PHYSICAL REVIEW E VOLUME 55, NUMBER 5 MAY 1997

Solitons in coupled waveguides with quadratic nonlinearity

William C. K. Mak,[1] Boris A. Malomed,[2,*] and P. L. Chu[1,†]

[1]*Optical Communications Group, School of Electrical Engineering, University of New South Wales, Kensington, New South Wales 2052, Australia*

[2]*Department of Interdisciplinary Studies, Faculty of Engineering, Tel Aviv University, Tel Aviv 69978, Israel*

(Received 27 November 1996)

We consider a model of two linearly coupled second-harmonic-generating waveguides. The analysis is focused on the case of no walkoff and full matching. We demonstrate existence of a bifurcation that transforms obvious symmetric soliton states into nontrivial asymmetric ones. The bifurcation point is found exactly, while a full analytical description of the asymmetric solutions is obtained by means of the variational approximation. Comparing this with numerical results generated by the shooting method, we conclude that, in a part of the range where the asymmetric states are predicted, the analytical approximation provides very good accuracy, while in another part, the asymmetric solitons disappear. Whenever they exist, however, direct partial differential equation simulations demonstrate that they are stable, while the symmetric ones are not. We also demonstrate that the asymmetric solitons remain stable if walkoff is added. The soliton states found here can be used for optical switching. [S1063-651X(97)11105-9]

PACS number(s): 42.65.Tg

I. INTRODUCTION

The study of solitons in waveguides with quadratic nonlinearity has recently attracted a lot of attention (see, e.g., [1–15]). The analogy with nonlinear optical fibers suggests that essentially new soliton states may be expected in parallel-coupled waveguides. Thus far, coupling effects were considered in terms of cw propagation in a quadratically nonlinear waveguide coupled to a linear one [13]. Very recently, some direct simulations of the pulse evolution in a pair of coupled waveguides with a mixed quadratic-cubic nonlinearity were reported in [14], but solitary-wave states were not considered there.

The objective of the present work is to initiate study of solitons in two linearly coupled second-harmonic-generating (SHG) two-dimensional waveguides. We will, chiefly, confine ourselves to the simplest case, when the waveguides are identical, and the beams in them are strictly parallel. However, we will also demonstrate that stable solitons found in this work survive if the spatial walkoff, produced by a misalignment between the beams, is added to the model, provided that it is not too strong.

Equations describing copropagation of the fundamental harmonic (FH) u and second harmonic (SH) v in the linearly coupled waveguides can be obtained as a straightforward generalization of the well-known equations for the single waveguide [1]:

$$iu_{1z}+i\delta u_{1x}+\frac{1}{2}u_{1xx}-qu_1+u_1^*v_1=-Qu_2, \tag{1}$$

$$2iv_{1z}+2i\delta v_{1x}+\frac{1}{2}v_{1xx}-v_1+\frac{1}{2}u_1^2=-Kv_2, \tag{2}$$

$$iu_{2z}-i\delta u_{2x}+\frac{1}{2}u_{2xx}-qu_2+u_2^*v_2=-Qu_1, \tag{3}$$

$$2iv_{2z}-2i\delta v_{2x}+\frac{1}{2}v_{2xx}-v_2+\frac{1}{2}u_2^2=-Kv_1, \tag{4}$$

where the subscripts 1 and 2 pertain to the first and second waveguides, z and x being, respectively, the propagation and the transverse coordinates in them (we refer to the more realistic case of spatial solitons), and δ is the walkoff parameter. The second derivatives and the nonlinear terms in Eqs. (1)–(4) account for, respectively, diffraction and FH-SH conversion. The parameter q measures the phase mismatch between the two harmonics, the system being fully matched at $q=1$. The terms on the right-hand sides represent the linear coupling between the waveguides, Q and K being the FH and SH coupling constants.

In this paper we will concentrate on the fully matched case, $q=1$, and, moreover, we will assume equal coupling constants, $K=Q$ (the latter may be justified if the separation between the waveguides is sufficiently small). Actually, it will be seen below that the latter condition, alongside $q=1$, is necessary to achieve full matching of the two harmonics in the coupled waveguides. A more general case (in particular, the case $K=0$, which corresponds to the case of a large separation between the waveguides) will be considered elsewhere. In this paper, we will consider mainly the no-walkoff case, $\delta=0$.

