COMMENTS

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Comment on ''Quantum suppression of chaos in the spin-boson model''

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We independently examine the case of a spin-j particle interacting with a boson field studied by Finney and Gea-Banacloche. We concentrate on their conclusion that in the case $j=1/2$ the correspondence between the quantum and the semiclassical case is damaged to the point that the increase of entropy and quantum uncertainty for regular motion exceeds that for the chaotic case. We demonstrate by direct calculation that this conclusion is the result of mistakenly identifying a regular evolution of the spin-boson system as being chaotic. [S1063-651X(97)01608-5]

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In a recent paper [1] Finney and Gea-Banacloche (FGB) have studied the problem of the correspondence between the chaotic behavior of the spin-boson system resulting from the so-called Hartree approximation [2–4], a factorization assumption also referred to as the ''semiclassical approximation,'' and the exact quantum behavior of the same system. In a striking contrast with the results reached by Bonci *et al.* [5] their main conclusion has been that the increase of entropy, S, and of quantum uncertainty, U, in the quantum case corresponding to the regular semiclassical case is larger than in the quantum case corresponding to the chaotic semiclassical case. They do not explain the physical reasons for this result but only vaguely allude to the fact that the spin $j=1/2$ is so dramatically quantum as to violate any correspondence between quantum and semiclassical mechanics. In fact they find that in the case $j=3/2$, the qualitative correspondence with the intuitive arguments of Bonci *et al.* [5] is recovered. They argue that this latter agreement is an indication that the predictions of classical physics, as well as those of the semiclassical approximation, should be restored in the limiting case of a spin j whose strength tends to infinity, but are strikingly violated in the case $j=1/2$.

The conclusions of FGB [1] that regular dynamics leads to an increase in S and U that is faster than that for chaotic dynamics should be viewed with skepticism for a number of reasons not discussed by these authors. First of all there are two mechanisms that yield increasing S and U. As clearly illustrated by Phoenix and Knight [6] the case of regular dynamics leads to increasing S when the effects of incommensurate fluctuations from an infinitely large number of photons are taken into account. Bonci *et al.* [5] discuss this alternative source of increase in S and U and have considered initial conditions canceling the contribution made by this source so as to isolate the increase produced by chaos alone. This was not done in [1]. In fact FGB [1] do not discuss this traditional mechanism for the increase in S and U at all in their paper, much less compare it with the increase associated to chaos.

Our main purpose here is to shed light on these important aspects, left unexplored by the work of FGB [1]. To accomplish this we not only do the calculations reported in [1], but we extend them to cases that are still more nonlinear. More precisely, we consider the key parameter defined by their equation (7):

$$\epsilon \equiv \frac{g\sqrt{\bar{n}}}{2\omega}, \tag{1}$$

where ω denotes the frequency of the radiation field. Following the calculations in [1] we focus on the case where $\omega=\omega_0$, with ω_0 being the frequency of the spin-1/2 system. We reproduce their calculation keeping the value of ϵ fixed, and changing the number of photons involved in the interaction. It is evident from the equation that decreasing the number of photons requires that the strength of the nonlinear interaction increases for constant ϵ. Consequently, the effect of semiclassical chaos is expected to become more and more important until it dominates the process. We study the cases of average photon number $\bar{n}$ equal to 100, 81, and 25. The first two cases correspond to dynamical configurations identified by FGB as regular and chaotic, respectively. The last case has the largest interaction strength and we shall discuss the effect of this nonlinearity subsequently. But first we follow the calculations also done in [1].

The time evolution is calculated corresponding to the factorized initial conditions given by $|\psi_{\pm}\rangle|\alpha\rangle$, where $|\alpha\rangle$ is a

