Control of noise-induced oscillations of a pendulum with a randomly vibrating suspension axis

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We consider the influence of an additional harmonic action on noise-induced oscillations of a pendulum with a randomly vibrating suspension axis. It is shown that these oscillations are intensified, or even initiated, if the frequency of the additional action is low, but that they are suppressed if it is high. Both intensification and suppression of the oscillations occur via ''on-off intermittency.'' In a certain range of the action frequencies, synchronization of a noise-induced pendulum's oscillations takes place in the sense that the mean frequency of the oscillations becomes close to the action frequency. Thus we demonstrate that both frequency and amplitude of noise-induced oscillations can be effectively controlled. Similarities and distinctions between these effects and classical phenomena of asynchronous excitation, asynchronous quenching, and synchronization are discussed. [S1063-651X(97)02208-3]

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I. INTRODUCTION

A noise-induced phase transition in a pendulum with a randomly vibrating suspension axis was considered in detail in Refs. [1,2]. This transition consists in the excitation of the pendulum's oscillations, and the birth of an induced attractor owing to the random vibration of the suspension axis. It takes place if the intensity of the random vibration exceeds some critical level. This transition is akin to that considered in Refs. [3,4], and is quite different from noise-induced phase transitions studied by a number of other researchers (see, e.g., Ref. [5]). In their works the appearance of additional peaks in the probability density under the influence of multiplicative noise, mainly in the systems with multistability, is called a noise-induced phase transition. For the pendulum under consideration, additional peaks in the probability density do not appear.

The equation for the description of noise-induced pendulum's oscillations is

$$\ddot{\varphi}+2\beta(1+\alpha\dot{\varphi}^2)\dot{\varphi}+\omega_0^2[1+\xi_1(t)+a\cos\omega_a t]\sin\varphi=k\xi_2(t), \quad (1)$$

where φ is the pendulum's angular deviation from the equilibrium position, $2\beta(1+\alpha\dot{\varphi}^2)\dot{\varphi}$ is the value proportional to the moment of the friction force which is assumed to be nonlinear, ω_0 is the natural frequency of small pendulum's oscillations, $\xi_1(t)$ is the acceleration of the suspension axis that is a comparatively wide-band random process with nonzero power spectrum density $\kappa(\omega)$ at the frequency $2\omega_0$, $k\xi_2(t)$ is the additive white noise, and a and ω_a are, respectively, the amplitude and frequency of the additional vibration of the suspension axis. (Unlike Refs. [1,2], in this equation additive noise and additional harmonic vibration of the suspension axis are taken into account.)

From the analysis of truncated equations obtained by the Krylov-Bogolyubov method and the corresponding Fokker-Planck equation, it was shown [1,2] that, as $\kappa(2\omega_0)$ is in excess of a certain critical value κ_{cr} proportional to the damping factor β, the mean values of the amplitude and the amplitude squared become different from zero. Taking into account that the truncated equation for the amplitude has the same form as for a noisy van der Pol generator, one can speak of the birth of a noise-induced attractor.

In Ref. [6] it was shown that the excitation of the pendulum's oscillations occurs via the so-called ''on-off intermittency'' [7]. (Intermittency of such a kind was first reported by Fujisaka and Yamada [8].) It is essential that this type of intermittency can occur not only in dynamical systems, but in stochastic ones as well [9]. In Ref. [9] the statistical properties of on-off intermittency were found from the analysis of a one-dimensional map. In particular, it was found that the mean duration of laminar phase has to be proportional to a^{-1}, where a is a bifurcation parameter. For the pendulum, we calculated the mean duration of laminar phase from the truncated equations for oscillation amplitude and phase and the corresponding Fokker-Plank equation [6]. Assuming that the pendulum oscillates in laminar phase if the oscillation amplitude A is less than a certain threshold ϵ, we obtained that the mean duration of laminar phase has to be proportional to ϵ and inversely proportional to $\kappa(2\omega_0)-\kappa_{\rm cr}$. This result, with $\kappa(2\omega_0)-\kappa_{\rm cr}$ taken as a bifurcation parameter, is in agreement with Ref. [9].

