Microstructure and microdynamics of uninterrupted traffic flow

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(Received 21 November 1996; revised manuscript received 12 May 1997)

In this paper, ''uninterrupted homogeneous traffic flow'' is viewed as a dynamic system of spontaneously aggregating and breaking clusters of cars. This approach gives qualitative and quantitative descriptions of density fluctuations and dynamics of free traffic flow in terms of car clusters, and their distributions with respect to the number of cars in a cluster. The distributions have a nontrivial shape and a nonlinear dependence on the density of cars on the expressway. Their shape and evolution as a function of car density are described adequately by a model proposed here. This one-parameter model based on a modified Smoluchowski equation predicts a critical density at which a phase transition occurs from free to synchronized traffic flow for a multilane expressway: $\rho_{cr}=16\pm3$ vehicles km^{-1} (lane)$^{-1}$, and that below this critical density traffic flow can be treated as a one-dimensional system. The model may also find application in the study of the low density pneumatic transport of granular and powdered materials. [S1063-651X(97)11009-1]

PACS number(s): 05.40.+j, 89.40.+k

INTRODUCTION

Traffic flow has been extensively studied because of interesting physical properties and practical importance for the design of transportation infrastructure [1]. When the number of cars is large, traffic flows can be modeled phenomenologically in terms of a one-dimensional compressible gas. Such a hydrodynamic approach predicts the appearance of traffic shock waves and traffic jams [2]. However, the hydrodynamic approach does not naturally describe the behavior of traffic flows in the low-density limit, where there are large heterogeneities in traffic density [3]. In addition, a very small dissipation in the microscopic dynamics can make the hydrodynamic approach incorrect [4]. For this situation, a microscopic model [3,5] will provide a more appropriate description, requiring, however, more studies of traffic flow at low and moderate car densities. Here I report results of such studies, and suggest and analyze a microscopic model describing the field data. This combination of experiment and theory provides an insight into a phase transition from free to synchronized traffic flow [6].

EXPERIMENTAL PROPERTIES OF FREE TRAFFIC FLOW

The uninterrupted traffic flow on a four-lane express section of the Dan Ryan Expressway in Chicago, Illinois has been studied (Fig. 1). Smoothly moving traffic flow is not perfectly uniform. Observations revealed the following structural features of the traffic flow.

(s1) Small clusters of several cars can be distinguished by the typical distance between cars, of the order of a car size. This distance is several times smaller then that between clusters. A cluster can consist of cars moving closely in the nearest lanes, or moving closely one after another in the same lane, or both, and not necessarily with the same speed.

(s2) Larger clusters, with number of cars $i\geqslant 8$, are not uniform and have an internal structure: they appear to be composed of closely packed smaller clusters of size $i\leqslant 4$. Though very rare at studied densities ($<0.5\%$ of the total number of clusters), these clusters are of interest as candidates to play a role of seed fluctuations for the development of even larger stable clusters and, ultimately, traffic jams [2]. It is interesting to note that cluster structure is present even in heavily congested traffic, but in this case the car clusters are considerably larger and consist of tens of cars each.

Besides these interesting structural features, the observations revealed a number of dynamic properties of traffic flow.

(d1) Due to differences in speed, car clusters typically break-up into clusters of smaller size;

(d2) Two car clusters, also, can constructively interact, yielding a larger cluster.

(d3) Clusters, composed of $i\geqslant 4$ cars, can be large enough to block all the lanes of the expressway (''blocking'' cluster) and to have a net synchronizing effect on cars speeds: a fast car can not pass the ''blocking'' cluster, slows, and ends up embedded within the cluster.

The described qualitative picture raises quantitative questions about the structure and dynamics of the traffic flow: What is the shape of a cluster distribution? How does the cluster distribution evolve as the density of cars on the expressway changes? With these questions in mind, a number of measurements have been carried out.

