Perturbation theory for the Rosenzweig-Porter matrix model

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We study an ensemble of random matrices (the Rosenzweig-Porter model) which, in contrast to the standard Gaussian ensemble, is not invariant under changes of basis. We show that a rather complete understanding of its level correlations can be obtained within the standard framework of diagrammatic perturbation theory. The structure of the perturbation expansion allows for an interpretation of the level structure on simple physical grounds, an aspect that is missing in the exact analysis [T. Guhr, Phys. Rev. Lett. **76**, 2258 (1996); T. Guhr and A. Müller-Groeling, J. Math. Phys. **38**, 1870 (1997)]. [S1063-651X(97)03308-4]

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I. INTRODUCTION

Random matrix ensembles were introduced into physics by Wigner, Dyson, and others [1] as phenomenological models of complex quantum systems. Such ensembles are defined so as to obey certain symmetries and constraints but are otherwise ''as random as possible.'' For instance, the Gaussian unitary ensemble (GUE) consists of all $N\times N$ Hermitian matrices H, the only constraint being that, on the average, $\mathrm{tr}\,H^2$ is a given constant. This leads to a probability density in the matrix space, $\mathcal{P}_0\sim\exp(-\mathrm{tr}\,H^2)$, which is invariant under unitary transformations.

Recently there has been some interest in various generalizations of the GUE and its orthogonal and symplectic counterparts [2]. One possible generalization amounts to breaking the U(N) symmetry of the GUE by introducing a parameter μ into the probability density function

$$\mathcal{P}(\{H_{ij}\})dH=\mathcal{N}\exp\left\{-\sum_{i=1}^{N}H_{ii}^2-2(1+\mu)\times\sum_{i<j}\left[(\mathrm{Re}H_{ij})^2+(\mathrm{Im}H_{ij})^2\right]\right\}dH. \quad (1)$$

Here H_{ij}, with $i\leqslant j$, designate the independent matrix elements of an $N\times N$ Hermitian matrix, $dH=\Pi_{i=1}^{N}dH_{ii}\Pi_{i<j}d(\mathrm{Re}H_{ij})d(\mathrm{Im}H_{ij})$ is the volume element in the matrix space, and $\mathcal{N}$ is a normalization constant. For $\mu=0$ the expression in the curly brackets is equal to $(-\mathrm{tr}\,H^2)$, so that the GUE is recovered. The parameter μ breaks the $U(N)$ symmetry and introduces a preferential basis. When $\mu\to\infty$, for fixed N, all matrices become diagonal in that basis. The ensemble thus exhibits a crossover from the Wigner-Dyson statistics of the standard random matrix theory ($\mu=0$) to the Poisson statistics of uncorrelated levels ($\mu=\infty$). Such an ensemble (for real symmetric matrices) was introduced by Rosenzweig and Porter [3] in their studies of complex atomic spectra, and more recently appeared in the field of quantum chaos and localization [2].

We shall be interested in the behavior of this ensemble in the $N\to\infty$ limit. In this limit, significant deviations from the GUE behavior can only occur if μ increases with N sufficiently fast. The local level statistics is controlled [4–7] by the parameter μ/N^2. Only when this parameter approaches infinity does the Wigner-Dyson statistics become completely obliterated and the Poisson limit of uncorrelated levels is reached. In the opposite case, i.e., when $(\mu/N^2)\to 0$, an arbitrary large sequence of levels will obey the Wigner-Dyson statistics of the GUE. The ''critical'' case corresponds to $\mu=cN^2$, with $c=\mathrm{const}$. The situation resembles the one which occurs in disordered electronic systems where, in the thermodynamic limit, three distinct types of statistics corresponding to insulator, metal, and a mobility-edge system [8,9] exist.

