Effect of ionization and recombination coefficients on the charge-state distribution of ions in laser-produced aluminum plasmas

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Various formulations for the ionization and recombination coefficients reported and used in the literature have been compared for laser-produced Al plasmas. Taking into account these formulations, the charge-state distribution of the plasma ions existing in various ionization states through their fractional densities and average charge state has been studied. The local thermodynamic, corona, and collisional-radiative equilibria are considered for the ionization model. Numerical results for Al plasmas with electron temperatures of 0.1–1.0 keV and electron densities of 10^{20}–10^{22} cm^{-3} are presented and discussed. It is observed that the several formulations for the ionization and recombination coefficients predict variously their rates. The consideration of these formulations in an ionization model is noted to significantly modify the charge-state distribution of the ions. It is further noted that the corona equilibrium model can be safely applied to laser plasmas with electron densities less than or equal to 10^{22} cm^{-3} for estimating the abundance of high-charge ions relevant to x-ray line radiation studies. [S1063-651X(97)02708-6]

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I. INTRODUCTION

The hot and dense plasma produced by irradiation of a solid target with high-power laser beams is of tremendous practical importance owing to its applications for inertial confinement fusion, x-ray sources, highly ionized ion sources and x-ray lasers. A significant proportion of the electromagnetic radiation emitted from such laser plasmas occurs in the x-ray spectral wavelength region. Hence x-ray emission from laser plasmas has been studied extensively both theoretically and experimentally [1]. Moreover, x-ray spectral line radiation from laser plasmas has been proved to be a useful diagnostic tool for characterizing plasma conditions [2]. For the interpretation of spectroscopic data, one requires a plasma ionization model to describe the ionization-state and ion-level populations in terms of electron temperature T_e and density n_e. As the reliability of the interpretations depends on the accuracy of the model, a knowledge of the suitable ionization model is important.

Various ionization models, namely, local thermodynamic equilibrium (LTE), corona equilibrium (CE), and collisional-radiative equilibrium (CRE), and their applicability to laser plasmas have been discussed by several workers [3–10]. These models require the ionization and recombination rate coefficients for calculating the ionization-state density and ion-level populations of different charge states of the ions. The adequacy of an ionization model depends on the accuracy of the available rate coefficients. Through the review of the literature on the subject one can find that several formulations for the rate coefficients of the collisional ionization, radiative two-body recombination, and collisional three-body recombination have been used in various works and there are still no universally accepted expressions for them. Salzmann and Krumbein [6] have considered four expressions for the ionization coefficient with a particular expression for the recombination coefficient in their calculation of the ionization-state density in a laser-produced Al plasma. The effect of various formulations of the ionization and recombination coefficients on the ionization-state density has not been reported so far.

In the present paper we have taken into account several formulations for the ionization and recombination coefficients and compared them for laser-produced Al plasmas. We have studied the effect of these formulations on the fractional densities and the average charge state of the ions with different charge states, as calculated from the LTE, CE, and CRE ionization models. We have discussed the results from our calculations. It is observed that the several formulations for the ionization and recombination coefficients predict variously their rates. The consideration of the formulations in an ionization model is noted to change significantly the values of the fractional densities and the average charge state of the ions. It is further noted from our results that the CE model can be safely applied to laser plasmas with $n_e \leqslant 10^{22}$ cm^{-3} for estimating the abundance of high-charge ions relevant to x-ray line radiation studies.

II. COLLISIONAL IONIZATION COEFFICIENT

The expression for the collisional ionization coefficient S is obtained by integrating the collisional ionization cross section over a Maxwellian electron velocity distribution. Various workers have obtained the different expressions for S by considering different approximations in the theoretical evaluation of the ionization cross section. We consider the ionization coefficient due to Bates, Kingston, and McWhirter [11], McWhirter [12], Seaton [13], Lotz [14], Wilson and White [15], and Landshoff and Perez [16], denoted by S^B, S^M, S^S, S^L, S^{WW}, and S^{LP}, respectively. These are expressed corresponding to the ionic charge state Z as

$$S^B(Z)=1.64\times10^{-6}\xi_Z T_{eV}^{-3/2}[\exp(-u)/u] \ \ \mathrm{cm^3/sec}, \quad (1)$$

$$S^M(Z)=2.43\times10^{-6}\xi_Z T_{eV}^{-3/2}[\exp(-u)/u^{7/4}] \ \ \mathrm{cm^3/sec}, \quad (2)$$

$$S^S(Z)=2.15\times10^{-6}\xi_Z T_{eV}^{-3/2}[\exp(-u)/u^2]\ \ \mathrm{cm^3/sec}, \quad (3)$$

$$S^L(Z)=3.00\times10^{-6}\xi_Z T_{eV}^{-3/2}[\exp(-u)/u]F_1(u)\ \ \mathrm{cm^3/sec}, \quad (4)$$

$$S^{\mathrm{WW}}(Z)=9.00\times10^{-6}\xi_Z T_{eV}^{-3/2}[\exp(-u)/u^2]F_2(u)\ \ \mathrm{cm^3/sec}, \quad (5)$$

$$S^{\mathrm{LP}}(Z)=1.24\times10^{-6}\xi_Z T_{eV}^{-3/2}[\exp(-u)/u^2]F_3(u)\ \ \mathrm{cm^3/sec}, \quad (6)$$

where

$$F_1(u)=E_1(u)\exp(u),$$

$$F_2(u)=(4.88+1/u)^{-1},$$

$$F_3(u)=0.915(1+0.64/u)^{-2}+0.42(1+0.5/u)^{-2},$$

$$E_1(u)\exp(u)=\begin{cases}\exp(u)(-\ln u-0.5772+u) & \text{for } u\leqslant 10^{-4}\\ \exp(u)(-\ln u-0.5772+1.0000u-0.2499u^2+0.0552u^3-0.0098u^4+0.0011u^5) & \text{for } 10^{-4}<u\leqslant 1\\ \dfrac{u+2.3347+0.2506/u}{u^2+3.3307u+1.6815} & \text{for } u>1.\end{cases}$$

