Ratchet motion induced by deterministic and correlated stochastic forces

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We investigate analytically and through stochastic simulations the motion of a particle in a periodic non-symmetric ratchet potential driven by time-correlated forces. We examine the extreme monochromatic deterministic case as well as the stochastic correlated noise case. Ratchet motion is found in both cases. In the correlated noise case we derive a single analytical expression for the induced current at large correlation times and compare it with numerical results. We also demonstrate numerically the occurrence of color-induced current reversal of a rigid dimer moving in a ratchet. [S1063-651X(97)06210-7]

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I. INTRODUCTION

The stochastic motion of a particle in a periodic but not symmetric potential has been the focus of attention of a number of authors in the last few years [1–5]. The interest in this problem stems from the fact that under rather general correlation conditions for the noise, the particle acquires net macroscopic motion in a specific direction. The noise correlations induce dynamical symmetry breaking that results in a macroscopic nonzero current. The original motivation for the interest in this particle ''ratchet effect'' arose in a biological context: When a microtubular associated protein (MAP) executes motion on a microtubule, its diffusive dynamics has a specified direction. This directionality in the protein motion was associated with the nonsymmetric form of the periodic potential of the microtubule and was thought to be induced by the correlated character of the ATP hydrolysis mechanism. Since the original work on the correlated ratchet effect there has been an increased interest in the phenomenon, both experimental and theoretical [5]. On the experimental front, optical, electrical, and mechanical systems have been shown to have the ratchet property. On the theoretical front, there have been several extensions of the ratchet effect such as in compound objects and solitons [6–8]. In the context of the original ratchet motivation that was related to protein motion on a microtubule it was shown that a simple extension of the original idea could even lead to current reversal. This was accomplished by the introduction of an additional length scale in the problem. The motivation for the study of the motion of a constrained two-particle system driven by dichotomous noise was the observation that different but similar proteins may move in opposite directions on the same microtubule.

In this paper we address the mechanisms that induce the ratchet effect in the presence of additive *correlated* noise. We consider first the limiting case where the ''noise'' has only one (nonzero) frequency, that is, we deal with the motion of a particle in a periodic non-symmetric potential driven by a sinusoidal field. We show in Sec. II that in this case of the ''deterministic ratchet'' the presence of true noise is not necessary for the occurrence of a macroscopic current. We present some analytical and numerical results in support of this assertion. The problem of the ratchet in the presence of additive correlated noise is addressed in Sec. III. We show that the ratchet current for very long correlation times of the noise can be calculated from a simple analytical formula that can be obtained straightforwardly through the use of Kramers's escape theory. We present extensive numerical simulations and compare the theoretical current expression with the simulation results. We find good agreement at large correlation times but poor quantitative agreement at small correlation times, as expected, even though the analytical expression recovers the correct qualitative features of the small correlation regime as well. In Sec. IV we report numerical results on stochastic simulations of a dimer on a ratchet. Under the action of additive correlated noise we recover the main result obtained earlier for dichotomous noise, namely, we observe current reversal for a range of dimer interparticle lengths [6]. Section V is a concluding summary.

II. DETERMINISTIC RATCHETS

We consider an overdamped particle under the influence of two forces: a spatial asymmetrical periodic force $f(x)$ of period L, $f(x+L)=f(x)$, and a time-periodic force $g(t)$ of period T, $g(t+T)=g(t)$. Both functions are assumed to be bounded, $|f(x)|\leqslant f_M$, $|g(t)|\leqslant g_M$. Furthermore, we take the constant Fourier component of $g(t)$ to be zero, that is,

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t g(t')dt' = 0, \tag{1}$$

so that $g(t)$ does not include a systematic forcing of the system in one direction or another.

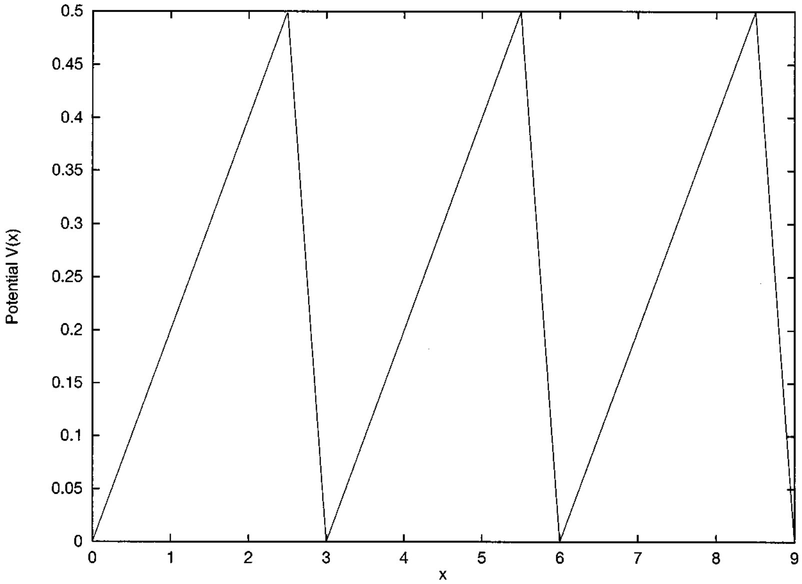

FIG. 1. Asymmetric piecewise linear ratchet potential (9), with $Q=0.5$, $d_1=2.5$, and $d_2=0.5$.

We seek solutions of the initial value problem

$$\dot{x}=f(x)+g(t) \quad \text{with} \quad x(0)=x_0. \tag{2}$$

The recent interest in ratchets has focused on systems of the form (2) where $g(t)$ is a random (in general unbounded) zero-centered noise of symmetric distribution, and it has been shown that a systematic drift of the process $x(t)$ [or of the overdamped particle whose position is $x(t)$] is induced for certain statistical properties of the noise even though there is no systematic force in either direction. In this section we take $g(t)$ to be deterministic and time symmetric, and our purpose is to show that even in this case one can induce a systematic drift along the ratchet for certain parameter combinations.