Thus we are dealing with the model controlled by the single parameter Q. First of all, we will consider stationary solutions by dropping the z-derivative terms and assuming all the variables real. The stationary version of Eqs. (1)–(4) (with $q=1$, $K=Q$, and $\delta=0$) has an obvious symmetric solution (corresponding to the classical solution obtained in [16]),

$$u_{1,2}=\pm\sqrt{2}v_{1,2}, \tag{5}$$

*Electronic address: malomed@eng.tau.ac.il

†Electronic address: p.chu@unsw.edu.au

(where the sign is the same for both values of the subscript), and

$$v_1 = v_2 = \frac{3}{2}(1-Q)\mathrm{sech}^2\left(\sqrt{\frac{1}{2}(1-Q)}x\right), \tag{6}$$

which exists provided that $Q<1$. Since special exact solitary-wave solutions to the SHG equations have always been the focus of attention [1], it may be relevant to mention that, in the general case ($q \neq 1$, $K \neq Q$), an exact sech^2 stationary solution to Eqs. (1)–(4) exists only at the mismatch values $q=1+Q-K$, when the solution is symmetric as above, and $q=1-Q-K$, when the solution is antisymmetric in its FH component, $u_1=-u_2$, $v_1=+v_2$.

A nontrivial issue is the search for asymmetric solutions. Asymmetric solitary waves have been studied in detail in a similar problem for nonlinear dual-core optical fibers (directional couplers) with the cubic (Kerr) nonlinearity, see, e.g., [17–24]. It was found that (using notation similar to that adopted here), at large values of the coupling constant, the symmetric solitons are unique and stable solutions. There is a critical value of the coupling constant, at which a bifurcation takes place: below this value, the symmetric solution is unstable, but there exist two nontrivial stable asymmetric solutions (which are mirror images of each other). The bifurcation was shown to be slightly subcritical [18], i.e., the asymmetric solutions appear at a value of the coupling constant which is slightly larger than the above-mentioned critical one. In a tiny region between these two values, both the symmetric and asymmetric solutions are stable (in this region, there also exists an additional pair of intermediate unstable asymmetric solutions).

In [18], it was shown that the bifurcation and the whole parametric plane of the soliton solutions could be obtained, with a fairly high accuracy (as compared to numerical results), in an analytical form by means of the variational approximation (VA). It is necessary to mention that VA was recently applied to description of stationary soliton solutions in the *single* SHG waveguide [15]; the corresponding analytical results were very close to the numerical ones. Thus the results of [15] and [18] strongly suggest applying VA to the present problem too. This will be done below, parallel to looking for stationary solitary-wave solutions by the shooting method. VA predicts asymmetric solitons in the region $-1<Q<3/8$. On the other hand, the shooting produces such solutions at (approximately) $-0.3<Q<0.35$. The shape of the numerically found asymmetric solitons is fairly close to the analytical prediction. However, for $Q<-0.3$, the shooting method fails to generate asymmetric solitary waves; in this region, we can find numerically only periodic waves.

As for the bifurcation point, it will be found both by means of VA, which yields the bifurcation (critical) value $Q_{\mathrm{cr}}=3/8$, and *exactly,* $Q_{\mathrm{cr}}=5/13$. Thus the error of VA in determining the bifurcation point is only 2.5%.

Stability of both asymmetric and symmetric solitons will be tested by direct simulations of Eqs. (1)–(4). We will see that, whenever the asymmetric solitons exist, they are stable. The symmetric solitons are always found to be unstable when they coexist with the asymmetric ones, which is a natural consequence of the bifurcation [18]. Moreover, by simulating development of the instability of the symmetric soliton over a long propagation distance, we observe a trend to its rearrangement into the stable asymmetric soliton. To the right of the bifurcation point (where the asymmetric solitons do not exist), the symmetric ones are stable. Thus this is similar to the situation in the dual-core optical fibers with the cubic nonlinearity [17,18]. Where the asymmetric solitary-wave solutions do not exist for $Q<-0.3$, the mode of instability of the symmetric solitons is oscillatory, showing a trend to their complete decay into dispersive radiation. Finally, we will also simulate the asymmetric solitons in the case when the walkoff parameter δ is nonzero, but small.

II. THE ANALYTICAL RESULTS AND NUMERICAL CHECK

In accordance with what was said above, we will concentrate on the most important case of full matching, $q=1$ and $K=Q$ (in this section, we will also set $\delta=0$). In this case, it is easy to see that the stationary version of Eqs. (1)–(4) allows the substitution $\pm v_{1,2}=u_{1,2}/\sqrt{2}$, cf. Eq. (5). Then, there remain two equations,

$$\frac{1}{2}u_1'' - u_1 + \frac{1}{\sqrt{2}}u_1^2 + Qu_2 = 0\,, \tag{7}$$

$$\frac{1}{2}u_2'' - u_2 + \frac{1}{\sqrt{2}}u_2^2 + Qu_1 = 0\,, \tag{8}$$

where the prime stands for d/dx.

