Bubble motion in a horizontal tube and the velocity estimate for curved flames

Vitaliy V. Bychkov
Department of Plasma Physics, Umea University, S-90187, Umea, Sweden
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Propagation of a semi-infinite bubble in a horizontal tube is considered. The velocity of a stationary moving bubble is calculated, which provides an estimate of the velocity of a flame front in a horizontal tube for the case of strong influence of gravity. [S1063-651X(97)02406-9]

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Interest in the problems of bubbles rising in a gravitational field was stimulated for several decades mainly by the studies of the Rayleigh-Taylor instability in inertial fusion [1–7]. Lately it has also been obtained that effects of bubble motion are of great importance for flame dynamics both in terrestrial and astrophysical conditions [8–12]. A flame front propagating in a cold heavy fuel converts it to hot products of burning at low density, which tend to move upwards in a gravitational field. Usually the rising burnt matter forms a bubble, with the flame front as the surface of the bubble. Following from the dimensional analysis, the velocity of the bubble motion depends upon the gravitational acceleration g and the bubble size D as $\sqrt{gD}$. For a sufficiently large bubble size the bubble velocity may be much larger than the normal velocity of a planar flame front. In this case the velocity of flame propagation is determined mostly by the gravity effects, and the flame front moves with the bubble velocity, as it happens, for example, for flames propagating upwards in vertical tubes of a large diameter $D \gg U_f^2/g$, U_f being the normal flame velocity [12].

A similar effect was observed experimentally for flames propagating in horizontal tubes [13]: the hot burnt gas tends to occupy the upper part of the tube, while the heavy fuel extends along the lower part. As a result the flame front acquires a curved shape and propagates stationary with a velocity exceeding the velocity of a planar flame by orders of magnitude. According to the experimental data the velocity of a curved flame in a horizontal tube increases with the tube diameter as $\sqrt{D}$, just like a bubble velocity. Up to now flame propagation in a horizontal channel has been studied theoretically only for the case of small differences in densities of the fuel and the burnt gas, a moderate gravity acceleration, and a moderate tube width [14]. In the scope of these assumptions a flame front remains almost planar, and the effects of gravity provide only small corrections to the flame velocity. Obviously, the model of small expansion of the burning matter ''yields a poor extrapolation to the important limit of large acceleration'' [14]. In the limit of a large acceleration the flame dynamics in a horizontal tube is dominated by the effects of bubble motion. Therefore, in order to estimate the velocity of the curved flame, one has to solve the problem of an inert semi-infinite bubble moving in a horizontal channel. Though the problem of the bubble motion does not include the process of burning and fuel consumption, it provides a very good approximation of the flame velocity for the case of a large acceleration $gD/U_f^2 \gg 1$. If one considers a solution of the complete set of the equations of flame dynamics in a strong gravitational field, then the velocity of a massless bubble represents the principal term of the expansion of the flame velocity into a power series of the small parameter $U_f^2/gD \ll 1$ for the typical laboratory case where the fuel density considerably exceeds the density of the burning products [12,15].

In the present paper we consider propagation of a massless semi-infinite bubble in a horizontal tube filled by an incompressible fluid. The velocity of the stationary moving bubble is calculated, which provides an estimate of the flame velocity in a horizontal tube for the case of the strong influence of gravity.

Let us consider a stationary flow caused by a semi-infinite bubble propagating in a horizontal channel of width D, as is shown schematically in Fig. 1. It follows from dimensional considerations that the bubble moves with the velocity $U = F\sqrt{gD}$; our purpose is to find the coefficient F. In the reference frame of the bubble ($x=0$, $y=0$ being the top of the bubble) the fluid at infinity flows toward the bubble with the velocity U, rounds the bubble and forms a thin jet of width d. The velocity of the fluid in the jet is $U_\infty = U/a$, where $a = d/D$. At the surface of a massless bubble the Bernoulli's equation takes the form

$$\tfrac{1}{2}\mathbf{v}^2 + gy = 0, \tag{1}$$

so that the velocity of the fluid in the jet may also be calculated as

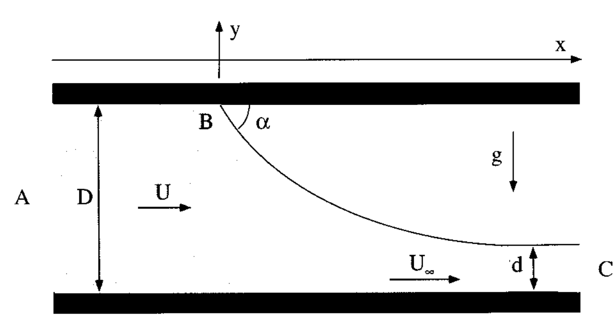

FIG. 1. The configuration of a bubble in a two-dimensional horizontal tube.