In this article we investigate the influence of an additional harmonic action upon the pendulum. It is shown that a low-frequency action can initiate the noise-induced phase transition if the intensity of the suspension axis random vibration is subthreshold, and intensify the noise-induced oscillations if this intensity exceeds its threshold; whereas a high-frequency action always suppresses the noise-induced pendulum's oscillations. It turns out that in a certain range of action frequencies, synchronization of noise-induced pendu-

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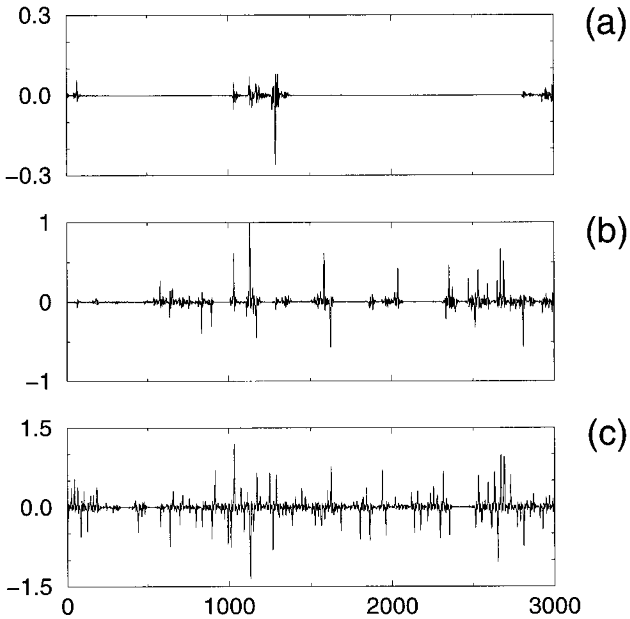

FIG. 1. Oscillations of the pendulum excited by the noise and additional periodic action. The dependencies of $\varphi(t)$ are shown for $k=0$, $\kappa(2\omega)/\kappa_{cr}=0.51$, $\omega_a=0.3$, and different values of the action amplitude: (a) $a=1.151$, (b) $a=1.3$, and (c) $a=1.5$.

lum's oscillations takes place in the sense that the mean frequency of the oscillations becomes close to the action frequency.

Our study was mainly incited by the known investigations on the intensification and suppression of turbulent pulsations in subsonic jets by slight acoustic field [10–12]. One of the purposes of our study is to emphasize some parallels between turbulent processes in nonclosed flows and noise-induced pendulum's oscillations [13–15].

Another purpose is to find similarities and differences in the response of systems with noise-induced and with ordinary attractors to an external action. In particular, we are interested in synchronization properties.

II. CONTROL OF THE INTENSITY OF THE PENDULUM'S OSCILLATIONS BY AN ADDITIONAL HARMONIC ACTION

In this section we discuss the problems of controlling the intensity of the pendulum's oscillations by an additional harmonic action, either multiplicative or additive. Because both types of the action result in the same qualitative effects, we dwell mainly on a multiplicative action. Such an action corresponds to additional harmonic vibration of the pendulum's suspension axis.

A. Initiation and intensification of the pendulum's oscillations by a low-frequency harmonic action

First we consider the case $k=0$, i.e., without additive noise. The results of numerical simulation of Eq. (1) for $\kappa(2\omega_0)<\kappa_{cr}$ and different values of a are represented in Fig. 1. In this case the excitation of oscillations is of a threshold character with the amplitude threshold value depending on $\kappa(2\omega_0)/\kappa_{cr}$. For $\omega_a=0.3$ and $\kappa(2\omega_0)/\kappa_{cr}=0.51$, the threshold value of a is equal to 1.15. For $a>a_{cr}$ the oscillations

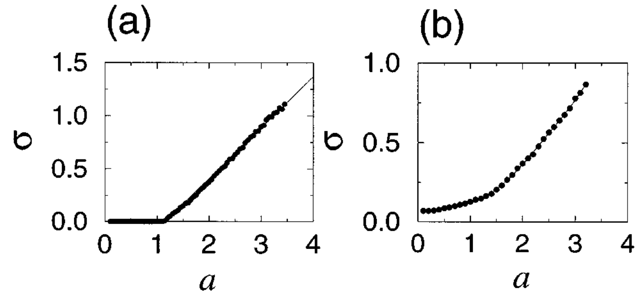

FIG. 2. The dependencies of the root-mean-square value of the pendulum's oscillations σ on a for (a) $\kappa(2\omega_0)/\kappa_{cr}=0.51$ and $\omega_a=0.3$ [the dependence $\sigma=0.48(a-1.1)$ is shown as a solid line], and (b) $\kappa(2\omega_0)/\kappa_{cr}(2)=1.89$ and $\omega_a=0.3$.

excited cannot practically be distinguished from those excited due to random vibration only.

