Dust acoustic waves in strongly coupled dusty plasmas

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Dust grains, or solid particles of μm to sub-μm sizes, are observed in various low-temperature laboratory plasmas such as process plasmas and dust plasma crystals. The massive dust grains are generally highly charged, and it has been shown within the context of standard plasma theory that their presence can lead to new low-frequency modes such as dust acoustic waves. In certain laboratory plasmas, however, the dust may be strongly coupled, as characterized by the condition $\Gamma_d = Q_d^2 \exp(-d/\lambda_D)/dT_d \geqslant 1$, where Q_d is the dust charge, d is the intergrain spacing, T_d is the dust thermal energy, and λ_D is the plasma screening length. This paper investigates the dispersion relation for dust acoustic waves in a strongly coupled dusty plasma comprised of strongly coupled negatively charged dust grains, and weakly correlated classical ions and electrons. The dust grains are assumed to interact via a (screened Coulomb) Yukawa potential. The strongly coupled gas phase (liquid phase) is considered, and a quasilocalized charge approximation scheme is used, generalized to take into account electron and/or ion screening of the dust grains. The scheme relates the small-k dispersion to the total correlation energy of the system, which is obtained from the results of published numerical simulations. Some effects of collisions of charged particles with neutrals are taken into account. Applications to laboratory dusty plasmas are discussed. [S1063-651X(97)04212-8]

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I. INTRODUCTION

Dust grains, or solid particles of μm to sub-μm sizes, are observed in many plasma environments, including space plasmas (e.g., planetary rings, comets, noctilucent clouds) [1–2], low temperature laboratory plasmas used for processing and tools [3–7], and "plasma crystals," or Coulomb lattices of charged dust grains in a plasma [8–11]. Dust grains that are immersed in plasma and/or radiative environments are electrically charged owing to processes such as plasma current collection, photoemission, or secondary emission. Due to the collection of electrons and ions from the plasma, for example, the grain acquires a negative surface potential ϕ_s (relative to the background plasma potential) much like a small floating probe in a plasma [12]: the charge on a grain of radius a is given by $Q_d = -Z_d e \sim a\phi_s$, where ϕ_s is determined by $e\phi_s/T_i \simeq 1 - (m_i T_e/m_e T_i)^{1/2} \exp(e\phi_s/T_e)$ [1]. Grains in a laboratory plasma are generally highly charged: for example, an "isolated" 1-μm grain in an argon plasma with $T_e \simeq 3$ eV has a charge state $Z_d \simeq 695 a(\mu\text{m})|\phi_s(\text{V})| \sim 10^4$. However, the dust grains are orders of magnitude heavier than ions: for example, the mass m_d of a 1-μm grain with a mass density of $\simeq 1$ g/cm^3 is about 10^{12} times the proton mass. It has been shown within the context of standard plasma theory that the presence of dust grains, which have very low charge-to-mass ratios, can give rise to new low-frequency waves associated with the dust grain dynamics, such as the dust acoustic wave [12–14].

The dust acoustic wave is the analog of the ion acoustic wave in a very low-frequency regime, where the dust mass provides the inertia and the electrons and ions provide the pressure to sustain the wave [13]. Dust acoustic wave frequencies are generally orders of magnitude smaller than typical ion wave frequencies, with $\omega \leqslant \omega_{pd} = (4\pi n_d Q_d^2/m_d)^{1/2}$, the dust plasma frequency (here n_d is the dust density), and with phase velocity $v_{\text{ph}} \ll$ the ion thermal speed (but much greater than the dust thermal speed). Further, in analogy with the ion acoustic instability, dust acoustic waves can be unstable in the presence of currents, and are excited primarily by ions drifting with speed $u_0 > v_{\text{ph}}$ relative to dust grains [15,16]. We note that recently the dust acoustic wave has been observed in a dedicated laboratory dusty plasma experiment [17], using a He-Ne laser beam to illuminate the dust, and a video camera. The wave was reported to have been excited by ion drift in the system, and had a wavelength of $\lambda \sim 0.6$ cm, a propagation speed of ~ 9 cm/s, and a frequency ~ 15 Hz [17]. There have been other observations of very low-frequency fluctuations in laboratory dusty plasmas which have been hypothesized to possibly be dust acoustic waves. For example, low-frequency fluctuations of order ~ 12 Hz have been reported under certain neutral gas pressure conditions in Coulomb solid experiments [18], and these fluctuations have been suggested to be dust acoustic waves [19]. Very low-frequency fluctuations of order of 10 Hz have been reported in a dusty plasma formed in a low-frequency discharge, and it was suggested that the waves could be either dust acoustic waves or ionization waves [20]. The latter paper also points out that dust in laboratory plasmas can often be strongly coupled, and calls for theory on waves in strongly coupled dusty plasmas. Recently, a theoretical study of dust lattice waves in the solid phase has been reported [21].

Indeed, in various laboratory dusty plasmas, the dust charge density can be sufficiently high so that the intergrain Coulomb potential energy can be larger than the dust thermal energy T_d. The Coulomb potential energy between dust grains is of the order of $(Q_d^2/d)\exp(-d/\lambda_D)$, where $d = (3/4\pi n_d)^{1/3}$ is the average intergrain spacing, and the exponential factor takes into account the screening of the dust charge by the plasma over a distance of the Debye length λ_D [8]. The dust grains are strongly coupled when

$$\Gamma_d = \frac{Z_d^2 e^2}{d T_d} \exp\left(-\frac{d}{\lambda_D}\right) = \Gamma \exp(-\kappa) \gg 1. \tag{1}$$

For example, using the following nominal values representative of certain process plasmas where dust is localized near plasma-sheath boundaries (e.g., Ref. [7]): $T_e \simeq 2$ eV, $n_i \simeq 3 \times 10^9$ cm^{-3}, $a \simeq 0.3$ μm, $Z_d \simeq 10^3$, and $n_d > 10^4$ cm^{-3}, and assuming the dust thermal energy is of the order of the ion or neutral temperature, $T_d \simeq 0.05$ eV, gives $\Gamma > 100$. When the dust grains are strongly correlated, the standard plasma theoretical description of dust acoustic waves needs to be modified to take into account the effect of strong correlations in analogy with analyses of ion acoustic waves in electron-ion plasmas [22,23] and liquid metals [24,25]. In the case of dusty plasmas, however, the electrons and ions are classical and weakly correlated since $n_{i,e}\lambda_D^3 \gg 1$ (here n_i and n_e are the ion and electron densities, respectively), and provide screening of the dust charge with a screening length given by the plasma Debye length λ_D. In this sense, the dusty plasma is similar to a strongly coupled colloidal suspension, where the classical charged colloidal spheres interact via a screened Coulomb potential, with the screening provided by the counterions (e.g., see Refs. [26,27]).