$$U_\infty=\sqrt{2g(D-d)}=U\ \frac{\sqrt{2}}{F}\ \sqrt{1-a}. \tag{2}$$

Equation (2) couples the dimensionless coefficient F to the ratio of the jet width and the tube width,

$$\tfrac{1}{2}\ F^2=a^2(1-a). \tag{3}$$

Since there are no sources of vorticity, the flow is potential everywhere. Close to the top of the bubble the velocity potential and the stream function may be written in polar coordinates in the following form [16]:

$$\varphi=Nr^n\cos(n(\Theta+\alpha)),\quad \psi=Nr^n\sin(n(\Theta+\alpha)),$$

$$n=\frac{\pi}{\pi-\alpha}, \tag{4}$$

with the velocity components

$$v_r=nNr^{n-1}\cos(n(\Theta+\alpha)),$$

$$v_\Theta=-nNr^{n-1}\sin(n(\Theta+\alpha)). \tag{5}$$

Substituting the velocities into the Bernoulli's equation, we obtain the equation for the coefficient N and the power exponent n:

$$\tfrac{1}{2}n^2N^2r^{2n-2}=gr\ \sin\alpha, \tag{6}$$

so that

$$n=\frac{3}{2},\quad \alpha=\frac{\pi}{3},\quad N^2=\frac{4\sqrt{3}}{9}g. \tag{7}$$

Then the complex potential $\phi=\varphi+i\psi$ at the bubble top becomes

$$\phi=UD\ \frac{2\sqrt[4]{3}}{3F}\left(\frac{z}{D}\right)^{3/2}\exp\left(i\frac{\pi}{2}\right), \tag{8}$$

where $z=x+iy$. An interesting point is that a horizontally propagating bubble touches the wall at a sharp angle $\alpha=\pi/3$, while the bubbles rising upwards always have a flat top.

In order to obtain the bubble velocity (i.e. the coefficient F) we have to find the solution of the Laplace equation $\Delta\phi=0$ for the whole flow, with the boundary conditions at the walls and the bubble surface. This is a very complicated mathematical problem which has not been solved yet even in the theory of bubbles rising upwards in spite of many attempts [1,5–7]. Instead we use a technique of approximate solution which proved to be a success in the theory of rising bubbles. According to this method one constructs an approximate solution reflecting the most important properties of the flow, and matches this solution with the solution at the bubble top, as has been done for spherical bubbles [5], cylindrical open bubbles [6], and plane open bubbles [7].

We introduce the dimensionless variables $u=u_x+iu_y=(v_x+iv_y)/U$, $\zeta=\xi+i\eta=z/D$, and $\Phi=\phi/UD$, so that the dimensionless complex potential at the top of the bubble takes the form

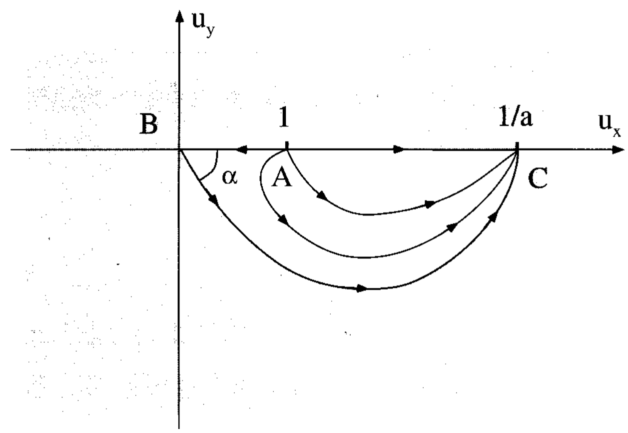

FIG. 2. The hodograph of a flow caused by a horizontally moving bubble. Points A, B, and C correspond to the uniform flow far ahead of the bubble, to the bubble top, and to the jet, respectively.