Equations (7) and (8) have an evident Lagrangian representation with the Lagrangian density

$$\mathcal{L} = \frac{1}{4}[(u_1')^2 + (u_2')^2] + \frac{1}{2}(u_1^2+u_2^2) - \frac{1}{3\sqrt{2}}(u_1^3+u_2^3) - Qu_1u_2\,. \tag{9}$$

To apply VA, we adopt the following ansatz (trial form) for the solitary-wave solution, which is suggested by the special exact solution (6), and also by analogy with VA for the solitons in the dual-core fiber with the cubic nonlinearity [18]:

$$u_1 = A\cos\theta\,\mathrm{sech}^2\left(\frac{x}{W}\right), \tag{10}$$

$$u_2 = A\sin\theta\,\mathrm{sech}^2\left(\frac{x}{W}\right), \tag{11}$$

where A, W, and θ are arbitrary amplitude, width, and asymmetry parameter of the soliton sought.

The next step is to insert Eqs. (10) and (11) into Eq. (9), and calculate the effective Lagrangian,

$$L \equiv \int_{-\infty}^{+\infty} \mathcal{L} dx = \frac{4}{15} A^2 W^{-1} + \frac{2}{3} A^2 W - \frac{16}{45\sqrt{2}} A^3 W (\cos^3\theta + \sin^3\theta) - \frac{2}{3} Q A^2 W \sin(2\theta). \tag{12}$$

Finally, equations that determine the unknown parameters A, W, and θ are obtained by demanding that variations of the Lagrangian with respect to each of them are zero.

After some algebra, we arrive at the following results. The variational equations have two different solutions, one of which exists at all $Q<1$ and has $\theta \equiv \pi/4$. It is easy to check that this solution coincides with the exact symmetric soliton (6). The other, asymmetric solution is

$$\theta = -\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\zeta, \tag{13}$$

$$A = 5Q \frac{\sin(\theta + \pi/4)}{\sin(2\theta)}, \tag{14}$$

$$W = \sqrt{-\frac{5}{6}\zeta^{-1} Q(1-\zeta)\left(1 + \frac{1}{2}\zeta\right)}, \tag{15}$$

where the auxiliary parameter

$$\zeta \equiv -\sin(2\theta) = \frac{5Q - 6 + \sqrt{3(12 - 20Q - 5Q^2)}}{2Q}. \tag{16}$$

Further straightforward consideration shows that this asymmetric solution exists in the interval of the coupling constant values

$$-1 < Q < \frac{3}{8} \tag{17}$$

[this limitation is imposed by the condition $|\sin(2\theta)|<1$], which should be compared to the existence range of the symmetric soliton (6), $Q<1$. It is also easy to check that at $Q=0$, when Eqs. (7) and (8) become decoupled, the variational solution goes over into the exact one for the single waveguide [16], while in the other waveguide the field is absent. At small values of $|Q|$ the solution is strongly asymmetric. In contrast with this, at the bifurcation point $Q=3/8$ the solution coincides with the exact symmetric solution (6) for the same value of Q, and in the opposite limit, $Q \to -1$ (though we will show below that this limit does not really exist), the variational solution describes an almost antisymmetric soliton with a vanishing amplitude A and a diverging width W.

The value $Q_{\rm cr}$ of the control parameter Q at the bifurcation point can be found *exactly*. Indeed, the bifurcation assumes the appearance of an unstable mode in the spectrum of small perturbations around the exact symmetric solution (6) when Q becomes smaller than $Q_{\rm cr}$ [18]. The change of stability of this mode at $Q=Q_{\rm cr}$, in turn, suggests existence of a *zero mode* $(\delta u, \delta v)$ at this value of Q. One can easily find that a nontrivial zero mode of the form

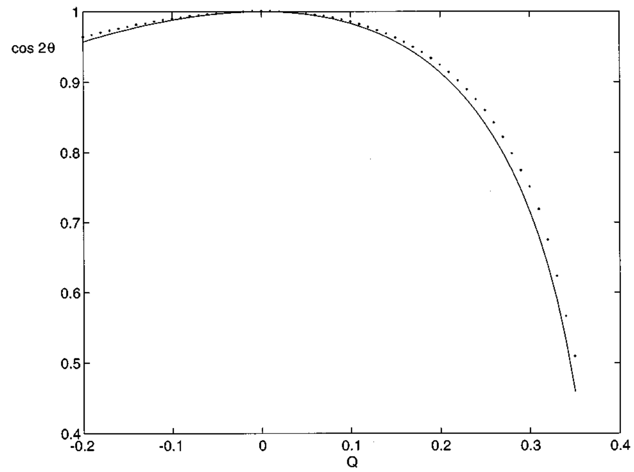

FIG. 1. The bifurcation diagram of the system (7), (8). The continuous line shows the analytical approximation, while the dots are results generated by the shooting method.

$$\delta u = -\delta v = a\,\mathrm{sech}^3\left(\frac{2}{\sqrt{13}}x\right), \tag{18}$$

where a is an infinitesimal perturbation amplitude, exists at $Q=5/13$. It is noteworthy that this zero mode, in contrast with the unperturbed symmetric solution, is antisymmetric, which indeed implies a transition to asymmetric solutions as a result of the bifurcation.