This paper investigates the dispersion relation for dust acoustic waves in a strongly coupled dusty plasma comprised of strongly coupled negatively charged dust grains and of weakly correlated classical ions and electrons. There is also a neutral gas background present, and collisions of charged particles with neutrals are taken into account through a phenomenological collision frequency in the dispersion relation. The dust grains are assumed to interact with each other via a screened Coulomb (Yukawa) potential. The strong coupling between the dust particles leads to crystallization at a sufficiently high coupling value [27–29]. For high coupling values, but below the crystallization limit, the dust plasma is in a ''liquid'' phase with a strong short-range order. It is this phase we focus on. Our approach is based on the quasilocalized charge approximation (QLCA), which was developed to describe the dynamics of such strongly coupled plasma systems. It has been applied to a number of one- and two-component situations [30,31]: here it is necessary to generalize it to the present scenario where the weakly coupled electrons and ions provide a polarizable background. We are interested in analyzing dust acoustic mode dispersion in the small-k domain. The QLCA relates the small-k dispersion to the total correlation energy of the system: this latter can be obtained from the results of numerical simulations of Yukawa systems in Refs. [28,29].

The plan of this paper is as follows. Section II describes the model plasma, gives the method of analysis, and derives the longitudinal dielectric response function based on the QLC analysis. The dispersion relation for dust acoustic waves in the small-k limit is derived and discussed in Sec. III; we also provide a careful analysis of the various energies involved in the description. The effects of dust-neutral collisions are considered in Sec. IV. Applications to laboratory dusty plasmas are discussed in Sec. V.

II. ANALYSIS

A. Model

We consider a partially ionized plasma comprised of negatively charged dust grains, electrons, and ions, in a background of neutrals. The density of dust grains is n_d, the density of the ions, electrons, and neutrals is n_i, n_e, and n_n, respectively, and the temperature (or thermal energy) of each species is T_α, where $\alpha = d, e, i$, and n refer to dust grains, electrons, ions, and neutrals, respectively. The model system comprises N strongly coupled dust grains ($n_d = N/V$, where V is the three-dimensional volume of the system) which interact with each other via a screened Coulomb potential energy

$$\phi(r) = \frac{Z_d^2 e^2}{r} \exp(-r/\lambda_D), \tag{2}$$

where Z_d is the dust charge state, and λ_D is the plasma Debye length. The exponential factor takes into account within the linear approximation the screening of the dust charge by the plasma electrons and ions, which are weakly correlated and classical, with the number of ions or electrons in a Debye sphere being very large. These plasma particles provide screening of the dust charge, like a Debye sheath which develops around a small probe in a plasma. The strength of the intergrain coupling in this system is characterized by Γ_d, as given in Eq. (1). Thus the system of strongly coupled dust grains is characterized by two parameters: the Coulomb coupling parameter $\Gamma = Z_d^2 e^2/dT_d$, and a parameter $\kappa = d/\lambda_D$, which is a measure of the magnitude of the dust grain charge screening by the plasma, as in Refs. [28,29]. The linear Debye length in a plasma with $T_e \neq T_i$ and no drift is approximately given by $\lambda_D = (1/\lambda_{Di}^2 + 1/\lambda_{De}^2)^{-1/2}$ [8]. However, in laboratory dusty plasma environments, the dust is often localized near or in plasma sheaths, where the electric field of the sheath provides a electrostatic force to levitate the grains. Since the ions come into the sheath with speeds of the order of the ion sound speed according to the Bohm sheath criterion for a dust-free discharge, it has been suggested that λ_D might be better modeled by $\lambda_D \sim \lambda_{De}$ [8].

The correlation between dust grains can be characterized by the equilibrium pair correlation function $g(r)$ [$g(r) \rightarrow 0$ as $r \rightarrow \infty$] which gives the probability that there is a grain at a distance r from a given grain.

B. Dielectric response function

The dispersion of the collective modes can be obtained from the dielectric function $\epsilon(\mathbf{k}\omega)$ of the system. In the case of strong correlations there is no standard method to calculate $\epsilon(\mathbf{k}\omega)$. However, for the domain we are interested in the quasilocalized charge approximation which has been developed and used to determine the dielectric response function and plasmon dispersion for various strongly coupled Coulomb systems [30,31] can be employed. The main physical feature of strongly coupled Coulomb systems upon which this model is based is that the charges are quasilocalized when $\Gamma \gg 1$, but smaller than the critical value for solidification. The QLC approximation describes the motions of the system around the average configuration represented through the equilibrium pair correlation function. The model resembles that of a disordered solid in which the particles occupy randomly located sites and undergo small amplitude oscillations around them. Inherent in the model is the assumption that the sites' positions change on a much longer diffusion time scale; ''direct thermal effects'' associated with this slow diffusion of the quasisites are thus neglected compared to correlational effects, which is a good approximation for $\Gamma_d \gg 1$. On the other hand, the effect of the dust and plasma temperature on the probability of particular configurations of quasisites is included via the Γ and κ dependence of the equilibrium pair correlation function. It should also be noted that since the diffusion time is of the order of d/v_d, where d is the average distance between dust grains, and v_d is the thermal speed of the grains, and $d \sim \lambda_D$ which is much larger than a ''dust Debye length'' defined via $\lambda_{\rm Dd} = (T_d/4\pi Z_d^2 e^2 n_d)^{1/2}$, the QLC approximation should be good for wave frequencies of the order of the dust plasma frequency $\omega_{\rm pd}$.

