega,k)=i/(\omega-kv_0-k^2\mu_0)$. In the DIA, we take $\phi_{k,\omega}^{(2)}\simeq\phi_{k,\omega}^{(0)}$. The terms proportional to $k$ and $k^2$ in the left-hand side act to modify $v_0$ and $\mu_0$. Thus, the nonlinear term of Eq. (5) represents an amplitude dependent correction to *both the velocity and dispersion coefficients*, and Eq. (5) is a recursive equation for the renormalized coefficients $v$ and $\mu$. The fixed point of this recursion relation gives the self-consistent values of these coefficients. Replacing the bare $v_0$, $\mu_0$ with their amplitude dependent counterparts $v$, $\mu$, we write

$$\begin{aligned}(\omega-vk-\mu k^2)&=\frac{\lambda^2}{(2\pi)^4}\int\int\int\int_{-\infty}^{\infty}d\omega'd\omega''dk'dk''|f_{k',\omega'}|^2|f_{k'',\omega''}|^2\\&\times\frac{k(k+k'+k'')}{|\omega'-vk'-\mu k'^2|^2|\omega''-vk''-\mu k''^2|^2}\\&\times\frac{[m_1+im_2\mathrm{sgn}(k+k')][m_1+im_2\mathrm{sgn}(k+k'')]}{\omega+\omega'+\omega''-v(k+k'+k'')-\mu(k+k'+k'')^2}.\end{aligned} \tag{6}$$

We should note that $v$ and $\mu$ will now assume complex values,

$$v=v_r+iv_i, \quad \mu=\mu_r+i\mu_i. \tag{7}$$

The real parts, $v_r$ and $\mu_r$, represent the amplitude dependent speed of a wave packet (note, there is a momentum transfer from waves to resonant particles in this model) and nonlinear dispersion (i.e., an amplitude dependent frequency shift in Fourier space), respectively. The imaginary parts, $v_i$ and $\mu_i$, correspond to damping processes. In particular, $v_i$ describes the exponential damping via phase mixing of a wave packet and $\mu_i$ describes turbulent, viscous dissipation. It is easily seen that for $m_2 \to 0$ the KNLS may be written in the comoving frame with $v_0 = 0$. Thus, no additional phase-mixing terms appear, since the terms $k'k$ and $k''k$ vanish upon integration over $-\infty < \{k', k''\} < \infty$, in the hydrodynamic limit. The collisionless damping breaks this symmetry of the $+k$ and $-k$ parts of the spectrum, thus resulting in the phase-mixing and phase velocity renormalization terms (analogous to nonlinear frequency shifts) encountered here.

We seek solutions in the hydrodynamic limit $\omega \to 0$, $k \to 0$. For simplicity, we assume for noise the white noise statistics, i.e., $f_{k,\omega} = f$. This assumption is not too artificial, since MHD waves are usually pumped at large scales (small-$k$) and the large-$k$ tail is heavily damped by collisionless dissipation, which is an increasing function of $k$. Ordered by powers of $k$, the nonlinear term in the integrals contains $k^3, k^4, \ldots$ contributions. However, the *hydrodynamic behavior* is completely determined by the small-$k, \omega$ limit. Thus, by omitting higher-$k$ terms, Eq. (6) naturally splits into two equations for $v$ and $\mu$, respectively. The $\omega', \omega''$ integrations can be easily performed in complex plane. It is convenient to introduce dimensionless variables $x' = k'\tilde{\mu}/\tilde{v}$, $x'' = k''\tilde{\mu}/\tilde{v}$. The $k', k''$ integrals in Eq. (6) diverge as $k', k'' \to 0$ (i.e. infrared divergence). The integrations can be performed consistently only in the limit where the infrared cutoffs satisfy the inequality $x'_c, x''_c \ll 1$. Quite lengthy, but straightforward complex integrations yield

$$v_r + i v_i = -f^4 \frac{\lambda^2}{(2\pi)^2} \frac{2\tilde{m}}{\tilde{v}^3} \left\{ \frac{\tilde{m}}{2} \ln^2 x_c + \tilde{m} \ln(1 - 2\overline{\mu}) + \frac{\tilde{m}}{2} \left( \frac{\pi^2}{3} + 3 \right) \right\}, \tag{8a}$$

$$\mu_r + i\mu_i = -f^4 \frac{\lambda^2}{(2\pi)^2} \frac{\tilde{m}\tilde{\mu}}{\tilde{v}^4} \left\{ \hat{m} \frac{\ln x_c}{x_c} - \frac{1}{x_c} [\hat{m}\, \overline{v}(\overline{\mu} - 3) + \tilde{m}\overline{v}(5 - 3\overline{\mu})] - \ln^2 x_c \tilde{m} [1 + 2\overline{v}(1 - \overline{\mu})] + \ln x_c [2\tilde{m}(\overline{\mu} - \overline{v}) - \hat{m}(4\overline{\mu} - 5\overline{v} + 1)] + F(\overline{v}, \overline{\mu}) \right\}, \tag{8b}$$

