Theory of magnetic field generation by relativistically strong laser radiation

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We consider the interaction of subpicosecond relativistically strong short laser pulses with an underdense cold unmagnetized electron plasma. It is shown that the strong plasma inhomogeneity caused by laser pulses results in the generation of a low frequency (quasistatic) magnetic field. Since the electron density distribution is determined completely by the pump wave intensity, the generated magnetic field is negligibly small for nonrelativistic laser pulses but increases rapidly in the ultrarelativistic case. Due to the possibility of electron cavitation (complete expulsion of electrons from the central region) for narrow and intense beams, the increase in the generated magnetic field slows down as the beam intensity is increased. The structure of the magnetic field closely resembles that of the field produced by a solenoid; the field is maximum and uniform in the cavitation region, then it falls, changes polarity and vanishes. In extremely dense plasmas, highly intense laser pulses in the self-channeling regime can generate magnetic fields $\sim$100 MG and greater. [S1063-651X(97)12701-5]

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Recently, considerable progress has been achieved in the development of compact terawatt laser sources [1]. Such laser sources generate subpicosecond pulses of electromagnetic (EM) radiation with focal intensities $I>10^{18}$ W/cm^2. One of the most powerful Neodymium-glass laser system, ''Vulcan'' at Rutherford Appleton Laboratory, delivers 35 TW to target at an intensity of $I=10^{19}$ W/cm^2 [2] in the short pulse mode. Preliminary reports from several other centers seem quite promising, and, in the very near future, it will be possible to design petawatt laser facilities which will produce even higher intensity ($\sim10^{21-23}$ W/cm^2) pulses [3]. In the field of such strong subpicosecond pulses, it is expected that the character of the nonlinear response of the medium would radically change [4].

These far-reaching developments have naturally led to an intense theoretical and experimental investigation of the interaction of ultrashort, relativistically strong pulses with plasmas. The strong pulse fields can impart to the electron an oscillation energy which could be comparable to or even larger than its rest energy. Thus the relativistic nonlinear effect, which is basically associated with the increase in the electron mass, will tend to determine the dynamics of EM pulses. At intensities of 10^{18} W/cm^2 and higher, the relativistic nonlinearities were predicted to cause a whole set of interesting phenomena; some of them have already been confirmed by experiments [5]. The bulk of the investigations were connected with (1) electrostatic wake-field generation due to the displacement of plasma electrons from the region occupied by the laser pulse under the action of the ponderomotive force [6], and (2) the relativistic self-focusing of the laser beam itself [7,8].

Among the various nonlinear effects which may occur in a plasma interacting with strong laser pulses, the generation of quasistatic magnetic fields (QSM's) is bound to be one of the most interesting and significant, particularly because the presence of these fields could have considerable influence on the overall nonlinear plasma dynamics. Although, in the recent past, much effort has been devoted to studying mechanisms leading to magnetic field generation in laser plasmas (for a review, see Ref. [9]), there still does not exist a well-established and satisfactory theory. Indeed, numerical simulations carried out by Wilks *et al.* [10] for the interaction of an ultraintense laser pulse with an overdense plasma target, predict extremely high self-generated magnetic fields $\sim$250 MG, these immense fields cannot be properly explained on the basis of existing theories. Sudan [11] suggested that the spatial gradients, and the nonstationary character of the ponderomotive force, may lie at the origin of the strong magnetic fields discovered in numerical simulations [10]. Several other analytical attempts have been made to understand the results of the simulation [12]. All these theoretical attempts use a hydrodynamical formulation. It must, however, be pointed out that the conditions prevalent in the simulation experiments (for example, the thermal speed $v_{\rm th}>v_p$, where v_p is a characteristic low-frequency phase speed) may not yield to a hydrodynamical description. The heat generated during the interaction further strengthens the above inequality as time goes on, and the transverse fields are pushed to the anomalous skin region, making it necessary to employ a kinetic treatment [13].

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The hydrodynamic treatment (which we will follow in this paper), however, can be quite adequate provided $v_p \gg v_{\rm th}$. In the problem of magnetic field generation in underdense plasmas, this condition is likely to prevail. We would like to point out here that relatively strong magnetic fields can also be generated in underdense plasmas; this has been definitively demonstrated in the numerical simulations of Askar'yan *et al.* [14], who studied the relativistic self-focusing of the laser beam in such plasmas. In Ref. [15], it was shown that due to the resonant excitation of plasma waves the generation of QSM fields occurs both in the body of the linearly polarized EM pulse, and also in its wake (region of the wakefield). The simulation, as well as experimental results, strongly indicate that the problem of the generation of QSM fields by EM pulses is ripe for a serious and careful theoretical investigation.