We use the QLC model adapted to the Yukawa system to determine the linear response of the system to a small perturbing external scalar potential $\hat{\Phi}$. This is done by first considering the microscopic equations of motion for the rapid oscillations of the dust charges about their slowly drifting equilibrium sites, and then calculating the linear response. Following Ref. [30], let $X_{i,\mu}(t) = x_{i,\mu}(t) + \xi_{i,\mu}(t)$ be the momentary position of the ith dust grain, with $x_{i,\mu}$ its quasiequilibrium site position, and $\xi_{i,\mu}$ the perturbed amplitude of its small excursion (the subscripts i and j denote dust particles, and μ and ν denote three-dimensional vector indices). The microscopic equation of motion of the ith particle can be obtained from the following Hamiltonian for the system:

$$H = \frac{1}{2m_d} \sum_i \pi_{i,\mu}\pi_{i,\mu} + \tfrac{1}{2} \sum_{i,j} K_{ij,\mu\nu}\xi_{i,\mu}\xi_{j,\nu} + \sum_i \xi_{i,\mu} \frac{\partial}{\partial x_{i,\mu}} \hat{\Phi}(\mathbf{x}_i, t). \tag{3}$$

Here $\hat{\Phi}(\mathbf{x}_i,t\rangle$ is the external potential, and $K_{ij,\mu\nu}$ is given by

$$K_{ij,\mu\nu} = (1-\delta_{ij}) \frac{\partial^2}{\partial x_{i,\mu}\partial x_{j,\nu}} \phi(|\mathbf{x}_i - \mathbf{x}_j|) - \delta_{ij} \sum_l (1-\delta_{il}) \frac{\partial^2}{\partial x_{i,\mu}\partial x_{l,\nu}} \phi(|\mathbf{x}_i - \mathbf{x}_l|), \tag{4}$$

where $\pi_{i,\mu} = m_d \dot{\xi}_{i,\mu}$ is the momentum conjugate to $\xi_{i,\mu}$, and $\phi(r)$ is the screened Coulomb potential given by Eq. (2). The diagonal terms in Eq. (4) arise from the displacement of a dust particle in a fixed environment of the other particles, while the off-diagonal terms arise from the fluctuating environment [30]. Introducing the collective coordinates $\xi_{\mathbf{k},\mu}$ and $\pi_{\mathbf{k},\mu}$

$$\xi_{i,\mu} = \frac{1}{(Nm_d)^{1/2}} \sum_{\mathbf{k}} \xi_{\mathbf{k},\mu} e^{i\mathbf{k}\cdot\mathbf{x}_i},$$

$$\pi_{i,\mu} = \left(\frac{m_d}{N}\right)^{1/2} \sum_{\mathbf{k}} \pi_{\mathbf{k},\mu} e^{i\mathbf{k}\cdot\mathbf{x}_i}, \tag{5}$$

with

$$\phi(k) = \frac{4\pi Z_d^2 e^2}{k^2 + k_D^2}, \tag{6}$$

the Fourier transform of the screened Coulomb potential (energy) Eq. (2), with $k_D = 1/\lambda_D$, the Hamiltonian becomes (for details, see Ref. [30])

$$\overline{H} = \tfrac{1}{2} \sum_{\mathbf{k}} \pi_{\mathbf{k},\mu}\pi_{-\mathbf{k},\mu} + \tfrac{1}{2} \sum_{\mathbf{k}} \left[D_{\mu\nu}(\mathbf{k}) + \omega_{\rm pd}^2 \frac{k_\mu k_\nu}{k^2 + k_D^2} \right] \xi_{\mathbf{k},\mu}\xi_{-\mathbf{k},\nu} - \frac{i}{V} \left(\frac{N}{m_d}\right)^{1/2} \sum_{\mathbf{k}} k_\mu \hat{\Phi}(-\mathbf{k},t) \xi_{-\mathbf{k},\mu}. \tag{7}$$

The central role is played by the dynamical matrix $D_{\mu\nu}(\mathbf{k})$,

$$D_{\mu\nu}(\mathbf{k}) = \frac{\omega_{\rm pd}^2}{V} \sum_{\mathbf{q}} \frac{q_\mu q_\nu}{q^2 + k_D^2} [g(\mathbf{k}-\mathbf{q}) - g(\mathbf{q})] \tag{8}$$

In order to arrive at this expression, the central assumption of the QLC model has been used, which consists of replacing the unperturbed microscopic dust density

$$n_{\mathbf{k}} = \sum_{i=1}^{N} e^{i\mathbf{k}\cdot\mathbf{x}_i}$$

by its ensemble average, using

$$\langle n_{\mathbf{k}} \rangle = N\delta_{\mathbf{k}},$$

$$\langle n_{\mathbf{p}} n_{\mathbf{q}} \rangle = N^2 \delta_{\mathbf{p}+\mathbf{q}} \left[\delta_{\mathbf{q}} + \frac{g(\mathbf{q})}{V} \right] + N\delta_{\mathbf{p}+\mathbf{q}},$$

where $g(\mathbf{q})$ is the Fourier transform of the equilibrium pair correlation function $g(\mathbf{r})$. Using Hamilton's equation $\ddot{\xi}_{\mathbf{k},\mu} = -\partial\overline{H}/\partial\xi_{-\mathbf{k},\mu}$, the resulting equation of motion for $\xi_{\mathbf{k},\mu}(\omega)$, the Fourier transform of $\xi_{\mathbf{k},\mu}(t)$ is then

$$\xi_{\mathbf{k},\mu}(\omega) = \frac{i}{V} \left(\frac{N}{m_d}\right)^{1/2} k_\nu \Gamma_{\mu\nu}^{-1} \hat{\Phi}(\mathbf{k},\omega), \tag{9}$$

where

$$\Gamma_{\mu\nu} = \omega^2 \delta_{\mu\nu} - \omega_{\rm pd}^2 \frac{k_\mu k_\nu}{k^2 + k_D^2} - D_{\mu\nu}(\mathbf{k}). \tag{10}$$

To calculate the dielectric response tensor, the relation between the equilibrium average of the perturbed microscopic dust density and the collective coordinate response of the system [Eq. (9)] to a perturbation ($\hat{\Phi}$) is used. This gives

$$\rho(\mathbf{k},\omega) = \frac{n_d}{m_d} k_\mu k_\nu \Gamma_{\mu\nu}^{-1} \hat{\Phi}(\mathbf{k},\omega). \tag{11}$$