The measurements have been done by visual counting and

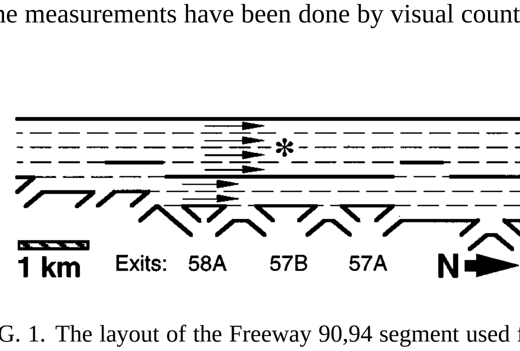

FIG. 1. The layout of the Freeway 90,94 segment used for measurements. The measurements were performed under daylight conditions, a temperature of about 20 °C, and a wind speed of about 16 km/h. The point on a bridge above the freeway, from which data were collected, is indicated with a star. Bold lines indicate concrete separations of express and local lines and exits.

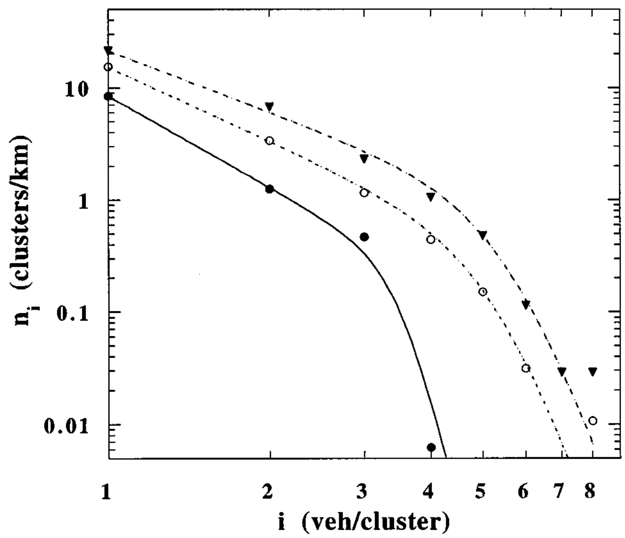

FIG. 2. Distribution of car clusters n_i, vs cluster size i, on a four-lane freeway for three car densities in vehicles km^{-1} (lane)$^{-1}$ (a log-log scale): ●, 3.1 ± 0.3; ○, (7.2 ± 0.8); ▼, (11.9 ± 1.1). To reflect the density of clusters per one kilometer, the distributions are normalized with coefficient $K=t_{\mathrm{tot}}v_{\mathrm{av}}$, where t_{tot} is a total time of data acquisition for a distribution, and v_{av} is an average cars speed measured by driving along with the traffic.

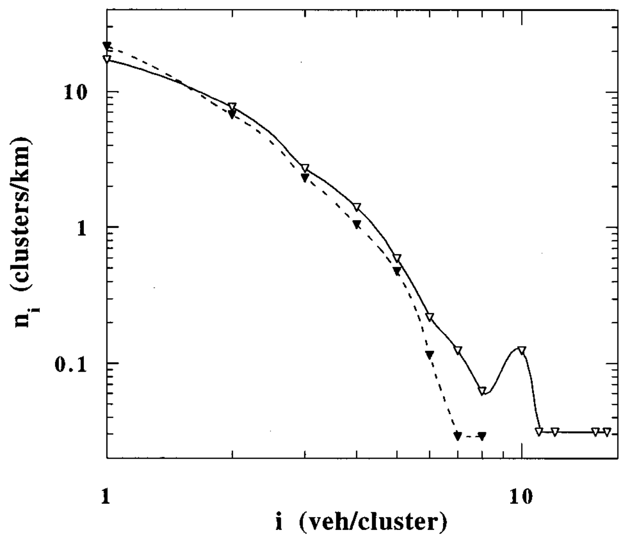

FIG. 3. Cluster distribution at $\rho\approx14.5$ vehicles km^{-1} (lane)$^{-1}$, ▽, demonstrates qualitative change of behavior with respect to the distribution at $\rho\approx11.9$, ▼: the depletion of the number of clusters for $i=1$, and an increment for $i\geqslant8$. The normalization procedure is the same as for Fig. 2.

