Parametric decays of a linearly polarized electromagnetic wave in an electron-positron plasma

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We study the parametric decays of a large amplitude, linearly polarized electromagnetic wave in an electron-positron plasma. We include harmonic generation, the ponderomotive force, and weakly relativistic effects. It is shown that when $v_s/c<c/v_p$ (v_s is the electroacoustic velocity, c is the speed of light, and v_p is the phase velocity of the electromagnetic wave), there are two instabilities. One is an ordinary decay instability, in which the pump wave decays into a sideband wave, propagating backward relative to the pump wave, and an electroacoustic mode propagating forward. The other is an essentially electromagnetic nonresonant modulational instability (which is due to higher order effects of the pump wave amplitude), in which the pump wave decays into two sideband waves. When $v_s/c \geqslant c/v_p$, there is a modulational nonresonant instability, and an ordinary modulational instability, in which the pump wave decays into a sideband wave and a forward propagating electroacoustic mode. [S1063-651X(97)03709-4]

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I. INTRODUCTION

Electron-positron plasmas are different from electron-ion plasmas in many aspects, because, in the absence of ion-electron mass difference, there are no high and low frequency scales [1].

The nonlinear decays of linearly and circularly polarized large amplitude electromagnetic waves in an electron-positron plasma have been thoroughly investigated (see [2] and references therein and [3]). In the case of linear polarization, the problem is more complicated because of harmonic generation and density perturbations due to the ponderomotive force of the electromagnetic wave. This problem has been addressed by a number of authors [4–9] in connection with the observed variability of spectral characteristics of active galactic nuclei and pulsars [10–14]. In order to account for the observations, Chian and Kennel [4] conjectured that the electromagnetic pulse could experience nonlinear modulation. Unfortunately, their results were proved to be wrong, because they omitted harmonic generation and ponderomotive effects [15]. A full treatment was then provided by Kates and Kaup [7,8], who showed that in a collisionless electron-proton plasma, within a narrow frequency interval near ω_p, the self-modulational instability is possible [8].

The subject of electron-positron plasmas is not only important in plasma astrophysics, but is also relevant in laboratory experiments [16,17].

Thus, we study the parametric decay of a large amplitude linearly polarized electromagnetic wave. Our treatment is similar to Ref. [9], but as we shall see, their treatment, in our opinion, is wrong in several aspects. They neglected the pressure gradient term in the longitudinal component of the force equation, but included it in their nonlinear treatment. This term introduces changes both in the expression of the dispersion relation of the electromagnetic wave and in the nonlinear dispersion relation that gives the coupling to sideband waves and electroacoustic modes.

Thus, this paper is organized as follows. In Sec. II, the model equations are analyzed, and the nonlinear dispersion relation is derived. In Sec. III, the nonlinear couplings of the pump wave to electroacoustic modes and to sideband waves are studied. In Sec. IV, the results are summarized and discussed.

II. THE MODEL

We assume that the electron-positron plasma is described by the following set of equations:

$$\frac{\partial n_j}{\partial t} = -\vec{\nabla}\cdot(n_j\vec{v}_j), \tag{1}$$

$$\left(\frac{\partial}{\partial t}+\vec{v}_j\cdot\vec{\nabla}\right)(\Gamma_j\vec{v}_j)=\frac{q_j}{m}\left(\vec{E}+\frac{1}{c}\,\vec{v}_j\times\vec{B}\right)-\frac{\gamma KT}{mn_0}\,\vec{\nabla}n_j, \tag{2}$$

$$\vec{\nabla}\cdot\vec{E}=4\pi\rho, \tag{3}$$

$$\vec{\nabla}\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t}, \tag{4}$$

$$\vec{\nabla}\times\vec{B}=\frac{4\pi}{c}\,\vec{J}+\frac{1}{c}\frac{\partial\vec{E}}{\partial t}, \tag{5}$$

$$\vec{J}=\sum_j q_j n_j\vec{v}_j, \tag{6}$$

$$\rho=\sum_j q_j n_j, \tag{7}$$

$$\Gamma_j=\left(1-\frac{\vec{v}_j^{\,2}}{c^2}\right)^{-1/2}, \tag{8}$$

where n_j is the density, and $\vec{v}_j$ is the bulk velocity of each fluid ($j=e$ for electrons and $j=p$ for positrons), $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields, respectively, K is the Boltzmann constant, $\vec{J}$ is the total current, γ is the adiabatic coefficient, T is the common temperature, m is the particle mass, and c is the speed of light.

Assuming that a linearly polarized electromagnetic wave propagates along the z axis, in absence of relativistic effects, linearizing Eqs. (1)–(8) we find the solution

$$\vec{E}_0=E_0\cos(k_0z-\omega_0t)\hat{x}, \tag{9}$$

$$\vec{B}_0=B_0\cos(k_0z-\omega_0t)\hat{y}, \tag{10}$$

$$B_0=\frac{ck_0}{\omega_0}E_0, \tag{11}$$

which satisfies the dispersion relation

$$\omega_0^2=c^2k_0^2+2\omega_p^2. \tag{12}$$

This wave induces a particle velocity given by

$$\vec{v}=-\frac{qE_0}{m\omega_0}\sin(k_0z-\omega_0t)\hat{x}. \tag{13}$$

When relativistic effects are considered, the component of Eq. (2) perpendicular to the direction of propagation of the plane electromagnetic wave can be written in terms of the vector potential as follows:

$$\frac{d}{dt}(\Gamma\vec{v})=-\frac{q}{mc}\frac{d\vec{A}}{dt}. \tag{14}$$

Assuming that the longitudinal component of the velocity is much less than the perpendicular velocity, integration of this equation yields

$$\vec{v}=-\frac{q\vec{A}}{mc}\left[1+\left(\frac{qA}{mc^2}\right)^2\right]^{-1/2}. \tag{15}$$