From the formalism of partial response functions [32], the longitudinal response of the system can be characterized through the ''external'' and ''total'' density response functions $\hat{\chi}(\mathbf{k},\omega),\chi(\mathbf{k},\omega)$, respectively, defined by

$$\rho(\mathbf{k},\omega)=\hat{\chi}(\mathbf{k},\omega)\hat{\Phi}(\mathbf{k},\omega)=\chi(\mathbf{k},\omega)\Phi(\mathbf{k},\omega), \tag{12}$$

where the total potential $\Phi(\mathbf{k}\omega)=\hat{\Phi}(\mathbf{k}\omega)+\Phi_{\text{ind}}(\mathbf{k}\omega)$ contains also the first-order potential response $\Phi_{\text{ind}}(\mathbf{k}\omega)$ to the $\hat{\Phi}$ perturbation. Since $\Phi_{\text{ind}}(\mathbf{k}\omega)=\phi(k)\rho(\mathbf{k}\omega)$, where $\phi(k)$ is the Fourier transform of the Yukawa potential as given by Eq. (6), it follows that $\hat{\chi}(\mathbf{k}\omega)=\chi(\mathbf{k}\omega)[1+\phi(k)\hat{\chi}(\mathbf{k}\omega)]$ [32]. Then, from Eqs. (11) and (12), for the longitudinal total response function we obtain

$$\chi(\mathbf{k}\omega)=\frac{n_d}{m_d}\frac{k^2}{\Delta(\mathbf{k}\omega)}, \tag{13}$$

where

$$\Delta(\mathbf{k}\omega)=\omega^2-\omega_{\text{pd}}^2\mathcal{D}(\mathbf{k}) \tag{14}$$

and

$$\omega_{\text{pd}}^2\mathcal{D}(\mathbf{k})=\frac{k_\mu D_{\mu\nu}(\mathbf{k})k_\nu}{k^2}=\frac{\omega_{\text{pd}}^2}{V}\sum_{\mathbf{q}}\frac{(\mathbf{k}\cdot\mathbf{q})^2}{k^2(q^2+k_D^2)}[g(\mathbf{k}-\mathbf{q})-g(\mathbf{q})]. \tag{15}$$

The longitudinal dielectric response function finally becomes

$$\epsilon_L(\mathbf{k}\omega)=1-\phi(\mathbf{k})\chi(\mathbf{k}\omega)=1-\frac{k^2}{k^2+k_D^2}\frac{\omega_{\text{pd}}^2}{\Delta(\mathbf{k}\omega)}. \tag{16}$$

III. DISPERSION RELATION

The dispersion relation for dust acoustic waves is obtained from Eq. (16) using $\epsilon_L(\mathbf{k}\omega)=0$, which gives

$$\omega^2(\mathbf{k})=\omega_{\text{pd}}^2\left(\frac{k^2}{k^2+k_D^2}+\mathcal{D}(\mathbf{k})\right). \tag{17}$$

In the following the small-k limit of $\mathcal{D}(\mathbf{k})$ is considered, in which case the dispersion can be related to the total correlation energy of the system. In order to expand $\mathcal{D}(\mathbf{k})$ for small k, we first change variables in the first term in the brackets in Eq. (15) to obtain

$$\mathcal{D}(\mathbf{k})=\frac{1}{V}\sum_{\mathbf{q}}\left(\frac{[\mathbf{k}\cdot(\mathbf{k}-\mathbf{q})]^2}{k^2[(\mathbf{k}-\mathbf{q})^2+k_D^2]}-\frac{(\mathbf{k}\cdot\mathbf{q})^2}{k^2[q^2+k_D^2]}\right)g(\mathbf{q}). \tag{18}$$

Then expanding for small k [e.g., expanding the denominator of the first term in Eq. (18) using $q^2+k_D^2\gg k^2-2\mathbf{k}\cdot\mathbf{q}$], retaining terms to order k^2 which are even in $\mathbf{k}\cdot\mathbf{q}=kq\cos\theta$, and integrating over $\cos\theta$, gives, after some algebra,

$$\mathcal{D}(k\to 0)\approx\frac{k^2}{V}\sum_{\mathbf{q}}\frac{1}{q^2+k_D^2}\times\left(1-\frac{5}{3}\frac{q^2}{[q^2+k_D^2]}+\frac{4}{5}\frac{q^4}{[q^2+k_D^2]^2}\right)g(q), \tag{19}$$

We now relate $\mathcal{D}$ to the dust correlation energy per particle E_c, which is defined as

$$\frac{E_c}{n_d}=\frac{1}{2V}\sum_{\mathbf{q}}\frac{4\pi Z_d^2e^2}{q^2+k_D^2}g(q), \tag{20}$$

We note that, in the limit $k_D=0$ (i.e., the case of a pure Coulomb interaction potential where $\kappa=d/\lambda_D=0$),

$$\mathcal{D}(k\to 0)\approx\frac{4}{45}k^2d^2\frac{E_c}{\Gamma T_d}, \tag{21}$$

which is the standard result for the OCP (one-component plasma) [30,31], for which there are available analytic fit expressions for the correlation energy. For a strongly coupled Yukawa system the energy relations are somewhat more complex. The total energy of the combined plasma-dust system consists of the kinetic (K) and potential (U) energies,

$$E=K_{pl}+K_d+U. \tag{22}$$

The latter can be decomposed as

$$U=U_{pl}^0+U_{pl,d}+U_d+\delta U_{pl}, \tag{23}$$

The four terms are, respectively, the potential energies associated with the interaction between the plasma particles in the homogeneous plasma, the interaction between the plasma and dust particles, the interaction between the dust particles, and the change in the plasma potential energy due to the inhomogeneity induced by the presence of the dust particles; all these terms are to be calculated, in principle, through the actual Coulomb interaction. In the so-called adiabatic approximation [33] the dust-dust Coulomb interaction is replaced by the screened Yukawa potential; the plasma loses its dynamical properties and ceases to contribute to the energy. As a result, U can be rearranged with reference to the dust particles only:

$$U=E_{\text{int}}+E_b=E_c+E_H+E_b=E_{\text{exc}}+E_H. \tag{24}$$

The meaning of the different terms in Eq. (24) is as follows (all energies are given per dust particle):

$$E_{\text{int}}=\tfrac{1}{2}n_d\int d^3r\ \phi(r)\{1+g(r)\} \tag{25}$$

is the total (positive) interaction energy between the dust particles, via the screened effective potential $\phi(r)$; it consists of the positive Hartree term

$$E_H=\tfrac{1}{2}n_d\int d^3r\ \phi(r)=+\frac{3}{2\kappa^2}\Gamma T_d, \tag{26}$$

and of the negative correlation term

$$E_c = \tfrac{1}{2} n_d \int d^3r\ \phi(r) g(r) = \tfrac{1}{2}\ \frac{n_d}{V} \sum_{\mathbf{q}} \frac{4\pi Z_d^2 e^2}{q^2 + k_D^2}\, g(q). \tag{27}$$