In the present paper we shall take a closer look at the eigenvalue statistics, with an emphasis on the ''critical'' case $\mu=cN^2$. We will show that a rather complete picture emerges already from diagrammatic perturbation calculations, along the lines of Refs. [10,11]. In this case the two-point correlation function $R(s)$ (smoothed out over few level spacings) differs substantially from both the GUE and the Poisson correlation functions. Here s denotes the energy difference in units of the average level spacing. For small c, $R(s)$ is approximately given by its Wigner-Dyson value, $-1/(2\pi^2s^2)$, as long as $s\ll 1/\sqrt{c}$ [12]. For larger s, however, $R(s)$ changes sign, reaches a maximum, and eventually decreases as $1/(\pi cs^2)$.

II. DIAGRAMMATIC ANALYSIS

To begin with, let us introduce a definition of the Rosenzweig-Porter (RP) model which is equivalent to Eq. (1) but more convenient for diagrammatic computation. We define

$$H=H_0+V,\quad (H_0)_{ij}=\varepsilon_i\delta_{ij},\quad V_{ii}=0, \quad (2)$$

where ε_i are independent real random numbers with Gaussian distribution

$$p(\varepsilon)=\pi^{-1/2}e^{-\varepsilon^2}. \quad (3)$$

The matrix elements V_{ij} of the Hermitian matrix V are independent complex random numbers with Gaussian distributed real and imaginary parts. The distribution is determined by

$$\langle V_{ij}\rangle = 0, \quad \langle |V_{ij}|^2\rangle = \frac{1}{2(1+\mu)}. \tag{4}$$

It is easy to see that the probability density function of the thus defined Hamiltonian is just Eq. (1).

The diagrammatic analysis amounts to a locator expansion of the full single-particle Green function

$$G^{\pm} = (E^{\pm} - H)^{-1}, \quad E^{\pm} = E \pm i\delta, \quad \delta \searrow 0, \tag{5}$$

with respect to the off-diagonal V. The unperturbed (bare) Green function $G_0 = G|_{V=0}$ is called the locator. We consider the density of states $\rho(E) = \mathrm{tr}\,\delta(E-H)$, its average value

$$\nu(E) = \langle\langle \rho(E)\rangle_V\rangle_\varepsilon, \tag{6}$$

and its correlation function

$$R(E,E') = \langle\langle \rho(E)\rho(E')\rangle_V\rangle_\varepsilon - \nu(E)\nu(E'). \tag{7}$$

Here $\langle\cdots\rangle_V$ ($\langle\cdots\rangle_\varepsilon$) stands for averaging with respect to the off-diagonal elements V_{ij} (the diagonal elements ε_i).

We concentrate on energy separations $\omega = E' - E$ for which both energies are close to the middle of the band, i.e., close to $E=0$ where $\nu(E)$ is maximal. In this region, $\nu(E)$ is approximately constant, $\nu(E) = \nu(E')$ up to $O(1/N)$ relative corrections. The density-density correlator $R(s) \equiv R(E,E')/\nu^2$ will then be a function of the dimensionless level separation $s = \omega/\Delta$ where the average level spacing is $\Delta = 1/\nu(E=0)$.

We next analyze the spectral correlation function $R(s)$ in the regime $s > 1$ where perturbative methods are applicable. To begin with, we decompose $R(s)$ according to

$$\begin{aligned} R(s) = \frac{1}{\nu^2}\{&\langle\langle \rho(E+\omega)\rangle_V\langle\rho(E)\rangle_V\rangle_\varepsilon - \langle\langle\rho(E+\omega)\rangle_V\rangle_\varepsilon \\ &\times\langle\langle\rho(E)\rangle_V\rangle_\varepsilon + \langle\langle\rho(E+\omega)\rho(E)\rangle_V \\ &-\langle\rho(E+\omega)\rangle_V\langle\rho(E)\rangle_V\rangle_\varepsilon\} \\ = R_1(s) + R_2(s), \end{aligned} \tag{8}$$

where $R_1(R_2)$ corresponds to the first (second) difference line contributing to the right hand side of Eq. (8). Note that the decomposition $R = R_1 + R_2$ is exact. The physical significance of the two functions $R_{1/2}$ will be discussed below. Here we merely note that R_1 measures correlations remaining in the GUE-averaged density of states whereas R_2 focuses on the GUE correlations as such.