The oscillation intensity is the greater, the larger is a. The dependence of the root-mean-square value of pendulum's angular deviation $\sigma=\overline{\varphi^2}^{1/2}$ on the difference between the amplitude a and its threshold value a_{cr} is found to be close to linear [Fig. 2(a)]. So we see that the low-frequency vibration initiates the noise-induced phase transition and the birth of the induced attractor.

It was found that the initiation of the pendulum's oscillations by a low-frequency additional action occurs via on-off intermittency. As a increases, the mean duration of laminar phases τ decreases. We have found that the dependence of τ on a [Fig. 3(a)] is in good accordance with the formula $\tau=0.44/(a^2-a_{cr}^2)$, where $a_{cr}=1.15$. We note that in the presence of additive noise on-off intermittency is not detectable and the threshold of the phase transition becomes fuzzy.

If the intensity of the suspension axis random vibration is in excess of its threshold value, i.e., $\kappa(2\omega_0)>\kappa_{cr}$, an additional low-frequency vibration significantly intensifies the noise-induced oscillations. The dependence of σ on a for $\kappa(2\omega_0)/\kappa_{cr}=1.89$ and $\omega_a=0.3$, is shown in Fig. 2(b). We see that small actions leave the pendulum's oscillations nearly unaffected. But, beginning with a certain action amplitude, the variance of the pendulum's oscillations rises steeply.

The initiation of noise-induced pendulum's oscillations and a noise-induced phase transition can be considered as

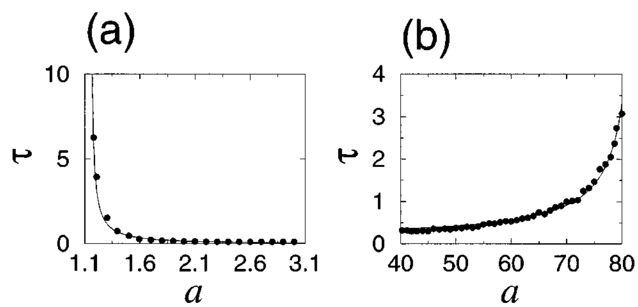

FIG. 3. The dependencies of the mean duration of ''laminar'' phases τ on the action amplitude a for the initiation (a) and suppression (b) of oscillations. The parameters are (a) $\epsilon=0.002$, $\kappa(2\omega_0)/\kappa_{cr}=0.51$, and $\omega_a=0.3$; and (b) $\epsilon=0.002$, $\kappa(2\omega_0)/\kappa_{cr}=5.6$, and $\omega_a=20$. The solid line shows the dependence $\tau=0.44/(a^2-a_{cr}^2)$ and $a_{cr}=1.15$ (a), and $\tau=1900/(a_{cr}^2-a^2)$ and $a_{cr}=83.5$ (b).

some analog of the well-known phenomenon of asynchronous excitation of self-oscillations [16–19], although there exist essential distinctions. Let us demonstrate this effect by the example of a generator with hard excitation and a harmonic external force described by the equation

$$\ddot{x}+(\eta-\alpha x^2+\beta x^4)\dot{x}+\omega_0^2 x=\omega_0^2 B\cos\omega t. \tag{2}$$

If the frequency of the external force is far from its resonance value then it is convenient to substitute into Eq. (2).

$$x=F\cos\omega t+y, \tag{3}$$

where $F=\omega_0^2 B/(\omega_0^2-\omega^2)$. In doing so we obtain the following equation for the variable y:

$$\ddot{y}+\omega_0^2 y=-[\eta-\alpha(y+F\cos\omega t)^2+\beta(y+F\cos\omega t)^4]\times(\dot{y}-F\omega\sin\omega t). \tag{4}$$