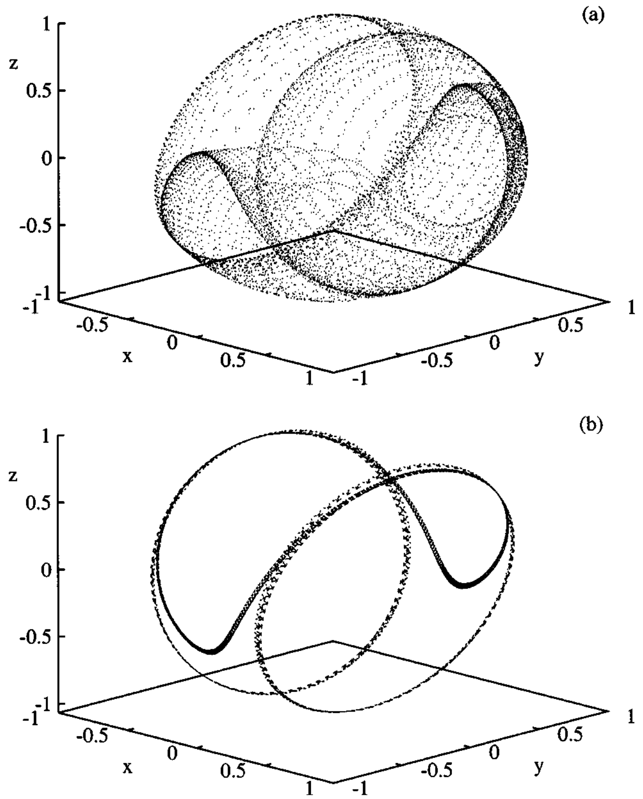

FIG. 1. Classical spin trajectories on the Bloch sphere for the case $\bar{n}=81$. The $(+)$ branch is illustrated in (a) and the $(-)$ branch in (b). The values of the parameters are $\omega=\omega_0=1$, $\epsilon=1$.

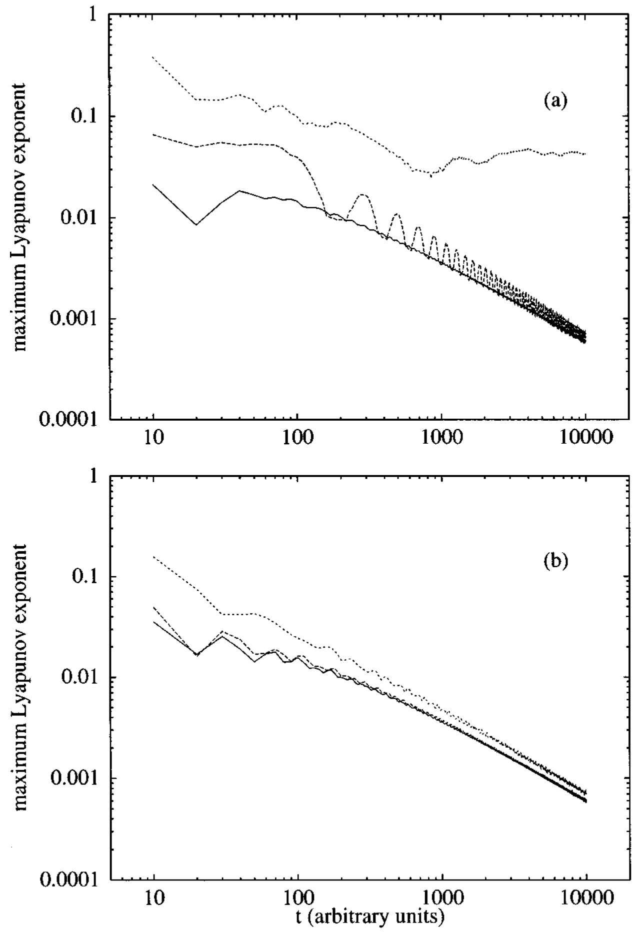

FIG. 2. Lyapunov exponents. (a) illustrates the results for the $(+)$ branch and (b) the results for the $(-)$ branch. The two sets of curves denote, from top to bottom, the cases $\bar{n}=25$, 81, and 100, respectively. The value of the parameters are $\omega=\omega_0=1$, $\epsilon=1$.

boson field coherent state with real amplitude α and $|\psi_{\pm}\rangle$ are two special initial states of the spin that, according to FGB, lead to a quantum evolution that preserves the initial factorization for long times (see Refs. [1,7] for details). Following FGB we denote as the $(+)$ and $(-)$ branch the trajectories stemming from the two respective initial conditions of the spin-1/2 system, $|\psi_{\pm}\rangle$.

Figure 1 illustrates the crucial case $\bar{n}=81$, discussed by FGB, and our results are indeed consistent with those depicted in their Fig. 1. We see that the trajectory corresponding to the $(-)$ branch is regular, whereas the trajectory corresponding to the $(+)$ branch looks very irregular. FGB identify the $(+)$ branch as being chaotic, however, we show below that it is at most quasiperiodic and certainly not chaotic.