We define the *mean velocity of the particle at time* t as

$$\sigma(t)=\frac{x(t,x_0)-x_0}{t}=\frac{1}{t}\left(\int_0^t \dot{x}(t')dt'-x_0\right) \tag{3}$$

and the *mean velocity at infinity* as

$$v_\infty=\lim_{t\to\infty}\sigma(t). \tag{4}$$

Our first goal is to show that the limit in Eq. (4) always exists and that it is either zero or finite.

The formal solution of Eq. (2) is

$$x(t)=x_0+G(t)+\int_0^t f[x(t')]dt', \tag{5}$$

where

$$G(t)=\int_0^t g(t')dt'. \tag{6}$$

Equation (5) in Eq. (3) provides a formal solution for $\sigma(t)$. In order to prove that the limit (4) exists, it is sufficient to prove that $\lim_{t\to\infty}|\sigma(t+a)-\sigma(t)|=0$ for every real finite number a. Using Eqs. (3) and (5) and the fact that $f(x)$ is bounded we find

$$|\sigma(t+a)-\sigma(t)|\leqslant\left|\frac{G(t+a)}{t+a}\right|+\left|\frac{G(t)}{t}\right|+\frac{2|a|f_M}{t+a}. \tag{7}$$

The limit as $t\to\infty$ of the right-hand side of Eq. (7) is zero [recall condition (1)], so that the existence of the limit (4) has been proved. Indeed, substituting the formal solution (5) into Eq. (3) and taking the absolute value it is easy to prove that

$$|v_\infty|=\lim_{t\to\infty}|\sigma(t)|\leqslant f_M. \tag{8}$$

The mean velocity at infinity is thus bounded by the value f_M.

The important point is this: if v_∞ is not zero, there is a net drift of the particle to the right or to the left (depending on the sign of v_∞). If, on the other hand, $v_\infty=0$ then the particle oscillates around x_0 (these oscillations could have an amplitude that increases with time but more slowly than linearly), and there is no net drift.

In order to display the behavior of the deterministic ratchet through numerical integration we choose the force $f(x)=-V'(x)$ where $V(x)$ is the asymmetric periodic potential of Fig. 1. The potential in the figure is defined as follows:

$$V(x)=\begin{cases} Q\left(\dfrac{x}{d_1}\right) & \text{if } 0\leqslant x\leqslant d_1 \\[2ex] Q\left(1+\dfrac{d_1-x}{d_2}\right) & \text{if } d_1\leqslant x\leqslant d_1+d_2. \end{cases} \tag{9}$$

This potential exerts two constant forces on the particle: one equal to $f_1=-Q/d_1$ on the shallower side of the potential and $f_2=Q/d_2$ on the steeper side.

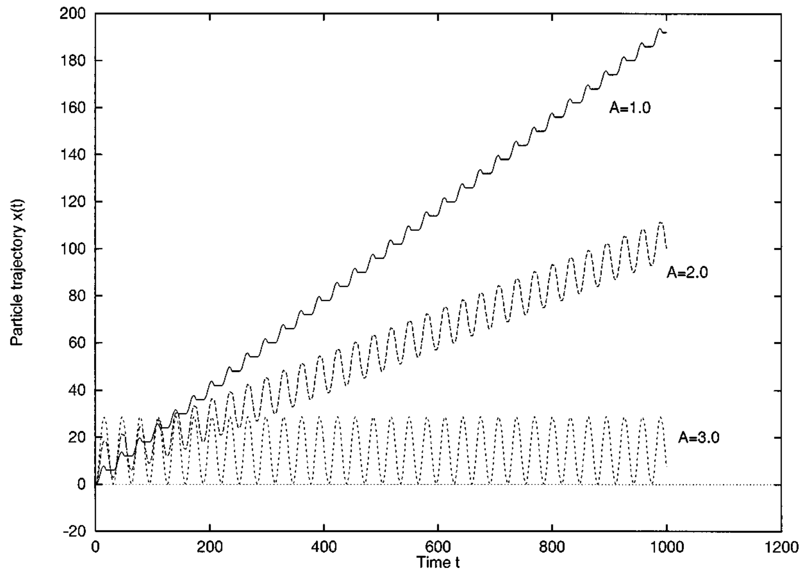

FIG. 2. Typical trajectories $x(t)$ for the ratchet potential of Fig. 1 and a sinusoidal driving force, with $Q=0.5$, $d_1=2.5$, $d_2=0.5$, $\omega=0.2$, and $A=1.0$, 2.0, 3.0.

A simple choice of a function $g(t)$ that satisfies condition (1) is

$$g(t)=A\sin(\omega t). \tag{10}$$

We present some typical trajectories $x(t)$ in Fig. 2 with initial condition $x_0=0$ and frequency $\omega=0.2$ and for different values of the amplitude A.

The shape of the trajectories confirms the existence of the limit (4). When $A=1.0$ and $A=2.0$ the particle acquires a finite drift to the right (the trajectory oscillates about the mean position, but the mean position clearly increases linearly with time). Thus there is a current to the right. The directionality is determined by the asymmetry of the ratchet. When $A=3.0$, on the other hand, the particle remains near the initial position, oscillating to the right of it and returning.