Setting $y=C\cos(\omega_0 t+\psi)$, for C and ψ we obtain the following truncated equations:

$$\dot{C}=-\frac{1}{2}\left(\eta-\frac{\alpha}{4}(C^2+2F^2)+\frac{\beta}{8}(C^4+6C^2F^2+3F^4)\right)C,$$

$$\dot{\psi}=0. \tag{5}$$

Hence the external force influences not only linear but nonlinear friction as well. The condition for asynchronous excitation of self-oscillations is obtained from Eq. (5):

$$\frac{\alpha F^2}{2}-\frac{3\beta F^4}{8}-\eta>0. \tag{6}$$

If the amplitude of the external force is fixed, then inequality (6) determines the region of relative mistunings $\Delta=(\omega_0^2-\omega^2)/\omega_0^2$ for which asynchronous excitation occurs:

$$\Delta_1^2<\Delta^2<\Delta_2^2, \tag{7}$$

where

$$\Delta_{1,2}^2=\frac{3\beta}{2\alpha}B^2\left[1\pm\left(1-\frac{6\beta\eta}{\alpha^2}\right)^{1/2}\right]^{-1}.$$

Consequently, asynchronous excitation is possible only for $\eta<\alpha^2/6\beta$, i.e., only for sufficiently small values of the coefficient η.

For mistunings $|\Delta|<\Delta_1$, only forced oscillations with frequency ω can exist, whereas for $|\Delta|>\Delta_2$, depending on the initial conditions, either forced oscillations or beats exist. We see from inequality (7) that the condition of asynchronous excitation of self-oscillation is independent of the sign of mistuning, whereas for the pendulum this effect can take place only for low frequencies of the external action.

B. Suppression of the pendulum's oscillations by a high-frequency harmonic action

If the frequency of additional harmonic action is sufficiently high, then a suppression of noise-induced pendulum's oscillations occurs instead of their intensification. From re-

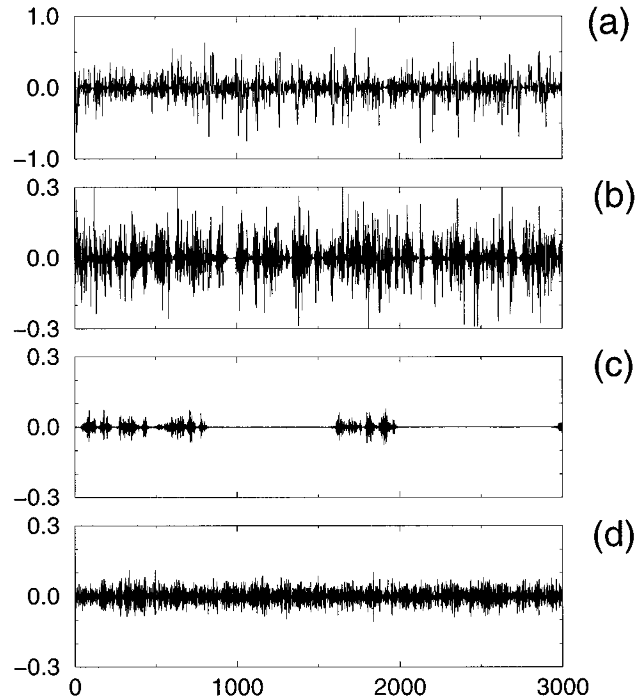

FIG. 4. The dependencies of $\varphi(t)$ for $\kappa(2\omega_0)/\kappa_{\rm cr}=5.6$ and $\omega_a=20$; (a) $k=0$ and $a=10$, (b) $k=0$ and $a=30$, (c) $k=0$ and $a=70$, and (d) $k^2\overline{\xi_2^2}=0.05\overline{\xi_1^2}$ and $a=70$. It is seen that in the presence of additive noise the pendulum's oscillations cannot be suppressed entirely.

sults of numerical simulation of Eq. (1) for $\omega_a>2\omega_0$, we see that, for small amplitudes of the high-frequency action, this action has little or no effect on the existing noise-induced oscillations (Fig. 4). As the action amplitude increases, the intensity of the noise-induced oscillations decreases rapidly, and in the absence of additive noise for a certain value of the action amplitude the oscillations are suppressed entirely. For example, for $\kappa(2\omega_0)/\kappa_{\rm cr}=5.6$ and $\omega_a=20$, this amplitude value is equal to 83.5. It is evident that in the presence of additive noise, the noise-induced pendulum's oscillations cannot be entirely suppressed, but the suppression can be impressive [Fig. 4(d)].