It is this latter that enters into the expansion of the dynamical matrix, Eq. (19). Finally E_b can be identified [28] as the free energy associated with the binding of the Debye sheath surrounding the dust particle

$$E_b = -\tfrac{1}{2} \int d^3r\ \delta\rho_{\mathrm{pl}}(r)\ \frac{Z_d e}{r} = -\frac{\kappa}{2}\,\Gamma T_d, \tag{28}$$

where $\delta\rho_{pl}(r)$ is the deviation of the plasma charge density from its uniform value around a dust particle. It can also be regarded as a reduction in the dust self-energy due to the screening,

$$E_b = \delta E_{\mathrm{self}} \equiv \tfrac{1}{2}\{\phi(r\to 0;\kappa) - \phi(r\to 0, \kappa=0)\}.$$

The ''excess energy'' E_{exc} is the portion of U which remains finite in the $\kappa\to 0$ (unscreened) limit; it is this energy that is extracted from the molecular dynamics simulation studies of strongly coupled Yukawa systems in Ref. [28]. The results of Ref. [28] are valid in the regime $\kappa \leqslant 1$. Thus we consider this limit $\kappa = k_D d \leqslant 1$, where the average intergrain spacing d $(=[3/4\pi n_d]^{1/3})$ is less than or comparable to the plasma Debye length; this may be applicable to the dusty plasmas in process plasmas [28] or to certain dusty plasma experiments [34]. We note that, in contrast to the OCP expression (21), the expansion of $\mathcal{D}(k)$, Eq. (19), is not immediately expressed in terms of E_c. Nevertheless, as long as $g(q)$ does not vary significantly with κ, $\mathcal{D}(k\to 0)$ can still be related to E_c and its derivatives. In fact, in the regime of interest this seems to be the case, as can be gleaned from Fig. 6 of Ref. [28], which shows the correlation functions for both the OCP and for the Yukawa systems. Then using the relations (here $y=\kappa^2$)

$$\frac{q^2}{(q^2+k_D^2)^2} = d^2\ \frac{\partial}{\partial y}\left(\frac{y}{q^2 d^2 + y}\right),$$

$$\frac{q^4}{(q^2+k_D^2)^3} = d^2\ \frac{1}{2}\ \frac{\partial^2}{\partial y^2}\left(\frac{y^2}{q^2 d^2 + y}\right),$$

we obtain, from Eq. (19),

$$\mathcal{D}(k\to 0) \approx \tfrac{2}{3}(kd)^2\ \frac{1}{\Gamma T_d}\left[E_c - \frac{5}{3}\ \frac{\partial}{\partial y}\,(yE_c) + \frac{2}{5}\ \frac{\partial^2}{\partial y^2}\,(y^2 E_c)\right]. \tag{29}$$

Analytical expressions for the dust correlation energy can be obtained from Refs. [28,29], that give expressions for the excess energy which fit their results. The excess energy E_{exc} calculated in those studies also includes, as noted above, the term $\delta E_{\mathrm{self}} = -(\kappa/2)\Gamma T_d$, so that this term has to be subtracted from the analytical expressions given in Refs. [28, 29] in order to obtain the dust-dust correlation energy. Thus, we use the following formula for the dust correlation energy [29]:

$$\frac{E_c}{T_d} = a(\kappa)\Gamma + b(\kappa)\Gamma^{1/3} + c(\kappa) + d(\kappa)\Gamma^{-1/3}. \tag{30}$$

Here $a(\kappa) = E_{\mathrm{bcc}}(\kappa) + \kappa/2 + \tilde{a}(\kappa)$, where the bcc Madelung energy is, to order κ^4,

$$E_{\mathrm{bcc}}(\kappa) = -0.896 - 0.104\kappa^2 + 0.003\kappa^4 \tag{31}$$

and

$$\tilde{a}(\kappa) = -0.003 + 0.001\kappa^2. \tag{32}$$

The coefficients are given, to order κ^4, by

$$\begin{aligned}
-\frac{\kappa}{2} + a(\kappa) &= -0.899 - 0.103\kappa^2 + 0.003\kappa^4 \\
&= a_0 + a_2\kappa^2 + a_4\kappa^4, \\
b(\kappa) &= 0.565 - 0.026\kappa^2 - 0.003\kappa^4 = b_0 + b_2\kappa^2 + b_4\kappa^4, \\
c(\kappa) &= -0.207 - 0.086\kappa^2 + 0.018\kappa^4 = c_0 + c_2\kappa^2 + c_4\kappa^4, \\
d(\kappa) &= -0.031 + 0.042\kappa^2 - 0.008\kappa^4 = d_0 + d_2\kappa^2 + d_4\kappa^4.
\end{aligned} \tag{33}$$

It is interesting to note that the term linear in κ in E_c plays no role in Eq. (35) below, and, therefore, for the purpose of this computation, E_c can be, in fact, replaced by E_{exc}.

Thus the dispersion relation for the dust acoustic wave in this approximation becomes

$$\frac{\omega^2}{\omega_{\mathrm{pd}}^2} = k^2 d^2\left[\frac{1}{k^2 d^2 + \kappa^2} + f(\kappa,\Gamma)\right], \tag{34}$$

where, for $\kappa \leqslant 1$, $f(\kappa,\Gamma) < 0$ is given by

$$\begin{aligned}
f(\kappa,\Gamma) \approx \frac{4}{45}\ \frac{1}{\Gamma}\Bigg[&\left(a_0 + a_2\ \frac{\kappa^2}{2} + 6a_4\kappa^4\right)\Gamma + \left(b_0 + b_2\ \frac{\kappa^2}{2} + 6b_4\kappa^4\right)\Gamma^{1/3} + \left(c_0 + c_2\ \frac{\kappa^2}{2} + 6c_4\kappa^4\right) \\
&+ \left(d_0 + d_2\ \frac{\kappa^2}{2} + 6d_4\kappa^4\right)\Gamma^{-1/3}\Bigg].
\end{aligned} \tag{35}$$

In the regime $kd \ll \kappa$ (i.e., $k\lambda_D \ll 1$), Eq. (34) gives

$$\omega^2 \approx k^2 c_{\mathrm{sd}}^2[1 + f(\kappa,\Gamma)\kappa^2], \tag{36}$$

where $c_{\mathrm{sd}} = \omega_{\mathrm{pd}}\lambda_D$ is the dust acoustic speed.