In the present work we deal with the generation of QSM fields by relativistically strong EM pulses propagating in an underdense plasma. Laser pulses are assumed to be short, with a time duration (T_l) less than the characteristic time for the ion response ω_i^{-1} (ω_i is the ion Langmuir frequency), so that the ion motion can be neglected. At the same time we assume that the pulse is sufficiently long, i.e., $T_l \gg \omega_e^{-1}$ (ω_e is the electron Langmuir frequency), that the complications due to the excitation of Langmuir waves are absent. For simplicity, the analysis is restricted to beams with a narrow cross section, i.e., $L_\parallel(\sim cT_l) \gg L_\perp$, where $L_\perp$ and $L_\parallel$ respectively, are the characteristic transverse and longitudinal spatial dimensions of the beam. This assumption is not particularly restrictive, and holds for the parameters pertinent to the experiment, for example, by Borisov *et al.* [5], where the propagation of relativistic high-intensity, linearly polarized pulses is explored. In fact, Ref. [5] reports the observation of self-channeled propagation of EM pulses from a subpicosecond KrF* ($\lambda = 0.248$ μm, $T_l \sim 500$ fs) excimer laser over a distance up to 2 mm which is two orders longer than the corresponding diffraction (Rayleigh) length ($\sim \pi r_0^2/\lambda$, where $r_0 \sim 3.5$ μm is an initial focal radius of the EM beam). The diameter of the channel ($\sim L_\perp$) was ~ 1 μm, and the peak intensity of the channeled radiation reached $I \sim 10^{19}$ W/cm^2. Note that the generation of QSM fields was not reported in Ref. [5].

Although it is not definite that linearly polarized pulses do not generate a magnetic field, it is likely that the effect may be small. In this paper, therefore, we concentrate on the circularly polarized pulses for which QSM fields should appear due to the inverse Faraday effect. The mechanism (originally found in Ref. [16] using a phenomenological approach) of excitation is the rotation of the polarization vector of the external radiation. In several later papers, the evolution of QSM fields was studied using the hydrodynamic approach for both the weak as well as relativistically strong pulses. The basic approach consists in using a relation which describes the conservation (at each instant) of the generalized vorticity, and then calculating a low-frequency (LF) drag current excited by the EM radiation [17–19]. However, there are several inconsistencies, and contradictions in the final expressions of the drag current obtained in these publications. We believe that these contradictions stem from the following fact: in the cold plasma limit (i.e., when the characteristic phase velocities of LF perturbations are much larger than the electron thermal velocity) the expression for the LF drag current is derived by taking the time average (over the fast scale associated with the laser frequency) of the product of two high-frequency quantities, one of which is $\sim \nabla \cdot \tilde{\mathbf{E}}$, where $\tilde{\mathbf{E}}$ is the high-frequency (HF) part of the EM field. It is well known that the laser field in a plasma is predominantly transverse ($\tilde{\mathbf{E}}_\perp$), i.e., the longitudinal field $\tilde{\mathbf{E}}_\parallel \ll \tilde{\mathbf{E}}_\perp$ for $k \gg L^{-1}$, where k is the wave number and L is the characteristic spatial spread of the pulse. Since $\nabla \cdot \tilde{\mathbf{E}} \sim \nabla_\perp \cdot \tilde{\mathbf{E}}_\perp + ik\tilde{\mathbf{E}}_\parallel$ is nearly zero (is proportional to the high-frequency density perturbation), its replacement by $\nabla \cdot \tilde{\mathbf{E}}_\perp$ (which most of these references do) can lead to a gross overestimate of the drag current. We shall derive a correct expression for the drag current for arbitrary amplitude laser pulses. We show that QSM field generation takes place due to the strong plasma inhomogeneity caused by the intense laser beam itself, and that the amplitude of the QSM field increases in the ultrarelativistic case. We also discuss the possibility of electron cavitation, and its influence on magnetic field generation.

We use Maxwell equations, which, under the above-mentioned assumptions, can be written as

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} - \frac{4\pi e}{c} n \frac{\mathbf{p}}{m\gamma}, \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{2}$$

$$\nabla \cdot E = 4\pi e(n_0 - n), \tag{3}$$

where $-e$, m, n, and $\mathbf{p}$ are the electron charge, mass, density, and momentum, respectively, c is the speed of light, n_0 is the ion background density, and $\gamma = (1 + \mathbf{p}^2/m^2c^2)^{1/2}$ is the relativistic factor.