Clearly, dealing with weak relativistic effects means $qA/mc^2\ll 1$, so that expanding the square root, the transverse velocity is then given by

$$v_{0x}=-\frac{qE_0}{m\omega_0}\sin(k_0z-\omega_0t)\left[1-\frac{\alpha^2}{2}\sin^2(k_0z-\omega_0t)\right], \tag{16}$$

$$\alpha=\frac{eA}{mc^2}=\frac{eE_0}{mc\omega_0}. \tag{17}$$

The transverse velocity induces a longitudinal velocity, δv_z, through the term $\vec{v}\times\vec{B}$ in the Lorentz force, and, consequently, a density perturbation, δn. We shall calculate this effect by using Eqs. (1)–(3) as a perturbation on a steady state in which the density is a constant, n_0, and the longitudinal velocity is $v_{0z}=0$. Thus, for each fluid, we have

$$\frac{\partial}{\partial t}\delta n=-n_0\frac{\partial}{\partial z}\delta v_z, \tag{18}$$

$$\frac{\partial}{\partial z}\delta E_z=4\pi e(\delta n_p-\delta n_e), \tag{19}$$

$$\frac{\partial}{\partial t}\delta v_z=\frac{q}{m}\left(\delta E_z+\frac{1}{c}v_{0x}B_{0y}\right)-\frac{\gamma KT}{mn_0}\frac{\partial}{\partial z}\delta n. \tag{20}$$

We have neglected the factor Γ in the left side of Eq. (20). In fact, since the longitudinal velocity is much less than the transverse velocity, and $v_{0x}\ll c$, $\Gamma\simeq(1-v_{0x}^2/c^2)^{-1/2}\simeq 1+v_{0x}^2/(2c^2)$, then in the left side of Eq. (20) there would be terms of order δv_z and $E_0^2\delta v_z$. This last term can be neglected as compared to the first one. For the same reason, we will neglect the relativistic correction to v_{0x} in Eq. (20), since it contributes a term of order α^4, which is negligible as compared to α^2, the order of the leading term. Moreover, as we shall see, δv_z is of the same order as the first order relativistic corrections, α^2. Since we shall keep only the leading corrections, we can neglect terms of the form $\delta v_z v_{0x}^2/c^2$ and the relativistic correction to $v_{0x}B_{0y}$.

Writing Eqs. (18)–(20) for each species, and combining them, yields

$$\left(\frac{\partial^2}{\partial t^2}-v_s^2\frac{\partial^2}{\partial z^2}\right)\delta n_p+\omega_p^2(\delta n_p-\delta n_e)=\left(\frac{eE_0}{m\omega_0}\right)^2 n_0k_0^2\cos[2(k_0z-\omega_0t)], \tag{21}$$

$$\left(\frac{\partial^2}{\partial t^2}-v_s^2\frac{\partial^2}{\partial z^2}\right)\delta n_e-\omega_p^2(\delta n_p-\delta n_e)=\left(\frac{eE_0}{m\omega_0}\right)^2 n_0k_0^2\cos[2(k_0z-\omega_0t)], \tag{22}$$

where $v_s=(\gamma KT/m)^{1/2}$ and $\omega_p=(4\pi e^2n_0/m)^{1/2}$ is the electron (positron) plasma frequency. It is easy to see that these equations admit an oscillatory solution induced by the electromagnetic wave given by

$$\delta n_p=\delta n_e=-n_0\alpha^2\tilde{q}\cos[2(k_0z-\omega_0t)], \tag{23}$$

and the corresponding longitudinal velocity [derived from the continuity equation (1)],

$$\delta v_{pz}=\delta v_{ez}=-\alpha^2\frac{\omega_0}{k_0}\tilde{q}\cos[2(k_0z-\omega_0t)], \tag{24}$$

where

$$\tilde{q}=\frac{1}{4}\frac{c^2k_0^2}{\omega_0^2-v_s^2k_0^2}. \tag{25}$$

The term $\vec{v}\times\vec{B}$ in the force equation (2) implies that an electromagnetic wave of frequency ω_0 induces a longitudinal oscillation of frequency $2\omega_0$. This is a nonlinear effect (harmonic generation), of order α^2, which couples the transverse and longitudinal motion of the particles.

Equations (23) and (25) are different from the corresponding expressions given in Ref. [9], namely,

$$\delta n_p = \delta n_e = -n_0 \alpha^2 \tilde{q} \cos[2(k_0 z - \omega_0 t)],$$

$$\tilde{q} = \frac{\omega_0^2 - \omega_p^2}{4\omega_0^2 - \omega_p^2}.$$

The differences arise because their expressions correspond to density perturbations due to harmonic generation in a cold ion-electron plasma. Indeed, following a similar procedure to the one followed here, it is easy to show that their expressions are valid for an ion-electron plasma where ion motion can be neglected and $\omega_0^2 = c^2 k_0^2 + \omega_p^2$.

Combining Eqs. (4)–(6) we find a wave equation for the electric field of the wave E_{0x}. Using Eqs. (23) and (24), to lowest order in α, we obtain

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) E_0 = -2\frac{\omega_p^2}{\omega_0 c^2} E_0 \frac{\partial}{\partial t}\left\{\sin(k_0 z - \omega_0 t) - \alpha^2 \tilde{q} \sin(k_0 z - \omega_0 t)\cos[2(k_0 z - \omega_0 t)] - \frac{1}{2}\alpha^2 \sin^3(k_0 z - \omega_0 t)\right\}. \tag{26}$$

Assuming that the wave electric field is still of the form (9) and, since

$$\sin(\phi)\cos(2\phi) = \tfrac{1}{2}[\sin(3\phi) - \sin(\phi)],$$

$$\sin^3(\phi) = -\tfrac{1}{4}[\sin(3\phi) - 3\sin(\phi)],$$

the resonant contributions to Eq. (26) give the dispersion relation

$$\omega_0^2 = c^2 k_0^2 + 2\omega_p^2 (1 - \tfrac{1}{2} q \alpha^2), \tag{27}$$

$$q = \tfrac{3}{4} - \tilde{q}. \tag{28}$$

This is the dispersion relation of linearly polarized electromagnetic waves in an electron-positron plasma when harmonic generation, weakly relativistic effects, and thermal effects are taken into account.

Equation (27) differs from the results in Ref. [9], not only in the aforementioned incorrect expression for $\tilde{q}$, but also in the factor $\frac{1}{2}$ before q, which, according to them, should be $\frac{1}{4}$.

In Fig. 1 we show the dispersion relation of the pump wave, $y = ck_0/\omega_p$ versus $x = \omega_0/\omega_p$, for $\alpha = 0.01$, $v_s/c = 0.1$. The straight lines correspond to the electroacoustic modes, and the parabolas to the electromagnetic modes. In Fig. 2, the same dispersion relation is shown, but for $\alpha = 0.5$. We see that starting at $\omega_0 = 0$ and $k_0 = 0$, there is an electrostatic instability. However, since the small parameter is α, from Eq. (17) it follows that the dispersion relation is not valid for ω_0 values close to the origin.