Now we can discuss the effect of the strong dust-dust correlations on the dust acoustic wave dispersion relation. There appear to be three effects. The first is a softening of the mode dispersion [Eq. (36)], with the phase speed decreasing since $f(\kappa,\Gamma) < 0$. Figures 1 and 2(a) show this softening of the dispersion relation for fixed Γ and various values of κ. Note that the region of the curves where $\kappa > 1$, that show this effect for $\Gamma = 5$ and 150, are unreliable because (a) the analytical expressions used for E_c are only valid for $\kappa \leqslant 1$ (although there appears to be only a few percent difference between the analytical values and the numerical results given in Ref. [29] for $\kappa = 1.2$ and 1.4), and (b) the assump-

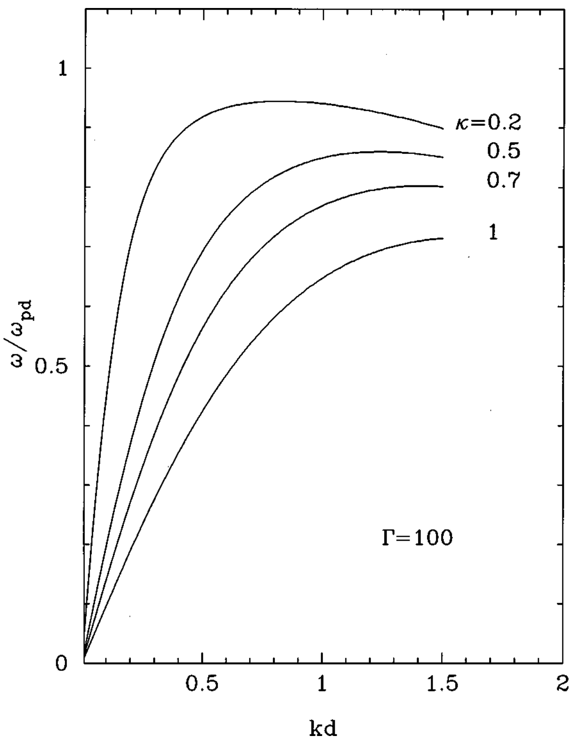

FIG. 1. Dispersion relation of the dust acoustic wave in the strongly coupled gas phase: solutions of Eq. (34) for frequency ω normalized to $\omega_{\rm pd}$ vs kd. Here $\Gamma=100$, and different values of κ label the curves.

tion that $g(r)$ does not vary with κ is probably not valid. In the regime $kd \gg \kappa$ (i.e., $k\lambda_D \gg 1$), the dispersion relation becomes

$$\omega^2 \approx \omega_{\rm pd}^2[1+f(\kappa,\Gamma)k^2d^2]. \tag{37}$$

From here it can be seen that the second effect of strong dust-dust correlations is also to reduce the effective dust plasma frequency, as shown also in Figs. 1 and 2(b). Figure 2(b) shows the effective dust plasma frequency obtained by evaluating $\partial\omega/\partial k=0$ from Eq. (34) to obtain the value of k where the maximum value of ω occurs, denoted by k_M and ω_M, respectively. (Again, the region of the curves where $\kappa > 1$ are unreliable, as discussed above). Figure 3 also shows a third effect, namely, that strong correlations lead to negative dispersion, that is, $\partial\omega/\partial k<0$, in the domain $kd>\kappa$. These properties of the dust acoustic wave in a strongly coupled dusty plasma are analogous to those of ion acoustic waves in a strongly coupled electron-ion plasma analyzed in Ref. [22]. It should be noted, however, that in the calculations of Ref. [22] the κ-dependent second and third terms of Eq. (31) are completely ignored, which can be interpreted as the neglect of the κ dependence of E_c.

IV. EFFECTS OF COLLISIONS

The effects of collisions of charged dust particles with neutrals can be included in the formalism summarized in Sec. II via the *ad hoc* introduction of a collisional damping term in the equation of motion for $\xi_{\mathbf{k},\mu}$, viz.