The motion of the cold unmagnetized electron fluid is described by the standard relativistic hydrodynamic equations. These consist of the equation of motion

$$\frac{\partial \mathbf{p}}{\partial t} = mc^2 \nabla \gamma = -e\mathbf{E}, \tag{4}$$

and the continuity equation

$$\frac{\partial n}{\partial t} + \nabla \cdot \left(n \frac{\mathbf{p}}{m\gamma} \right) = 0. \tag{5}$$

The absence of the magnetic part of the Lorentz force in Eq. (4) is due to the assumption that the generalized vorticity is zero in the body of the electron fluid; this assumption relates the magnetic field with the electron momentum (the London equation of superconductivity)

$$B = \frac{c}{e} \nabla \times p. \tag{6}$$

For laser plasma interactions, the hydrodynamic equations in this form were displayed in Ref. [20];a more complete discussion can be found in Ref. [21]. Equations (4)–(6) are in an extremely convenient form for further manipulation. Substituting $\mathbf{E}$ from Eq. (4) into Eqs. (1) and (3), and using

Eq. (6), we obtain our first equation relating $\mathbf{p}$ (γ is just a function of $\mathbf{p}^2$) and the density n,

$$c^2\nabla\times\nabla\times\mathbf{p}+\frac{\partial^2\mathbf{p}}{\partial t^2}+mc^2\frac{\partial}{\partial t}\nabla\gamma+\omega_e^2\frac{n}{n_0}\frac{\mathbf{p}}{\gamma}=0. \quad (7)$$

The second equation relating n and $\mathbf{p}$ is derived by a combination of Eqs. (3) and (4),

$$\frac{n}{n_0}=1+\frac{1}{m\omega_e^2}\left(\frac{\partial}{\partial t}\nabla\cdot\mathbf{p}+mc^2\Delta\gamma\right). \quad (8)$$

Equations (7) and (8) are a closed set to which the system of Maxwell and hydrodynamical equations has been reduced [22]. Note that this very set of equations was derived in our recent publication [23] dealing with the problem of wakefield generation in semiconductor plasmas. Before proceeding further, it is worthwhile to remark that the continuity equation (5) was totally ignored in the derivation of Eqs. (7) and (8). It is evident, however, that Eq. (5) is not really independent, and is readily derived by taking the divergence of Eq. (7) and using Eq. (4). In fact, any two of the set of Eqs. (5), (7), and (8) can be used as independent equations for n and $\mathbf{p}$. Our goal in this paper is to calculate the relatively slow-varying (quasistatic) magnetic field induced by a specified high-frequency laser pulse. In response to the laser field, all the fields in the plasma will contain both slow and fast time dependences (with characteristic time $\tau\sim\omega^{-1}$). Therefore, we may decompose each of the variables $A\equiv(\mathbf{E},\mathbf{B},\mathbf{p},n,\gamma)$ into averaged and varying parts,

$$A=\langle A\rangle+\tilde{A} \quad (9)$$

where the brackets $\langle\ \rangle$ denote averaging over the time interval τ. With this prescription, the averaged equation for $\langle n\rangle$ and $\langle\mathbf{p}\rangle$ becomes

$$\frac{\langle n\rangle}{n_0}=1+\frac{1}{m\omega_e^2}\left(\frac{\partial}{\partial t}\nabla\cdot\langle\mathbf{p}\rangle+mc^2\Delta\langle\gamma\rangle\right) \quad (10)$$

and

$$c^2\nabla\times\nabla\times\langle\mathbf{p}\rangle+\frac{\partial^2}{\partial t^2}\langle\mathbf{p}\rangle+mc^2\frac{\partial}{\partial t}\nabla\langle\gamma\rangle+\omega_e^2\frac{\langle n\rangle}{n_0}\frac{\langle\mathbf{p}\rangle}{\langle\gamma\rangle}=-\frac{\omega_e^2}{n_0\langle\gamma\rangle}\langle\tilde{n}\tilde{\mathbf{p}}\rangle \quad (11)$$

where it is assumed (to be justified below) that $\langle\gamma\rangle=\gamma$. The averaged Eq. (6),

$$\langle\mathbf{B}\rangle=\frac{c}{e}\nabla\times\langle\mathbf{p}\rangle, \quad (12)$$

allows us to relate the generated magnetic field with the averaged momentum. The electric field of the HF radiation can be written in the form

$$\tilde{\mathbf{E}}=[(\mathbf{x}+i\mathbf{y})E_\perp(\mathbf{r},t)+\mathbf{z}E_\parallel(\mathbf{r},t)]\exp(-i\omega t+ikz)+\text{c.c.}, \quad (13)$$

where the transverse ($E_\perp$) and longitudinal ($E_\parallel$) amplitudes are slowly varying.