We now perturb the system assuming that it consists of electrons, positrons, and a linearly polarized electromagnetic wave satisfying Eq. (27)—the pump wave. The zeroth order solution is

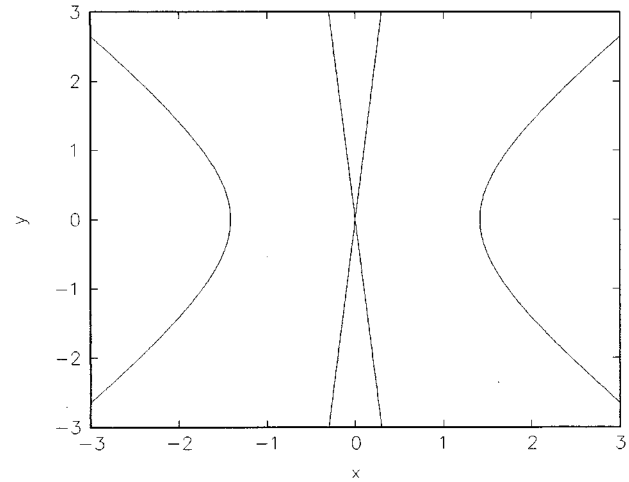

FIG. 1. Dispersion relation of the pump wave, Eq. (27). Normalized frequency, $x = \omega_0/\omega_p$, vs normalized wave number, $y = ck_0/\omega_p$, for $v_s/c = 0.1$, and $\alpha = 0.01$.

$$\vec{E}_0 = E_0 \cos(k_0 z - \omega_0 t)\hat{x}, \tag{29}$$

$$\vec{B}_0 = B_0 \cos(k_0 z - \omega_0 t)\hat{y}, \tag{30}$$

$$B_0 = \frac{ck_0}{\omega_0} E_0, \tag{31}$$

$$v_{j0x} = -\frac{q_j E_0}{m\omega_0}\sin(k_0 z - \omega_0 t)\left[1 - \frac{\alpha^2}{2}\sin^2(k_0 z - \omega_0 t)\right], \tag{32}$$

$$v_{j0z} = -\alpha^2 \frac{\omega_0}{k_0} \tilde{q} \cos[2(k_0 z - \omega_0 t)], \tag{33}$$

$$n_{j0} = n_0 + n_{jh} = n_0\{1 - \alpha^2 \tilde{q} \cos[2(k_0 z - \omega_0 t)]\}. \tag{34}$$

Longitudinal and transverse perturbations are of the form

$$\delta C_z = \mathrm{Re}[\tilde{C} e^{i(kz - \omega t)}], \tag{35}$$

and

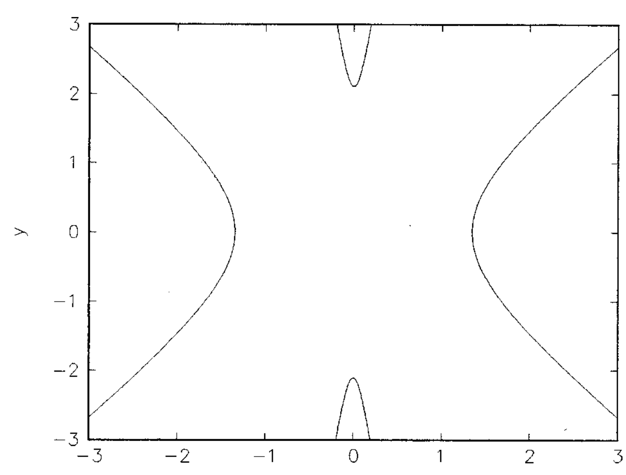

FIG. 2. Same as Fig. 1, but $\alpha = 0.5$.

$$\delta C_\perp = \mathrm{Re}[c_+ e^{i(k_+ z - \omega_+ t)} + c_- e^{i(k_- z - \omega_- t)}], \tag{36}$$

respectively, with $k_\pm = k_0 \pm k$, $\omega_\pm = \omega_0 \pm \omega$.

To lowest order,

$$\Gamma_0 \simeq 1 + \frac{1}{2}\frac{v_{0x}^2}{c^2},$$

and

$$\delta(\Gamma v_i) = \delta\Gamma v_{0i} + \Gamma_0 \delta v_i, \quad i = x,z,$$

so that

$$\delta(\Gamma v_x) = \left(1 + \frac{3}{2}\frac{v_{0x}^2}{c^2}\right)\delta v_x,$$

$$\delta(\Gamma v_z) = \left(1 + \frac{1}{2}\frac{v_{0x}^2}{c^2}\right)\delta v_z.$$

Defining

$$\delta V = \delta v_{px} - \delta v_{ex},$$

$$\delta U = \delta v_{pz} + \delta v_{ez},$$

$$\delta N = \delta n_p + \delta n_e,$$

the perturbed equations (1)–(8) are

$$\frac{\partial}{\partial t}\left[\left(1 + \frac{3}{2}\frac{v_{p0x}^2}{c^2}\right)\delta V\right] + v_{p0z}\frac{\partial}{\partial z}\delta V + \delta U \frac{\partial}{\partial z}\left[\left(1 + \frac{1}{2}\frac{v_{p0x}^2}{c^2}\right)v_{p0x}\right] = \frac{e}{m}\left[2\,\delta E_x - \frac{1}{c}(\delta U B_{0y} + 2 v_{p0z}\delta B_y)\right], \tag{37}$$

$$\frac{\partial}{\partial t}\left[\left(1 + \frac{1}{2}\frac{v_{p0x}^2}{c^2}\right)\delta U\right] + \frac{\partial}{\partial z}(v_{p0z}\delta U) = \frac{e}{mc}(\delta V B_{0y} + 2 v_{p0x}\delta B_y) - \frac{v_s^2}{n_0}\frac{\partial}{\partial z}\delta N, \tag{38}$$

$$\frac{\partial}{\partial t}\delta N = -\frac{\partial}{\partial z}[(n_0 + n_{ph})\delta U + \delta N v_{p0z}], \tag{39}$$

$$\frac{\partial}{\partial z}\delta E_x = -\frac{1}{c}\frac{\partial}{\partial t}\delta B_y, \tag{40}$$

$$-\frac{\partial}{\partial z}\delta B_y = \frac{4\pi e}{c}[(n_0 + n_{ph})\delta V + \delta N v_{p0x}] + \frac{1}{c}\frac{\partial}{\partial t}\delta E_x. \tag{41}$$