Here $u=\chi_Z/T_{eV}$, χ_Z is the ionization potential in eV, T_{eV} is the value of electron temperature T_e in eV, and ξ_Z is the number of electrons in the outermost (n,l) subshell with n the principal quantum number and l the azimuthal quantum number. S^B is used in the work of Kolb and McWhirter [17] by assuming hydrogenic ions ($\xi_Z=1$) for nonhydrogenic ions. S^M is used in the works of Salzmann and Krumbein [6], Eidmann [9], Sinha [18], Itoh, Yabe, and Kiyokawa [19], and Gupta and Sinha [20,21]. It is important to note that the numerical coefficient in the expression for S^M considered in Refs. [6,19] with T_e in eV is incorrect as the chosen value (2.34×10^{-7}) of the numerical coefficient is the one given by McWhirter [12] with T_e in degrees Kelvin. Accordingly, as shown in Ref. [20], the conclusion in Refs. [6,19] that the widely used McWhirter formula (S^M) predicts in the lowest ionization rate is incorrect. S^S is used in the works of Duston and Davis [7], De Michelis and Mattioli [8], Brunner and John [10], Gupta and Sinha [20], Davis and Whitney [22], Duston and Duderstadt [23], and Sasaki *et al.* [24]. S^L is used in the works of Salzmann and Krumbein [6], De Michelis and Mattioli [8], and Itoh, Yabe, and Kiyokawa [19]. S^{WW} is used in the works of Colombant and Tonon [3] and Gupta and Sinha [20]. S^{LP} is used in the works of Salzmann and Krumbein [6] and Itoh, Yabe, and Kiyokawa [19].

III. RADIATIVE RECOMBINATION COEFFICIENT

There is no universal formula for the radiative recombination coefficient α. A simple, approximate formula α^S, given by Seaton [25], is derived for H-like ions and is generally applied for all ions irrespective of their number of bound electrons. This is expressed corresponding to the ionic charge state $Z+1$ as

$$\alpha^S(Z+1)=5.2\times10^{-14}(Z+1)u^{1/2}(0.429+0.5\ \ln u+0.469u^{-1/3})\ \ \mathrm{cm^3/sec}. \quad (7)$$

This formula is used in the works of Duston and Davis [7], Brunner and John [10], and Duston and Duderstadt [23]. One similar formula, given by Kolb and McWhirter [17] and used by Colombant and Tonon [3] and Gupta and Sinha [20,21], is expressed by α^{KM} as

$$\alpha^{\mathrm{KM}}(Z+1)=5.2\times10^{-14}(Z+1)u^{1/2}(0.429+0.5\ \ln u+0.469u^{-1/2})\ \ \mathrm{cm^3/sec}. \quad (8)$$

We further consider some other formulations for α reported in the literature such as those due to McWhirter [12], Pert [26], and Griem [27], denoted by α^M, α^P, and α^G, respectively, and expressed as

$$\alpha^M(Z+1)=1.9\times10^{-14}T_{eV}^{1/2}u\ \ \mathrm{cm^3/sec}, \quad (9)$$

$$\alpha^P(Z+1)=5.2\times10^{-14}(Z+1)^4T_{eV}^{-1/2}F_1(u)\ \ \mathrm{cm^3/sec}, \quad (10)$$

$$\alpha^G(Z+1)=5.2\times10^{-14}(Z+1)u^{3/2}F_1(u)\ \ \mathrm{cm^3/sec}. \quad (11)$$

α^M is used in the works of Eidmann [9] and Sinha [18]. α^G is used in the works of Brunner and John [10] and Sasaki *et al.* [24].

IV. COLLISIONAL RECOMBINATION COEFFICIENT

The collisional recombination coefficient β is calculated by using the principle of detailed balance between ionization and recombination due to electron collisions. A general formula for β is given by Salzmann and Krumbein [6], which is expressed by β^{SK} as

$$\beta^{\mathrm{SK}}(Z+1)=\{3\times10^{21}[2g(Z+1)/g(Z)]T_{eV}^{3/2}\times\exp(-u)\}^{-1}n_eS(Z)\ \ \mathrm{cm^3/sec}, \quad (12)$$

where n_e is the electron density and g is the statistical weight. This relation leads to the Saha equation in the LTE model for a given S. From this general relation one may obtain various expressions of β for different expressions of

S. Duston and Duderstadt [23] have given an expression for β corresponding to Seaton's ionization coefficient S^S and we denote this as β^{DD},

$$\beta^{\mathrm{DD}}(Z+1)=8.05\times10^{-28}n_e\xi_Z[g(Z)/2g(Z+1)]\times T_{eV}^{-3}/u^2 \ \mathrm{cm^3/sec}. \tag{13}$$