recording the number and sizes of car clusters, traversing in the same direction a fixed cross section of the expressway (Fig. 1). To quantify the rate of the traffic flow, the traffic data were recorded for a sequence of time intervals, varying in length from 10 to 70 s. The time of the measurements varies typically from 20 to 50 min. The traffic data were initially recorded with a voice recorder, and later transferred into a computer for processing.

The distributions of clusters, n_i, with respect to the number, i, of cars in a cluster were measured for different flow rates (Fig. 2). The distributions demonstrate two regimes of behavior: a weaker dependence on i for the clusters of small size, and a steeper exponential dependence on i for the clusters of larger size. The distributions evolve nonlinearly as a function of car density: when the car density increases, the number of small-size $(i<4)$ clusters increases for less then 50%, while the number of large-size clusters $(i>4)$ can increase by two orders of magnitude.

When the car density grows, one can expect the process (d3) to become more important, and to cause depletion in the number of fast small $(i<4)$ clusters through their embodiment into large $(i\geqslant4)$ ''blocking'' clusters. The number of clusters with $i\geqslant8$ will grow correspondingly. This effect, first a hypothesis based on (s2) and (d3), indeed has been observed: a cluster distribution at the density $\rho=14.5$ vehicles km^{-1} (lane)$^{-1}$ demonstrates a qualitative change of its behavior with respect to the cluster distribution at $\rho=11.9$ vehicles km (lane)$^{-1}$ (Fig. 3). Instead of an anticipated increment in the number of all clusters the distribution for $\rho=14.5$ shows $\sim30\%$ depletion for $i=1$, no change for $i=2$, and a fourfold growth for $i\geqslant8$. This observation, together with a recent observation [6] of phase transition from free to synchronized traffic flow at $\rho=17.5\pm3$ vehicles km^{-1} (lane)$^{-1}$ suggest that the nonlinear preferred growth of the number of large clusters and the phase transition might be closely connected. This suggestion is discussed further in the text.

The observed complex structure of free uninterrupted flow raises a question about the completeness of the formulated set, (d1)–(d3), of dynamic properties: Is this set sufficient to describe the complexity? To answer this question Monte Carlo simulations of traffic flow have been performed.

MODEL OF FREE TRAFFIC FLOW

Traffic flow was supposed to consist of car clusters. The properties of interaction between clusters, (d1) and (d2), can be described in generalized form by means of the following reactions:

$$c_i+c_j\rightarrow c_{i+j}\quad\text{for the aggregation of the clusters,}\tag{1a}$$

$$c_i\rightarrow c_j+c_{i-j}\quad\text{for the breakup of the clusters,}\tag{1b}$$

where c_i and c_j stand for a cluster of size i and j, respectively. The process (d3), which is essential for a three-cluster interaction $\sim A_3n_in_jn_k$, was not included because the rate of this interaction estimated for $(i+j+k\leqslant6)$ from experimental data is more than 20 times smaller than the rate of two-cluster interaction. However, for large ''blocking'' clusters the three-cluster interaction becomes important and, as it will be discussed below, sets an upper limit of car densities to which the following model is applicable.