Representing the density of states in terms of the single-particle Green function

$$\rho(E) = -\pi^{-1}\mathrm{Im}\,\mathrm{tr}\,G^+(E),$$

and making use of the fact that correlations (of any type) between products of purely retarded or advanced Green's functions vanish for $N\to\infty$: $\langle G^{+n}\rangle = \langle G^+\rangle^n$, we obtain

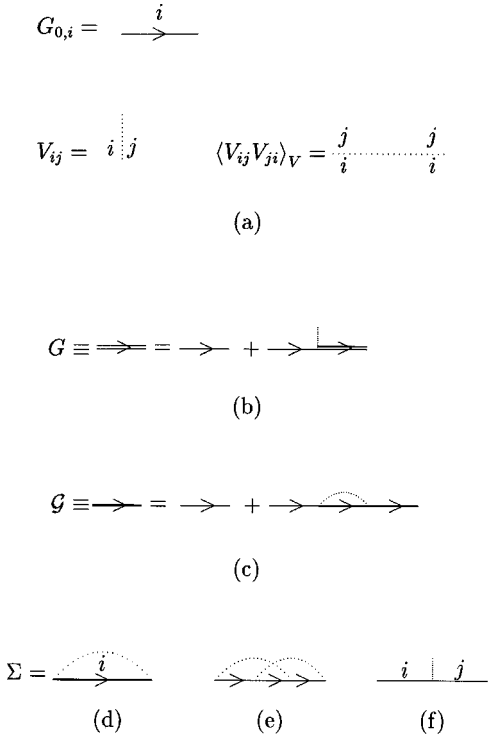

FIG. 1. Graphic representation of the Green function G_0, the perturbation matrix elements V_{ij}, and its correlations (a). The Dyson equation for the full Green function is shown in (b), for the averaged Green function in (c), and the graphical representation of the self-energy is shown in (d). (e) contains a diagram with crossed GUE lines. (f) for an explanation see text.

$$R_1 = \frac{\Delta^2}{2\pi^2}\mathrm{Re}\langle \mathrm{tr}\langle G^+(E+\omega)\rangle_V \mathrm{tr}\langle G^-(E)\rangle_V\rangle_{\varepsilon,c},$$

$$R_2 = \frac{\Delta^2}{2\pi^2}\mathrm{Re}\langle\langle \mathrm{tr}G^+(E+\omega)\,\mathrm{tr}G^-(E)\rangle_{V,c}\rangle_\varepsilon, \tag{9}$$

where $\langle\cdots\rangle_c$ denotes the connected average, $\langle XY\rangle_c = \langle XY\rangle - \langle X\rangle\langle Y\rangle$. Before turning to the actual calculation of these functions let us make a few methodological remarks and introduce some building blocks that will be of importance throughout. The whole approach will be based on a perturbative expansion of the Green functions in powers of V. It is instructive to visualize the structure of the expansion scheme graphically. To this end we introduce the notation of Fig. 1(a) where i and j represent matrix indices (which will not be indicated explicitly unless necessary). As a first step of our perturbative analysis (cf. the definition of the correlation function R_1 above) we have to calculate the V average of the Green function G. In a diagrammatic language, the expansion of the Green function can be visualized as shown in Fig. 1(b). The subsequent diagrammatic analysis of this equation is simplified drastically by two observations that hold to leading order in N^{-1}.

(i) Contributions with ''crossed GUE lines'' [see Fig. 1(e)] are negligible [10].

(ii) Diagram segments which are separated from each other by vertices V_{ij} are statistically independent with respect to the average over the on-site distribution functions $\rho(\varepsilon)$.

The second statement is based on the fact that the indices i and j in a diagram like the one shown in Fig. 1(f) are eventually summed over independently of each other. (All contributions where one of the summations is constrained will be of higher order in N^{-1}.) On the other hand, the variables ε_i at different sites are statistically independent. Put together, these two facts imply that (ii) holds to leading order in N^{-1}.