This expression is used in the works of Duston and Davis [7] and Sasaki *et al.* [24]. It leads to the Saha equation in the LTE case only when it is used along with S described by S^S. Another expression for β is given by Kolb and McWhirter [17] and is used in the works of Refs. [3,20,21]. This expression, denoted by β^{KM}, is written as

$$\beta^{\mathrm{KM}}(Z+1)=2.97\times10^{-27}n_e\xi_Z/[T_{eV}^3u^2(4.88+1/u)] \ \mathrm{cm^3/sec}. \tag{14}$$

We consider two other approximate formulations for β due to Elwert [28] as β^E and due to Brunner and John [10] as β^{BJ} and express them as

$$\beta^E(Z+1)=3.9\times10^{-28}n_e\xi_Z/(u^{7/4}T_{eV}^3) \ \mathrm{cm^3/sec}, \tag{15}$$

$$\beta^{\mathrm{BJ}}(Z+1)=8.75\times10^{-27}n_e(Z+1)^3/T_{eV}^{9/2} \ \mathrm{cm^3/sec}. \tag{16}$$

Expressions (14)–(16) do not lead to the Saha equation in the LTE model for any S described in Sec. II, although Eidmann [9], using β^E along with S^M in the LTE model, incorrectly refers to the LTE result as the LTE (Saha) result.

V. DIELECTRONIC RECOMBINATION COEFFICIENT

The dielectronic recombination coefficient D is difficult to calculate properly owing to the involvement of doubly excited states. Many workers [3,10,18,24] neglected the dielectronic recombination in the ionization model, whereas it has been taken into account in several other works [6–9,20,21,26] by using various approximate formulations. Following the work of Eidmann [9], we account for the dielectronic recombination by considering $D(Z)=d\alpha(Z)$, with d a dielectronic recombination parameter for all charge states except the fully stripped one, where the process is not possible.

VI. ION DENSITIES AND IONIZATION MODELS

Ions in a laser plasma generally exist in various ionization charge states and their relative densities are evaluated from a balance between ionization and recombination assuming plasma electrons having a Maxwellian velocity distribution with a temperature T_e. Considering only collisional ionization from the ground-state ion and recombination from the continuum into the ground state, the steady-state ion densities in two consecutive charge states are related as [6,9]

$$\frac{n(Z+1)}{n(Z)}=\frac{S(Z)}{\alpha(Z+1)+D(Z+1)+\beta(Z+1)}, \tag{17}$$

where $n(Z)$ is the ion density of the charge state Z. From this expression one can obtain the charge-state distribution through the fractional densities $[n(Z)/n_i,\ n_i=\Sigma_{Z=1}^{Z_a}n(Z)$ being the total ion density] of the ions in different charge states Z and the average ionic charge state $\bar{Z}$ $[=\Sigma_{Z=1}^{Z_a}Zn(Z)/n_i,\ Z_a$ being the atomic number of the target element]. This evaluation is generally carried out by using one of the LTE, CE, and CRE ionization models.

The LTE model is applicable to the high-density regime where $\beta(Z)\gg\alpha(Z)+D(Z)$. In this model, Eq. (17) reduces to

$$\frac{n(Z+1)}{n(Z)}=\frac{S(Z)}{\beta(Z+1)}. \tag{18}$$

Using β^{SK} for β in Eq. (18), one obtains

$$\frac{n_en(Z+1)}{n(Z)}=3\times10^{21}\left[\frac{2g(Z+1)}{g(Z)}\right]T_{eV}^{3/2}\exp(-u), \tag{19}$$

which is the Saha equation. McWhirter [12] has laid down a necessary, but not sufficient, condition for the LTE to hold as

$$n_e\gtrsim1.4\times10^{14}T_{eV}^{1/2}\chi^3(i,j) \ \mathrm{cm^{-3}}, \tag{20}$$

where $\chi(i,j)$ is the energy difference between levels i and j in eV of an ion. This equation provides the validity condition for two energy levels of an ion. By summing the condition for all levels and charge states, Eliezer, Krumbein, and Salzmann [5] have given a generalized validity condition for the LTE that predicts a lower limit by about an order of magnitude as compared to that from Eq. (20). From the work of Salzmann and Krumbein [6] it is inferred that the LTE condition is satisfied only at low-Z states in an Al plasma produced by a Nd-glass laser. Hence the LTE model is not strictly applicable for a laser plasma.

The CE model is applicable to the low-density regime where $\alpha(Z)+D(Z)\gg\beta(Z)$. In this limit Eq. (17) reduces to

$$\frac{n(Z+1)}{n(Z)}=\frac{S(Z)}{\alpha(Z+1)+D(Z+1)}. \tag{21}$$

Cooper [29] has derived the applicability condition for the CE model as

$$n_e\leqslant1.4\times10^{14}Z^7\left(\frac{T_{eV}}{Z^2E_{\mathrm{H}}}\right)^4\left(\frac{Z^2E_{\mathrm{H}}}{\chi_{Z-1}}\right) \ \mathrm{cm^{-3}}, \tag{22}$$

where E_{H} is the ionization energy of the hydrogen atom in eV. Salzmann and Krumbein [6] have shown the validity of the CE model to electron densities somewhat higher than the limit obtained from Eq. (22), which predicts the validity condition as $n_e\leqslant2.2\times10^{18}\ \mathrm{cm^{-3}}$.

The CRE model is a generalized model taking into account all the recombination processess. In this model the charge-state abundances are evaluated using Eq. (17). It reduces to the CE model when the radiative and dielectronic recombination is dominant over the collisional recombination and goes to the LTE model if the collisional recombination is dominant over the radiative and dielectronic recombination. The validity limit of the CE and LTE models can be better judged by comparing the results from these models with those from the CRE model.

