Renormalized field theory of the Gribov process with quenched disorder

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Using field theory, I show that the renormalization-group flow of the Gribov process (Reggeon field theory or directed percolation) equipped with quenched randomness does not reach a stable fixed point, and has only runaway solutions in the physical domain. This result supports recent findings of Moreira and Dickman from Monte Carlo simulations of the two-dimensional contact process with random dilution, namely, logarithmic critical spreading and no power laws. [S1063-651X(97)10804-2]

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The investigation of the formation and properties of random structures has been an exciting topic in statistical physics for many years. When the formation of such structures obeys local rules, these processes can often be expressed in the language of population growth. It is well known that the Gribov process [1,2] (Reggeon field theory [3–5], stochastic Schlögl model [6]) is a stochastic multiparticle process that describes the essential features of a vast number of growth phenomena of populations near their extinction threshold, and without exploitation of the environment. The transition between survival and extinction of the population is a nonequilibrium continuous phase transition phenomenon and is characterized by universal scaling laws. The Gribov process belongs to the universality class of local growth processes with absorbing states [7,8] such as directed percolation [9–11], the contact process [12–14], and certain cellular automata [15,16], and is relevant to a vast range of models in physics, chemistry, biology, and sociology.

Some time ago Noest [17,18] studied the critical behavior of disordered one- and two-dimensional stochastic automata (belonging to the Gribov class), and found critical exponents that are qualitatively consistent with results of a field-theoretic study by Obukhov [19]. Recently Moreira and Dickman [20] reported Monte Carlo simulations of the two-dimensional contact process with random dilution. They found logarithmic critical spreading and no power laws. In this Brief Report I will show that the calculation of Obukhov is incorrect. The field theory of the disordered Gribov process does not exhibit a stable fixed point for the renormalization flow of the coupling constants. Thus no scaling behavior in the usual sense is found. My results therefore support the findings of Moreira and Dickman.

The failure in Obukhov's calculation is easily located. In his paper he shows six one-loop diagrams which lead to renormalizations. But, unfortunately, two ladder-type diagrams are neglected. These diagrams with four external legs become relevant, because the double interaction with the quenched impurity leads to a time-delocalized vertex. The relevant part of this vertex function can be transformed into an additional renormalization of the impurity interaction vertex. The disregard of the ladder diagrams is understandable, because the method of Obukhov is not quite straightforward for the disordered problem. I use the well-established method of renormalized field theory [22,23] of nonlinear stochastic processes [24–27], which I presented for the Gribov process without impurities some time ago [7]. Here the theory of the Gribov process with impurities is developed using the technique introduced by DeDominicis [21] for the N-vector model dynamics with random dilution.

The Gribov process is characterized by the following four principles: (1) Self-reproduction (''birth'') and annihilation (''death'') of particles. (2) Interaction (''competition'') between the particles. (3) Diffusion (''motion'') of the particles in a d-dimensional space. (4) The state where the particles are locally extinct is absorbing.

A continuum description in terms of a particle density $n(\mathbf{r},t)$ typically arises from a coarse-graining procedure where a large number of microscopic degrees of freedom are averaged out. Their influence is simply modeled as Gaussian noise terms in a Langevin equation that have to respect the absorbing state condition. Then the stochastic reaction-diffusion equation for the particle density $n(\mathbf{r},t)$ is constructed in accordance with the four principles given above:

$$\partial_t n(\mathbf{r},t) = \lambda \nabla^2 n(\mathbf{r},t) + R(n(\mathbf{r},t))\, n(\mathbf{r},t) + \zeta(\mathbf{r},t). \qquad (1)$$

Here the first term on the right-hand side models the (diffusive) motion, and R is the sum of the production and annihilation rates of the particles. The deterministic parts of (1) are constructed proportional to n in order to ensure the existence of an absorbing state. Expanding the rate R in powers of n, one obtains, up to subleading terms,

$$R(n(\mathbf{r},t)) = -\lambda[\tau + \psi + \tfrac{1}{2} g\, n(\mathbf{r},t)]. \qquad (2)$$