$$\Phi=\frac{2\sqrt[4]{3}}{3F}\zeta^{3/2}\exp\left(i\frac{\pi}{2}\right). \tag{9}$$

The important features of the flow under consideration are that the fluid moves from infinity with the dimensionless velocity $u=1$, rounds the bubble top of an angle $\alpha=\pi/3$, and forms a jet with the dimensionless velocity $u=1/a$. A typical hodograph of such a flow is shown in Fig. 2 with the points A, B, and C corresponding to the incoming flow, to the bubble top and to the outgoing jet, respectively. A flow like that shown in Fig. 2 may be obtained from the flow in Fig. 3 by the linear conformal transformation

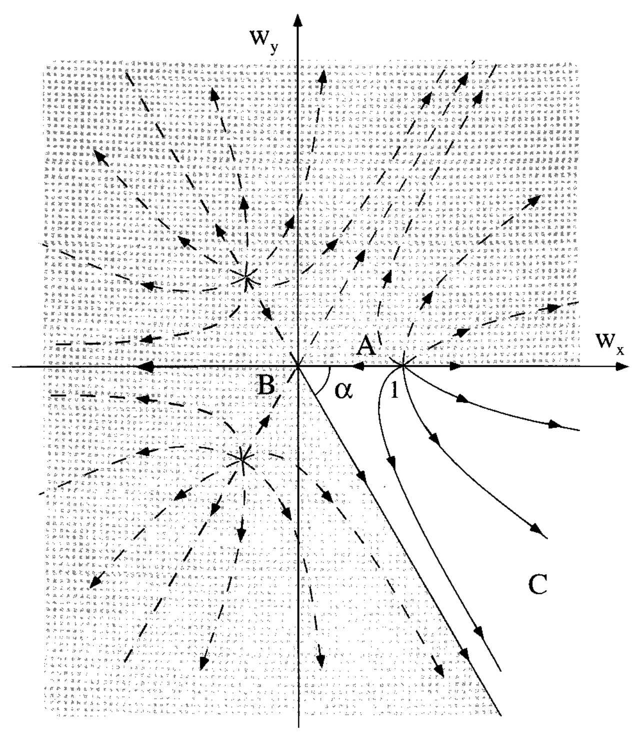

FIG. 3. A flow caused by three symmetrically placed sources. Points A, B, and C correspond to the uniform flow far ahead of the bubble, to the bubble top, and to the jet according to the transformation Eq. (10).

$$u=\frac{w}{aw+1-a}, \tag{10}$$

so that the points $w=1$, $w=0$, and $w=\infty$ correspond to the points $u=1$, 0, and $1/a$, respectively. The flow in Fig. 3 is produced by three sources placed symmetrically in the points $w=1$, $w=-1/2+(\sqrt{3}/2)i$, and $w=-1/2-(\sqrt{3}/2)i$ and the complex potential of the flow has the form

$$\Phi=\frac{1}{\pi}\ln|w-1|+\frac{1}{\pi}\ln\left|w+\frac{1}{2}-\frac{\sqrt{3}}{2}i\right|+\frac{1}{\pi}\ln\left|w+\frac{1}{2}+\frac{\sqrt{3}}{2}i\right|$$
$$=\frac{1}{\pi}\ln|w^3-1|. \tag{11}$$

The strength 2 of the sources is chosen in order to satisfy the boundary conditions in the incoming flow: it provides the unit mass flux for the flow marked by white in Fig. 3. By the definition, the complex velocity is the derivative of the complex potential $u=d\Phi/d\zeta$, therefore it follows from Eq. (11) that

$$u=\frac{3}{\pi}\frac{w^2}{w^3-1}\frac{dw}{d\zeta}. \tag{12}$$

Taking into account Eq. (10), we obtain a differential equation for the variable w,

$$\frac{1}{aw+1-a}=\frac{3}{\pi}\frac{w}{w^3-1}\frac{dw}{d\zeta}. \tag{13}$$

Though Eq. (13) may be integrated in terms of standard analytical functions, the result is rather complicated. At the same time we are interested only in the complex potential at the bubble top B $(z=0)$, where $u\to 0$, $w\to 0$. Close to the bubble top, Eq. (13) is reduced to the equation