Since we are using $\tilde{\mathbf{p}}$, rather than $\tilde{\mathbf{E}}$ as our dynamical variable, let us find the corresponding expression for $\tilde{p}$. It can be shown that if $r_E/\lambda\ll1$, where r_E is a characteristic displacement of the oscillating electrons due to the HF field, and λ is the EM field wavelength, the relation between $\tilde{\mathbf{p}}$ and $\tilde{\mathbf{E}}$ has the form

$$\frac{\partial\tilde{\mathbf{p}}}{\partial t}=-e\tilde{\mathbf{E}}, \quad (14)$$

which, coupled with Eq. (13), yields

$$\tilde{\mathbf{p}}=[(\mathbf{x}+i\mathbf{y})p_\perp(\mathbf{r},t)+\mathbf{z}p_\parallel(\mathbf{r},t)]\exp(-i\omega t+ikz)+\text{c.c.} \quad (15)$$

after a simple integration over the fast time (ω^{-1}). The slowly varying amplitudes (kept constant during the integration) $p_\perp$ and $p_\parallel$, are given by

$$p_\perp=-\frac{ie}{\omega}E_\perp,\quad p_\parallel=-\frac{ie}{\omega}E_z. \quad (16)$$

Our next order of business is to evaluate the driving term proportional to $\langle\tilde{n}\,\tilde{p}\rangle$ in Eq. (11). For this we must begin by deriving an expression for $\tilde{n}$ in terms of $\tilde{p}_\perp$, which we are assuming is a ''given'' quantity. We could use the high-frequency version of either Eqs. (5) or (8) to accomplish this. We choose to use Eq. (8) primarily to show, in a very perspicuous manner, how our treatment differs from, and corrects earlier treatments. From Eq. (8), we find (γ has only an averaged part)

$$\frac{\tilde{n}}{n_0}=\frac{1}{m\omega_e^2}\frac{\partial}{\partial t}(\nabla\cdot\tilde{\mathbf{p}}), \quad (17)$$

Since $\nabla\cdot\tilde{\mathbf{p}}$ for a basically transverse wave is very small, extreme care must be taken in its evaluation. It is conventional to replace $\nabla\cdot\tilde{\mathbf{p}}$ by $\nabla\cdot\tilde{\mathbf{p}}_\perp$ because $|p_\parallel|$ is much smaller than $|p_\perp|$. This, in our opinion, is a serious mistake. Although $|p_\parallel|\ll|p_\perp|$, $|\nabla\cdot\tilde{\mathbf{p}}_\perp|\sim|p_\perp|/R$ may be (and is) of the same order as $|\nabla\cdot\tilde{\mathbf{p}}_\parallel|\sim k|p_\parallel|$ because $kR\gg1$, where R is the transverse scale length associated with the laser pulse. Replacing $\nabla\cdot\tilde{\mathbf{p}}$ with $\nabla\cdot\tilde{\mathbf{p}}_\perp$ results in a gross overestimate of $\tilde{n}$ and hence of the driving term. There is a general lesson to be learned here: whenever the end results depend on $\nabla\cdot\tilde{\mathbf{E}}(\sim\nabla\cdot\tilde{\mathbf{p}})$, as they do in the magnetic field generation problem, one must not neglect the contributions from $p_\parallel$, and one must find an appropriate (generally indirect) way of evaluating this small quantity.

We now calculate $\nabla\cdot\tilde{\mathbf{p}}$ by taking the divergence of the high frequency version of Eq. (7), and obtain, for a transparent plasma ($\omega>\omega_e$) [it should be mentioned that relation (18) is the relativistic version of well-known equation from the continuous media electrodynamics $\epsilon\nabla\cdot\mathbf{E}=-(\mathbf{E}\cdot\nabla)\epsilon$, where ϵ is the dielectric permittivity of a medium]

$$\nabla\cdot\tilde{\mathbf{p}}=\frac{\omega_e^2}{\omega^2}\tilde{\mathbf{p}}_\perp\cdot\nabla_\perp\left(\frac{\langle n\rangle}{n_0\gamma}\right), \quad (18)$$

where we have used the fact that, for circularly polarized radiation, the particle energy does not depend on the ''fast'' time, and there is no generation of high harmonics of the EM field (note that effects of high harmonic generation, which take place due to the longitudinal part of the HF field, are negligibly small). This indeed is the reason for the equality $\gamma=\langle\gamma\rangle$, which is approximately given by