where $\tilde{v} = v - v^*$, $\tilde{\mu} = \mu - \mu^*$, $m = m_1 + i m_2$, $\tilde{m} = [m - m^*]\mathrm{sgn}(v_i \mu_i)$, $\hat{m} = m + m^*$, $\overline{v} = v/\tilde{v}$, $\overline{\mu} = \mu/\tilde{\mu}$, and the dimensionless infrared cut-off is $x_c = k_{min}\mu_i/v_i$. The function $F(\overline{v}, \overline{\mu})$ is positive definite and contains no explicit divergences $\sim x_c$. Since we are concerned with the hydrodynamic limit, where $x_c \ll 1$, the detailed structure of this function is not significant. Returning to standard notation, extracting real and imaginary parts, and keeping the leading, divergent (in $x_c$) terms, we have from Eq. (8a)

$$v_r v_i^3 = f^4 [\lambda^2/(2\pi)^2](m_2^2/2) 2\pi\sigma, \tag{9a}$$

$$v_i^4 = f^4 [\lambda^2/(2\pi)^2](m_2^2/2) \ln^2(k_{min}\mu_i/v_i), \tag{9b}$$

where $\sigma = \mathrm{sgn}(\mu_r/\mu_i)$. As can be easily seen, $v_r/v_i \sim 1/(\ln^2 k_{min}) \to 0$ as $k_{min} \to 0$, so we obtain from Eq. (8b)

$$\mu_r v_i^3 = f^4 \frac{\lambda^2}{(2\pi)^2} \frac{m_2}{8k_{min}} \left[ 4m_1 \ln\left( \frac{k_{min}\mu_i}{v_i} \right) + 3m_2 \frac{\mu_r}{\mu_i} \right], \tag{9c}$$

$$\mu_i v_i^3 = f^4 [\lambda^2/(2\pi)^2](m_2/8k_{min}) m_1 (\mu_r/\mu_i). \tag{9d}$$

For the coefficients $v$ and $\mu$, we may now write

$$v_r \sim v_i \mathrm{sgn}(\mu_r/\mu_i) \ln^{-2} x_c, \quad v_i \sim -f\sqrt{\lambda m_2 \ln x_c}, \qquad \mu_r \sim \mu_i \left( \frac{4m_1}{3m_2} \right), \quad \mu_i \sim -\left( \frac{m_1}{m_2} \right)^2 \frac{f}{k_{min}} \sqrt{\frac{\lambda m_2}{\ln x_c}}. \tag{10}$$

Note that the factor $\ln x_c = \ln(k_{min}\mu_i/v_i) \sim \ln(\ln k_{min})$ makes an insignificant cut-off correction. Nonzero $v_r$ arises due to wave momentum loss via interaction with resonant particles and reflects the process whereby a nonlinear wave accelerates in the direction of steepening (i.e., $v_r > 0$ for $\beta \ll 1, T_e = T_i$), an effect that is observed in numerical solutions of the KNLS equation [7]. This effect is logarithmic for $k_{min} \to 0$. Negative $v_i$ corresponds to exponential damping due to phase mixing, and is proportional to the dissipation rate $m_2$. The coefficient $\mu_r$ represents turbulent dispersion, and the coefficient $\mu_i < 0$ corresponds to turbulent viscous damping. By analogy with noisy Burgers equation [12], Eqs. (10) for the turbulent transport coefficients yield the pulse propagation scaling exponents for the hydrodynamic regime, which are defined by divergences at the cutoff. For diffusion term, we have $\delta x^2/\delta t \sim \mu_i \sim |\delta x|$ (i.e., $\sim 1/k_{min}$), so that $|\delta x| \sim \delta t$. This corresponds to symmetric ballistic *dispersion* of the shocklet waveform. For the velocity term, we write (as $v_r \to 0$ when $k_{min} \to 0$) $x/t \sim v_i \sim \sqrt{\lambda \ln x_c} \sim \mathrm{const}$, that is $x \sim t$. This corresponds to ballistic *translation* of the shocklet.

We now construct the quantity $x_c = k_{min}\mu_i/v_i$ from Eqs. (9) to determine when our cutoff approximation $x_c \ll 1$ is valid. Note that $x_c \ll 1$ must be satisfied for a self-consistent, hydrodynamic regime solution. Dividing Eqs. (9b) and (9c) by Eq. (9d), we derive a system of equations that is easily simplified to give the condition

