Migration control in two coupled Duffing oscillators

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In this paper we study the migration from one attractor to another coexisting attractor in two coupled Duffing oscillators by an open-plus-closed-loop control method and adaptive control algorithm. Suppression of chaos by these methods is also investigated. [S1063-651X(97)06904-3]

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Even though chaos is a robust phenomenon exhibited by nonlinear systems, recent investigations clearly show that chaotic motion can be controlled or directed towards a desired regular orbit by means of preassigned and small perturbations, either to the system parameters or through the addition of weak external forces. The control methods can be broadly classified into (i) feedback [1–5] and (ii) nonfeedback [6–10] methods. The feedback methods mainly aim to stabilize the suitable unstable periodic orbit (UPO) embedded in the chaotic attractor of the system. Very recently, Jackson [11] showed that directing the system from one attractor to another coexisting attractor is possible by an open-plus-closed-loop (OPCL) control. This is called migration.

In this paper we study the migration control in the two coupled Duffing oscillators

$$\dot{x}=y, \tag{1a}$$

$$\dot{y}=-dy+\alpha_1 x-\beta_1 x^3-\delta x u^2+f\cos(\omega t), \tag{1b}$$

$$\dot{u}=v, \tag{1c}$$

$$\dot{v}=-dv+\alpha_2 u-\beta_2 u^3-\delta u x^2+f\cos(\omega t). \tag{1d}$$

Equation (1) has been used to model Soret-Bénard convection [12], vibration of a stretched string [13], motions of nonlinear circular plates [14], and so forth.

In Eq. (1) coexistence of multiple attractors are found for a range of values of the parameters. We illustrate the migration from one periodic orbit to another coexisting periodic orbit, and chaotic orbit to a coexisting periodic orbit. We study the migration control using OPCL and adaptive control algorithms.

For a system of the form

$$\dot{X}=F(X), \tag{2}$$

where $X=(x_1,x_2,\ldots,x_N)$ and $F=(F_1,F_2,\ldots,F_N)$, the OPCL control is given by

$$\dot{X}=F(X)+K(g,X,t), \tag{3a}$$

with

$$K(g,X,t)=S(t)\{\dot{g}-F(g)+[F'(g)+A](X-g)\}, \tag{3b}$$

where the prime denotes differentiation with respect to g and $S(t)$ is the switching function, $S(t)=0$ $(t<0)$ and, for example, $S(t)=1$ $(t>0)$. The time $t=0$ refers to the time at which control is activated. The matrix A is a constant whose eigenvalues all have negative real parts. For simplicity one can choose its elements as $a_{ij}=a\delta_{ij}$. The function $g(t)$ is a goal dynamics towards which $X(t)$ would tend, that is, with the control in the long time limit we have

$$\lim_{t\to\infty}\|X(t)-g(t)\|=0. \tag{4}$$

If $g(t)$ is an attractor of Eq. (2) then $\dot{g}-F(g)$ in Eq. (3b) vanishes and, consequently, the factor $F'(g)=dF/dg$ alone, is specifically related to the system.

The two coupled Duffing oscillators with OPCL control are written as

$$\dot{x}=y+K_x, \tag{5a}$$

$$\dot{y}=-dy+\alpha_1 x-\beta_1 x^3-\delta x u^2+f\cos(\omega t)+K_y, \tag{5b}$$

$$\dot{u}=v+K_u, \tag{5c}$$

$$\dot{v}=-dv+\alpha_2 u-\beta_2 u^3-\delta u x^2+f\cos(\omega t)+K_v, \tag{5d}$$

where K_x, K_y, K_u, and K_v are the perturbations given by Eq. (3b), introduced for migration. When the external periodic force is included, for small values of amplitude f, of the force two orbits with period $T=2\pi/\omega$ coexist. For example, $\alpha_1=1$, $\beta_1=1$, $\alpha_2=0.114$, $\beta_2=0.1$, $\delta=0.05$, $\omega=1$, $d=0.4$, and $f=0.25$ for two period-T orbits to coexist. Figure 1 shows the transfer of the system dynamics from the limit cycle X_+ to X_-, where $X=X(x,y,u,v)$.