According to rule (i), the diagrammatic expression for the V-averaged Green function $\mathcal{G}_i\delta_{ij}\equiv\langle G_{ij}\rangle_V$ reads as shown in Fig. 1(c). Statement (ii) implies that to leading order in N^{-1}, the self-energy part $\Sigma^{\pm}$ [as shown in Fig. 1(d)] can be replaced by the ϵ_i-averaged one. We thus obtain

$$\mathcal{G}_i^{\pm}\simeq\frac{1}{G_{0,i}^{\pm\,-1}-\overline{\Sigma^{\pm}}},\quad \overline{\Sigma^{\pm}}\equiv\langle\Sigma^{\pm}\rangle_{\varepsilon}=\frac{N}{2(1+\mu)}\langle\mathcal{G}^{\pm}\rangle_{\varepsilon}.$$

In order to solve this equation self-consistently, we have to compute the energy average of $\mathcal{G}$. Anticipating that (a) the self-energy will be largely imaginary, $\Sigma^{\pm}\simeq\mp i\Gamma$, and (b) $\Gamma\ll 1$, we obtain

$$\langle\mathcal{G}^{\pm}\rangle_{\varepsilon}=\frac{1}{\pi^{1/2}}\int d\varepsilon e^{-\varepsilon^2}\frac{1}{E-\varepsilon\pm i\Gamma}$$
$$\simeq\mp\frac{1}{\pi^{1/2}}\int d\varepsilon e^{-\varepsilon^2}\frac{i\Gamma}{(E-\varepsilon+i\Gamma)(E-\varepsilon-i\Gamma)}=\mp i\pi^{1/2}.$$

The second equality is based on the assumption that the energy argument $E\ll 1$ is close to the middle of the band. As a consequence the real part of the integral is of $O(E)$ and negligible in comparison with the imaginary part $O(1)$. This justifies assumption (a) above. Collecting everything so far we obtain the V-averaged Green function

$$\mathcal{G}_i^{\pm}\simeq\frac{1}{E-\varepsilon_i\pm i\Gamma},\quad \Gamma=\frac{N\pi^{1/2}}{2(1+\mu)}. \tag{10}$$

[Note that assumption (b) above holds for all $\mu\sim N^x, x>1$, i.e., Eq. (10) indeed represents the self-consistent solution of the Dyson equation in Fig. 1(c).] We next insert this expression into the defining equation of the correlation function R_1 and obtain

$$R_1=\frac{\Delta^2 N}{2\pi^2}\mathrm{Re}\frac{1}{\pi^{1/2}}\int d\varepsilon e^{-\varepsilon^2}\frac{1}{E+\omega-\varepsilon+i\Gamma}\frac{1}{E-\varepsilon-i\Gamma}$$
$$\simeq\frac{\Delta^2 N}{\pi^{3/2}}\frac{2\Gamma}{\omega^2+4\Gamma^2}.$$

Noting that the level spacing $\Delta=\pi^{1/2}/N$, we arrive at the final result

$$R_1(s)=\frac{1}{\pi c}\frac{1}{s^2+c^{-2}},\quad c=\frac{\Delta}{2\Gamma} \tag{11}$$

for the first of the above introduced correlation functions. We postpone the discussion of this equation until the complementary correlation function R_2 has been calculated. In principle one might compute R_2 via a straightforward perturbative expansion of the Green function G. However, experience gained from the analysis of similar correlation functions [11] has shown that it is advantageous to represent the Green functions according to