$$\gamma=\left(1+\frac{|p_\perp|^2}{m^2c^2}\right)^{1/2}. \tag{19}$$

The LF drag current, which appears in the right-hand side of Eq. (11), can now be computed using Eqs. (17) and (18). One can already see that for circularly polarized radiation, it is nonvanishing provided the quantity $\langle n\rangle\gamma^{-1}$ depends on the radial variable $r_\perp$. When a radially inhomogeneous beam propagates in an initially uniform plasma, the ponderomotive force of the EM radiation ($\sim\nabla\gamma$) pushes out the plasma electrons from the region of its localization, and creates an effective plasma density inhomogeneity. Because $\langle n\rangle$ and γ have the same characteristic radial scale lengths, their contributions in the creation of the drag current are equally important. At this junction it is worthwhile to mention that if the EM beam has a spatially constant amplitude, an initial inhomogeneity of plasma density will be required. Thus in a homogeneous plasma, contrary to a statement made in Ref. [17], the constant amplitude EM beam cannot generate the QSM field by ''magnetization currents.'' It was shown in Ref. [24] that the physical reason for the absence of the QSM field generating source in the homogeneous case is the mutual compensation of the circular electron currents.

Now, for simplicity, we consider an axisymmetric electromagnetic pulse propagating along the z axis: $p_\perp=p_\perp(r,z-v_gt,t)$, where $v_g=c(1-\omega_e^2/\omega^2)^{1/2}$ is the group velocity of the laser radiation. Using Eqs. (17)–(19), the ϕ component of Eq. (11) can be written as

$$\frac{2}{c}\frac{\partial^2\langle p_\phi\rangle}{\partial\xi\partial\tau}+\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial\langle p_\phi\rangle}{\partial r}-\left(\frac{\langle n\rangle}{n_0}\frac{k_e^2}{\gamma}+\frac{1}{r^2}\right)\langle p_\phi\rangle$$
$$=-2\frac{\omega}{mc^2}\frac{\omega_e^2}{\omega^2}\frac{|p_\perp|^2}{\gamma^2}\left[\frac{\partial}{\partial r}\left(\frac{\langle n\rangle}{n_0}\right)-\left(\frac{\langle n\rangle}{n_0}\right)\frac{\partial}{\partial r}\ln\gamma\right], \tag{20}$$

where $\xi=z-v_gt$ and $k_e=\omega_e/c$.

For a narrow laser beam, within the approximations used in this paper, Eq. (8) can be approximated by

$$\frac{\langle n\rangle}{n_0}=1+\frac{1}{k_e^2}\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial\gamma}{\partial r}\right). \tag{21}$$

It is now clear that, using Eqs. (19) and (21), Eq. (20) can be viewed as an inhomogeneous differential equation (the driving term is fully known because $p_\perp$ is supposed to be specified) for $\langle p_\phi\rangle$. If we can solve Eq. (20) for $\langle p_\phi\rangle$, then the required components of the QSM field are readily determined by

$$\frac{e}{c}B_z=\frac{1}{r}\frac{\partial}{\partial r}r\langle p_\phi\rangle, \tag{22}$$

$$\frac{e}{c}B_r=-\frac{\partial}{\partial z}\langle p_\phi\rangle. \tag{23}$$

The explicit form for the driving term (drag current) in Eq. (20) allows us to reexpress the discussion following Eq. (17) in clearer terms: For the nonrelativistic case ($\mathbf{p}^2\ll m^2c^2$), an equation similar to Eq. (20) was derived in Ref. [19]. However they used the relation $\nabla\cdot\tilde{\mathbf{p}}\sim\nabla\cdot\tilde{\mathbf{p}}_\perp$ and, consequently, the source term came out to be proportional to $|p_\perp|^2$. This is in marked contrast to our result; the nonrelativistic limit of our source term is, in fact, proportional to $|p_\perp|^4$, because both the terms in the square brackets of the right hand side of Eq. (20) are also proportional to $|p_\perp^2|^2/m^2c^2\ll1$. Thus the magnetic field strength will be considerably smaller than what was found in Ref. [19].