Combining these equations, we obtain the following set of fundamental equations:

$$\frac{\partial}{\partial t}\left[\left(1 + \frac{3}{2}\frac{v_{p0x}^2}{c^2}\right)\delta V\right] + v_{p0z}\frac{\partial}{\partial z}\delta V + \delta U \frac{\partial}{\partial z}\left[\left(1 + \frac{1}{2}\frac{v_{p0x}^2}{c^2}\right)v_{p0x}\right] = \frac{e}{m}\left[2\,\delta E_x - \frac{1}{c}(\delta U B_{0y} + 2 v_{p0z}\delta B_y)\right], \tag{42}$$

$$\left[\frac{\partial^2}{\partial t^2}\left(1 + \frac{1}{2}\frac{v_{p0x}^2}{c^2}\right) - v_s^2\frac{\partial^2}{\partial z^2}\right]\delta U = \frac{e}{mc}\frac{\partial}{\partial t}(\delta V B_{0y} + 2 v_{p0x}\delta B_y), \tag{43}$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)\delta B_y = \frac{4\pi e}{c}\frac{\partial}{\partial z}[(n_0 + n_{ph})\delta V + \delta N v_{p0x}]. \tag{44}$$

Replacing in Eqs. (42)–(44) the assumed space and time dependence [Eqs. (35) and (36)], and selecting the resonant terms, yields

$$\mp i\omega_\pm\left(1 + \frac{3}{4}\alpha^2\right)V_\pm \pm i\frac{\alpha^2}{2}\left(\frac{3}{4}\omega_\pm + \frac{\omega_0}{k_0}\tilde{q}k_\mp\right)V_\mp = \frac{e}{m}\left(2e_\pm + \frac{1}{c}\alpha^2\frac{\omega_0}{k_0}\tilde{q}b_\mp\right), \tag{45}$$

$$\left[\omega^2\left(1 + \frac{\alpha^2}{4}\right) - v_s^2 k^2\right]\tilde{U} = \frac{e}{mc}i\omega\left[\frac{B}{2}(V_+ + V_-) - i\left(\frac{eE_0}{m\omega_0}\right)\left(1 - \frac{3}{8}\alpha^2\right)(b_+ - b_-)\right], \tag{46}$$

$$(-\omega_\pm^2 + c^2k_\pm^2)\frac{b_\pm}{c^2k_\pm} = \pm i\frac{4\pi e}{c}\left[n_0\left(V_\pm - \frac{1}{2}\alpha^2\tilde{q}V_\mp\right) \pm i\frac{\tilde{N}}{2}\left(\frac{eE_0}{m\omega_0}\right)\left(1 - \frac{3}{8}\alpha^2\right)\right]. \tag{47}$$

From the continuity equation and Faraday's law, (1), (4),

$$\widetilde{N}=n_0\,\frac{k\widetilde{U}}{\omega},\tag{48}$$

$$e_\pm=\frac{\omega_\pm}{ck_\pm}\,b_\pm\,.\tag{49}$$

Thus from Eq. (45),

$$V_\pm=\pm\frac{ie}{mc}\left[2\,\frac{b_\pm}{k_\pm}-\frac{3}{2}\,\alpha^2\left(\frac{b_\pm}{k_\pm}+\frac{1}{2}\,\frac{b_\mp}{k_\mp}\right)\right].\tag{50}$$

Replacing Eqs. (48)–(50) in Eqs. (46)–(47), yields

$$\left[c^2k_\pm^2-\omega_\pm^2+2\omega_p^2\left(1-\frac{1}{2}\,q\alpha^2\right)-\omega_p^2\alpha^2\left(\frac{3}{4}+\widetilde{q}\right)\right]\frac{b_\pm}{k_\pm}-\omega_p^2\alpha^2q\,\frac{b_\mp}{k_\mp}=-\omega_p^2\,\frac{E_0c}{\omega_0}\,\frac{1}{2}\,\frac{\widetilde{N}}{n_0}\left(1-\frac{3}{8}\,\alpha^2\right),\tag{51}$$

$$\left[\omega^2\left(1+\frac{1}{4}\,\alpha^2\right)-v_s^2k^2\right]\frac{\widetilde{N}}{n_0}=\frac{e}{m}\,k^2\,\frac{eE_0}{mc\omega_0}\left(1-\frac{3}{8}\,\alpha^2\right)\left(\frac{b_+}{k_+}+\frac{b_-}{k_-}\right).\tag{52}$$

Upon elimination of all quantities $(\widetilde{N},b_\pm)$, from Eqs. (51) and (52), we obtain the following nonlinear dispersion relation:

$$0=S[D_+D_--\omega_p^2\alpha^2(\tfrac{3}{4}+\widetilde{q})(D_++D_-)]+\tfrac{1}{2}\omega_p^2\alpha^2c^2k^2[(1-\tfrac{3}{4}\alpha^2)(D_++D_-)-4\omega_p^2\alpha^2\widetilde{q}],\tag{53}$$

where

$$D_\pm=c^2k_\pm^2-\omega_\pm^2+2\omega_p^2(1-\tfrac{1}{2}q\alpha^2),\tag{54}$$

$$S=\omega^2(1+\tfrac{1}{4}\alpha^2)-v_s^2k^2.\tag{55}$$

Clearly, when $\alpha=0$, $D_\pm=0$ and $S=0$, which correspond to the electromagnetic waves and the electroacoustic modes. There are two electroacoustic modes, one propagating in the direction of the electromagnetic wave, and the other in the opposite direction. They are denoted by S_+ and S_-, respectively. When $\alpha\neq 0$, the modes are coupled. A necessary condition for wave coupling is that $n\omega_0=\omega_1+\omega_2$, with $n=0,1,2,3,\ldots$, where ω_1 and ω_2 are the frequencies of the daughter waves.