The ''temperature'' variable τ measures the mean difference of the rate of death and birth. Thus this parameter may be positive ore negative. One considers the case where $\tau \approx 0$ (up to fluctuation corrections), defining the critical region. Under these conditions the population lives on the border of extinction. The quenched randomness ψ is introduced by local deviations from the mean death and birth rates, and has a Gaussian distribution

$$\langle \psi(\mathbf{r}) \psi(\mathbf{r}') \rangle = \tfrac{1}{2} f \delta(\mathbf{r} - \mathbf{r}'), \qquad (3)$$

with $f > 0$. The Gaussian noise $\zeta(\mathbf{r},t)$ must also respect the absorbing state condition, thus

$$\langle \zeta(\mathbf{r},t) \zeta(\mathbf{r}',t') \rangle = \lambda g' n(\mathbf{r},t) \delta(\mathbf{r} - \mathbf{r}') \delta(t - t') \qquad (4)$$

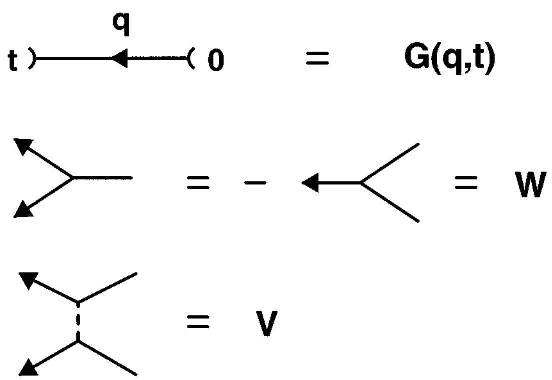

FIG. 1. Elements of the perturbation expansion.

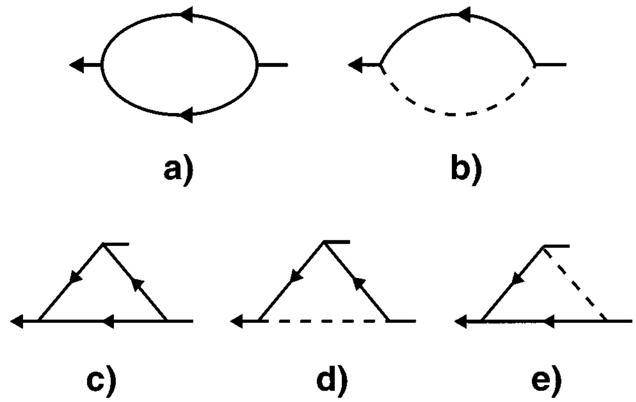

FIG. 2. One-loop self-energy and three-vertex diagrams.

up to subleading contributions.

In order to develop a renormalized field theory it is convenient to recast the Langevin equation (1) as a dynamic functional [28,29]

$$\mathcal{J}_{\psi}[\tilde{s},s]=\int dt\, d^dr\{\tilde{s}\,[\partial_t+\lambda(\tau+\psi-\nabla^2)+\tfrac{1}{2}\lambda g(s-\tilde{s})]s\}, \qquad (5)$$

where $s(\mathbf{r},t)\sim n(\mathbf{r},t)$ is the rescaled particle density which ensure that $g'=g>0$. The $\tilde{s}(\mathbf{r},t)$ denotes the response field (the Martin-Siggia-Rose [30] auxiliary variable). Correlation and response functions can now be expressed as functional averages (path integrals) of monomials of s and $\tilde{s}$ with weight $\exp\{-\mathcal{J}_{\psi}\}$. The responses are defined with respect to an additional local particle source $\tilde{h}(\mathbf{r},t)\geqslant 0$ in the equation of motion (1). By this source a further term $-\tilde{h}\tilde{s}$ is added to the integrand of the dynamic functional $\mathcal{J}_{\psi}$. The so-called prepoint discretization that sets the step function $\theta(t)$ equal to zero for $t=0$ is used in the interpretation of the functional integrals. As a consequence there is no normalization factor in these integrals, and the weight $\exp\{-\mathcal{J}_{\psi}\}$ can be easily averaged over the quenched randomness ψ. One obtains the dynamical functional for the calculation of the path integrals averaged over the randomness,

$$\mathcal{J}[\tilde{s},s]=\int d^dr\left\{\int dt\,\tilde{s}\left[\partial_t+\lambda(\tau-\nabla^2)+\frac{1}{2}\lambda g(s-\tilde{s})\right]s-\frac{1}{2}\lambda^2 f\left[\int dt\,\tilde{s}s\right]^2\right\}. \qquad (6)$$