$$\frac{1}{1-a}=-\frac{3}{\pi}w\frac{dw}{d\zeta}, \tag{14}$$

so that the variable w, the velocity, and the complex potential at the bubble top are

$$w^2=-\frac{2\pi}{3}\frac{\zeta}{1-a}, \tag{15}$$

$$u=\frac{1}{(1-a)^{3/2}}\left(\frac{2\pi}{3}\zeta\right)^{1/2}\exp\left(i\frac{\pi}{2}\right), \tag{16}$$

$$\Phi=\frac{2}{3}\left(\frac{2\pi}{3}\right)^{1/2}\frac{\zeta^{3/2}}{(1-a)^{3/2}}\exp\left(i\frac{\pi}{2}\right). \tag{17}$$

Comparing Eqs. (17) and (9), we obtain the second equation relating the dimensionless parameters F and a,

$$F^2=\frac{3\sqrt{3}}{2\pi}(1-a)^3. \tag{18}$$

From Eqs. (3) and (18), one finds the equation for the parameter a,

$$\frac{3\sqrt{3}}{2\pi}(1-a)^2=2a^2, \tag{19}$$

and calculates $a\approx 0.391$ and $F\approx 0.43$. Thus a bubble in a horizontal two-dimensional channel of width D propagates with the velocity $U\approx 0.43\sqrt{gD}$. The same velocity is expected for a flame in a horizontal two-dimensional tube in the case of sufficiently wide tubes $D\gg 5.4U_f^2/g$. The above estimate of the flame velocity and the coefficient F is smaller than the estimate $F=1/\sqrt{2}$ which follows from Eq. (6.3) of [14]. At the same time a curved flame front does not reproduce all features of the bubble shape. For example, the semi-infinite jet shown in Fig. 1 is impossible for a flame front because of the ''fire-polishing effect'': flame consumes the cold fuel in the jet.

The obtained velocity of a horizontally moving bubble is quite close to the velocity of open bubbles rising upwards in ideal two-dimensional vertical tubes $F_1\sqrt{gD}$, $F_1=0.3$–0.4 [1]. Thus, similar to the case of vertical tubes, the dimensionless coefficient F is rather small, which implies a weaker influence of gravity on flame dynamics than is generally believed [8]. The gravity effects become especially important for slow flames like a flame in a mixture 6% CH_4 with the normal velocity $U_f=5$ cm/s [16]. For such a flame the condition $D\gg 5.4U_f^2/g\approx 0.13$ cm is satisfied for any reasonable laboratory installation and the velocity of flame propagation is always determined by the effects of bubble motion. If a flame has a larger normal velocity, then the gravity effects become important for wider tubes. For example, the flame in the mixture 10% CH_4 propagates normally with the velocity $U_f=43$ cm/s typical of the laboratory flames, and for such a flame the bubble velocity is equal to the normal flame velocity for the tube width $5.4U_f^2/g\approx 10$ cm. The experiments [13] show that in a wide horizontal tube $D=90$ cm, the curved flame in the mixture 10% CH_4 propagates with the velocity $U_w=245$ cm/s, considerably exceeding the normal velocity of the flame. The theoretical analysis of the present paper predicts the bubble velocity $U\approx 130$ cm/s for a two-dimensional tube of this width. The difference between the theoretical and experimental results is presumably caused by the well-pronounced three-dimensional nature of the experimental flow. If we suppose that a three-dimensional bubble moves approximately 1.5 times faster than a two-dimensional one (as takes place for bubbles in vertical tubes [1]), then we obtain the flame velocity $U\approx 200$ cm/s which is only slightly lower than the experimental result $U_w\approx 245$ cm/s. It is quite natural that the bubble velocity somewhat underestimates the velocity of a gravity-influenced curved flame: the joint effect of the bubble motion and the fuel consumption by the flame front provides a larger flame velocity compared to the velocity of an inert bubble. An additional increase of the flame front velocity may be caused by the Landau-Darrieus hydrodynamic instability inherent to any gaseous flame [16].

Another interesting feature of a flame front in a horizontal tube is the sharp angle $\alpha\approx\pi/3$ at the top of the flame front, which is seen in the snapshot photographs [13], in agreement with the present theoretical results. The sharp angle can beclearly distinguished on the bright part of the flame front, where effects of thermal conduction and losses to the walls do not influence the flame structure.

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