We can see from Fig. 4 that the suppression of noise-induced oscillations, like their excitation in the absence of an external action and initiation by a low-frequency action, occurs via on-off intermittency. As the action amplitude increases, the duration of ''laminar'' phases also increases. The dependence of the mean duration of ''laminar'' phases τ on the action amplitude a found numerically for $\epsilon=0.002$, $\kappa(2\omega_0)/\kappa_{\rm cr}=6.25$ is shown in Fig. 3(b). The solid line shows the dependence $\tau=1900/(a_{\rm cr}^2-a^2)$ and $a_{\rm cr}=83.5$. We see that this dependence fits the experimental data rather well.

The dependencies of the intensity of the pendulum's oscillations on the amplitude and frequency of the action are shown in Fig. 5. It is seen from Fig. 5(a) that in the case of moderately high frequencies of the action the intensity of noise-induced pendulum's oscillations with increasing action amplitude first decreases to a certain minimal value and then increases; for sufficiently large amplitudes of the action it can become even more than in the absence of the action. It

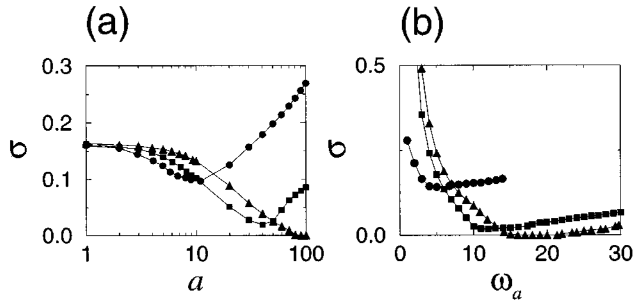

FIG. 5. (a) The dependencies of σ on a for $\omega_0=1$, $\beta=0.1$, $\alpha=100$, $\kappa(2\omega_0)/\kappa_{\rm cr}=5.6$, $\omega_a=3.5$ (circles), $\omega_a=11$ (squares), and $\omega_a=20$ (triangles). (b) The dependencies of σ on ω_a for $a=3$ (circles), $a=40$ (squares), and $a=80$ (triangles).

should be noted that this minimal value is smaller the greater the action frequency is, and it is attained for larger amplitudes of the action, the higher the action frequency is. If the action frequency is sufficiently high, then, as indicated above, a complete suppression occurs.

In the case of a change of the frequency of the action for a fixed value of its amplitude [Fig. 5(b)], the intensity of the noise-induced pendulum's oscillations with increasing action frequency first decreases to a certain minimal value, which is smaller the greater the action amplitude is, and then increases. This minimal value is attained for higher frequencies of the action, the larger its amplitude is. Thus we can conclude that there exist some optimal value of the controlling parameter (either a or ω_a) for which the suppression is most effective.

Similar to the initiation of the pendulum's oscillations by means of an additional harmonic action, the suppression of these oscillations is much different from the known phenomenon of asynchronous suppression (or quenching) of self-oscillations [18,19]. Let us consider a van der Pol generator with a harmonic external force described by the equation

$$\ddot{x}-\mu(1-\alpha x^2)\dot{x}+\omega_0^2 x=\omega_0^2 B\cos\omega t. \tag{8}$$

By substituting Eq. (3) into Eq. (8), for y we obtain the equation

$$\ddot{y}+\omega_0^2 y=\mu[1-\alpha(F\cos\omega t+y)^2](\dot{y}-F\omega\sin\omega t). \tag{9}$$

Setting $y=C\cos(\omega_0 t+\psi)$, for C and ψ we obtain the truncated equations

$$\dot{C}=\frac{\mu}{2}\left(1-\frac{C^2}{A_0^2}-\frac{2F^2}{A_0^2}\right)C, \tag{10}$$

$$\dot{\psi}=0,$$

where $A_0=2/\sqrt{\alpha}$ is the amplitude of free self-oscillations. Equation (10) shows that under the action of an external force a decrease in the effective increment of the self-oscillation component occurs. If the external force amplitude exceeds a certain critical value

$$B_{\rm cr}=A_0\frac{|\omega_0^2-\omega^2|}{\sqrt{2}\omega_0^2},$$

then the self-oscillations become suppressed entirely, and only forced oscillations at the frequency ω remain. The amplitude of these oscillations is smaller the greater the absolute value of the mistuning $\omega^2-\omega_0^2$ is.