As an interesting case we next consider the migration from chaotic motion to a coexisting periodic motion. For $\alpha_1=-1$, $\beta_1=-4$, $\alpha_2=-1.1$, $\beta_2=-3.9$, $d=0.4$, $\delta=0.002$, $f=0.1147$, and $\omega=0.526$ both chaotic and periodic attractors coexist. Suppose the system is in the chaotic state. We select the goal dynamics $g(t)$ as the coexisting periodic orbit and fix $a=-0.5$. Figure 2 illustrates the migration from chaotic motion to the chosen goal orbit. In the absence of the control the system is integrated using a fourth-order Runge-Kutta method with time step $t=(2\pi/\omega)/100$ with the initial condition $X(0)=(0,0.35,0,0.3)$. The system is allowed to evolve in a

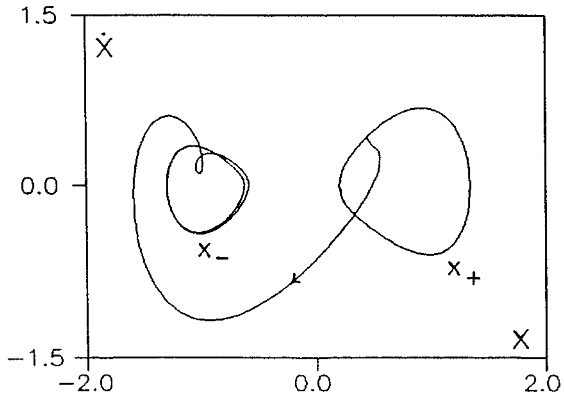

FIG. 1. Migration dynamics from the limit cycle X_+ to X_- of the two coupled Duffing oscillators by the OPCL method.

chaotic state. Control is switched on at $t=80(2\pi/\omega)$, with $S(t)=1$. Figures 3(a) and 3(b) show the required perturbations. The perturbations are found to vanish once the migration to $g(t)$ is achieved. This is because the goal orbit is a particular solution of the uncontrolled system.

The system dynamics is studied with the switching function

$$S(t)=1-\exp(-\lambda t), \tag{6}$$

where λ is a constant parameter. Desired migration is achieved for $\lambda>0$. The efficacy of the OPCL control has been studied by calculating the recovery time $R_T=t_0'-t_0$, where t_0 and t_0' are the times at which control is initiated and after which $||X(t)-g(t)||$ is always less than 10^{-3}, respectively. R_T is calculated for 200 initial conditions chosen on the chaotic attractor and then its average value is obtained. Figure 4 shows the dependence of R_T on λ. As λ is increased from zero R_T decreases rapidly and approaches a constant value for higher values of λ. Migration from one attractor to another attractor can also be achieved by the

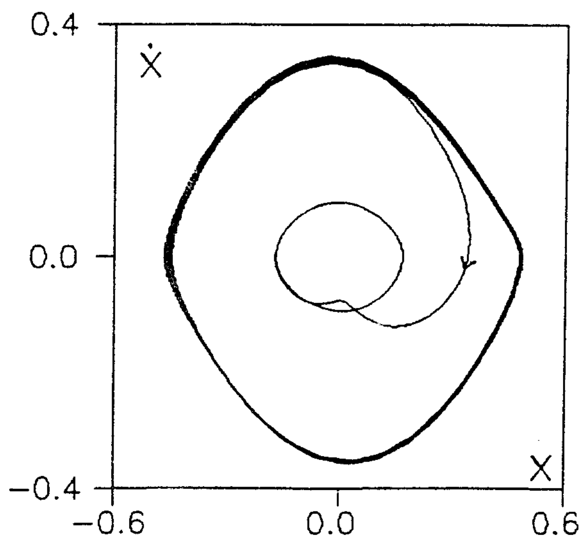

FIG. 2. Migration from chaotic motion to a coexisting periodic orbit in Eq. (1). The controlled equation is Eq. (5).