The usual scaling by the convenient length and time scales μ^{-1} and $(\lambda\mu^2)^{-1}$, respectively, shows $\tilde{s}\sim s\sim\mu^{d/2}$ and $g^2\sim f\sim\mu^{\varepsilon}$, where $\varepsilon=4-d$ which signals $d_c=4$ to be the upper critical dimension. The expansion of the path integrals about the Gaussian part of $\mathcal{J}$ generates the (diagrammatic) perturbation series, which has to be regularized and renormalized because the series are UV divergent at the upper critical dimension. The propagator is $G(\mathbf{q},t)=\theta(t)\exp(-\lambda(\tau+q^2)t)$, and the vertices are given by $W=\lambda g$ and $V=\lambda^2 f$ (Fig. 1). A look at the naive dimensions of the generated diagrams shows that primitive divergencies arise only in the one-particle irreducible vertex functions $\Gamma_{1,1}$, $\Gamma_{1,2}=-\Gamma_{2,1}$, and $\Gamma_{2,2}$. Here the first and last indices denote the number of amputated external $\tilde{s}$ and s legs, respectively, and the vertex functions are considered as functions of the external momenta and frequencies. It is easily seen that the primitive divergencies of $\Gamma_{2,2}$ result only from time-separated parts of the diagrams. Therefore the model is renormalizable by the scheme

$$s\to\mathring{s}=Z_s^{1/2}s, \qquad \tilde{s}\to\mathring{\tilde{s}}=Z_s^{1/2}\tilde{s},$$

$$\lambda\to\mathring{\lambda}=Z_s^{-1}Z_{\lambda}\lambda, \qquad \tau\to\mathring{\tau}=Z_{\lambda}^{-1}Z_{\tau}\tau, \qquad (7)$$

$$f\to\mathring{f}=A_{\varepsilon}\mu^{\varepsilon}Z_{\lambda}^{-2}Z_v v, \qquad g^2\to\mathring{g}^2=A_{\varepsilon}\mu^{\varepsilon}Z_{\lambda}^{-2}Z_s^{-1}Z_u u.$$

$A_{\varepsilon}=(4\pi)^{d/2}\Gamma(1+\varepsilon/2)^{-1}$ is a suitable constant. The Z factors are determined by dimensional regularization and minimal renormalization.

The associated renormalization-group functions are defined by

$$\gamma_i=\mu\partial_{\mu}|_0 \ln Z_i, \qquad i=s,\lambda,\tau,u,v, \qquad (8)$$

and

$$\beta_u=\mu\partial_{\mu}|_0 u=[-\varepsilon+2\gamma_{\lambda}+\gamma_s-\gamma_u]u, \qquad (9a)$$

$$\beta_v=\mu\partial_{\mu}|_0 v=[-\varepsilon+2\gamma_{\lambda}-\gamma_v]v, \qquad (9b)$$

where $\partial_{\mu}|_0$ means a derivative at fixed bare variables. If there exists a physical infrared-stable fixed point u_*,v_* of β_u,β_v, the critical exponents can be expressed in terms of values γ_{i*} of the functions (8) by using familiar renormalization-group arguments. In particular the anomalous scaling exponent of the fields would be $\eta=\gamma_{s*}$, the dynamic exponent $z=2+\gamma_{s*}-\gamma_{\lambda*}$, and the correlation length exponent $\nu=(2+\gamma_{\tau*}-\gamma_{\lambda*})^{-1}$ (in directed percolation one often uses the transversal and longitudinal correlation length exponents $\nu_{\perp}=\nu$ and $\nu_{\parallel}=z\nu_{\perp}$).

To one-loop order the primitively divergent diagrams are shown in Figs. 2 and 3. The calculation of the renormalized vertex functions and their expansion in momentum q and frequency ω up to the relevant order leads to

$$\Gamma_{1,1}=i\omega\left\{Z-\frac{1}{\varepsilon}\left[\frac{u}{4}-2v\right]\left(\frac{\mu^2}{\tau}\right)^{\varepsilon/2}\right\}$$
$$+\lambda\tau\left\{Z_\tau-\frac{1}{\varepsilon}\left[\frac{u}{2}-2v\right]\left(\frac{\mu^2}{\tau}\right)^{\varepsilon/2}\right\}$$
$$+\lambda q^2\left\{Z_\lambda-\frac{u}{8\varepsilon}\left(\frac{\mu^2}{\tau}\right)^{\varepsilon/2}\right\}, \tag{10a}$$

$$(\Gamma_{1,2})^2=(\lambda g)^2\left\{Z_u-\frac{1}{\varepsilon}[2u-12v]\left(\frac{\mu^2}{\tau}\right)^{\varepsilon/2}\right\}, \tag{10b}$$

$$\Gamma_{2,2}=\lambda^2 f\left\{Z_v-\frac{1}{\varepsilon}[u-8v]\left(\frac{\mu^2}{\tau}\right)^{\varepsilon/2}\right\}. \tag{10c}$$