We note that the suppression of self-oscillations in a system under an asynchronous external action occurs only when cubic nonlinearity, resulting in the appearance of nonlinear positive friction, plays a dominant role. It is important that this effect is independent of the sign of the frequency mistuning; whereas for the pendulum the suppression of noise-induced oscillations is possible only for sufficiently high frequencies of the external action.

To conclude this section, we emphasize that the intensification and suppression of a noise-induced pendulum's oscillations can be obtained by means of both multiplicative and additive harmonic action. In the last case the efficiency of the action is essentially higher. For example, to increase σ from 0.18 to 0.58, the amplitude of the low-frequency action ($\omega_a=0.3$) should be equal to 1, if it is additive, but 7.8, if it is multiplicative. Similarly, the decrease of σ from 0.18 to 0.13 due to a high-frequency action ($\omega_a=20$) can be obtained if its amplitude is equal to 1 in the case of the additive action, and to 8 in the case of the multiplicative one.

III. SYNCHRONIZATION OF THE PENDULUM'S OSCILLATIONS BY AN EXTERNAL HARMONIC FORCE

In a certain range of action frequencies, a synchronization of pendulum's oscillations takes place in the sense that the mean frequency of noise-induced pendulum's oscillations becomes close to the action frequency. Such an approach to the problem of synchronization was used in Refs. [20–22], where the effects of phase and frequency locking in chaotic systems have been studied numerically. Here we demonstrate that synchronization of such a kind can be observed in systems with noise-induced oscillations as well.

We can explain the occurrence of synchronization and its character analytically. In the case of an additive harmonic action without additive noise, the equation of pendulum's oscillations can be written as

$$\ddot{\varphi}+2\beta(1+\alpha\dot{\varphi}^2)\dot{\varphi}+\omega_0^2[1+\xi_1(t)]\sin\varphi=\omega_0^2 B\cos\omega t. \tag{11}$$

Setting $\varphi\approx A\cos(\omega t+\phi)$ and solving Eq. (11) approximately by the Krylov-Bogolyubov method, we obtain the following truncated equations for the amplitude A and phase ϕ:

$$\dot{A}=\frac{\omega^2 K_{1\omega}}{4}\left(\eta_\omega-\frac{3\beta\alpha}{K_{1\omega}}A^2\right)A-\frac{\omega B}{2}\sin\phi+\frac{\omega}{2}A\zeta_1(t), \tag{12}$$

$$\dot{\phi}=\Delta_0+\omega M_\omega-\tfrac{1}{16}\omega A^2-\frac{\omega B}{2A}\cos\phi+\omega\zeta_2(t), \tag{13}$$

where $\eta_\omega=1-4\beta/\omega^2 K_{1\omega}$, $K_{1\omega}=\kappa(2\omega)/2$, $\zeta_1(t)$ is a random process with zero mean value and intensity $K_{1\omega}$, $\Delta_0=(\omega_0^2-\omega^2)/2\omega$ is the frequency mistuning, $M_\omega=\overline{\langle\xi\cos^2(\omega t+\phi)\rangle}$, and $\zeta_2(t)$, much like to $\zeta_1(t)$, is a random process with zero mean value and the intensity

$K_{2\omega}=[\kappa(0)+K_{1\omega}]/4$. The value of M_ω can be calculated in a similar way to that in Refs. [1,14].

If the action amplitude is sufficiently small, then the synchronization region is narrow. In this case the phase varies slowly, and we can assume that the oscillation amplitude attains a steady-state value considerably faster than the phase, and its steady-state value is close to that without harmonic action [18]. Then Eq. (13) can be rewritten in the form

$$\dot{\phi}=\Delta(A_0^2)-\Delta_s(A_0)\cos\phi+\omega\zeta_2(t), \tag{14}$$

where $\Delta(A_0^2)=\Delta_0+\omega M_\omega-\frac{1}{16}\omega A_0^2$ is the effective frequency mistuning depending on the steady-state value of amplitude A_0 in the absence of an additional action, and $\Delta_s(A_0)=\omega B/2A_0$ is the effective half-width of the synchronization region which also depends on A_0.