The dispersion relation (53) differs from the result given in Ref. [9]. In order to compare their results with ours, we now write Eqs. (25), (26), and (38) in Ref. [9], which, assuming (without loss of generality) $\vec{A}_0$ real, can be expressed as

$$(D_+-\tfrac{1}{2}\omega_p^2q\alpha^2)\,\delta\widetilde{A}-\tfrac{1}{2}\omega_p^2q\alpha^2\,\delta\widetilde{A}^*=-\omega_p^2\delta\widetilde{N}A(1-\tfrac{1}{4}q\alpha^2),\tag{56}$$

$$(D_--\tfrac{1}{2}\omega_p^2q\alpha^2)\,\delta\widetilde{A}^*-\tfrac{1}{2}\omega_p^2q\alpha^2\,\delta\widetilde{A}=-\omega_p^2\delta\widetilde{N}A(1-\tfrac{1}{4}q\alpha^2),\tag{57}$$

$$(\omega^2-v_s^2k^2)\,\delta\widetilde{N}=\left(\frac{e}{mc}\right)^2Ak^2(\delta\widetilde{A}+\delta\widetilde{A}^*).\tag{58}$$

In order to compare these equations with our results, we have repeated our calculations in terms of the vector potential, using their notation, and following their procedure, properly including relativistic effects. Then one obtains the zeroth order dispersion relation (27), and Eqs. (56)–(58) are replaced by

$$[D_+-\omega_p^2\alpha^2(\tfrac{3}{4}+\widetilde{q})]\,\delta\widetilde{A}-\omega_p^2q\alpha^2\,\delta\widetilde{A}^*=-\omega_p^2\delta\widetilde{N}A(1-\tfrac{3}{8}\alpha^2),\tag{59}$$

$$[D_--\omega_p^2\alpha^2(\tfrac{3}{4}+\widetilde{q})]\,\delta\widetilde{A}^*-\omega_p^2q\alpha^2\,\delta\widetilde{A}=-\omega_p^2\delta\widetilde{N}A(1-\tfrac{3}{8}\alpha^2),\tag{60}$$

$$[(1+\tfrac{1}{4}\alpha^2)\omega^2-v_s^2k^2]\,\delta\widetilde{N}=\tfrac{1}{2}\left(\frac{e}{mc}\right)^2k^2A(1-\tfrac{3}{8}\alpha^2)\times(\delta\widetilde{A}+\delta\widetilde{A}^*).\tag{61}$$

We note that our quantities are related to those in Ref. [9] through

$$\frac{b_+}{k_+}\rightarrow\delta\widetilde{A},\tag{62}$$

$$\frac{b_-}{k_-}\rightarrow\delta\widetilde{A}^*,\tag{63}$$

$$\frac{1}{2}\,\frac{\widetilde{N}}{n_0}\rightarrow\delta\widetilde{N},\tag{64}$$

consistent with $\nabla\times\vec{A}=\vec{B}$ and the respective definitions of Fourier transform of the potential and density perturbations.

Taking these identifications into account, Eqs. (59)–(61) are exactly equivalent to our equations (51) and (52), and,

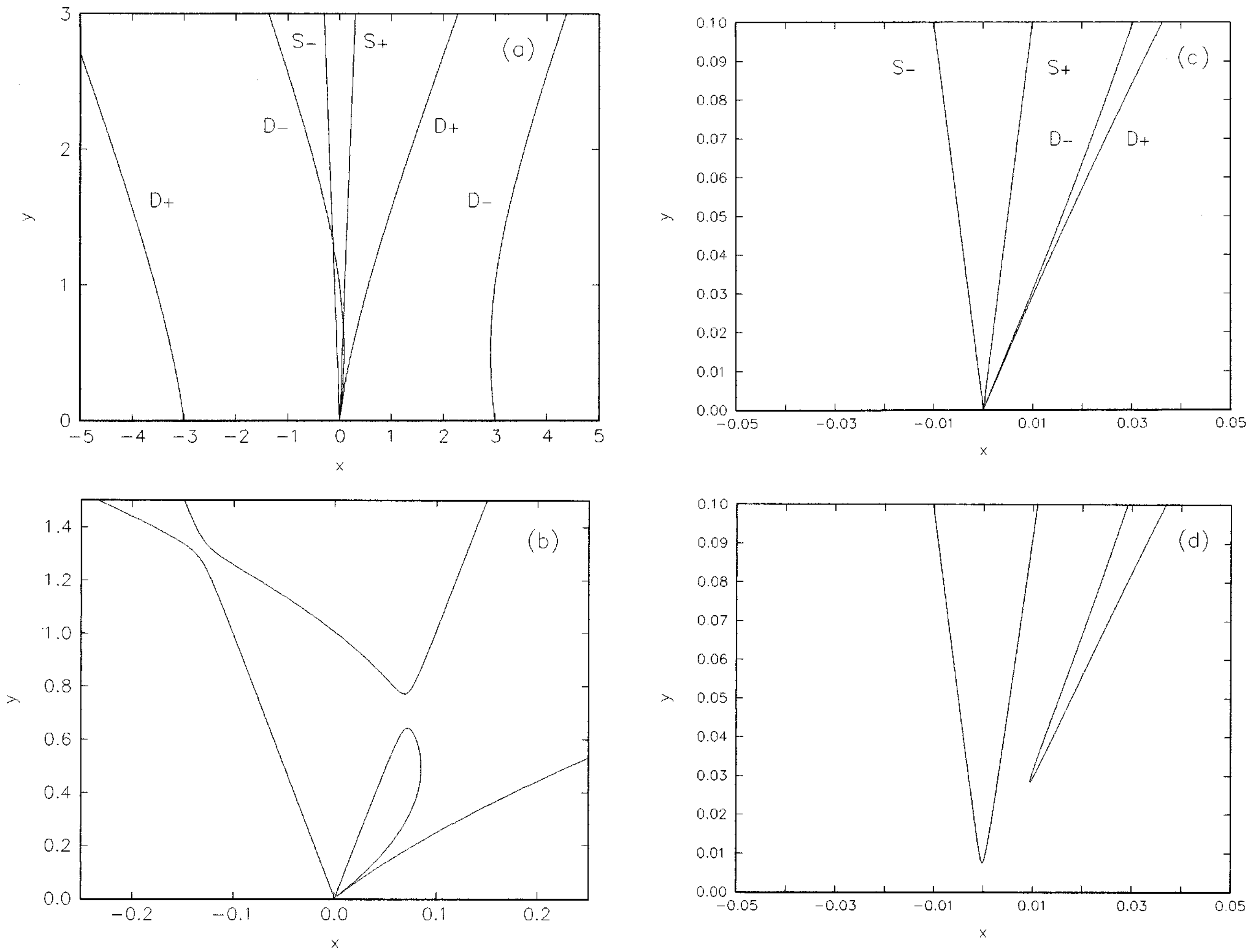

FIG. 3. (a) Nonlinear dispersion relation, Eq. (53). Normalized frequency, $x=\omega/\omega_p$, vs normalized wave number, $y=ck/\omega_p$, for $v_s/c=0.1$, $\omega_0/\omega_p=1.5$, and $\alpha=0$. (b) Same as (a) for $\alpha=0.01$. (c) Enlargement of the origin of (a) for $\alpha=0$. (d) Same as (c), but for $\alpha=0.03$.

therefore, give the same dispersion relation, Eq. (53), we obtained by using an independent method.