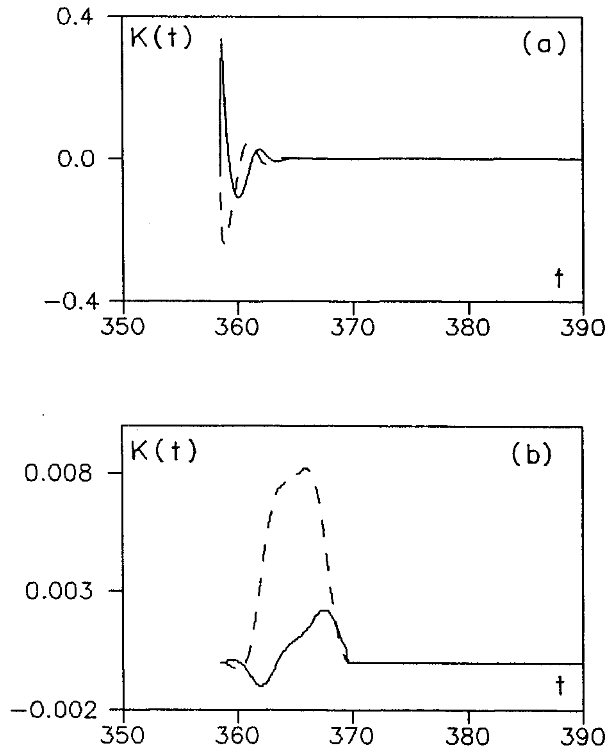

FIG. 3. Variation of the required perturbations in the controlled two coupled Duffing oscillators (5) for migration from chaotic motion to a periodic motion. In (a) continuous and dashed curves represent the perturbations K_x and K_y respectively. In (b) they represent K_u and K_v, respectively.

adaptive control algorithm (ACA) [2,9]. The two coupled Duffing oscillators equation with the ACA is written as

$$\dot{x}=y, \tag{7a}$$

$$\dot{y}=-dy+\alpha_1 x-\beta_1 x^3-\delta x u^2+f\cos(\omega t)+p(t), \tag{7b}$$

$$\dot{u}=v, \tag{7c}$$

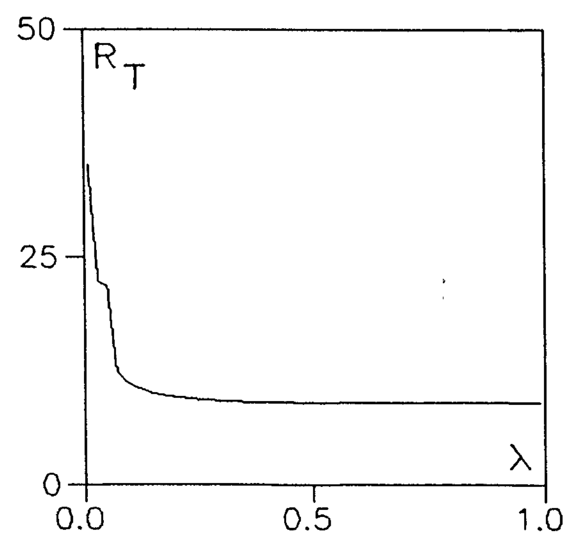

FIG. 4. Recovery time R_T vs λ for the OPCL method.

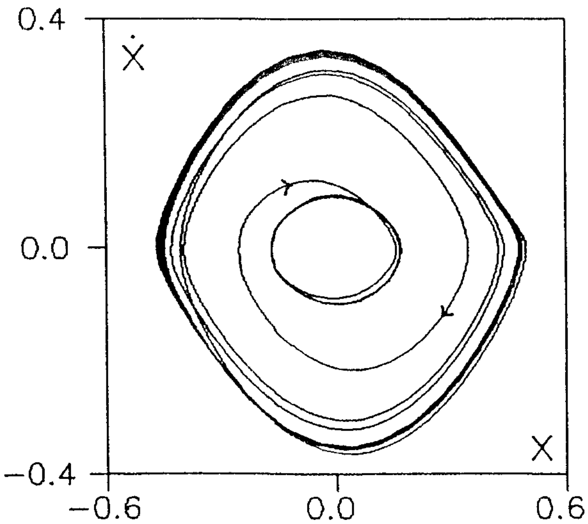

FIG. 5. Migration from chaotic motion to a coexisting periodic orbit by the ACA.

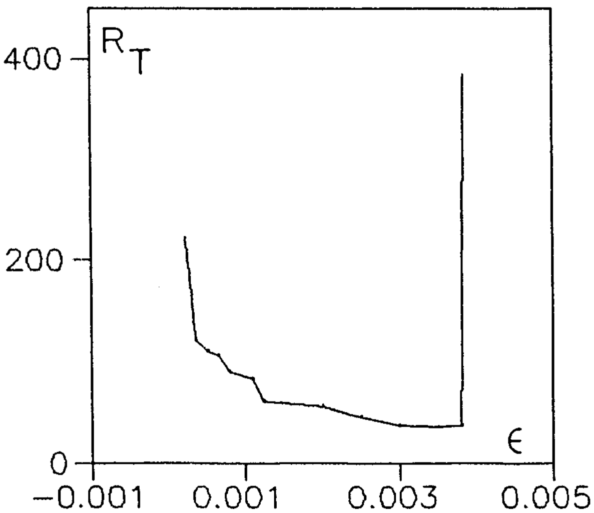

FIG. 7. Recovery time R_T vs λ for the ACA.