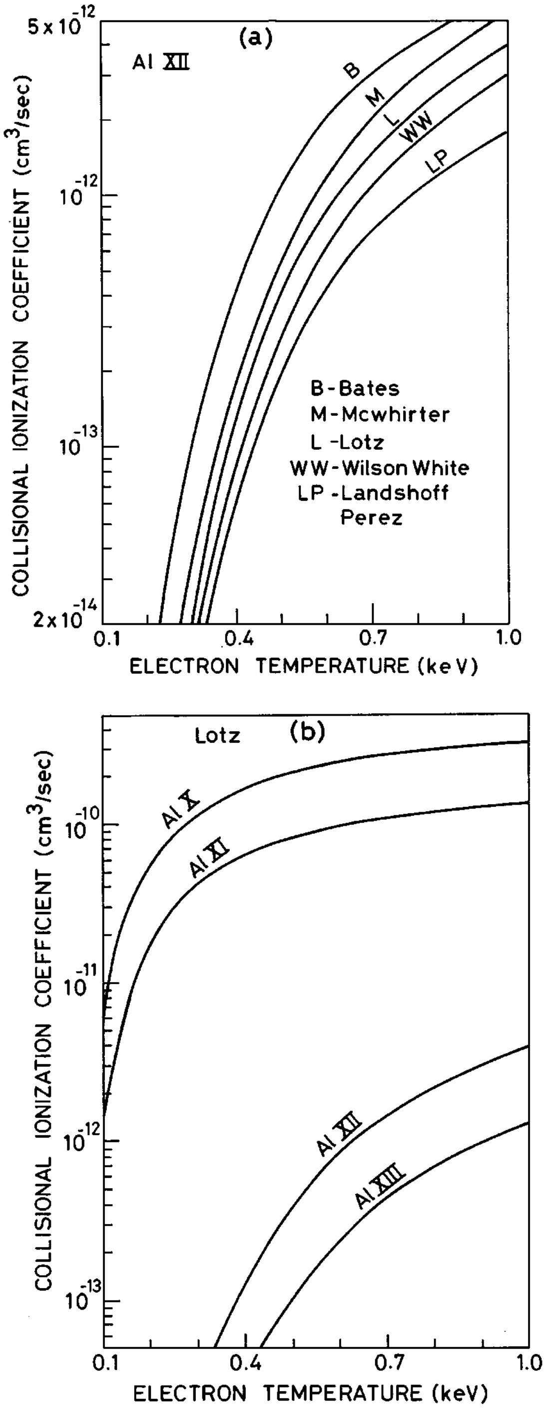

FIG. 1. (a) Collisional ionization coefficient as a function of electron temperature for Al XII ions by using various formulations due to Bates, Kingston, and McWhirter (curve B), McWhirter (curve M), Lotz (curve L), Wilson and White (curve WW), and Landshoff and Perez (curve LP). (b) Collisional ionization coefficient as a function of electron temperature for Al X–Al XIII ions by using the Lotz formula.

VII. NUMERICAL RESULTS AND DISCUSSION

In order to study the effect of ionization and recombination coefficients on the charge-state distribution, we have calculated the ionization and recombination coefficients for a laser-produced Al plasma by using various formulations for them, as mentioned earlier, and have shown the results in Figs. 1–3. The atomic data used in the calculation are taken from available tabulations [30]. The value of χ_Z for Al XIII is taken as 2304 eV, obtained on extrapolation of the tabulated values given up to Al XII in Ref. [30]. Figure 1(a) shows the variation of the collisional ionization coefficient with T_e for Al XII ions by using various formulations. The curves B, M, L, WW, and LP correspond to the formulations for S due to Bates, Kingston, and McWhirter [11], McWhirter [12], Lotz [14], Wilson and White [15], and Landshoff and Perez [16], respectively. The values of S, as calculated by using the formula due to Seaton [13], are found to be close to those obtained from the formula due to Lotz [14] and hence are not shown in the figure. As seen from the figure, the formula S^{LP} yields the lowest ionization rate and the expression S^B gives the highest one. The ratio of these two coefficients changes from 21.9 to 3.4 as T_e increases from 0.1 to 1.0 keV. At $T_e = 0.5$ keV, the values of S from S^B, S^M, S^L, and S^{WW} are, respectively, 5.4, 2.7, 2.0, and 1.4 times higher than that from S^{LP}. It is thus clear that the widely used McWhirter formula S^M does not predict the lowest ionization rate, which is in contrast to that reported in Ref. [6].

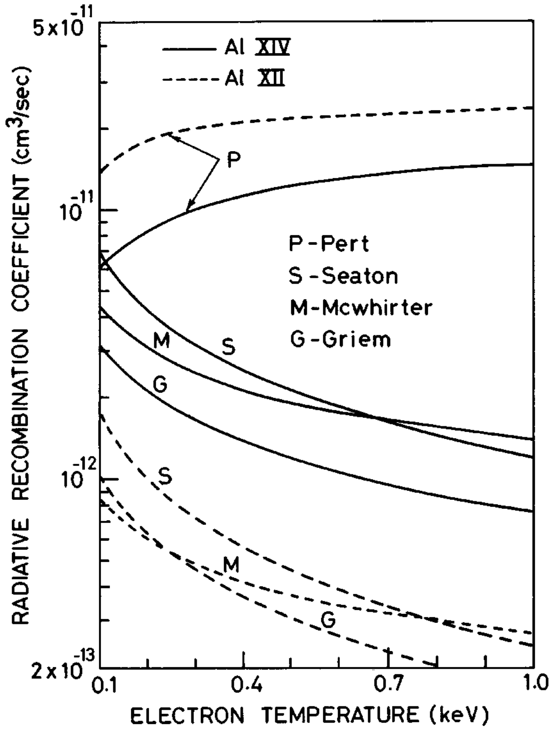

FIG. 2. Radiative recombination coefficient as a function of electron temperature for Al XII and Al XIV ions by using various formulations due to Pert (curves P), Seaton (curve S), McWhirter (curves M), and Griem (curves G).