In Figs. 3 and 4 we have plotted the limit v_∞ as a function of the parameters A and ω, respectively. In Fig. 3 we have chosen the frequency $\omega=0.2$ and in Fig. 4 we have fixed the amplitude at $A=2.0$. We notice in both figures that the limit (4) is either zero or finite and that it indeed never exceeds the value $f_M=Q/d_2=1.0$ in agreement with condition (8). The interesting point, as noted earlier, is that for some parameter values the symmetric zero-average deterministic force $g(t)$ induces a current to the right, while for others it does not. The sensitivity of the behavior to the particular parameter values is evident in the figures. The nonlinearity of Eq. (2), even with the simple sawtooth potential of Fig. 1, does not allow the analytic prediction of the gaps and more generally of the highly irregular behavior that is evident in Figs. 3 and 4. This information appears accessible only via numerical integration. However, it should be noted that the gaps (that is, the sets of parameter values for which there is no net current) are not a general feature of all deterministic ratchets but are in fact due to the ''peculiarity'' of the piecewise linear potential. For instance, consider instead the smoother

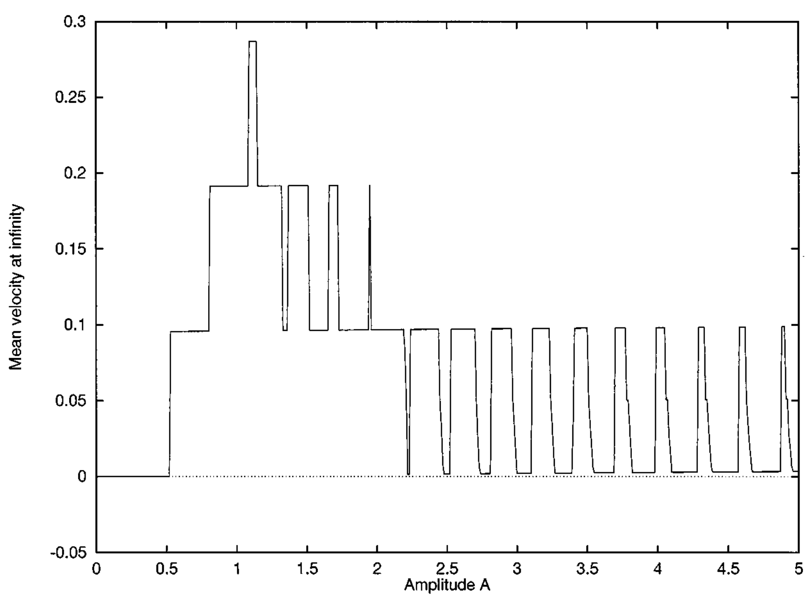

FIG. 3. v_∞ versus A, with $\omega=0.2$ and the ratchet potential of Fig. 1.

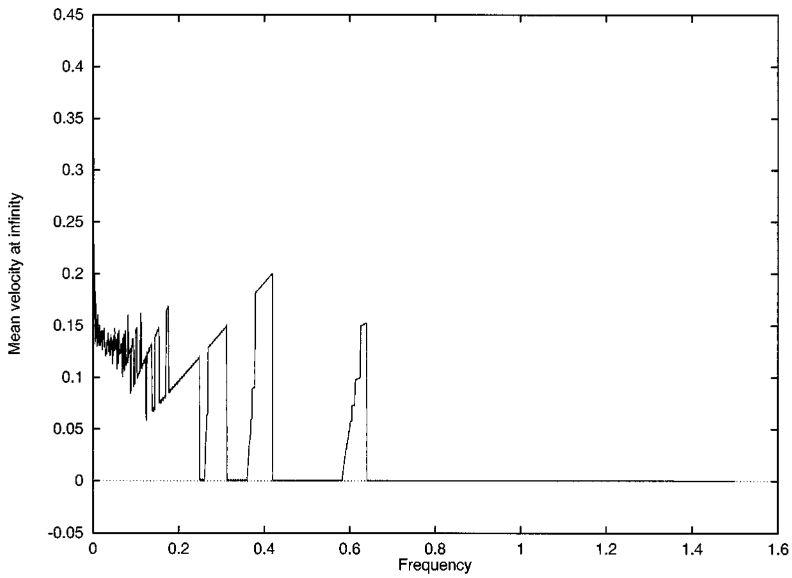

FIG. 4. v_∞ versus frequency ω, with $A=2.0$ and the ratchet potential of Fig. 1.

L-periodic asymmetric potential

$$V(x)=\frac{1}{2}Q\left(1+\frac{\sin(2\pi x/L-\phi_c)+b\sin[2(2\pi x/L-\phi_c)]}{\sin(\phi_c)+b\sin(2\phi_c)}\right) \tag{11}$$

of height Q, where $\phi_c=\arccos[(-1+\sqrt{1+32b^2})/8b)$. This potential is a generalization to adjustable height Q, period L, and asymmetry b of the potential found in [9]. For $|b|<0.5$ the potential has exactly one minimum and one maximum within a period, and its asymmetry increases with increasing b. For $|b|>0.5$ the potential is also asymmetric but it acquires a second minimum and maximum within each period. Therefore the most asymmetric choice with a single minimum and maximum per period corresponds to $b=0.5$, and this is the case that is shown in Fig. 5.

Integration of Eq. (2) using the potential (11) leads to the results displayed in Figs. 6 and 7; these should be compared with Figs. 3 and 4, respectively, for the sawtooth potential. The gaps are now gone, and although still quite irregular, the behavior of v_∞ is now somewhat smoother (but still rather irregular). Again, the interesting point is that a current to the right is induced provided the amplitude of $g(t)$ is sufficiently large and/or the frequency of $g(t)$ is not too high. We have not established the specific bounding relations between the two that are required to produce a current.