III. WAVE COUPLING

We now solve Eq. (53) graphically, using the method first derived by Longtin and Sonnerup [18] (see also [19–22]). To this end, we plot the dispersion relation, Eq. (53), in the upper half of the ω-k plane. The lower half plane can be obtained by rotating the upper half plane through an angle of 180° through the origin. There are four lines corresponding to $D_\pm=0$ (labeled on Fig. 3 as $D_\pm$) and two straight lines symmetric around the origin, corresponding to the electroacoustic modes, one propagating backwards relative to the pump wave (S_-), and the other in the same direction as the pump wave (S_+). Notice that the dispersion relation is of order six in ω and k. In Fig. 3(a), we illustrate Eq. (53) for $\omega_0/\omega_p=1.5$ with a corresponding $ck_0/\omega_p=0.5$, for $\alpha=0$. The other parameters are indicated in the figure. There are some crossings between the lines. These are, from left to right, a crossing between (D_-,S_-), another at the origin between D_-, D_+, and S_+, and one between (D_-,S_+). Only the last two satisfy the conditions given above. In order to see whether they lead to coupling, we switch on the pump wave. Thus, taking for the pump wave an α value of $\alpha=0.01$, we obtain Fig. 3(b). From Fig. 3(b), it follows that the crossing between (D_-,S_-) is an avoided crossing, as expected, because it does not satisfy energy conservation. The next crossing, namely, the one between (D_-,S_+), leads to a coupling. In fact, at this crossing there is now a gap, which means that two roots of Eq. (53) have become complex conjugate. Therefore, at this crossing there is an instability that corresponds to an ordinary decay instability where the pump wave decays into a sideband wave corresponding to a solution of $D_-=0$ and an electroacoustic mode, S_+.

Next, in Fig. 3(c) we investigate the crossing at the origin of Fig. 3(a), for $\alpha=0$. In Fig. 3(d) the pump wave amplitude has been raised to $\alpha=0.03$. We see from the figure that at the crossing between (D_+,D_-) there is now a gap. This gap is a modulational instability corresponding to a nonresonant decay of the pump wave into two sideband waves, of frequencies ω_+ and ω_-. This decay is essentially electromagnetic, but is mediated by plasma oscillations that are not normal modes of the system [23] and it is of higher order, $O(\alpha^4)$.

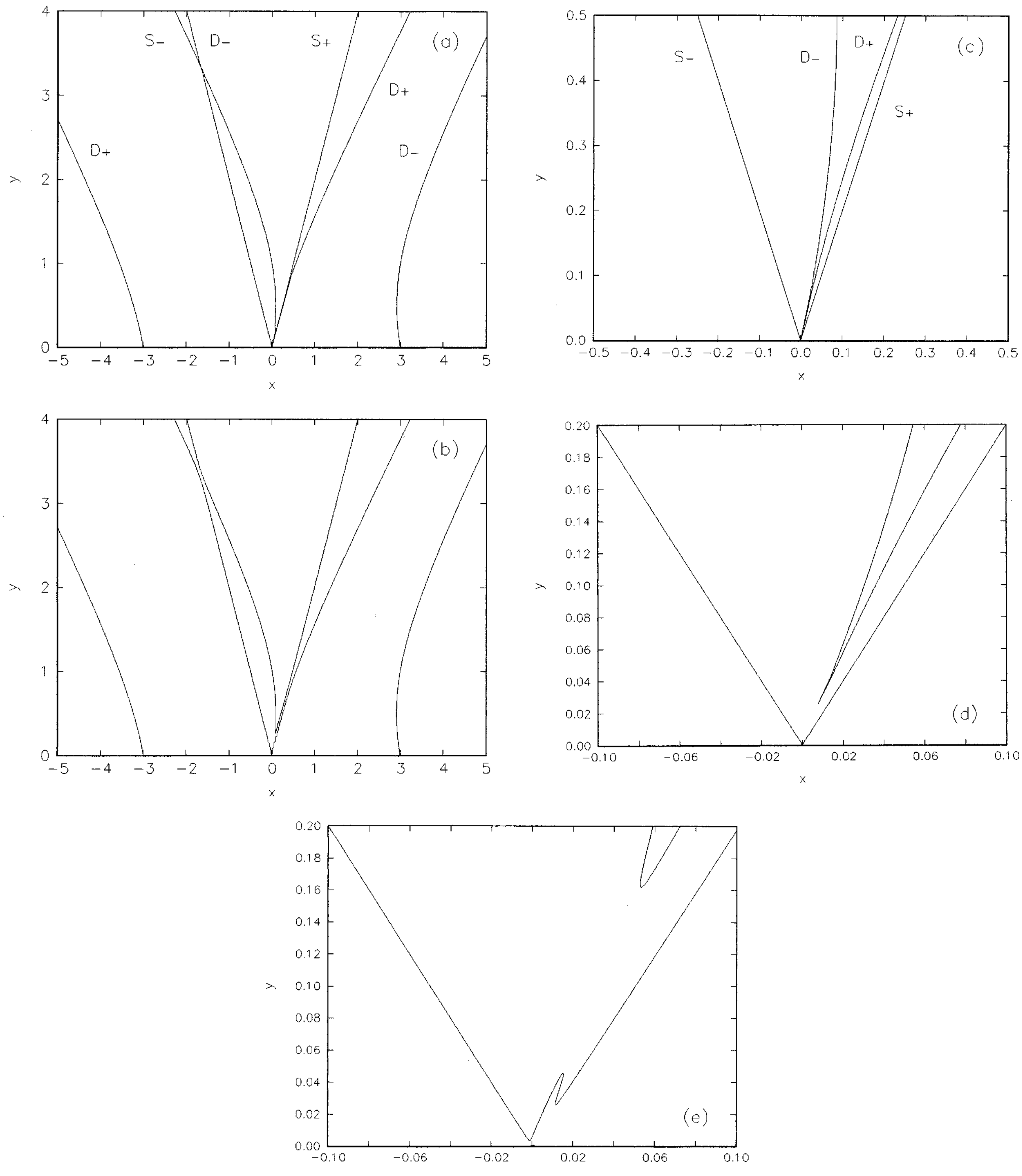

FIG. 4. (a) Nonlinear dispersion relation, Eq. (53). Normalized frequency, $x=\omega/\omega_p$, vs normalized wave number, $y=ck/\omega_p$, for $v_s/c=0.5$, $\omega_0/\omega_p=1.5$, and $\alpha=0$. (b) Same as (a), but for $\alpha=0.08$. (c) Enlargement of the origin of (a). (d) Same as (c), but for $\alpha=0.01$. (e) Same as (d), but for $\alpha=0.05$.