Thus to one-loop order minimal renormalization yields the Z factors

$$Z_s=1+\frac{1}{\varepsilon}\left[\frac{u}{4}-2v\right],\quad Z_\tau=1+\frac{1}{\varepsilon}\left[\frac{u}{2}-2v\right], \tag{11a}$$

$$Z_u=1+\frac{1}{\varepsilon}[2u-12v],\quad Z_\lambda=1+\frac{u}{8\varepsilon}, \tag{11b}$$

$$Z_v=1+\frac{1}{\varepsilon}[u-8v], \tag{11c}$$

and the one-loop renormalization-group functions follow as

$$\gamma_s=2v-\frac{u}{4},\quad \gamma_\tau=2v-\frac{u}{2}\ ,\quad \gamma_\lambda=-\frac{u}{8},$$
$$\gamma_u=12v-2u,\quad \gamma_v=8v-u \tag{12}$$

and

$$\beta_u=\left(-\varepsilon+\frac{3}{2}u-10v\right)u, \tag{13a}$$

$$\beta_v=\left(-\varepsilon+\frac{3}{4}u-8v\right)v. \tag{13b}$$

Now it is easily seen that the flow equations (9a) and (9b) have only runaway solutions for $\mu\to 0$ in the physical region $u>0$, $v>0$. The fixed point of the pure system $u_*=2\varepsilon/3$, $v_*=0$ is unstable just as the nonphysical one $u_*=-4\varepsilon/9$, $v_*=-\frac{1}{6}$. The stable fixed point is $u_*=0$, $v_*=-\frac{1}{8}$, but this one is nonphysical also.

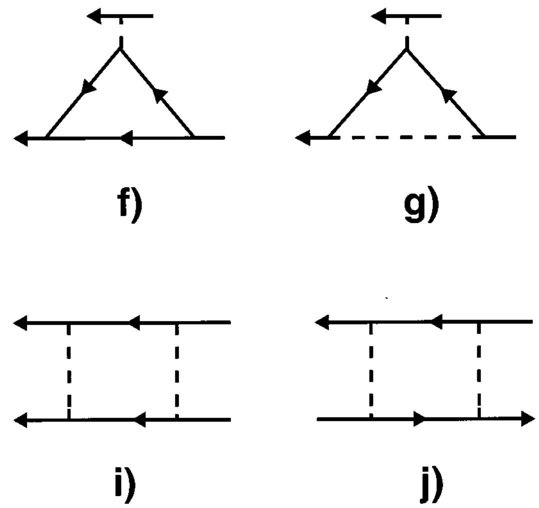

FIG. 3. One-loop time-delocalized four-vertex diagrams.

If one neglects the ladder diagrams, Figs. 3(i) and 3(j) the numerical factor 8 before v in the bracket after the ε pole in Eq. (10c) changes to 4. As a consequence also the factor 8 in (11c) and (13b) is replaced by 4. Then a stable physical fixed point exists, and eventually one finds the critical exponents given by Obukhov [19].

Summarily I have shown that the large distance and long-time behavior of the Gribov process are crucially disturbed by the quenched randomness. In the space of the renormalized coupling constants the behavior with respect to these scales is characterized by runaway solutions, making a direct perturbation theory useless, and, perhaps, producing more complicated critical behavior than simple power laws. The simulations of Moreira and Dickman [20] indeed show logarithmic critical spreading instead of power laws. Their results are therefore in agreement with my findings. I have shown that a study found in the literature that leads to scaling laws is incorrect because it neglects the ladder diagrams. This may provide a hint to find a better solution of the problem: the instability induced by the ladder diagrams can be ascribed to bound states between the particles. Indeed, if the mean production rate of particles is zero, a long-lived density greater than zero is possible in localized regions with a positive rate. The particles seem to be bound together. Thus one has to find the correct order parameter and dynamic functional to describe this phenomenon. But this is not the concern of this Brief Report.

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