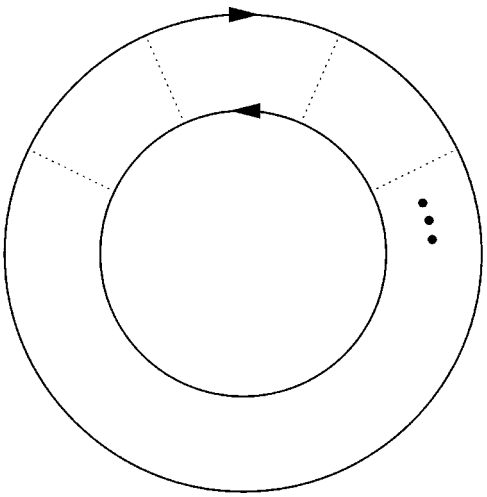

FIG. 2. Graphical representation of $S_n(E,E')$ appearing in Eq. (12).

$$G^{\pm}(E)=\partial_E\ln(E^{\pm}-H)$$

prior to the perturbative expansion. In this way we are led to consider

$$R_2=\frac{\Delta^2}{2\pi^2}\partial^2_{E',E}\Big|_{E'=E+\omega}$$
$$\times\mathrm{Re}\langle\langle\mathrm{tr}\ln(E'^{+}-H)\mathrm{tr}\ln(E^{-}-H)\rangle_{V,c}\rangle_{\varepsilon}.$$

Expanding the logarithms in powers of V and applying the noncrossing rule (i) we obtain

$$R_2=\frac{\Delta^2}{2\pi^2}\partial^2_{E',E}\Big|_{E'=E+\omega}\mathrm{Re}\sum_{n=2}\frac{1}{n}\langle S_n(E,E')\rangle_{\varepsilon}, \tag{12}$$

with $S_n(E,E')$ graphically represented as shown in Fig. 2. There the segments on the outer (inner) ring correspond to the Green function $\mathcal{G}^{+}(E')$ $[\mathcal{G}^{-}(E)]$ and the two rings are connected by n V lines. (Note that an $n=1$ contribution is absent because the potential V is off-diagonal in the site indices.) Rule (ii) implies that each segment of the ''wheel'' above can be averaged individually over the on-site energies ε_i. As a result, the diagram S_n factorizes, $S_n=\gamma^n$, $\gamma=[N/2(1+\mu)]\langle\mathcal{G}^{+}(E')\mathcal{G}^{-}(E)\rangle_{\varepsilon}$, and $\Sigma_{n=2}n^{-1}\langle S_n\rangle_{\varepsilon}=-\ln(1-\gamma)-\gamma$. Computing the energy average (cf. the computation of the correlation function R_1 above)

$$\langle\mathcal{G}^{+}(E')\mathcal{G}^{-}(E)\rangle_{\varepsilon}=\frac{2\pi^{1/2}i}{E'-E+2i\Gamma},$$

and collecting constants we obtain

$$R_2(s)=\frac{1}{2\pi^2}\partial_s^2\mathrm{Re}\left[\ln\left(\frac{s}{s-ic^{-1}}\right)-\frac{ic^{-1}}{s-ic^{-1}}\right]. \tag{13}$$

We finally carry out the differentiation and add the R_1 contribution (11) to arrive at the final result

$$R(s)=\frac{1}{c\pi}\frac{1}{s^2+c^{-2}}+\frac{1}{2\pi^2}\left\{\frac{-1}{s^2}+\frac{s^2-3c^{-2}}{(s^2+c^{-2})^2}+\frac{8s^2c^{-2}}{(s^2+c^{-2})^3}\right\}. \tag{14}$$

Equation (14) is applicable when the energy $s\gg 1$ and fine structure scales $s\approx 1$ are inessential. Let us conclude this section with a brief discussion of this result.

The contribution R_1 [cf. Eq. (11)] has the following interpretation: The V-averaged Green function $\mathcal{G}$ is similar to G_0 except for the fact that a finite width Γ has been attached to each of the uncorrelated levels ε_i. This ''smearing'' implies that the corresponding correlation function R_1 is Lorentzian, i.e., it is a broadened version of the δ function that would be obtained for sharply defined autocorrelated levels. The complementary term R_2 describes correlations between the V degrees of freedom. After combining the two contributions three qualitatively different regimes can be identified.

(1) For $s\gg c^{-1}$ the dominant contribution comes from R_1 and we obtain $R(s)\approx(\pi c s^2)^{-1}$.