In the case when the plasma temperature is increased to $v_s/c=0.5$ [Fig. 4(a)], there are crossings between (D_-,S_-), (D_+,S_+), and one at the origin. In Fig. 4(b) we have raised the pump wave amplitude to $\alpha=0.08$ in order to show that the crossings between (D_-,S_-) and (D_+,S_+) are now avoided crossings. It is easy to see that for $v_s<c^2k_0/\omega_0$, S_+ will always cross D_-, thus giving rise to a decay instability, and when $v_s\geqslant c^2k_0/\omega_0$, S_+ will cross D_+, an avoided crossing when the pump is switched on. In Fig. 4(c), we have enlarged the origin for $\alpha=0$. In Fig. 4(d), we have raised the amplitude of the pump wave to $\alpha=0.01$. One can see that there is a modulational instability

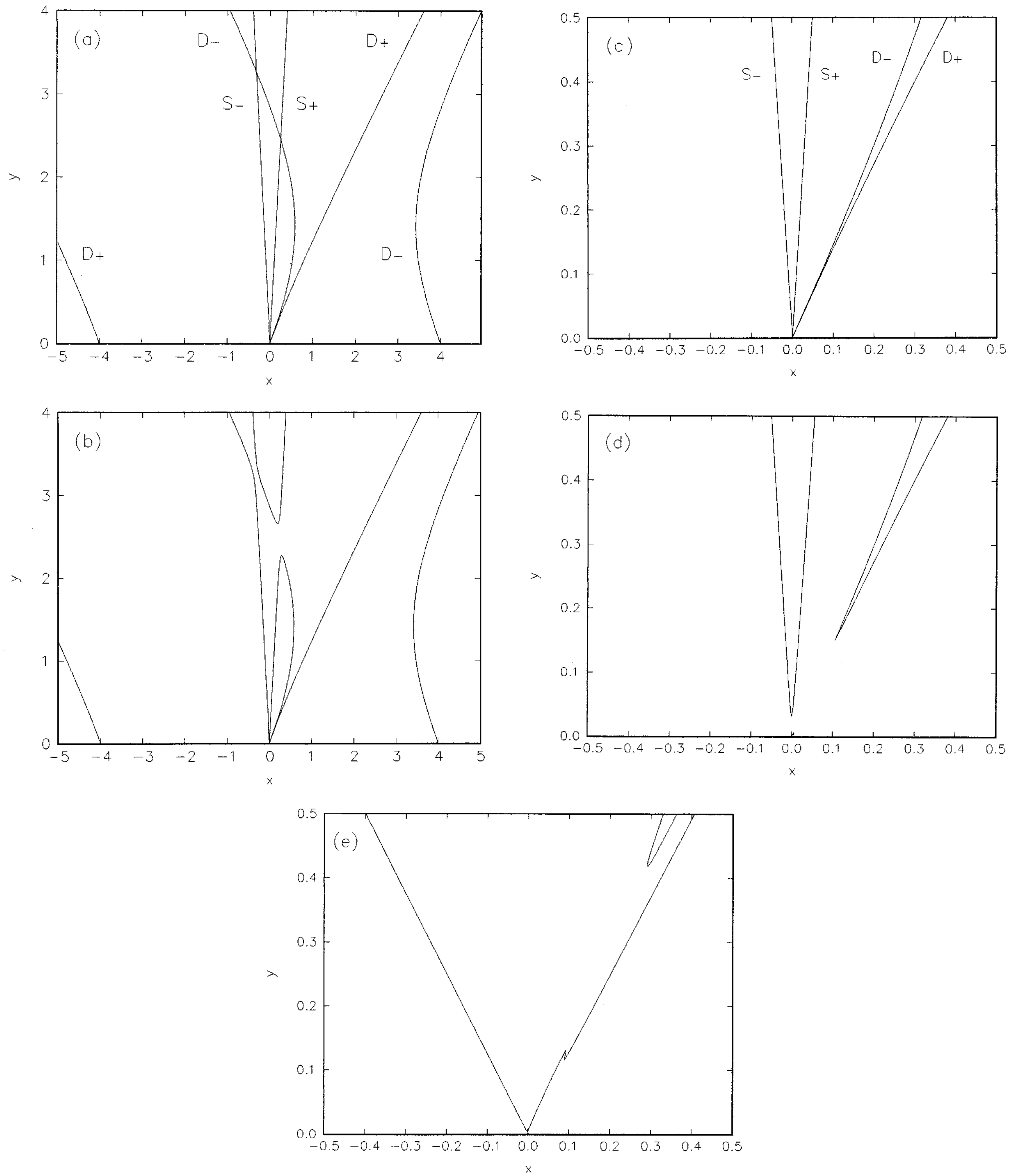

FIG. 5. (a) Nonlinear dispersion relation, Eq. (53). Normalized frequency, $x=\omega/\omega_p$, vs normalized wave number, $y=ck/\omega_p$, for $v_s/c=0.1$, $\omega_0/\omega_p=2$, and $\alpha=0$. (b) Same as (a), but for $\alpha=0.05$. (c) Enlargement of the origin of (a). (d) Same as (c), but for $\alpha=0.1$. (e) Same as (c), but for $v_s/c=0.8$ and $\alpha=0.1$.

between (D_-,D_+) that is mainly electromagnetic, in the sense that the pump wave decays into two sideband waves, through nonresonant plasma oscillations. If we continue to increase $\alpha=0.05$, from Fig. 4(e) it follows that there is a new modulational instability between (S_+,D_-). This last one is an ordinary modulational instability in which the pump wave decays into an electroacoustic mode and a sideband wave. The latter instability is always present for $v_s \geqslant c^2k_0/\omega_0$.