Figure 1(b) depicts the collisional ionization coefficient as a function of T_e for Al X–Al XIII ions by using the Lotz formula. It is observed that at a given temperature S decreases with increasing charge state of the ion, the decrease being substantially larger (about two orders of magnitude) for Al XII ions relative to Al XI ions as compared to the decrease by a factor of about 3 for Al XIII ions relative to Al XII ions. It is worth noting that Fig. 5 of Ref. [9] shows the values of S for Al XIII to be about two orders of magnitude greater than those for Al XII ions, which are in disagreement with the results shown in Fig. 1(b). A comparison of Fig.

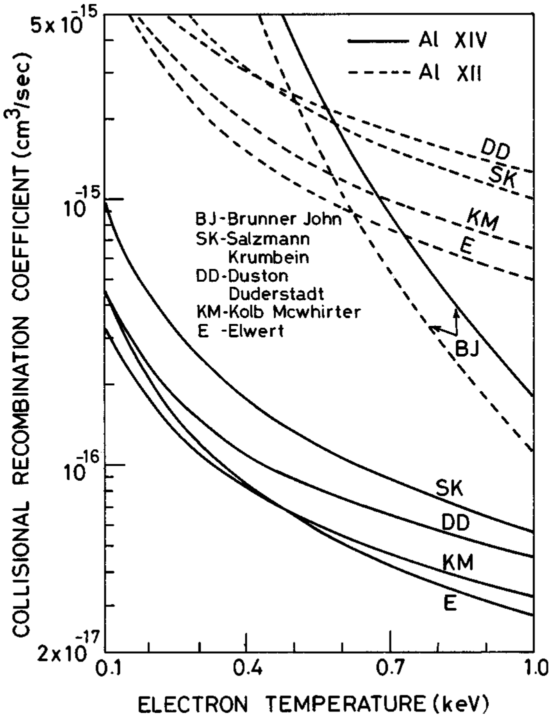

FIG. 3. Collisional recombination coefficient as a function of electron temperature at an electron density of 3×10^{20} cm^{-3} for Al XII and Al XIV ions by using various formulations due to Brunner-John (curves BJ), Salzmann and Krumbein (curves SK), Duston and Duderstadt (curves DD), Kolb and McWhirter (curves KM), and Elwert (curves *E*).

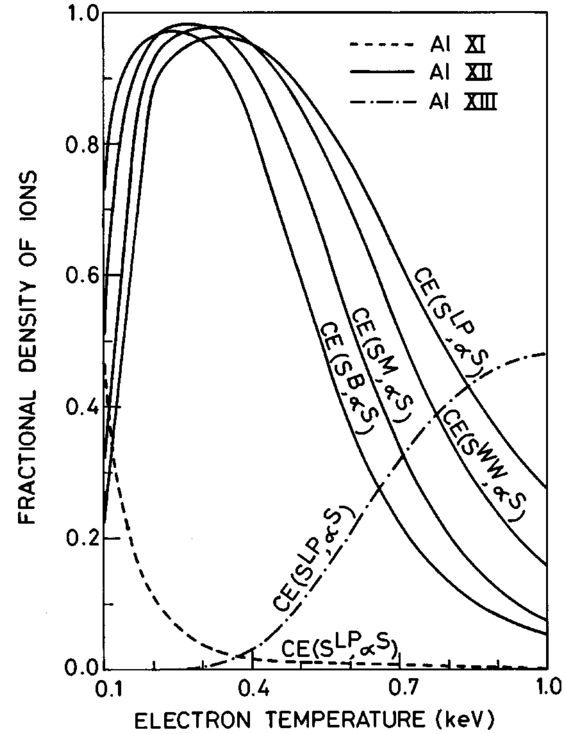

FIG. 4. Fractional density of Al XI–Al XIII ions as a function of electron temperature, calculated from the CE model using various formulations (S^{LP}, S^{WW}, S^M, and S^B) for S and a particular formula (α^S) for α. The formulations used for the rate coefficients are shown in parentheses along with the model. Dielectronic recombination is not considered.

1(b) with Fig. 5 of Ref. [9] shows that the results quoted in Ref. [9] for Al XIII ions are actually for Al XI ions.

Figure 2 shows the radiative recombination coefficient as a function of T_e for Al XII and Al XIV ions by using various formulations due to McWhirter [12], Seaton [25], Pert [26], and Griem [27] as curves M, S, P, and G, respectively. The formula α^{KM} given by Kolb and McWhirter [17] predicts the values of α that are very close to those given by the formula α^S (curves S) and hence are not shown in the figure. As seen from the figure, the nature of the variation of α with T_e and Z predicted by the Pert formula is opposite that predicted by the other formulations. Moreover, the Pert formula gives values of α that are about an order of magnitude larger for Al XIV ions and about two orders of magnitude larger for Al XII ions as compared to those given by the Griem formula. The values of α predicted by Griem's, Seaton's, and McWhirter's formulations are found to be within a factor of 2 from each other.