III. PROBABILISTIC RATCHETS

It has been proposed recently that the unidirectional motion of certain proteins such as kinesin on cellular networks can be understood in terms of a mechanism involving a biological ratchet operating at finite temperatures [1,2], where the protein acts as a Brownian particle executing a certain random walk in this ratchet potential. The process has been modeled by an equation of the form of Eq. (2) where the deterministic force $g(t)$ is now replaced by a zero-centered noise. This is the Langevin picture of such a process. Under conditions of thermal equilibrium, the second law does not permit net particle motion in the ratchet. In the Langevin

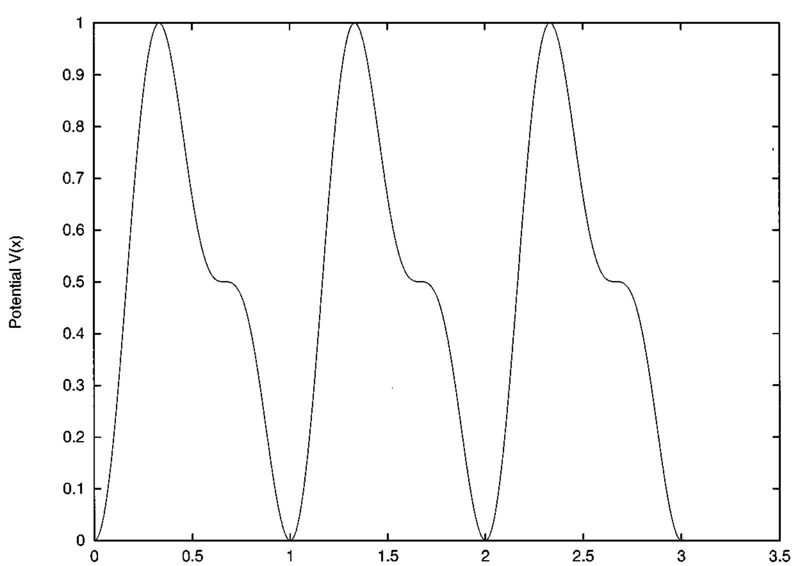

FIG. 5. Asymmetric ratchet potential (11), with $Q=1$, $L=1$, and $b=0.5$.

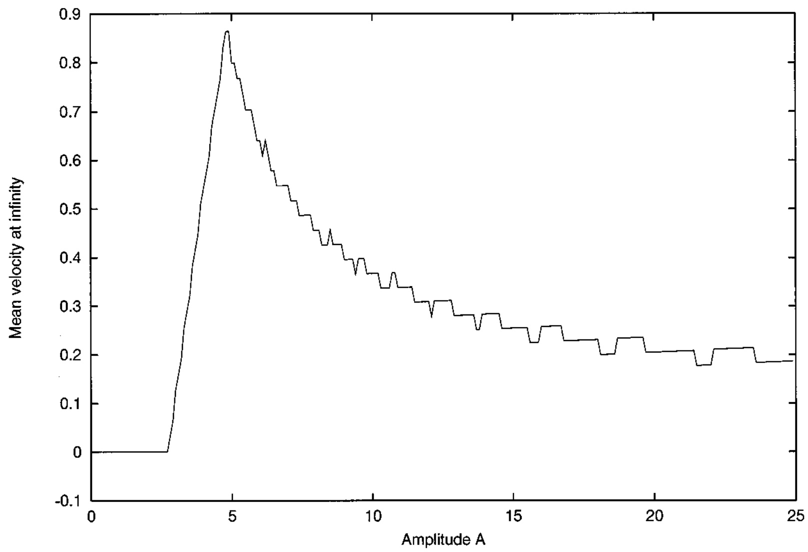

FIG. 6. v_∞ versus A, with $\omega=0.2$ and the ratchet potential of Fig. 5.

picture, this is equivalent to the fact that white, uncorrelated fluctuations cannot induce symmetry breaking and thus are not able to induce a macroscopic current — the Langevin picture with white noise rests on a fluctuation-dissipation balance that prevents such nonequilibrium effects. When there are time correlations in the noise, on the other hand, this balance is destroyed, the system becomes ''open'' in the thermodynamic sense, and a nonzero current can arise due to the asymmetry of the periodic potential. As a result the particle can move in a specified direction even when the driving fluctuations are completely symmetric. The particle effectively acts as an engine that operates in the Brownian regime. The engine consumes energy extracted from the nonequilibrium fluctuations of the environment and transforms it into mechanical work, manifested by its average net velocity in a given direction. The efficiency of this engine is determined mainly by two characteristics of the system: the asymmetry of the ratchet potential, and the ''environmental'' features such as the correlation properties of the nonequilibrium fluctuations and the ambient temperature. It is clear that the induced particle motion is a finite-temperature phenomenon that disappears at very high temperatures since then the details of the asymmetric ratchet potential are drowned out by the noise. If one stays away from this regime, one finds that the crucial environmental property lies in its temporal correlation properties, which are in turn modeled by assuming that the noise driving the particle is colored. In this section we analyze the properties of the Brownian current as a function of the color of the noise.