(2) $c^{-1/2}\ll s\ll c^{-1}$: Still R_1 dominates but now $R_1\approx c/\pi$.

(3) $s\ll c^{-1/2}$: The R_2 contribution becomes the dominant one and we obtain the GUE result $R_2\approx-(2\pi^2 s^2)^{-1}$ corrected by a small term $R_1\approx c/\pi$.

In summary, Eq. (14) essentially represents a superposition of a GUE correlation function and a smeared Poissonian autocorrelation function.

III. NONPERTURBATIVE RESULTS

The diagrammatic treatment is incapable of describing structures on the energy scale of $O(\Delta)$. For $c\gg 1$ an alternative perturbation technique, applicable over the whole energy axis, can be used. Within this approach spectral correlations are described in terms of stochastic evolution equations [13] (see also Ref. [14]). In this way one obtains a spectral correlation function $R(s)$ that depends only on the combination $s\sqrt{c}$. For large energies, $s\gg 1/\sqrt{c}$, the result coincides with ours, i.e., $R(s)\approx(\pi c s^2)^{-1}$, and for small energies, $s\ll 1/\sqrt{c}$ level repulsion sets in, i.e., $R(s)+1\propto cs^2$.

The complementary regime of $c\ll 1$ can be treated by Efetov's nonperturbative supersymmetry technique [15] (for a recent review see [16]), where averages of Green's functions are obtained from a generating functional. The generating functional corresponding to the RP model is similar to the one described in [5,6]. In these works the problem of a random banded matrix with additional diagonal disorder was addressed. Taking the bandwidth equal to the matrix size N leads to the RP model. From the generating functional one can obtain the correlation function of retarded and advanced Green's functions $K^{12}=\langle \mathrm{tr}\, G^+(E+\omega)\mathrm{tr}\, G^-(E)\rangle$ where the average is taken with respect to Eq. (1). The final integrations can be carried out within a saddle-point expansion the validity of which is controlled by $N^2/\mu\gg 1$ or equivalently by $c\ll 1$, and by $\omega\ll N/\mu$ or equivalently by $s\ll c^{-1}$. In the present work we skip the technical details and concentrate on the discussion of the results.

In the limit $N\to\infty$ the function K^{12} is given by

$$K^{12}(E,\omega)=\langle \mathrm{tr}\, G^+(E)\rangle\langle \mathrm{tr} G^-(E)\rangle+\left(1+\frac{c}{\pi}\right)\frac{-2i}{s^2\Delta^2}e^{-i\pi s}\times\sin(\pi s)+\frac{2ic}{\Delta^2 s}+\frac{2\pi c}{\Delta^2}. \tag{15}$$

The first term is the entirely disconnected part and the terms of $O(c)$ describe deviations from a pure GUE behavior. These terms represent the analog of the contribution R_1 that appeared in the diagrammatic analysis. They result from the correlation between the on-site energies ε_i. [Note that in principle correlations of this type exist in the pure GUE as well. In that case, however, they represent negligible $O(1/N)$ effect.] From Eq. (15) we obtain the correlation function

$$R(s)=\left(1+\frac{c}{\pi}\right)\left\{-\left(\frac{\sin(\pi s)}{\pi s}\right)^2\right\}+\delta(s)+\frac{c}{\pi} \tag{16}$$