We mentioned earlier that if the pulse amplitude $p_\perp$, and therefore γ, has a strong space dependence, then the inhomogeneity of $\langle n\rangle\gamma^{-1}$ will always lead to the generation of QSM fields. Thus the system of Eqs. (20)–(23) with Eq. (19) is an acceptable self-consistent model for describing the magnetic field generation process by narrow relativistic short laser beams. In what follows we assume that, during the interaction time of interest, the laser beam profile remains unchanged, and can be presumed to be a Gaussian (this assumption can be justified until the pulse passes the self-focusing length or the Rayleigh length in the case of diffraction spreading):

$$|p_\perp|^2=p_0^2\exp\left[-\frac{r^2}{R^2}-\frac{\xi^2}{L^2}\right], \tag{24}$$

where R and L are the transverse and longitudinal dimensions of the pulse ($R\ll L$). For the pulse shape represented by Eq. (24), we find

$$N=\frac{\langle n\rangle}{n_0}=1-\frac{1}{k_e^2R^2}\frac{(\gamma^2-1)}{\gamma}\left(2-\frac{r^2}{R^2}\frac{(\gamma^2+1)}{\gamma^2}\right). \tag{25}$$

From Eq. (25) one can see that the plasma electrons are expelled from the central part of the pulse ($r\approx0$), creating a density hump away from the beam axis ($r\sim R$); with a final ($r\to\infty$) exponential decay to the equilibrium value n_0. However, as it was shown in Ref. [8], under certain conditions, the electrons can be fully expelled from the central part of the EM beam (electron cavitation). To derive this condition, let us first define a critical radius

$$R_c^2=\frac{1}{k_e^2}\frac{2(\gamma_0^2-1)}{\gamma_0}, \tag{26}$$

where $\gamma_0=\gamma(r=0)=(1+p_0^2/m^2c^2)^{1/2}$. For $R>R_c$, Eq. (25) reveals that $N>0$ for all r, and, consequently, the electron cavitation does not occur. However if $R=R_c$, the density does vanish at $r=0$. Thus R_c defines the minimum beam radius for the beginning of cavitation. It is evident that, within the framework of the current model equations (which are being widely exploited for the problem of relativistic self-focusing of EM beams), one cannot prevent the occurrence of unphysical, negative values for the electron density when $R<R_c$. This failure of the hydrodynamical model of a plasma is generally corrected by putting $N=0$ in the entire spatial region where $N<0$ [8]. For the current paper, we will follow this arbitrary, though workable, ansatz. In future, more detailed work, we will examine whether this unphysical feature is basic to the hydrodynamical description or is a consequence of the approximations made. [It has also been argued (Ref. [25]) that, in the case of cavitation, an alternative to the fluid models, the particle in cell (PIC) technique should be used.]

Algebraic complications prohibit a general analytical solution of Eq. (20). In the nonrelativistic case ($p_0^2 \ll m^2c^2$), Eq. (20) reduces to an equation which can be solved by taking a Fourier-Bessel transform. However, the fields produced are uninterestingly small, and are not presented here. The interested reader can consult Ref. [19], remembering that they have overestimated the fields by a factor $m^2c^2/p_\perp^2 \gg 1$.

Concentrating on the relativistic case, we first neglect the first term on the left-hand side (because $c^{-1}\partial^2/\partial\tau\partial\xi \ll \partial^2/\partial r^2$) of Eq. (20), arriving at an ordinary differential equation in r. For this ordinary differential equation, we can obtain an analytical solution in two different limits, for arbitrary amplitudes. Indeed, in the limit of a smoothly inhomogeneous laser beam, $k_e^2R^2 \ll \gamma_0$, for which the electron cavitation does not occur, Eq. (20) yields (derivative terms are neglected)

$$\langle p_\phi \rangle = 2\frac{mc^2}{\omega}\frac{(\gamma^2-1)}{\gamma}\frac{\partial}{\partial r}\ln\left(\frac{N}{\gamma}\right). \tag{27}$$

The profile for the magnetic field $B_z(r)$ can be calculated using Eqs. (22) and (27); $B_z(r)$ has a maximum on the beam axis, then it decreases with increasing r, changing polarity near the beam edge ($\sim R$), and decaying rapidly to zero when $r \to \infty$. The central maximum can be conveniently expressed as

$$\Omega_c(0) = 4\frac{\omega_e^2}{\omega}\frac{\gamma_0}{k_e^2R^2}\left(1-\frac{1}{\gamma_0^2}\right)^2, \tag{28}$$

where $\Omega_c(r) = eB_z(r)/mc$ is an effective cyclotron frequency. Remembering that Eq. (28) is valid only for $\gamma_0 \ll k_e^2R^2$, we conclude that $\Omega_c < \omega_e$ even in the relativistic case.