The collisional recombination coefficient as a function of T_e at $n_e = 3\times10^{20}$ cm^{-3} for Al XII and Al XIV ions is plotted in Fig. 3. The curves SK, BJ, KM, DD, and E refer to the formulations given by Salzmann and Krumbein [6], Brunner and John [10], Kolb and McWhirter [17], Duston and Duderstadt [23], and Elwert [28], respectively. In the calculation using β^{SK} we have considered $S=S^M$ in Eq. (12). As seen from the figure, the curves BJ differ substantially from the other curves, which are slightly different from each other. For Al XIV ions, the values of β from β^{BJ} are substantially larger than those from β^{SK}, β^{DD}, β^{KM}, and β^E in the considered temperature regime, although the difference decreases with increasing T_e. For Al XII ions, the values of β from β^{BJ} are higher up to a certain electron temperature and thereafter decrease as compared to the values from any other formula. The formulations β^{SK}, β^{DD}, β^{KM}, and β^E predict significantly larger values of β for Al XII ions with respect to those for Al XIV ions, whereas the formula β^{BJ} yields smaller values for Al XII ions as compared to those for Al XIV ions.

In Fig. 4 we have plotted the fractional density of Al XI–Al XIII ions versus T_e as calculated from the CE model using various formulations (S^{LP}, S^{WW}, S^M, and S^B) for S and a particular formula (α^S) for α, showing the effect of using the various ionization coefficients. Dielectronic recombination is not considered here. The curve CE (S^{LP} and α^S) refers to the results obtained from the CE model incorporating the formula S^{LP} for S and the formula α^S for α in Eq. (21). As seen from the figure, Al XII ions form the majority of Al ions in the plasma in the temperature range 0.3–0.5 keV. The value of the optimum T_e corresponding to the maximum abundance of Al XII ions changes by using different formulations for S. The fractional density of Al XII ions is also substantially modified at temperatures away from the optimum T_e by using different formulations for S. For example, at $T_e = 0.5$ keV the values of the fractional density of Al XII ions are about 60%, 75%, 85%, and 90%, corresponding to the ionization coefficient formulations S^B, S^M, S^{WW}, and S^{LP}, respectively.

Figure 5 shows the fractional density of Al XII ions as a function of T_e as calculated from the CE model using various formulations (α^P, α^S, and α^G) for α and a particular

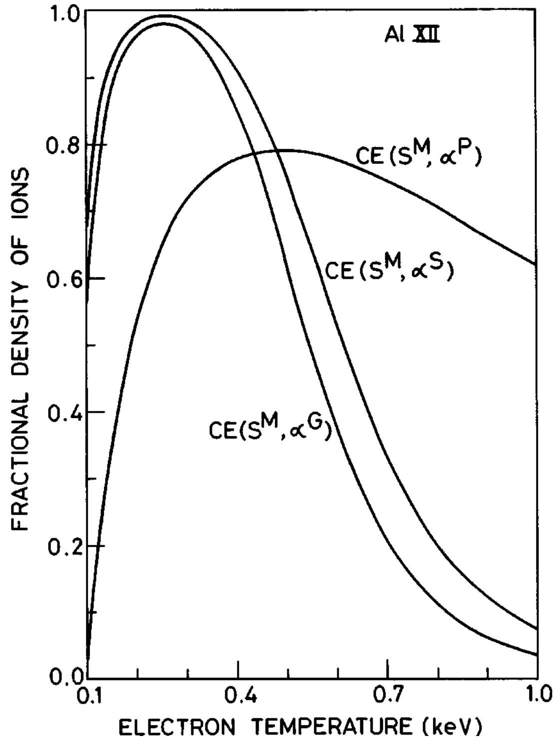

FIG. 5. Fractional density of Al XII ions as a function of electron temperature, calculated from the CE model using various formulations (α^P, α^S, and α^G) for α and a particular formula (S^M) for S. Dielectronic recombination is not considered.

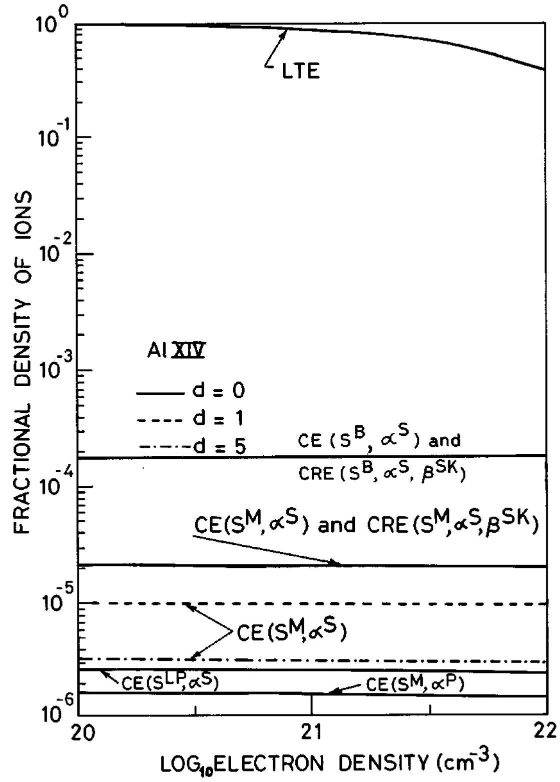

FIG. 6. Fractional density of Al XIV ions as a function of electron density at an electron temperature of 0.3 keV, calculated from the LTE, CE, and CRE models. Dielectronic recombination is considered through the parameter d.