describing the spectral behavior in the regime $s\ll c^{-1}$, $c\ll 1$. We next turn to the discussion of this result. We first observe that the term c/π equals the leading contribution of the smeared autocorrelation $R_1(s)$ for $s\ll c^{-1}$. For very small level separations $R(s)$ behaves as $-1+(1+c/\pi)(\pi s)^2/3$, i.e., apart from a slightly modified prefactor we obtain generic GUE behavior. For larger values $1\ll s\ll c^{-1}$ the leading terms are identical with those obtained in the diagrammatic treatment, as expected. [By ''leading'' we mean the first order terms of an expansion in the parameter $1/s\ll 1$ after the oscillatory structure in Eq. (16) has been averaged out.] In particular, the GUE behavior is only valid up to $s\ll 1/\sqrt{c}$. Thus the nonperturbative results underline the conclusion drawn from the diagrammatic analysis: $R(s)$ is essentially a superposition of a GUE correlation with a smeared Poissonian autocorrelation. A conclusion to be drawn from this observation is that the analogy between the Wigner-Dyson-to-Poisson transition in the RP model and disordered electron systems, respectively, is not complete. In the latter case the critical correlation function can hardly be interpreted as a simple superposition of two terms. This qualitative difference manifests, e.g., in the behavior of the level compressibility, $\chi=\lim_{S\to\infty}\int_{-S}^{S}ds R(s)$ (where it is essential that the limit $N\to\infty$ is taken first). The two extremes GUE (Poisson) correspond to values $\chi=0$ ($\chi=1$). In the case of a disordered metal at criticality the compressibility takes an intermediate value $0<\chi<1$ [17]. In the critical RP model, however, $\chi=1$, i.e., perturbing a Poisson ensemble by a GUE ensemble does not change the level compressibility [13].

Finally, we would like to comment on the analysis [18,19]. In these references, the RP model was solved *exactly* for arbitrary values of the parameters μ and s. As a result of a sophisticated combination of supersymmetry and group-theoretical concepts Guhr [18] obtained nontrivial double integral representations for the correlation functions which turned out to be difficult to evaluate. In order to derive closed expressions for $R(s)$ the integral was analyzed in the two limiting cases $c\gg 1$ [18] and $c\ll 1$ [19] by means of asymptotic expansion schemes. The price to pay for the

mathematical rigor of Guhr's approach is that the physical origin of the various ingredients to $R(s)$ is hard to identify. For this reason we believe that a more conventional analysis like the one discussed above was called for.

IV. CONCLUSIONS

We have studied the density of states correlation function $R(s)$ (s measures the energy difference in units of the average level spacing) of the Rosenzweig-Porter model. This random matrix model contains a parameter μ which allows us to interpolate between GUE ($\mu=0$) and Poisson statistics ($\mu=\infty$). In the thermodynamic limit $N\to\infty$ the model shows three different types of universal functions $R(s)$ depending on how μ scales with N. From a diagrammatic analysis (locator expansion) assisted by nonperturbative methods we draw the following conclusions: Parameter values scaling as $\mu(N)/N^2\to 0$ [$\mu(N)/N^2\to\infty$] lead to GUE (Poisson) statistics. In the borderline case $\mu(N)/N^2\equiv c$, however, a novel universal type of spectral behavior is observed. The corresponding correlation function $R(s)$ has the following features: As in the GUE case levels repel each other, i.e., $R(s)\to -1$ for $s\to 0$. At some c-dependent value s_0, $R(s)$ changes sign, then reaches a maximum and decreases as $(\pi c s^2)^{-1}$ for large s. For $c\ll 1$ we find that the spectrum shows GUE-type statistics up to values $s\sim 1/\sqrt{c}$. For larger values of s, a different type of statistics is observed, this being a consequence of the non-GUE correlation of the diagonal matrix elements in the RP model. These large energy correlations can be interpreted as the tails of a widely ''smeared'' energy autocorrelation of Poissonian type. We thus conclude that the RP model in the critical case $\mu(N)/N^2\equiv c$ essentially leads to a linear superposition of Wigner-Dyson and Poissonian behavior. Let us finally comment on the aspect of symmetries. In this paper we have considered the Rosenzweig-Porter model in its unitary version. It is a straightforward matter to extend both the diagrammatic and the nonperturbative analysis to the case of orthogonal, respectively, symplectic symmetry. On the other hand, none of our main conclusions on the structure of the model's eigenvalue statistics did depend in a *conceptual* way on symmetry aspects. We thus expect the level statistics of the models of higher symmetry to be qualitatively similar but did not embark on any kind of quantitative analysis.

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