Note that the final value of Ω_c [Eq. (28)] does not depend on the equilibrium plasma density n_0. However, for this calculation to be valid, certain ($\omega > \omega_e$, $\gamma_0 \ll k_e^2R^2$) constraints on the density have to be imposed. Let us now estimate the strength of the magnetic field for a relativistically strong pulse. For this purpose we choose the wavelength and intensity in the experimentally relevant range (see Borisov *et al.* in Ref. [8]), $\lambda = 0.248$ μm, $I = 1.3\times10^{20}$ W/cm^2 ($\gamma_0 = 2$). For representative values of $R = 1$ μm to -3 μm, the maximum value of the magnetic field is found to be 3–0.4 MG. Corresponding plasma densities needed to satisfy the aforementioned constraint must lie in the range $n_0 = 5\times10^{19}$–10^{21} cm^{-3} for $R = 3$ μm, and $n_0 = 5\times10^{20}$–10^{21} cm^{-3} for $R = 1$ μm.

In the opposite case of a narrow pulse, $K_e^2R^2 \ll 1$, the differential term dominates, and Eq. (20) can be readily integrated to give

$$\Omega_z(r) = 2\frac{\omega_e^2}{\omega}\int_r^\infty dr'\left(\frac{\gamma^2-1}{\gamma}\right)\frac{\partial}{\partial r'}\left(\frac{N}{\gamma}\right), \tag{29}$$

from which, with the aid of Eq. (25), one can obtain the radial structure of $B_z(r)$ for given R and γ_0. However, in the relativistic case when $R_c^2k_e^2 > 1$, electron cavitation occurs ($R < R_c$). In order to incorporate this phenomena, we must put $N = 0$ for $0 < r < r_c$ where r_c is the solution of $N(r_c) = 0$, and use Eq. (25) for N for $r > r_c$. Thus, for $r < r_c$, we obtain a constant magnetic field (the source is zero). For this case it is straightforward to see that the strength of the magnetic field has a maximum on the beam axis $r = 0$, remains unchanged up to $r = r_c$, then drops down, changes polarity, and rapidly tends to zero as $r \to \infty$. This behavior closely resembles the field produced by a solenoid. Indeed, the induced current is located on the ''wall'' of the cavitating plasma cylinder with radius $r \approx r_c (< \sqrt{2}R)$; there is no current in the body of the cylinder ($r < r_c$). The magnetic field, created by this current formation, remains uniform inside the ''cylinder.'' The maximum value of $B_z = B_z(0)$ can be found from Eq. (29) by replacing the lower limit r by r_c. For $\lambda = 0.248$ μm, $\gamma_0 = 2$, $R = 3$ μm, and plasma density $n_0 = 10^{17}$ cm^{-3} ($k_e^2R^2 = 0.03$), the magnetic field comes out to be $B_z(0) \approx 0.1$ MG. We would like to emphasize that, if we were to neglect the effects of cavitation, and try to obtain $B_z(0)$ by integrating from $r = 0$, we could severely overestimate the strength of the generated magnetic field.

A caveat is in order here. For the narrow beam (with cavitation and $\gamma_0 > 1$) case, the term proportional to N/γ [in the left-hand side of Eq. (20)] is not smaller than the differential term for all r. Equation (29), therefore, should be just viewed as a very approximate indicator of the field strength and structure. For a proper and accurate evaluation, Eq. (20) should be solved numerically.

The above Eqs. (28) and (29) provide us with estimates at the focal spot area. The radiation pulse, after it has passed the focal area, either diffracts (in the case of narrow beam $R^2k_e^2 \ll 1$) or enters the self-focusing regime (for $R^2k_e^2 \gg 1$), provided that the laser radiation power exceeds the critical value [$\sim 17(\omega/\omega_e)^2$ GW]. In the latter case the beam intensity is concentrated in a narrow channel of radius $R \sim 2k_e^{-1}$. Note that the approximate solutions given by Eqs. (28) and (29) cannot be used for the evaluation of the magnetic field inside the channel, since all the terms on the left-hand side of Eq. (20) are, now, of the same order. In order to evaluate the magnetic field in the channel we have solved Eq. (20) numerically. In Fig. 1, typical behavior of $B_z(r)$ is displayed for $p_0/mc = 3$. One can see that, in the region of the electron cavitation, the magnetic field is uniform. Outside of this region, the magnetic field changes polarity, and then it vanishes.