An unambiguous way to assess whether or not a trajectory is chaotic or not is to determine if its Lyapunov exponent is positive. We see from Fig. 2(b) that the time averaged Lyapunov exponent of the trajectories corresponding to the $(-)$ branch tends to vanish like $1/t$ with increasing time. It is therefore safe to consider these trajectories as being regular for the values of the average photon number considered. On the other hand, the Lyapunov exponents depicted in Fig. 2(a) for the $(+)$ branch tell a very different story. There is no doubt that the case $\bar{n}=25$ implies chaos, since the Lyapunov exponent converges to a finite value $\lambda\sim 0.04$. For the other two cases, we find the same behavior as in Fig. 2(b), i.e., a Lyapunov exponent vanishing within the computational accuracy. In other words, if the authors of Ref. [1] refer to $\bar{n}=100$ as regular the case $\bar{n}=81$ also should be considered regular.

This error in the identification of the nature of a trajectory would account for the counterintuitive result [1], namely, that the regular case produces a faster increase of S and U than does the chaotic case. The authors would have in fact assigned different dynamical properties to two orbits of the same type. Had these authors judged to be chaotic a trajectory that is actually regular, the discrepancy between the conclusions of [1] and those of [5] would be resolved. Thus, to double check this possibility, in Fig. 3 we plot the Poincaré sections corresponding to the ''erratic'' and regular trajectories of Fig. 1. We see that in this representation the apparently chaotic trajectory looks completely regular. To strengthen our conclusions we changed a little bit the initial conditions around the two branches and again we found regular behavior as shown in Fig. 4.

We can thus conclude that the reason for the ''erratic'' behavior of Fig. 1(a) is that the trajectories plotted concern the motion of $x(t)$,$y(t)$, and $z(t)$, which represent the mean value of the Pauli spin operators. In fact these variables are not canonical and there is no reason to conclude that the projection on the Bloch sphere of a regular trajectory is regular too.

We are therefore forced to conclude that FBG [1] were deceived by the representation of the trajectories on the Bloch sphere, which makes the regular motion of the trajec-

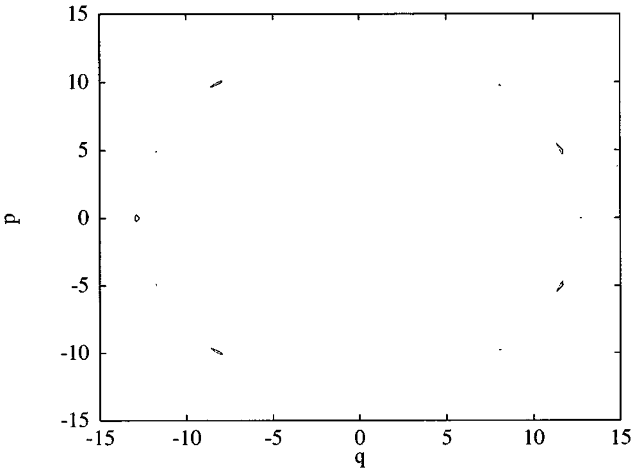

FIG. 3. Poincaré sections in the qp space of the boson system for the $(+)$ branch, the five small islands, and the $(-)$ branch, the five points. The sections are defined by $y \equiv \langle \sigma_y \rangle = 0$ with $dy/dt \leqslant 0$; see Ref. [8]. Note that within the semiclassical approximation the radiation field becomes a classical oscillator and q and p denote its coordinate and momentum, respectively. The values of the parameters are $\overline{n} = 81$, $\omega = \omega_0 = 1$, $\epsilon = 1$.