Consider the basic ''engine'' equations in the Langevin picture,

$$\frac{dx}{dt}=f(x)+\xi(t),$$

$$\frac{d\xi}{dt}=-\frac{1}{\tau}\xi+\frac{1}{\tau}\eta(t). \tag{12}$$

Here x again denotes the position of the Brownian particle in the overdamped limit, and, as before, $f(x)\equiv -V'(x)$ where $V(x)$ is a periodic nonsymmetric potential. The auxiliary variable ξ represents the coupling of the particle to the environment; if the noise variable $\eta(t)$ is Gaussian and δ corre-

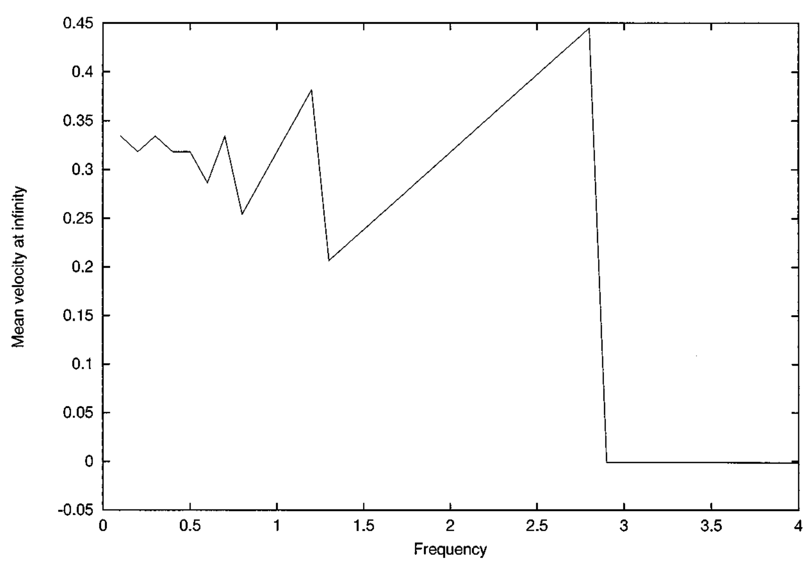

FIG. 7. v_∞ versus frequency ω, with $A=3.5$ and the ratchet potential of Fig. 5.

lated, $\langle \eta(t)\eta(t')\rangle = D\delta(t-t')$, then $\xi(t)$ is an Ornstein-Uhlenbeck process, that is, $\xi(t)$ is Gaussian and exponentially correlated,

$$\langle \xi(t)\xi(t')\rangle = \frac{D}{2\tau} e^{-|t-t'|/\tau}. \tag{13}$$

The white noise strength D is proportional to the ambient temperature. In order to study the effects of the color of the noise $\xi(t)$, that is, the effects of τ on the induced Brownian motion, we will again use the periodic asymmetric piecewise linear potential shown in Fig. 1 — most studies of noise-induced Brownian motion on ratchets have used this as a model potential [1–3].

When τ is exactly zero there is no current; in the limit $D \to 0$ and for τ very small the induced current also essentially vanishes [3,4]. In the other limit, that is, when $\tau \gg 1$, we can evaluate an asymptotic expression for the current using the following argument [10]. Let us initially place the Brownian particle at the bottom of the potential. For extremely correlated noise, i.e., when $\tau \gg 1$, the effect of the white noise $\eta(t)$ in the evolution of ξ, given in Eq. (12), is negligible. Thus in this limit we can set $\dot{\xi} \approx 0$. The net force acting on the particle is then simply the quasistatic force $f+\xi$. The force ξ does fluctuate, but it does so extremely slowly. The particle escapes to the next well, left or right, when a fluctuation leads to a (quasiconstant) value of ξ of the appropriate size to cancel the force due to the potential. The particle is thus brought to the ''top of the barrier.'' The value of ξ must remain essentially constant for the time that it takes the particle to reach this position. Once there, the particle can immediately ''roll down'' to the next minimum in a time that is small compared to the time it has waited for the appropriate fluctuation. In this picture, the average time that the particle waits before passing from one well to another is given by the mean first passage time for the noise ξ to reach the appropriate critical value ξ_c to cancel the effects of the potential. For the particle to escape to the right, the noise must reach the critical value $\xi_1^c = Q/d_1$, and to escape to the left the critical value of the noise is $\xi_2^c = -Q/d_2$. The mean first passage time for the Ornstein-Uhlenbeck process $\xi(t)$ to reach a value ξ_i^c is [10,11]

$$T_i(\xi_i^c) = \frac{\sqrt{2\pi D\tau}}{|\xi_i^c|} \exp\left(\frac{\xi_i^{c2}\tau}{2D}\right). \tag{14}$$

The color-induced current is proportional to the net rate R of escape from a well. Inserting the appropriate critical values of ξ, this rate is then given by

$$R = \frac{1}{2}\left[\frac{1}{T_1(\xi_1^c)} - \frac{1}{T_2(\xi_2^c)}\right] = \frac{Q}{2\sqrt{2\pi D\tau}}\left[\frac{1}{d_1}\exp\left(-\frac{Q^2\tau}{2Dd_1^2}\right) - \frac{1}{d_2}\exp\left(-\frac{Q^2\tau}{2Dd_2^2}\right)\right]. \tag{15}$$

The rate (15) exhibits a maximum at a finite value of τ (whose specific value depends on the other parameters) even though it is an asymptotic expression strictly valid only in the large-τ regime. The formula thus qualitatively correctly captures the finite-τ dynamics: it is known, mainly from numerical simulations, that there is indeed a maximum in the current at intermediate correlations. We note that Eq. (15) has a different τ dependence than the small τ result calculated in [4]. Our result (15) is consistent with the large τ expression obtained in [4].

Our next order correction of the rate can be obtained if we also take into account the ''rolling time'' of the Brownian particle from the top of the barrier to the next potential minimum [12]. From the condition that the correlation time of the noise must be at least equal to the total rising plus rolling time, we obtain new critical fluctuation values for ξ:

$$\xi_1^\tau = \xi_1^c + \frac{d_1}{\tau} = \frac{Q}{d_1} + \frac{d_1}{\tau},$$

$$\xi_2^\tau = \xi_2^c - \frac{d_2}{\tau} = -\frac{Q}{d_2} - \frac{d_2}{\tau}. \tag{16}$$

Upon substitution into Eq. (14) we obtain a modified expression for the net rate out of the well:

$$R = \frac{1}{2}\left[\frac{1}{T_1(\xi_1^\tau)} - \frac{1}{T_2(\xi_2^\tau)}\right]. \tag{17}$$

In Fig. 8 we plot the net rate given by Eq. (17) as a function of the correlation time τ for different values of the noise strength D.