formulation (S^M) for S, representing the effect of using the various radiative recombination coefficients. Dielectronic recombination is not considered here. As seen from the figure, the Pert formula for α, represented by the results of the curve CE (S^M and α^P), predicts substantially different fractional densities of Al XII ions as compared to those from the other formulations for α used in the model. It is also noted that the use of the formulations α^S, α^{KM}, and α^M in the model results in similar values of the fractional density of Al XII ions, which are considerably different from those predicted from the Griem formula (α^G) in the model. At $T_e = 0.7$ keV, the resulting ionic density is about 75%, 20%, and 35%, respectively, from the consideration of α^G, α^P, and the other formulations in the model.

Figure 6 shows the fractional density of Al XIV ions as a function of $\log_{10} n_e$ at $T_e = 0.3$ keV, calculated from the LTE, CE, and CRE models with and without dielectronic recombination in the CE and CRE models. The LTE results are the LTE (Saha) results obtained from Eq. (19). The electron density range shown is 10^{20}–10^{22} cm^{-3}, although we have done calculations beyond this range also. As seen from Fig. 6, the results from the CE model are the same as those from the corresponding CRE model, which uses the same expressions for S and α along with β^{SK} for β, whereas the CE and CRE results are significantly different for different formulations for S and α in the model. Moreover, the LTE results are observed to be substantially different from those obtained using the CE and CRE models. These results suggest that the CE model can be safely applied to laser plasmas with $n_e \leqslant 10^{22}$ cm^{-3} for high-Z ions. The effect of the consideration of dielectronic recombination is observed to decrease the ion density with the decrease being larger for larger dielectronic recombination coefficients.

From the comparison of the calculated results from the CRE and LTE models for high electron densities, it is noted that the results from the CRE (S^B, α^S, and β^{SK}) model are closer to the LTE results as compared to those from the CRE model incorporating other formulations for S and α. For example, at $n_e = 10^{22}$, 10^{24}, and 10^{25} cm^{-3} the values of the fractional density of Al XIV ions from the LTE model or 3.8×10^{-1}, 3.2×10^{-4}, and 2.3×10^{-7}, whereas those are 1.8×10^{-4}, 5.1×10^{-5}, and 1.7×10^{-7} from the CRE (S^B, α^S, and β^{SK}) model and 2.1×10^{-5}, 1.1×10^{-5}, and 1.1×10^{-7} from the CRE (S^M, α^S, and β^{SK}) model. The corresponding values from the CRE model using other combinations of S and α are lower. The CRE results follow the LTE results for $n_e > 10^{25}$ cm^{-3}, with the former one in the case of using S^B, α^5, and β^{SK} formulations earlier than the corresponding ones in the case of using other combinations of the rate coefficients. Thus the formulations S^B, α^S, and β^{SK} represent the most suitable expressions for S, α, and β, respectively.

Figure 7 depicts the average charge state of Al ions as a function of T_e at $n_e = 3\times10^{20}$ cm^{-3}, calculated from the LTE and CE models with various rate coefficients shown in parentheses. Dielectronic recombination in the CE model is not considered here. The CRE results are close to the CE predictions and hence are not shown in the figure. The LTE results are noted to be significantly different from the CE results. One may further note that for a given formula for S the CE results with α^S are considerably higher than those

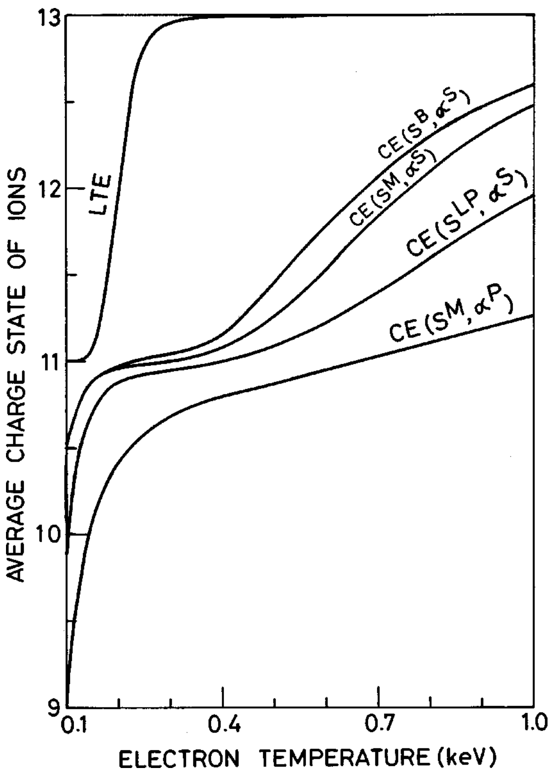

FIG. 7. Average charge state of Al ions as a function of electron temperature at an electron density of 3×10^{20} cm^{-3}, calculated from the LTE and CE models with various rate coefficients. Dielectronic recombination in the CE model is not considered.

with α^P. Moreover, for a given formula for α the CE results are also affected by the use of different formulations for S.