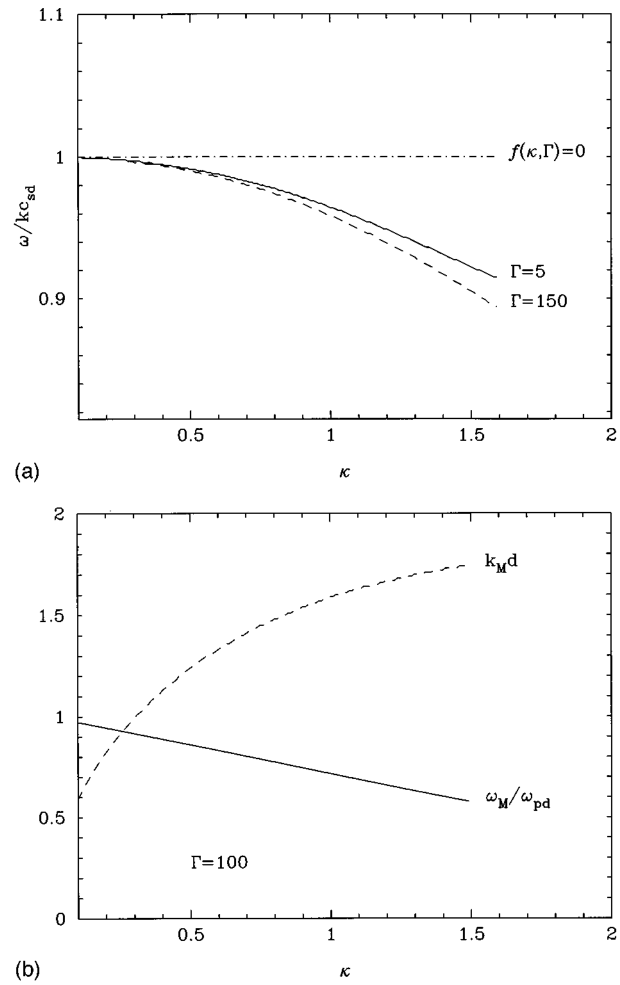

FIG. 2. (a) Phase speed of the dust acoustic wave, ω/k, normalized to $c_{\rm sd}$ from Eq. (36) as a function of κ for two values of Γ; the weak-coupling result is the curve labeled by $f(\kappa,\Gamma)=0$. (b) Maximum frequency ω_M (normalized to $\omega_{\rm pd}$) as a function of κ from Eq. (34), with the corresponding value of kd where the maximum occurs, k_Md. Here $\Gamma=100$. (Note that the regions of the curves beyond $\kappa=1$ are unreliable owing to the approximations used in this paper which are for $\kappa \leqslant 1$.)

$$\ddot{\xi}_{\mathbf{k},\mu}+\nu_d\dot{\xi}_{\mathbf{k},\mu}=-\frac{\partial \overline{H}}{\partial \xi_{-\mathbf{k},\mu}}, \tag{38}$$

where $\nu_d \sim \pi a^2 n_n v_n m_n/m_d$ is the hard sphere dust-neutral collision frequency (here a is the grain radius, m_n and v_n are the neutral mass and thermal speed, and m_d is the dust mass). The resulting equation of motion for $\xi_{\mathbf{k},\mu}(\omega)$, the Fourier transform of $\xi_{\mathbf{k},\mu}$, then has the form of Eq. (9), but with the replacement $\omega^2 \rightarrow \omega(\omega+i\nu_d)$ in the matrix $\Gamma_{\mu\nu}$ given in Eq. (10). The derivation of the dielectric response function then proceeds as in Sec. II but with the replacement $\omega^2 \rightarrow \omega(\omega+i\nu_d)$ in $\Delta(\mathbf{k}\omega)$ in Eq. (14). Then the net result is that $\omega^2(\mathbf{k}) \rightarrow \omega(\mathbf{k})[\omega(\mathbf{k})+i\nu_d]$ in Eq. (17).

We again consider the small-k limit as in Sec. III, and assume that the dust-dust correlation energy is given by Eq. (30). This assumes that the presence of a background neutral gas does not affect the correlation energy of the Yukawa system, via, for example, Brownian motion effects [35], but we are unaware of any calculations on the thermodynamics

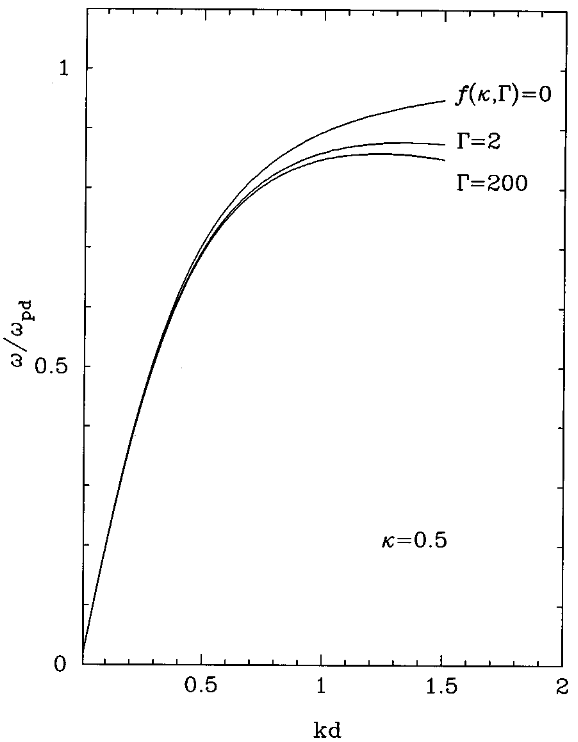

FIG. 3. Dispersion relation of the dust acoustic wave: solutions of Eq. (34) for frequency ω normalized to $\omega_{\rm pd}$ vs kd. Here the curve labeled by $f(\kappa,\Gamma)=0$ is the standard weak-coupling result, while curves labeled $\Gamma=2$ and 200 show results for strongly coupled gas phase.

of Yukawa systems including such neutral gas effects, and note that this may be an interesting topic for further study. Proceeding as in Sec. III, the inclusion of dust-neutral collisional damping results in the replacement $\omega^2 \to \omega(\omega+i\nu_d)$ in the dispersion relation given in Eq. (34). We rewrite this in the following form:

$$k^2\lambda_D^2 = \frac{\omega(\omega+i\nu_d)}{\omega_{\rm pd}^2[1+f(\kappa,\Gamma)\kappa^2(1+k^2\lambda_D^2)]-\omega(\omega+i\nu_d)}. \qquad (39)$$

This has the form of the fluid dispersion relation for dust acoustic waves including collisions as given in Ref. [34], but with a reduction in the effective dust plasma frequency arising from strong correlation between dust grains, as discussed in Sec. III. Solving for real ω and real and imaginary wave number k_r and k_i, Fig. 4(a) shows a solution retaining strong correlation effects, while Fig. 4(b) shows the solution obtained by setting $f(\kappa,\Gamma)=0$ in Eq. (39). As discussed in Ref. [34], without collisional damping and without correlation effects $[f(\kappa,\Gamma)=0]$ the wave number k would be purely real for $\omega<\omega_{\rm pd}$, resonant $(k\to\infty)$ for $\omega=\omega_{\rm pd}$, and purely imaginary for $\omega>\omega_{\rm pd}$. With collisional damping, the resonance at the dust plasma frequency disappears, while the real part of the wave number k_r rolls over at higher $\omega>\omega_{\rm pd}$, corresponding to a transition from a damped propagating wave to an evanescent wave. With strong correlation, this rollover occurs at a lower value of ω, corresponding to a reduced effective dust plasma frequency, and the maximum value of k becomes smaller, as shown in Fig. 4. The reason