We note that Eq. (17) leads to a current that is always to the right on the ratchet as drawn in Fig. 1. The current increases with increasing correlation time τ, reaches a maximum, and then decreases back to zero at large values of the correlation time. The maximum current increases with increasing noise intensity parameter D (but only for sufficiently small D; as noted earlier, if the noise is too intense it swamps out the ratchet effect entirely). It is noteworthy that the result (17) leads to the same general important features, in particular, the nonmonotonic dependence of the current on τ as in [4], even though our result was obtained from a very simple *asymptotic* argument. This same argument has been found to be useful in a number of cases in regimes earlier than the asymptotic for which it was designed [13,14]. Other features to be noted about our result (17) include the relatively mild (albeit exponential) decay of the rate with τ because the dominant term in the exponent is proportional to only the first power of τ. Although there is no reason to expect the result to be accurate as $\tau \to 0$, the expression does remain analytic and continuous even in that limit.

Our results lead directly to a simple description of the effect of the color of the noise on the Brownian particle in the ratchet. In the limit of large correlation time, $\tau \to \infty$, the noise acts like a constant force that at times (when it takes on the appropriate values) opposes the action of the ratchet potential. This effect was described earlier in this section. Since it is more likely for the noise to attain the value needed to counteract the smaller force exerted on the particle (on the right-hand side of the potential minima in our particular ex-

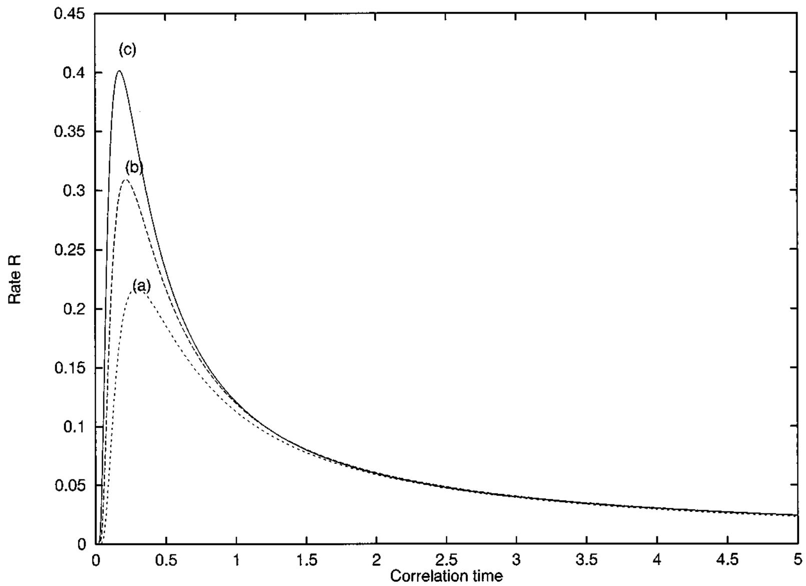

FIG. 8. Color-induced rate R, which is proportional to the current, as a function of the correlation time τ given by Eq. (17). The parameter values are $Q=0.5$, $d_1=2.5$, $d_2=0.5$, and (a) $D=0.3$, (b) $D=0.4$, (c) $D=0.5$.

ample), the particle is moved by the noise more often to the right than to the left, thus leading to a net current. As the correlation time τ becomes smaller, two ''opposite'' effects begin to play a role. On the one hand, as the noise changes value more rapidly, the difference between the number of trajectories going to the right and going to the left diminishes. On the other hand, this same more rapid change leads to a reduction in the mean exit time from a well (in either direction) because larger values of the noise are more frequently attained (albeit retained for a shorter time) within a given time period. The maximum current occurs when these two effects are optimized together in the way typical of opposing tendencies. As τ becomes smaller the current again diminishes until, in the white noise limit $\tau\rightarrow 0$, there is no net current since the rates of escape to the left and to the right become equal.

Ours is an asymptotic result whose validity for decreasing τ is less than certain. We show in Fig. 9 a typical simulation result together with that obtained from our theory for comparison. The curves clearly agree well for large τ. Both curves start from the origin and both display a maximum for finite correlation times. However, our theory does not lead to the quantitatively correct location of the maximum obtained from the simulations (our value of τ being too small), and our result leads to a current that is too large in this region. This quantitative disparity is not surprising; what is more noteworthy is that our results lead to the correct qualitative behavior for all correlation times, particularly in its prediction of a maximum in the current as a function of τ, even though it relies only on rather simple large-τ asymptotic arguments.

Our simulation results (both that in Fig. 9 and those to follow) were obtained by solving Eq. (12) N times for different realizations of the colored noise $\xi(t)$ and computing and plotting