Then, an equation for the concentration of the clusters can be written by recourse to the Smoluchowski equation [7], modified to include two terms describing cluster breaking [8]:

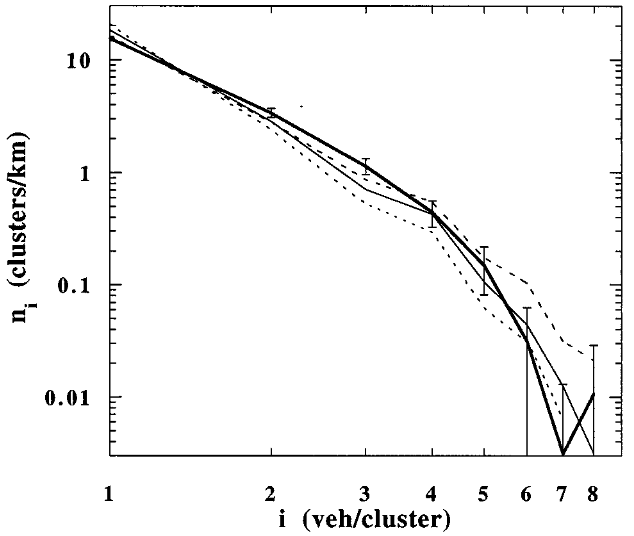

FIG. 4. Results of Monte Carlo simulations based on one-parameter Smoluchowski equation: —, experimental data for the car density 7.2 (veh km^{-1} lane^{-1}); ----, Monte Carlo simulations with $A_0/b_0=0.0048$ km; —, $A_0/b_0=0.0064$ km; ––––-, $A_0/b_0=0.0080$ km.

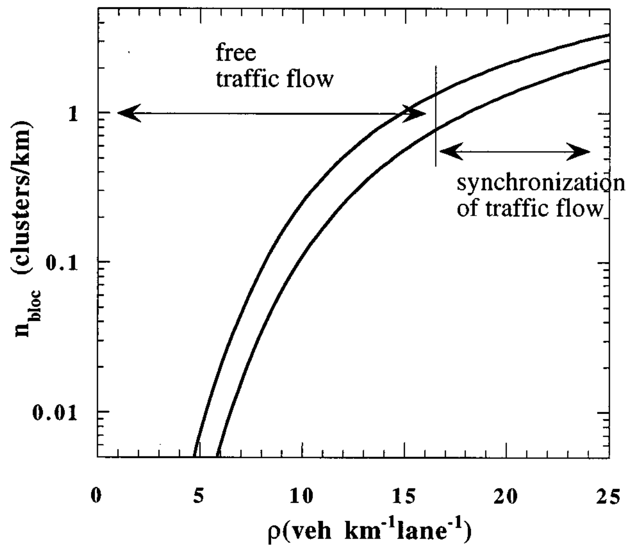

FIG. 5. Total number of ''blocking'' clusters, n_{bloc}, on a four-lane freeway as a function of car density, ρ. The uncertainty of n_{bloc} (indicated by two lines) is due to an uncertainty in the probability of a ''blocking'' configuration for clusters of sizes $4\leqslant i\leqslant 7$.

$$\frac{dn_i}{dt}=\frac{\partial n_i}{\partial t}+\frac{1}{2}\sum_{j=1}^{i-1}A(j,i-j)n_jn_{i-j}+\sum_{j=i+1}^{i_{\max}}B(i,j-i)n_j$$
$$-n_i\sum_{j=1}^{i_{\max}}A(i,j)n_j-(1-\delta_{1i})n_i\sum_{j=1}^{i-1}B(j,i-j), \qquad (2)$$

for $\forall i\geqslant 1$, where δ_{1i} is kroneker's index. Here $A(i,j)$ and $B(j,i-j)$ are constants of reactions (1a) and (1b), respectively.

Stationary solutions: $dn_i/dt=0$, $\partial n_i/\partial t=0$, $\forall i\geqslant 1$, of the above equation have been studied with several assumptions about the constants of the reactions: (a1) $A(i,j)=A_0$, $\forall i,j\geqslant 1$ (equal probability of association); (a2) $B(i,j)=b_0p(i,j)$, where b_0 is the constant for decay of a cluster of size i, and $p(i,j)=1/(i-1)$ is a probability of decay into the (c_j+c_{i-j}) channel. These assumptions about the constants make of Eq. (2) a one-parameter (A_0/b_0) model. This model has been used to fit the distributions of clusters with respect to their size (Fig. 4). The fit yields the value $A_0/b_0=0.0064\pm 0.0016$ km, the same, within error, for all studied car densities: 3–16 vehicles km^{-1} $(\mathrm{lane})^{-1}$. This value of A_0/b_0 is in agreement with a theoretical estimate given by a formula, which I derived assuming reaction constants being reciprocal of characteristic times for Eqs. (1a) and (1b):