$$4x_c^2 \ln^3 x_c - 3x_c \ln x_c - (m_1/m_2)^2 = 0. \tag{11}$$

Again, for $x_c \ll 1$, we may omit the small $x_c^2$ term. This equation has maximum at $(x_c)_{max} = e^{-1} < 1$, i.e., a solution of this equation for small $x_c$ exists only when $x_c \leq (x_c)_{max}$. When $x_c > (x_c)_{max}$, Eq. (11) does not have a small-$x_c$ solution, so no stationary state is possible in the hydrodynamic limit. To clarify the physical meaning of the control parameter $x_c$, we write it as $x_c = (k_{min}^2 \mu_i)/(k_{min} v_i)$. Obviously, $x_c$ is just a measure of the efficiency of turbulent *viscous* damping $\sim k^2$ relative to *collisionless* (Landau) damping (distributed in all scales). Smallness of $x_c$ indicates a situation of stronger Landau damping and weaker linear turbulent (viscous) dissipation. The two cases of $x_c$ lesser or greater $(x_c)_{max}$ thus correspond to different states of turbulence. The regime of hydrodynamic turbulence [i.e., $x_c \leq (x_c)_{max}$] corresponds to strong damping, $|m_2| \gg |m_1|$, which dominates nonlinear steepening. Large-scale wave-form structures are possible, consistent with the notion of a hydrodynamic regime. The turbulent viscous damping dominated nonlinear dispersion in this case, $\mu_r/\mu_i \simeq 4m_1/3m_2 \ll 1$. The opposite

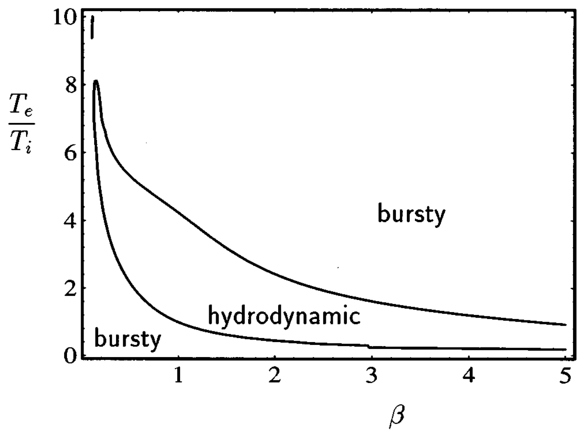


FIG. 1. $\beta$-$T_e/T_i$ diagram of state. The region inside the curve corresponds to highly damped turbulence. No steep fronts appear. There is wave steepening in the region outside the curve.

regime of ''shock'' turbulence (i.e., $x_c \geqslant (x_c)_{max}$) corresponds to weakly damped Alfvén waves, $|m_2| \ll |m_1|$ (however, Landau damping still dominates the small-scale dissipation), where a stationary, hydrodynamic regime is not possible. In this case, nonlinear steepening is balanced by turbulent dispersion, resulting in a state of small-scale coherent nonlinear structures, steep fronts and discontinuities. The bifurcation point can easily be found from $(x_c)_{max} \simeq e^{-1}$ and Eq. (11) as

$$|m_1|/|m_2||_{bif} \simeq \sqrt{3/e} \simeq 1.1 \tag{12}$$

(the exact numerical solution yields $\simeq 1.3$). The coefficients $m_1, m_2$ depend on plasma parameters, i.e., on $\beta$ and $T_e/T_i$. We plot the condition Eq. (12) in the form of a $\beta$ vs $T_e/T_i$ diagram in Fig. 1. The region inside the curve corresponds to $|m_1/m_2| < 1$, i.e., a phase of hydrodynamic turbulence. The outer region corresponds to a phase of bursty turbulence of steep nonlinear Alfvén waves.

For completeness, the perturbation parameter $\lambda$ must be renormalized, because the KNLS is not Galilean invariant. Corrections to $\lambda$ follow from the third order expansion and are so laborious that they are left for a future publication. We can, however, estimate the renormalized $\lambda$ as follows. The energy spectrum is

$$\tilde{B}^2(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, \frac{f_{k_\omega} f^*_{k_\omega}}{|\omega - vk - \mu k^2|^2}$$
$$= -(f^2/2)[1/(v_i k + \mu_i k^2)]. \tag{13}$$

The fluctuation level

$$\tilde{B}^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \tilde{B}^2(k) = -\frac{f^2}{4v_i} \tag{14}$$

should be independent of the cut-off $x_c$, thus $\lambda \sim \ln^{-1} x_c$. Of course, the fluctuation level may only depend on the noise strength, $f$, and the dissipation rate, $m_2$. As is expected, $\tilde{B}^2$ varies as $m_2^{-1}$ while noise is constant.

To conclude, we have presented the first analytical theory analysis of a noisy KNLS (and DNLS) model. The noisy KNLS describes turbulence of kinetically damped (at $\beta \simeq 1$) nonlinear Alfvén wave turbulence, i.e., a turbulence of dissipative structures [7], discontinuities, and shock waves. The renormalized wave velocity and dispersion coefficients, as well as the pulse propagation exponents, were calculated. Two different phases of turbulence were identified, depending on the nonlinearity-to-dissipation coefficient ratio, $m_1/m_2$. For $|m_1/m_2| < 1$ a stationary state of hydrodynamic ($k \to 0, \omega \to 0$) turbulence (with noise) is predicted, while for $|m_1/m_2| > 1$ such a state is precluded and small-scale bursty, spikey turbulence is indicated. A phase diagram in the space of $\beta$ and $T_e/T_i$ is given. These findings may be pertinent to recent observations of multiple states in solar wind plasma turbulence.

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