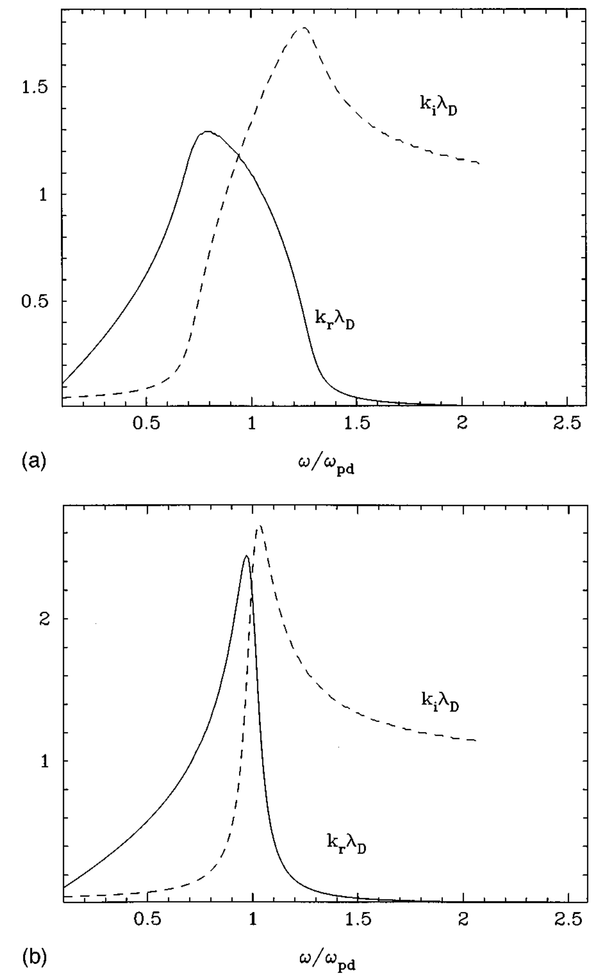

FIG. 4. Dispersion relation of the dust acoustic wave including dust-neutral collisions: the imaginary part of wave number k_i (dashed line), normalized to λ_D^{-1} vs real ω, normalized to $\omega_{\rm pd}$. Parameters are $\nu_d/\omega_{\rm pd}=0.1$, $\Gamma=150$, and $\kappa=1$. (a) Solution of Eq. (39) for the strongly coupled gas phase. (b) Solution of Eq. (39) in the weak-coupling limit, with $f(\kappa,\Gamma)=0$.

for the decrease in the maximum value of k can be seen by considering Eq. (39) at $\omega=\omega_{\rm pd}$, where, roughly, $k_r \sim k_i$:

$$k^2\lambda_D^2(\omega=\omega_{\rm pd}) = \frac{(1+i\nu_d/\omega_{\rm pd})}{[f(\kappa,\Gamma)\kappa^2(1+k^2\lambda_D^2)-i\nu_d/\omega_{\rm pd}]}. \qquad (40)$$

From here it can be seen that the term arising from strong correlations in the denominator $[\propto f(\kappa,\Gamma)\kappa^2]$ has the effect of reducing both the real and imaginary parts of k. For the parameters chosen for Fig. 4, the magnitudes of $f(\kappa,\Gamma)\kappa^2(1+k^2\lambda_D^2)$ and $\nu_d/\omega_{\rm pd}$ are comparable, and this reduction of the maximum values of k_r and k_i are apparent. This will be discussed further in Sec. V.

V. APPLICATION AND DISCUSSION

We consider the application of these results to the experimental observations of dust acoustic waves in the strong-coupling regime reported by Pieper and Goree [34], since that is the only reference we are aware of that measures the dispersion relation in this regime. In the above paper [34], fitting the experimental dispersion relation yielded the parameter $\kappa<1$, ranging from $\kappa\sim0.15-0.6$, while $\Gamma\gg1$, and ranged from about 11 to 10^4. The collisionality parameter $\nu_d/\omega_{\rm pd}$ ranged from about $\frac{1}{2}$ to near unity. (Note that the approximations used in our present paper correspond to $\kappa<1$, and to the fluid phase with $\Gamma\lesssim170$.) In the experiment, both the real and imaginary parts of the complex wave number were measured as a function of the real driving frequency ω, and it was reported that the results fit a dispersion relation of the form of Eq. (39) without strong-coupling effects [i.e., Eq. (39), above with $f(\kappa,\Gamma)=0$] [34].

The theoretical results presented in this paper appear to be be consistent with the above discussed experimental observations for the following reasons. For the parameters of the experiment, the magnitude of the collisionality parameter $\nu_d/\omega_{\rm pd}$ is considerably larger than the magnitude of the strong-coupling term $f(\kappa,\Gamma)\kappa^2$ in the denominator of Eq. (39). That is, $f(\kappa,\Gamma)$ from Eq. (35) is of the order of -0.1 using the experimental parameters $\Gamma\gg1$ and $\kappa\lesssim0.6$, so that the magnitude of $f(\kappa,\Gamma)\kappa^2$ is at most of the order of 0.035. On the other hand, $\nu_d/\omega_{\rm pd}>\frac{1}{2}$, which is considerably larger than the magnitude of the strong-coupling term. Thus, according to the results of this paper, we would expect the strong correlation effects to be relatively small in the wave dispersion in this parameter regime, essentially washed out by collisional effects. It is not surprising then that the measured results approximately fit a fluid-based dispersion relation ignoring strong coupling, since that is what Eq. (39) would also yield approximately for this set of parameters.

However, we point out that it may be possible to investigate in more detail the effects of strong coupling on wave dispersion in somewhat lower collisionality experimental regimes. From the results of this paper, we would expect that correlation effects would dominate collisional effects at lower values of $\nu_d/\omega_{\rm pd}$ and larger values of κ than those reported in Ref. [34]. For example, with the parameters used for Fig. 4(a) one would expect to see the effects of strong correlation in a reduction of the maximum values of k_r and k_i. For the parameters used in Fig. 4(a), namely, $\nu_d/\omega_{\rm pd}=0.1$, $\Gamma=150$, and $\kappa=1$, we find a reduction in the maximum values of k_r and k_i by a factor of about 0.7 compared with the solutions of the fluid dispersion relation in the weak-coupling limit shown in Fig. 4(b).

We note also that the strong correlation term in Eq. (39) leads to a rollover in k_r versus ω even without the collisional term. This is in contrast to theoretical results for a damped lattice-wave dispersion relation [21] which apparently does not exhibit the observed rollover [34].

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