Equation (14) has the same form as the equation describing the phase evolution of a van der Pol generator driven by an external harmonic force in the presence of additive noise [23,24,18]. The only distinction is that Δ and Δ_s here depend on A_0. Therefore the results obtained in Refs. [23,24,18] must be averaged over the amplitude A_0 with using the steady-state probability distribution found in Refs. [1,2]. As a result, the region of the frequency mistunings $\Delta_0(\varepsilon)$, within which the mean frequency of the pendulum's oscillations differs from the action frequency by a value smaller than ε, can be found. We call this region of the frequency mistuning synchronization region.

Numerically the mean frequency is calculated as $\Omega=\omega+\langle\dot{\phi}\rangle\equiv\langle\dot{\psi}\rangle$, where the instantaneous phase ψ is determined by means of the analytical signal approach based on the Hilbert transform (the description of the technique and references can be found in Refs. [20,22]).

In Fig. 6(a), the differences between the mean frequency of pendulum's oscillations Ω and the frequency of external force ω are plotted versus ω for different values of the force amplitude B. This shows that, if B is large enough, then $\Omega\approx\omega$ in a certain range of ω, i.e., we can speak about frequency entrainment. These dependencies are similar to those known for synchronization of the van der Pol generator in the presence of noise [23,24,18]. As already noted, we assume a system to be synchronized if $|\Omega-\omega|<\varepsilon$, where ε is a certain small value. In this way we obtain the synchronization regions in the plane (ω,B) for different values of the intensity of the suspension axis random vibration. These synchronization regions are illustrated in Fig. 7. We see that, as the random vibration intensity increases, the synchronization regions are shifted to the lower-frequency region, and the threshold value of the force amplitude increases.

It should particularly be emphasized that within the synchronization region the oscillations of the pendulum remain irregular, i.e., only the frequency of these oscillations is partially entrained by the external action, whereas their amplitude remains random. This is demonstrated in Fig. 8, where an example of the time dependence of φ in synchronous regime (a) and the dependence $\varphi(t)$ in the absence of the external action (b) are given. Comparing Figs. 8(a) and 8(b) we see that the external force intensifies the pendulum's oscillations and makes them more ordered.

We note that in contrast to synchronization of chaotic systems [20,22], frequency locking of the pendulum's oscillations occurs only for rather strong external action (the amplitude of the external force is of the same order or larger than the mean amplitude of the pendulum's oscillations in the absence of the action). It is very interesting that, as dif-

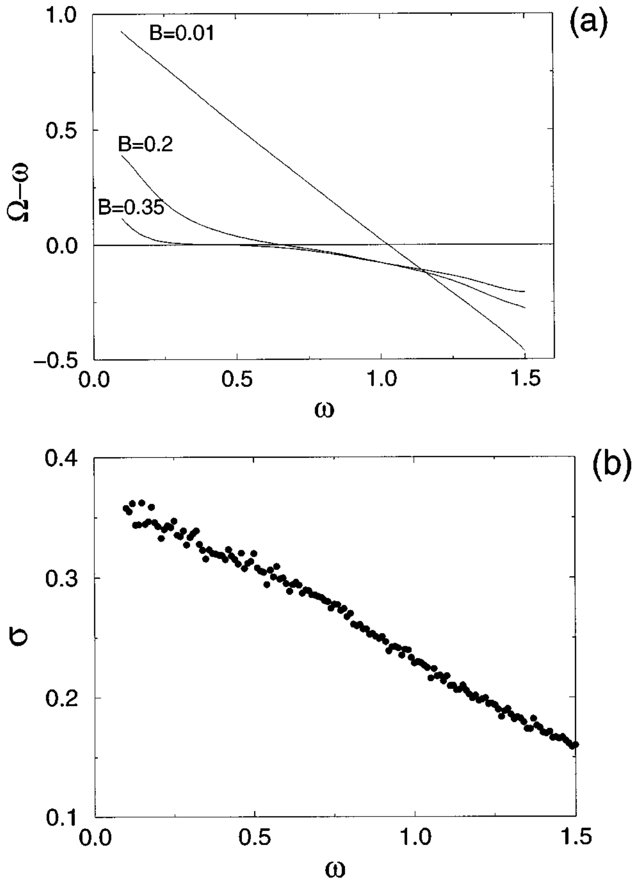

FIG. 6. (a) The difference between the mean frequency of the pendulum's oscillations Ω and the external force frequency ω vs ω for different values of the force amplitude B (the values of B are indicated next to the corresponding curve). (b) The plot of σ vs ω.