$$A_0/b_0\approx(1/N_{\mathrm{tot}}\tau_{\mathrm{ass}})/(1/\tau_{\mathrm{br}})=l_{\mathrm{cl}}, \qquad (3)$$

where N_{tot} is the density of the clusters per mile, $\tau_{\mathrm{br}}\approx l_{\mathrm{cl}}/\Delta v$ is a characteristic time for cluster breaking, $\tau_{\mathrm{ass}}=1/(\Delta v N_{\mathrm{tot}})$ is a characteristic time for clusters association, and Δv and l_{cl} are an average speed dispersion and an average cluster length in traffic flow. Thus A_0/b_0 depends only on average cluster length, which was found to be $l_{\mathrm{cl}}\approx 6^{+4}_{-1}$ m, and expression (3) yields $A_0/b_0\approx 0.0061^{+0.004}_{-0.001}$ km, in close agreement with the above fit value. To test the predictive power of this model further, I have used it to describe earlier obtained traffic data for different road conditions: a no-passing zone in one-lane tunnel [9]. The fit parameter for the data, $A_0/b_0=0.0048\pm 0.0016$ km, is in a good agreement with its estimate above. Thus the set of dynamic properties deduced from the observed data is sufficient to describe the complex structure and evolution of free traffic flow, and allows one to formulate a model for the prediction of density fluctuations in the traffic flow.

The Monte Carlo simulations of Eq. (2) predict that n_1 reaches its maximum value $\sim$40 at $\rho\sim 0.8(A_0/b_0)^{-1}=30$ vehicles km^{-1} $(\mathrm{lane})^{-1}$ and remains approximately constant for the density range 30–50 vehicles km^{-1} $(\mathrm{lane})^{-1}$. This results, along with the observed decrement in n_1 (Fig. 3) and a possible connection of the decrement with the transition to synchronized flow, motivated a use of this model to determine a critical density for the transition: assuming that such transition is caused by ''blocking'' clusters, one can expect that the synchronizing effect becomes important when the total number of ''blocking'' clusters, n_{bloc}, exceeds 1. Calculation of this total number n_{bloc} (Fig. 5) gives the critical density value 16 ± 2 vehicles km^{-1} $(\mathrm{lane})^{-1}$, which agrees with its experimental value for a five-lane expressway [6]: 17.5 ± 3 vehicles km^{-1} $(\mathrm{lane})^{-1}$. These results taken together allow one to conclude that an external geometrical factor, the expressway width, determines the change in dynamics from free to synchronized flow. From this, a formula for the critical density on a multilane expressway can be deduced: $\rho_{\mathrm{cr}}=16\pm 3$ vehicles km^{-1} $(\mathrm{lane})^{-1}$. Below the critical density traffic flow can be treated as a one-dimensional system. Thus the phase transition from free to synchronized flow indeed can be linked to the evolution of cluster distribution as a function of car density.

The similarity between traffic and granular flows suggests that the model may be relevant to study transport of granular and powdered materials highly utilized in industry [10]. Developing of solid plugs during pneumatic conveying, ''choking'' and ''saltation,'' is analogous to the development of traffic jams. Then Eq. (2) may describe a stationary material state at densities below the ''choking'' limit. In this case, energy for the clusters' breakup would be provided by the air flow. It would be also interesting to study if the cluster structure of the flowing material remains preserved during the material densification. Recent experimental [11] and theoretical [12] results support such direction of questioning.

CONCLUSIONS

The results presented here suggest the following.