In order to study the dependence of the parametric decays on the pump wave frequency, we set $\omega_0/\omega_p=2$ and $v_s/c=0.1$. In Fig. 5(a) the structure of the crossings is shown for

$\alpha=0$. They are the same as in the previous cases, except that ;the scale is now much larger. In Fig. 5(b), the pump wave amplitude has been raised to $\alpha=0.05$. As before, there is an avoided crossing between (S_-,D_-), and a decay instability between (S_+,D_-). In Fig. 5(c) we have enlarged the origin. As before, there is an essentially electromagnetic modulational instability between (D_+,D_-), shown in Fig. 5(d). For $v_s \geqslant c^2 k_0/\omega_0$, there is a new modulational instability of the decay type. This is illustrated in Fig. 5(e) for $v_s/c=0.8$, and $\alpha=0.1$.

Finally, in Fig. 6(a) $\omega_0/\omega_p=4$, $v_s/c=0.1$, and $\alpha=0$. From Figs. 6(b) and 6(c), it follows that the decays are the same as in the previous cases, except that the unstable frequency range is much larger. In other words, the system is much more modulationally unstable for larger pump wave frequencies.

IV. SUMMARY

We have studied parametric decays of a linearly polarized electromagnetic wave in an electron-positron plasma. This has been done taking into account all relevant effects such as harmonic generation, the ponderomotive force, and relativistic effects on the motion of the particles in the wave field. However, relativistic temperature effects have been neglected [24].

We have followed a procedure similar to Ref. [9]. However, their treatment has several mistakes, such as, for example, the neglect of finite temperature in the longitudinal component of the force equation [see Eqs. (2), (27), and (28)], and we believe the nonlinear procedure followed by them is also wrong at various levels, as discussed in details in Sec. II.

We have shown that for $v_s \leqslant c^2 k_0/\omega_0$ (see Fig. 3), there are two instabilities: an ordinary decay instability, and an essentially electromagnetic modulational instability. The latter instability may be responsible for the observational variability of the radiation coming from pulsars and active galactic nuclei. It is important to note that, for small pump wave frequencies, the modulational instability has a very short frequency range [see Figs. 3(d) and 4(d)]. However, the frequency range increases with increasing pump wave frequencies [see Figs. 5(d) and 6(c)].

In the case when $v_s \geqslant c^2 k_0/\omega_0$ (see Fig. 4), there are also two instabilities. Both of them are of the modulational type. As the pump wave amplitude increases, the first to appear is the nonresonant one between (D_-,D_+). As the pump wave intensity continues to increase, a new instability develops in which the pump wave decays into a forward propagating electroacoustic mode and a sideband wave [see Fig. 4(e)]. However, since we have not included relativistic thermal effects [24], the sound speed must be much smaller than the speed of light. Therefore, our treatment is only valid for $v_s/c \ll 1$.

Finally, we want to stress the fact that our treatment is based on the fluid theory. Therefore, important kinetic effects such as Landau damping, do not appear in the model. Kinetic effects may lead to threshold effects, which may change the present results. Consequently, an approach based on kinetic theory is lacking.

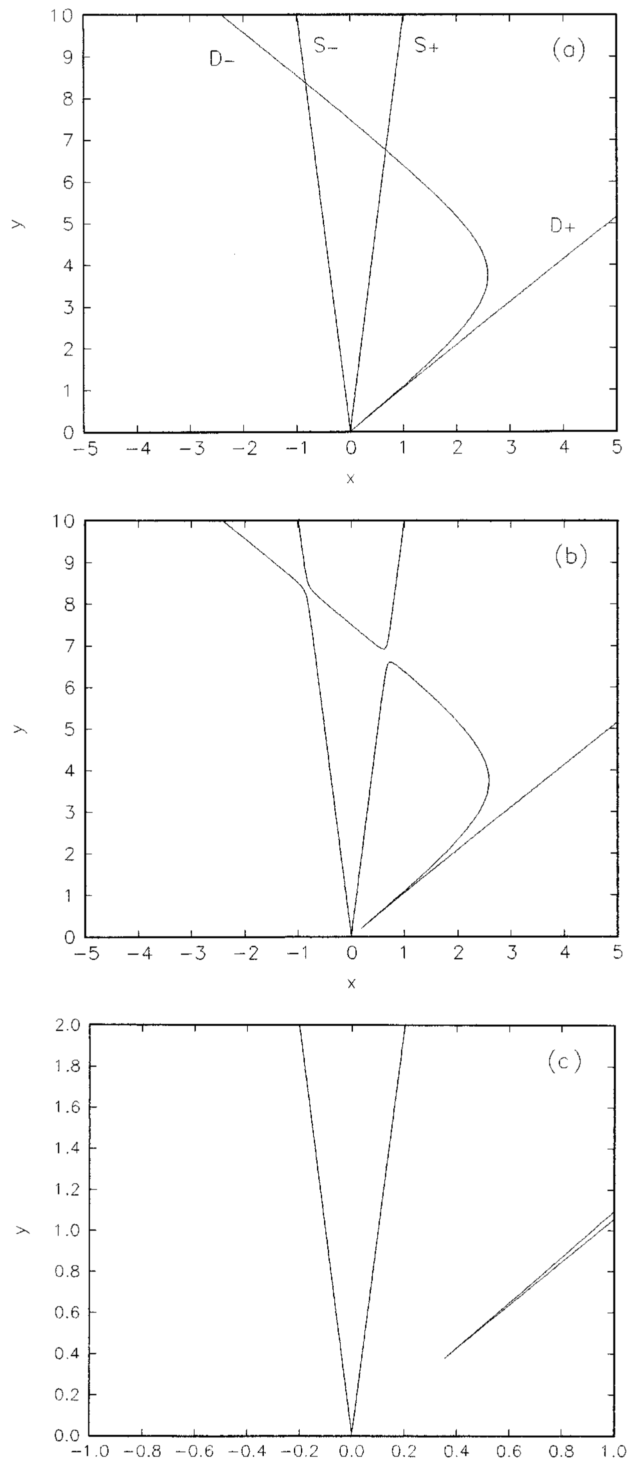

FIG. 6. (a) Nonlinear dispersion relation, Eq. (53). Normalized frequency, $x=\omega/\omega_p$, vs normalized wave number, $y=ck/\omega_p$, for $v_s/c=0.1$, $\omega_0/\omega_p=4$, and $\alpha=0$. (b) Same as (a), but for $\alpha=0.05$. (c) Enlargement of the origin of (a), but for $\alpha=0.1$.

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