$$J=\frac{1}{N}\sum_{n=1}^{N} J_i, \quad J_i=\frac{x_i(t,x_0)-x_0}{t} \tag{18}$$

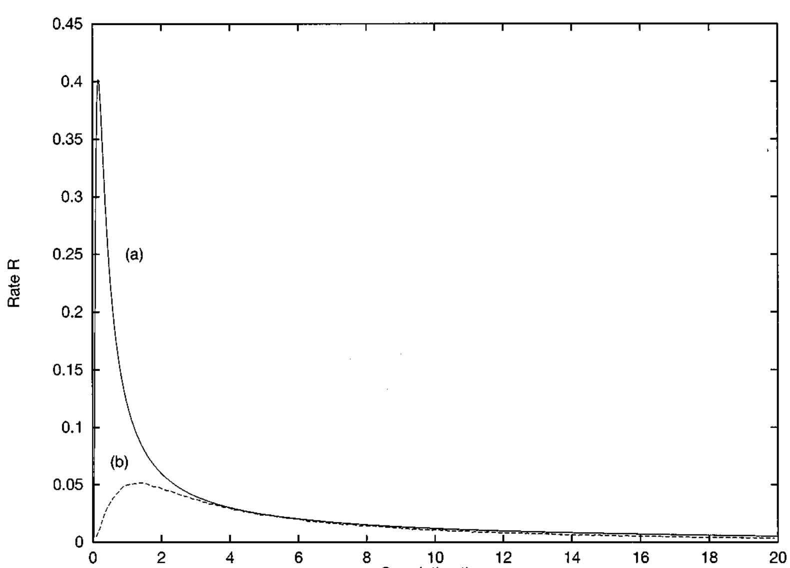

FIG. 9. Comparison of our theory (a) and numerical simulations (b) for the net rate R of the Brownian particle as a function of the correlation time τ. The parameter values are $Q=0.5$, $d_1=2.5$, $d_2=0.5$, and $D=0.5$.

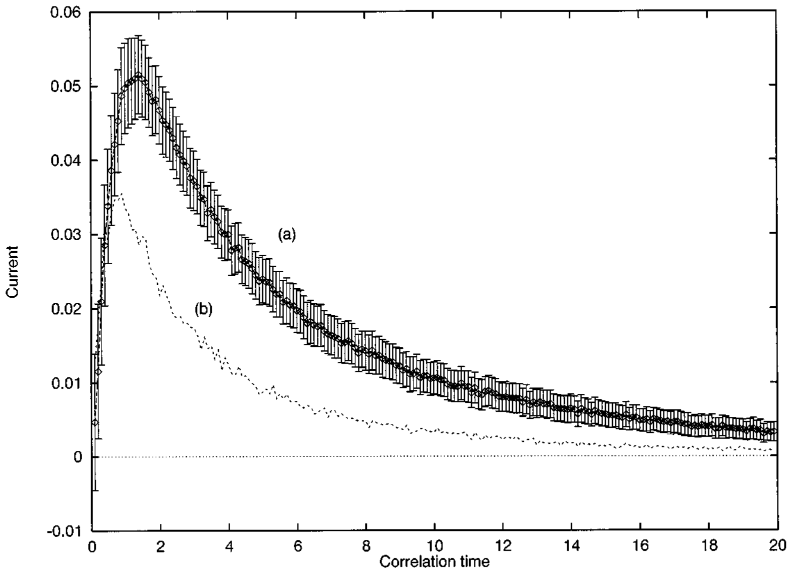

FIG. 10. Simulation results for the current on the sawtooth ratchet as a function of the correlation time τ of the noise, for two different values of the noise intensity parameter D. Parameter values are $Q=0.5$, $d_1=2.5$, $d_2=0.5$, and (a) $D=0.5$, (b) $D=0.3$. Simulation error bars are explicitly indicated for one of the simulations.

[cf. Eq. (4)]. Ideally $N\rightarrow\infty$ and $t\rightarrow\infty$, but of course we have to be content with large but finite N and t. Simulation results for the current obtained with different values of the noise intensity parameter D are shown in Fig. 10.

For $D=0.5$ we have explicitly indicated the numerical error to convey the limits of accuracy of our simulations. It is not difficult to decrease the error by simulating each trajectory for a longer time and/or repeating the simulation for a larger number of trajectories, but it is very time costly to do so. For example, to decrease the error by a factor of $1/\sqrt{10}$ requires that we run each trajectory for ten times as long as we have done. Our comparison with theory does not require a finer level of accuracy than we have implemented. Note that as with our theory (and as is to be expected), the current for these relatively small values of D increases with increasing D. To confirm that strong noise does drown out all ratchet effects we show in Fig. 11 simulation results for the current as a function of the noise parameter D. For small values of D (such as those used in our theory and simulation results shown earlier) the current rises, but it then decreases when D becomes comparable to the potential barrier height. The fluctuations in this figure could of course also be reduced by using larger ensembles and/or longer trajectories but, again, as displayed it adequately conveys the intended information.

We note that the behavior of the current as shown in Figs. 10 and 11 displays the same features as those found in [9], where the results of simulations for superpositions of white and colored noise are presented. We also note that it appears very difficult to improve on the analytical prediction (17).

An exact analytic solution of the two-dimensional Fokker- Planck equation corresponding to the Langevin equations Eq. (12) seems impossible [9,15–17]; the ''effective Fokker Planck equation'' for small τ to order τ^2 gives expressions that are not finite in some regions due to the local nonexistence of the first and second derivatives of the potential of Fig. 1. The expression of Millonas and Dykman [4] for small τ obtained using path integral methods breaks down for this potential for the same reason.