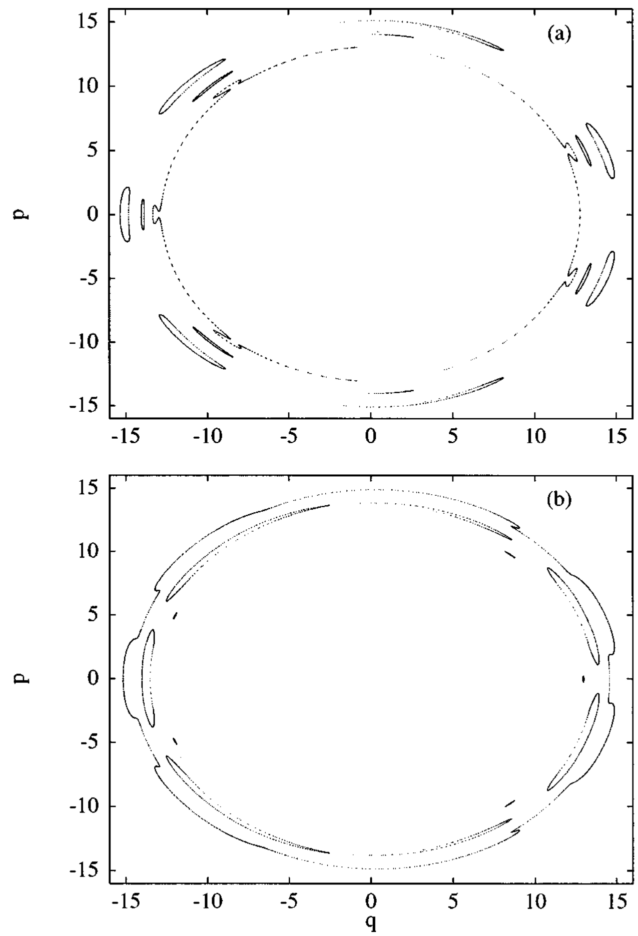

FIG. 4. Same as Fig. 3, for two sets of three distinct trajectories close to the $(+)$ branch (a) and to the $(-)$ branch (b).

tories appear erratic. Thus, in both cases explored by them in their paper, the only source of the S and U increase is given by the incommensurate fluctuations of the infinite number of photons necessary to realize a coherent state [6] not by any chaotic sea in phase space. These fluctuations in photon number were in fact eliminated by Bonci *et al.* [5] in order for their calculation to reveal the true effects of chaos on S and U.

Before ending this Comment, we would like to warn the reader against another incorrect conclusion that could be derived from the paper by FGB [1]. FGB gives the impression that it is possible to think of the quantum dynamics in terms of single classical trajectories, some stable and regular, and others unstable and chaotic. Applied to the case $\overline{n} = 25$, the FGB analysis of the quantum result would be based on two single trajectories, one corresponding to the $(+)$ branch and the other to the $(-)$ branch, with totally different dynamical properties: the former would be chaotic and the latter regular, although imbedded in the chaotic sea.

We judge this physical condition to be very difficult to realize. The explanation of our conviction lies in an important aspect already pointed out in [4] and [5], and it has to do with the joint action of deterministic chaos and the uncertainty principle. For a proper comparison between the quantum and the semiclassical treatment, not conflicting with the uncertainty principle, we cannot rest on an individual trajectory but we have to consider a set of distinct trajectories departing from a spread of initial conditions. In our previous work [5] this allowed us to reproduce with remarkable precision the quantumlike dynamical properties of the ''regular'' case. In the case $\overline{n} = 25$ this quantum mechanical constraint might result in a spreading of initial conditions larger than the distance from the stable branch within which the initial condition must lie for the trajectory to remain regular. Thus, the mere fulfillment of the uncertainty principle requirement might have the effect of abolishing the distinction between the chaotic and the regular branch. This physical condition is in fact illustrated by Fig. 5 where we plot the Poincaré section of two sets of three different trajectories with initial conditions close to the $(+)$ branch, (a), and to the $(-)$ branch, (b). Of course, within the rigorous quantum mechanical treatment this chaotic phase space structure becomes a broadened quantum mechanical wave function.

The consequences of this property on the growth of the quantum mechanical uncertainty U, as given by both a rigorous quantum mechanical treatment and the semiclassical approximation of [5], are illustrated in Fig. 6. We see that in the quantum case the distinction between the branches is almost abolished and that, as in the original papers of Ref. [5], in the semiclassical case the increase of U is slower than in the quantum case. This result shows that we cannot reach a reliable conclusion on the effect of chaos on quantum dynamics if we do not properly refer to the semiclassical dynamics, by focusing on the global properties of the corresponding phase space. In fact, Fig. 6 shows that it is misleading to interpret the results of an exact quantum mechanical calculation as the signature of a single classical trajectory. We see that in the case of the regular $(-)$ branch the uncertainty U grows with a speed comparable to if not faster than that of the chaotic $(+)$ branch. This seemingly counterintuitive property is made still more pronounced by the semiclassical calculation yielding a growth of U distinctly faster in the former case. Actually, all this only proves that the