(1) ''Uninterrupted homogeneous traffic flow'' can be viewed as a dynamic system of spontaneously aggregating and breaking clusters of cars. This approach allows one to give qualitative and quantitative descriptions of the structure and dynamics of the traffic flow in terms of car clusters and their distributions with respect to the number of cars in a cluster.

(2) The distributions of car clusters, n_i, with respect to the number i of cars in a cluster have a nontrivial shape and demonstrate a complex nonlinear dependence on the density of cars on the expressway. When the density of cars increases, the slope of the distributions for small i decreases and the exponential ''cutoff'' shifts to larger values of i. The dependence of n_i on the density of cars described above brakes down around the density 14.5 ± 0.8 vehicles km^{-1} (lane)$^{-1}$: the number of clusters, large enough to block the expressway, increases considerably, and the number of small clusters decreases. This density value corresponds to a phase transition from the free to synchronized traffic flow on a multilane expressway.

(3) The evolution of the cluster distribution and, hence, density fluctuations, measured in the regime of free traffic flow at car densities $3-16$ vehicles km^{-1} (lane)$^{-1}$ can be qualitatively and quantitatively described by the one-parameter model proposed here, based on a modified Smoluchowski equation. The model parameter has a value of 0.0064 ± 0.0016 km for the studied densities of free traffic flow.

The results also suggest that the transition to synchronized regime on a k-lane expressway occurs at: $\rho_{cr}=16\pm3$ vehicles km^{-1} (lane)$^{-1}$, below which traffic flow, even on a multilane expressway, can be treated as a one-dimensional system of car clusters. A correct microscopic description of traffic flow at synchronized regime may require the inclusion of a many-cluster interaction.

ACKNOWLEDGMENTS

The author thanks his scientific advisor, Professor Riccardo Levi-Setti, Professor L. P. Kadanoff, Professor J. D. Cowan at the University of Chicago, Dr. Eli Ben-Naim at Los Alamos National Laboratory, and Professor M. Shlesinger at the Navy Research Laboratory for their encouragement and helpful discussions during my work on this manuscript; and also E. G. Berezutskaya for her help with processing the data. This work was supported in part through the MRSEC program by the National Science Foundation under Grant No. DMR-94-000379.

[1] See, e.g., *Transportation and Traffic Theory*, edited by N. H. Gartner and N. H. M. Wilson (Elsevier, New York, 1987); W. Leutzbach, *Introduction to the Theory of Traffic Flow* (Springer-Verlag, Berlin, 1988).
[2] B. S. Kerner and P. Konhauser, Phys. Rev. E **50**, 54 (1994).
[3] E. Ben-Naim, P. L. Krapivsky, and S. Redner, Phys. Rev. E **50**, 822 (1994).
[4] Y. Du, H. Li, and L. P. Kadanoff, Phys. Rev. Lett. **74**, 1268 (1995).
[5] M. Schreckenberg, A. Schadschneider, K. Nagel, and N. Ito, Phys. Rev. E **51**, 2939 (1995); K. Nagel and M. Paczuski, *ibid.* **51**, 2909 (1995).
[6] B. S. Kerner and H. Rehborn, Phys. Rev. E **53**, R4275 (1996).
[7] M. von Smoluchowski, Z. Phys. Chem. (Munich) **92**, 129 (1917).
[8] P. G. J. van Dongen and M. H. Ernst, J. Stat. Phys. **37**, 301 (1984).
[9] E. W. Montroll, in *Theory of Traffic Flow*, edited by R. Herman (Elsevier, New York, 1961), p. 147.
[10] T. M. Knowlton, J. W. Carson, G. E. Klinzing, and Wen-Ching Yang, Chem. Eng. Prog. **90**, 44 (1994).
[11] J. B. Knight, C. G. Fandrrich, C. N. Lau, H. M. Jaeger, and S. R. Nagel, Phys. Rev. E **51**, 3957 (1995).
[12] G. C. Barker and Anita Mehta, Phys. Rev. A **45**, 3435 (1992).