$$\dot{v} = -dv + \alpha_2 u - \beta_2 u^3 - \delta u x^2 + f\cos(\omega t), \qquad (7d)$$

$$\dot{p} = \epsilon[(x+y-u-v) - (\overline{x}+\overline{y}-\overline{u}-\overline{v})]$$

$$\equiv \epsilon G(X-\overline{X}), \qquad (7e)$$

where $p(t)$ is the perturbation added for migration, $\overline{X}=(\overline{x},\overline{y},\overline{u},\overline{v})$ is the desired orbit, ϵ is the stiffness parameter of the control, and G is a function proportional to $(X-\overline{X})$. The function G can be linear or nonlinear. Here we consider the linear form of G. To illustrate the migration from chaotic dynamics to a coexisting periodic motion we choose $\alpha_1=-1$, $\beta_1=-4$, $\alpha_2=-1.1$, $\beta_2=-3.9$, $d=0.4$, $\delta=0.002$, $\omega=0.526$, and $f=0.11474$. Figure 5 shows the migration from chaotic attractor to the coexisting limit cycle for $\epsilon=0.002$. The variation of the perturbation $p(t)$ is plotted in Fig. 6. The control is switched on at $t=80(2\pi/\omega)$. The parameter $p(t)$ evolves according to Eq. (7e) and adjusts its value until the desired state is reached. Once the desired migration is achieved $p(t)$ vanishes and the control can be switched off if the condition $X=\overline{X}$ is realized.

In general, the control mechanism is sensitive to the value of ϵ and the form of the function G. In Eq. (7) stable control

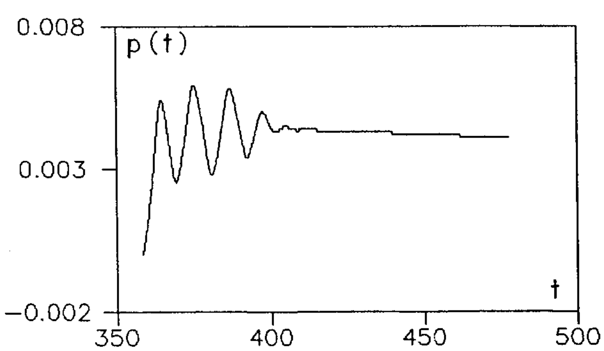

FIG. 6. Variation of the required perturbation $p(t)$.

to the coexisting limit cycle attractor is found to occur for ϵ values in the interval $(-0.0035, -0.0018)$ and $(0.00023, 0.004)$. Figure 7 shows the dependence of recovery time on ϵ. We add that the recovery time shows a different characteristic behavior in the ACA (Fig. 7) compared to the OPCL (Fig. 4) method. In the two coupled Duffing oscillators, instead of p, any other parameters can also be chosen for migration control.

In summary, we studied the transfer from one attractor to another coexisting attractor in the two coupled Duffing oscillators. Interestingly, migration from chaos to periodic motion is possible by both OPCL and ACA methods. Thus, the simultaneous presence of periodic orbits in a chaotic system is of great use for bringing the system from chaos to order. In the OPCL and ACA methods the required perturbation vanishes once the desired goal orbit is reached. The other existing feedback methods [1,2,4,5] are primarily designed to stabilize the unstable periodic orbits embedded in the chaotic attractor, where, as to implement the OPCL, the desired attractor need not be embedded in the chaotic attractor. As shown in the two coupled Duffing oscillator the actual dynamics can be directed towards a goal orbit which is far away from the actual orbit. In the OPCL method migration from one attractor to a desired coexisting attractor is always guaranteed. In the case of ACA and other feedback methods [2,4,5] stable control is possible only for certain range of values of the stiffness parameter ϵ, and it has to be determined either by linear stability analysis or experimentally before implementing the specific control algorithm. Further, in contrast to the linear feedback methods, where control function must be on forever, the migratory controls (OPCL and ACA) require control actions for only a limited time. That is, the control can be switched off once the system trajectory reaches the basin of attraction of the goal dynamics.

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