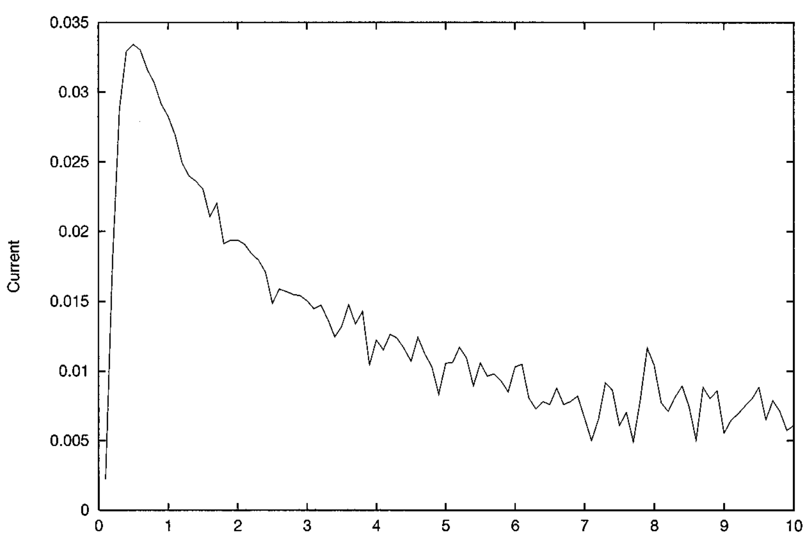

FIG. 11. Simulation results for the current as a function of the noise parameter D, with $Q=0.5$, $d_1=2.5$, $d_2=0.5$, and $\tau=0.5$.

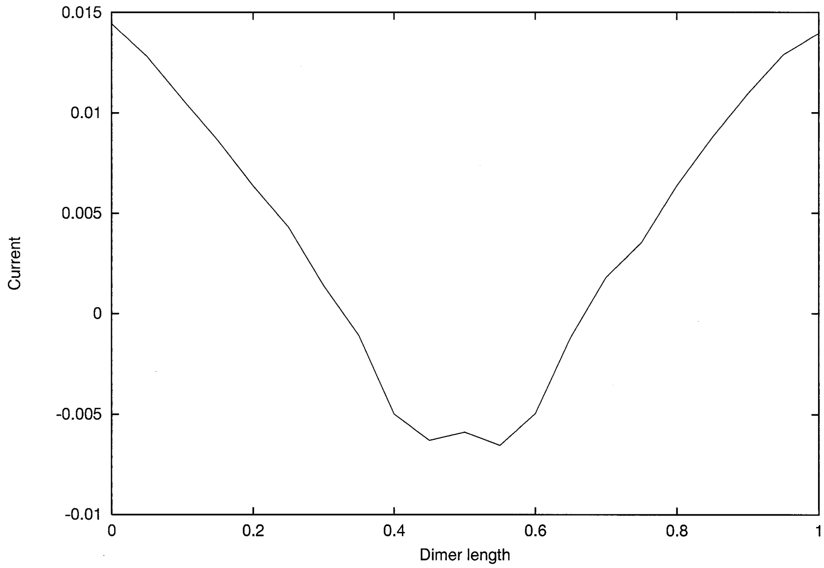

FIG. 12. Simulation of the current as a function of the length of the rigid Brownian dimer on the sawtooth ratchet. Parameter values are $Q=1.0$, $d_1=0.55$, $d_2=0.45$, $\tau=0.1$, and $D=0.5$.

IV. CURRENT REVERSAL

Current reversal that arises on ratchet systems as a consequence of a number of modifications that can be made to the problem as stated above has also been of interest. We mention in particular the current inversion found when in place of a point particle the Brownian particle is an extended rod (a rigid dimer). The dimer consists of a Brownian particle of length l that experiences a net potential $\widetilde{V}(x)$ that is a superposition of the periodic ratchet potential $V(x)$ evaluated at x and at $x+l$, i.e., $\widetilde{V}(x)=V(x)+V(x+l)$ where $V(x)$ is an asymmetric periodic potential such as that given in Eq. (9) or in Eq. (11). Current inversion has been observed for a dimer on the ratchet (9) when the driving noise is dichotomous [6]. The results shown in Fig. 12 confirm this behavior when the driving noise is Gaussian colored noise. For very short rods (not unlike point particles) and for long rods the current is to the right, but for rods of intermediate length there is a regime where the ratchet current, driven by the same Gaussian noise, is actually to the left. We have carried out the simulations here for the same rod lengths used in [6], and find the current inversion range to be approximately the same as in the case of dichotomous noise. In these simulations an increase in the number of simulations and/or of the lengths of the trajectories would again smooth out the results and would more clearly display the fact that the current is symmetric about a rod length of 0.5 for the potential parameters used in the figure.

V. SUMMARY

We have explored a number of issues related to the so-called ''ratchet effect,'' whereby a process in an asymmetric potential driven by colored noise can exhibit a net current although the noise is totally symmetric. Since our contribution here is an analytic theory valid in the limit of highly colored noise, we first considered a particle in an asymmetric potential under the influence of an external deterministic monochromatic force. We showed that such a system can also exhibit a net current, that is, that stochastic forces are not necessary in order to produce a net current. In the deterministic problem the direction of the current is the same as in a stochastic ratchet driven by, say, Gaussian colored noise: the particle moves in the direction in which the slope of the potential is milder.

We then studied the ratchet effect in the presence of highly correlated Gaussian noise. Based on straightforward asymptotic arguments, we derived a simple expression for the current in the large correlation time regime and compared it with stochastic simulations. We find good agreement between theory and simulations in the large correlation time regime. The analytical formula breaks down quantitatively at small correlation times but exhibits the correct qualitative properties throughout: if the noise is not too strong the current increases with increasing noise intensity, there is a maximum current at a particular correlation time, and the current vanishes in the small correlation time limit. The dependence of the maximum current and the correlation time at this maximum on the other parameter values is not predicted correctly by our theory, but it is noteworthy that its existence is included in our asymptotic result.

Finally, we investigated the effect of colored noise on a compound object consisting of two particles joined by a rigid rod, that is, a rigid dimer. In an earlier paper [6] we found that current reversal for such an object can occur for some range of parameters when the system is driven by dichotomous noise. Here we have shown that the same system driven by Gaussian correlated noise demonstrates similar current reversal, indicating that the effect is robust and not dependent on very specific noise statistics.

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