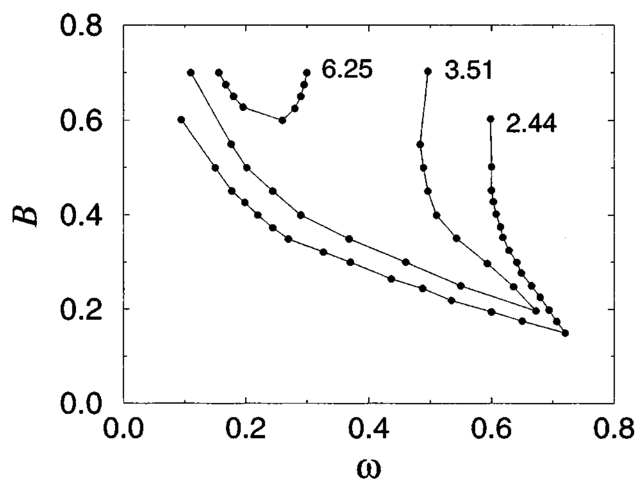

FIG. 7. The synchronization regions for different values of excess of the noise intensity over its critical value [the values of $\kappa(2\omega_0)/\kappa_{cr}$ are indicated next to the corresponding curves]. $\varepsilon=5\times10^{-3}$.

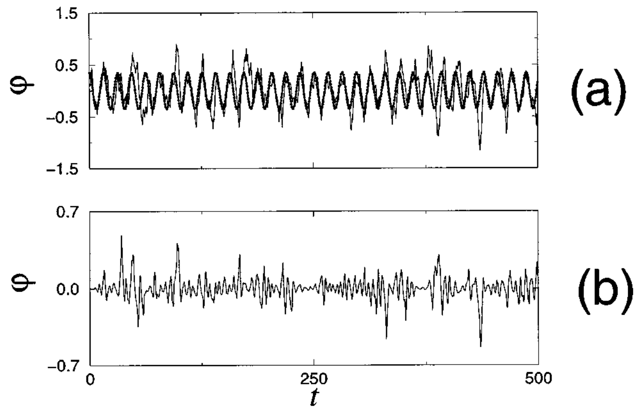

FIG. 8. (a) The time dependence of φ in a center of the synchronization region ($\omega=0.4$) for $\kappa(2\omega_0)/\kappa_{cr}=2.44$ and $B=0.35$ along with the external force. (b) The time dependence of φ for $\kappa(2\omega_0)/\kappa_{cr}=2.44$ in the absence of an additional action.

ferentiated from the ordinary synchronization of self-oscillatory systems, the intensity of oscillations at the center of synchronization region has no peak, but decreases monotonically as the external force frequency increases [see Fig. 6(b)].

IV. CONCLUSIONS

To summarize, we have studied the response of a pendulum with a randomly vibrating suspension axis to a harmonic external action, both multiplicative and additive. We have found two effects: a low-frequency action results in intensification, or even initiation, of the noise-induced pendulum's oscillations, whereas a high-frequency action suppresses them. These phenomena can be considered as analogies of classical effects of asynchronous excitation and quenching; however, there are essential differences. An interesting finding is that the initiation and suppression of the pendulum's oscillations, much like their excitation in the absence of an external action, occurs via on-off intermittency.

In a certain range of the action frequencies, a synchronization of the noise-induced pendulum's oscillation takes place. The mean frequency of these oscillations is entrained by the external force, while the amplitude remains random. In this sense the behavior of the system with noise-induced attractor resembles that of chaotic oscillators. This phenomenon is justified by the analysis of the truncated equation for the phase of oscillations. We conclude that intensity and frequency of noise-induced oscillations can be effectively controlled by an additional harmonic action.

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