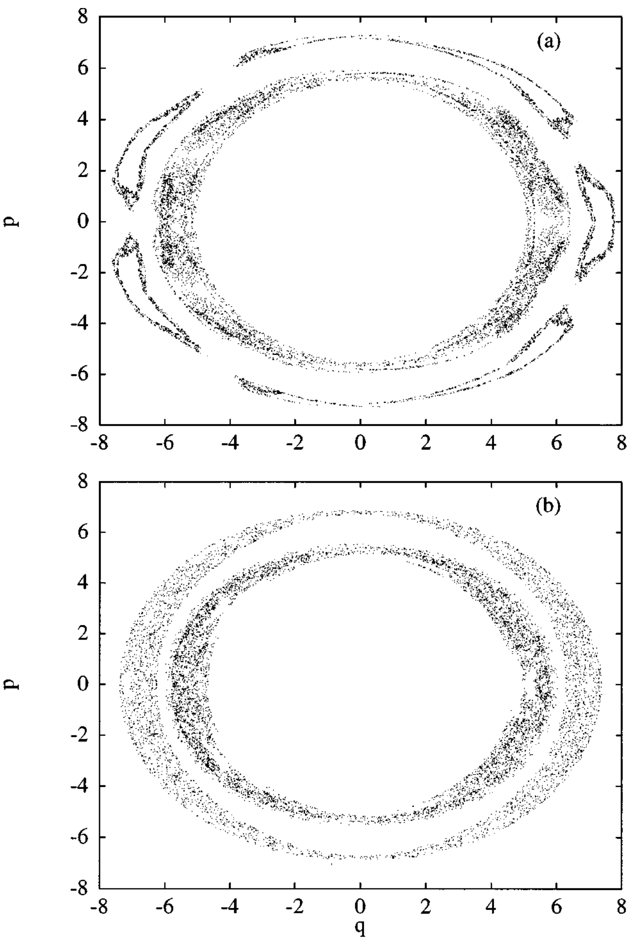

FIG. 5. Poincaré sections for three trajectories with initial conditions close to the (+) branch (a) and the (−) branch (b). The values of the parameters are $\overline{n}=25$, $\omega=\omega_0=1$, $\epsilon=1$.

speed of the quantum uncertainty growth cannot be connected to the local stability of the single trajectories. If the statistical prescription of Bonci *et al.* [5] is adopted, the paradoxical result might be explained, since it seems to depend on the fact that the phase space surrounding the (−) branch is probably more chaotic than that in which the (+) branch is imbedded, see Fig. 5.

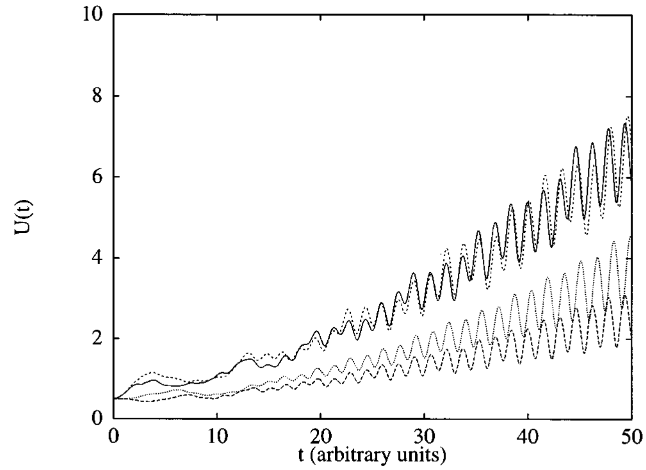

FIG. 6. The quantum uncertainty U as a function of time. The two upper curves refer to the quantum case and the two bottom curves to the semiclassical approximation. The solid and lower dashed line refer to the branch (+). The upper dashed line and the dotted one refer to the branch (−). The value of the parameters are $\overline{n}=25$, $\omega=\omega_0=1$, $\epsilon=1$.

In conclusion, the claim that the regular case produces an entropy and uncertainty increase faster than in the chaotic case is wrong, and it is essentially due to mistaking for chaotic a quite regular case. As far as the role of chaos is concerned, as already done in Ref. [5], we point out again that the result of Fig. 6 is produced by the joint action of two distinct sources of disorder, and that to see the effect of deterministic chaos alone we should proceed as in Ref. [5], and thus adopt initial conditions that cancel the effects of the incommensurate fluctuations produced by the broad photon distribution of the coherent state. We cannot rule out, however, the possibility that physical conditions might exist producing trajectories that are stable within a belt whose width is larger than the size of the quantum blurring strip that, according to the prescriptions of the uncertainty principle, must surround each trajectory. In this case, the perspective suggested by FGB could be sustained.

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