For arbitrarily strong laser radiation, it is revealing to write down the ratio of the maximum magnetic field [$B_m = B_z(0)$] generated in the self-guiding channel to the high-frequency pulse magnetic field B_p,

$$\frac{B_m}{B_p} = \frac{\omega_e^2}{\omega^2}b(p_0^2), \tag{30}$$

where $b(p_0^2)$ is a dimensionless function indicating the departure of the calculated field from what could be expected on simple estimates. In Fig. 2 we plot this important indicator b as a function of p_0^2/m^2c^2. We can see that b is, initially, a fast growing function of its argument. But as soon as

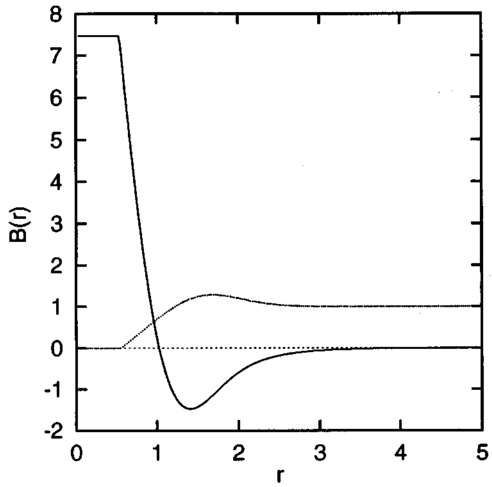

FIG. 1. The magnetic field $B(r)[=\Omega_c(r)\omega/\omega_e^2]$ (solid line), and the density $N(r)$ (dashed line) profiles as functions of the dimensionless radius $r[=r/R]$.

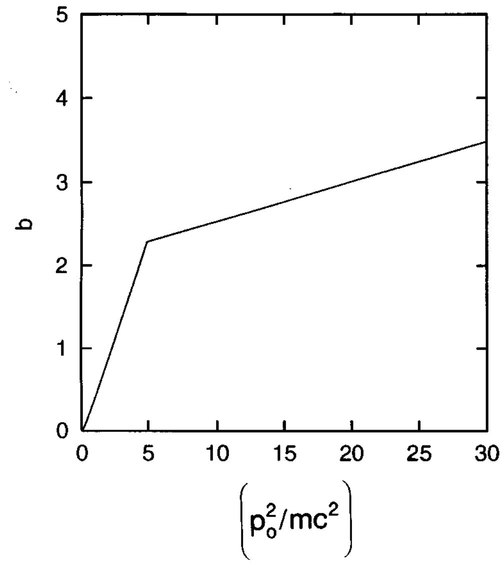

FIG. 2. The dimensionless measure b, indicating the excess of the calculated over the simply estimated field, vs p_0^2/m^2c^2.

p_0^2 reaches the value when electron cavitation occurs, its growth becomes considerably weaker. This latter part of this graph (the slower variation of b with p_0^2) shows that if we had neglected the effects of electron cavitation, we would have grossly overestimated the strength of the generated magnetic fields.

The maximum value of magnetic field B_m, as follows from Eq. (30) and Fig. 2, cannot be as high as the magnetic field of the laser radiation (at least for the EM field intensities which can be currently created in a channel). However, in a dense plasma, the fields can be quite strong for high-intensity laser radiation. Indeed, for $\lambda=0.248$ μm, $I=4\times10^{20}$ W/cm^2 in the channel (i.e., $p_0/mc=3$), and for the plasma density $n_0=10^{20}$–10^{21} cm^{-3} ($\omega_e/\omega=0.07$–0.24), the maximal value of the magnetic field turns out to be $B_m=17$–170 MG.

In the present paper, we have attempted to develop a systematic treatment of the phenomenon of the generation of quasistatic magnetic fields by relativistically strong circularly polarized laser pulses propagating in an initially uniform underdense cold electron plasma. We show that because of the strong plasma inhomogeneity caused by the intense laser beam, a low-frequency drag current is induced, which, due to the inverse Faraday effect, produces a quasistatic magnetic field in the beam propagation area. We derive an expression for the drag current valid for arbitrary amplitude laser pulses, and show that for the weakly relativistic (or nonrelativistic) laser radiation, the QSM field is smaller than what was found in previous publications. In the case of ultrarelativistic pulses, however, the generated QSM fields can reach considerable magnitudes. In all of these cases, the fields peak on the beam axis. We have also calculated the QSM field generation in the self-channeling regimes of intense laser pulses, and found that the electron cavitation makes the QSM field resemble closely the field produced by a solenoid. The maximum value of the generated magnetic field in the channel increases rapidly with the beam intensity, and when cavitation occurs the rate of growth of B_m with the intensity becomes slower. Finally, we show that for high-density plasma, the strength of the QSM field, which can be generated in the channel, can be ~100 MG and greater for currently available laser pulses.

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