VIII. CONCLUSION

We have noted that there are various formulations for the collisional ionization and collisional and radiative recombination coefficients reported and used in the literature and there are still no universally accepted formulations for them. We have compared several formulations for S, α, and β for laser-produced Al plasmas and studied the effect of the ionization and recombination coefficients on the fractional densities and the average charge state of the ions with different charge states. We have taken into account the LTE, CE, and CRE ionization models for the calculation of the charge-state distribution in the plasma. The formula of Landshoff and Perez [16] for S predicts the lowest ionization coefficient, whereas that of Bates, Kingston, and McWhirter *et al.* [11] gives the highest one. The ratio of these two ionization coefficients for Al XII ions changes from 21.9 to 3.4 as T_e increases from 0.1 to 1.0 keV. Among the considered formulations for α, the predictions of α from the formulations of McWhirter [12], Seaton [13], and Griem [27] are noted to be within a factor of 2 from each other, whereas those from the formula of Pert [26] differ by about an order of magnitude for Al XIV ions and about two orders of magnitude for Al XII ions from the corresponding values from the Griem formula. Among the chosen formulations for β, the values of β estimated from the formula of Brunner and John [10] are significantly different from those obtained from the other formulations, which give slightly different values of β from each other.

The fractional density of Al XII ions that are dominant ions in the plasmas with T_e of 0.3–0.5 keV gets substantially modified by using different formulations for S with a given expression for α in the CE model. For a given formula for S in the CE model, the consideration of the formula α^P predicts the fractional density of Al XII ions substantially different from that given by the other formulations for α, whereas the results from the formulations α^S, α^{KM}, and α^M being similar differ considerably from those from the formula α^G. From the comparison of the CE, CRE, and LTE results of the fractional ionic density, it is observed that the formulations S^B, α^S, and β^{SK} represent the most suitable expressions for S, α, and β, respectively, and the CE model can be safely applied to laser plasmas with $n_e \leqslant 10^{22}$ cm^{-3} for estimating the abundance of high-Z ions and the average ionic charge state.

[1] R. Kauffman, in *Handbook of Plasma Physics*, edited by M. N. Rosenbluth and R. Z. Sagdeev (North-Holland, Amsterdam, 1991), pp. 111–162.

[2] D. Giulietti, S. Bastiani, T. Ceccotti, A. Giulietti, L. A. Gizzi, and A. Macchi, Nuovo Cimento D **17**, 401 (1995).

[3] D. Colombant and G. F. Tonon, J. Appl. Phys. **44**, 3524 (1973).

[4] M. Galanti and N. J. Peacock, J. Phys. B **8**, 2427 (1975).

[5] S. Eliezer, A. D. Krumbein, and D. Salzmann, J. Phys. D **11**, 1693 (1978).

[6] D. Salzmann and A. Krumbein, J. Appl. Phys. **49**, 3229 (1978).

[7] D. Duston and J. Davis, Phys. Rev. A **21**, 1664 (1980).

[8] C. DeMichelis and M. Mattioli, Nucl. Fusion **21**, 677 (1981).

[9] K. Eidmann, Laser Part. Beams **12**, 223 (1994).

[10] W. Brunner and R. W. John, Laser Part. Beams **13**, 403 (1995).

[11] D. R. Bates, A. E. Kingston, and R. W. P. McWhirter, Proc. Phys. Soc. London, Sec. A **267**, 297 (1962).

[12] R. W. P. McWhirter, in *Plasma Diagnostic Techniques*, edited by R. H. Huddlestone and S. L. Leonard (Academic, New York, 1965), p. 210.

[13] M. J. Seaton, in *Atomic and Molecular Processes*, edited by D. R. Bates (Academic, New York, 1962), p. 375.

[14] W. Lotz, Z. Phys. **216**, 241 (1968).

[15] R. Wilson and R. H. White, in *Plasma Diagnostic Techniques* (Ref. [12]), p. 221.

[16] R. K. Landshoff and J. D. Perez, Phys. Rev. A **13**, 1619 (1976).

[17] A. C. Kolb and R. W. P. McWhirter, Phys. Fluids **7**, 519 (1964).

[18] B. K. Sinha, J. Phys. D **13**, 1253 (1980).

[19] M. Itoh, T. Yabe, and S. Kiyokawa, Phys. Rev. A **35**, 233 (1987).

[20] G. P. Gupta and B. K. Sinha, J. Appl. Phys. **77**, 2287 (1995).
[21] G. P. Gupta and B. K. Sinha, J. Appl. Phys. **79**, 619 (1996).
[22] J. Davis and K. G. Whitney, J. Appl. Phys. **45**, 5294 (1974).
[23] D. Duston and J. J. Duderstadt, J. Appl. Phys. **49**, 4388 (1978).
[24] A. Sasaki, H. Yoneda, K. Ueda, and H. Takuma, Laser Part. Beams **11**, 25 (1993).
[25] M. J. Seaton, Mon. Not. R. Astron. Soc. **119**, 81 (1959).
[26] G. J. Pert, Laser Part. Beams **12**, 209 (1994).
[27] H. Griem, *Plasma Spectroscopy* (McGraw-Hill, New York, 1964), p. 161.
[28] G. Elwert, Z. Naturforsch. **9A**, 637 (1954).
[29] J. Cooper, Rep. Prog. Phys. **29**, 35 (1966).
[30] C. E. Moore, *Atomic Energy Levels*, Natl. Bur. Stand. Ref. Data Ser., Natl. Bur. Stand. (U.S.), Circ. No. 35 (U.S. GPO, Washington, DC, 1971), Vol. 1, pp. 124–143.