One can now compare the approximate and exact values of $Q_{\rm cr}$, i.e., respectively, $3/8=0.3750$ and $5/13 \approx 0.3846$. The relative error of our simple VA in predicting the bifurcation point is 2.5%, which is quite acceptable.

Proceeding to numerical analysis of the bifurcation and asymmetric soliton states, we solved Eqs. (7) and (8) by means of the well-known shooting method, which was implemented in terms of the fourth-order Runge-Kutta numerical scheme. The analytical prediction and numerical results are summarized in Fig. 1, which represents a *bifurcation diagram*, i.e., a plot of the effective asymmetry parameter $\cos(2\theta)$ [18] vs the control parameter Q [the branch of the solution corresponding to the symmetric soliton is $\cos(2\theta)\equiv 0$; it is not specially marked in Fig. 1]. As one sees, the agreement between VA and the numerical results is fairly good in the interval $-0.3<Q<0.35$, cf. Eq. (17).

Very close to the bifurcation point, the shooting method becomes unstable because of large numerical fluctuations, but there is no doubt that the bifurcation takes place as predicted by VA (incidentally, this bifurcation is clearly supercritical, unlike the slightly subcritical one in the dual-core fiber with the cubic nonlinearity [18]). However, for $Q<-0.3$, the shooting method has never produced solitary-wave solutions. Instead, it generated periodic waves. We do not consider them here at large, as the subject of this work is the soliton, and periodic waves are usually unstable.

We have also directly compared dependences of the peak values of the variables $u_1(x)$ and $u_2(x)$ on the control pa-

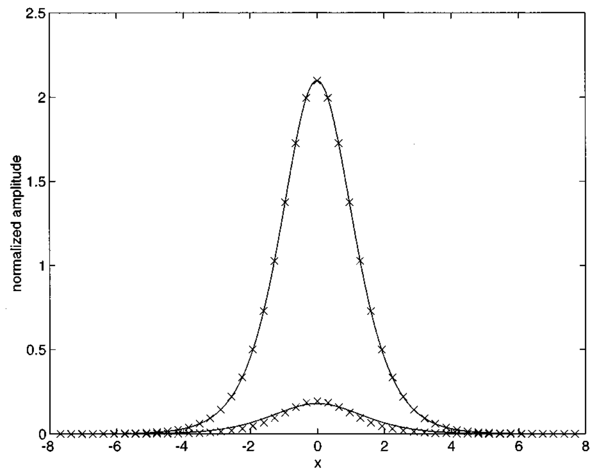

FIG. 2. The shape of the asymmetric soliton at $Q=0.1$. Shown are the FH components $u_{1,2}$: the analytical prediction (crosses) and the results obtained by means of the shooting method (solid).

rameter Q, as predicted by VA and as given by the shooting method. Within the same interval $-0.3<Q<0.35$, they prove to be fairly close. The worst case error is about 8%, which happens at the smaller of the peak values of u_1 and u_2 when they are strongly asymmetric; i.e., Q is close to 0. It is interesting to add that, in this case, the larger peak value achieves the best agreement between VA and the shooting method; the error is less than 0.04%. Finally, as a particular example, we display in Fig. 2 the analytically predicted and numerically found shapes of the asymmetric soliton at $Q=0.1$.

It remains unclear if another bifurcation is amenable for termination of the numerically found branch of the asymmetric soliton solution at Q close to -0.3. Nevertheless, the above results furnish a sufficiently complete description of the stationary asymmetric solitons in the underlying model of the linearly coupled SHG waveguides.

III. STABILITY OF THE ASYMMETRIC SOLITONS

To verify the stability of the stationary solitons found in the preceding section, we directly simulated the full system of partial differential equations (PDE's) (1)–(4). The split-step Fourier method (also called the beam propagation method) was employed. The method was implemented using the third-order Runge-Kutta scheme, together with the so-called transparent boundary condition algorithm [25], which, effectively, allows dispersive waves emitted by a perturbed soliton to be radiated away through the edges of the integration domain, and thus eliminates the aliasing problem, i.e., distortion of the picture by waves reflected from the edges. In order to control the accuracy of the simulations, selected runs (especially those which produced unexpected results) were repeated with a smaller step size in the propagation direction, and/or with a larger number of the points implementing the fast Fourier transform in the transverse direction. These changes in the numerical scheme never produced any conspicuous difference in the results.

The initial conditions used in the PDE simulations of the asymmetric solitons were slightly different from the stationary solutions found by the shooting method: the peak values of the waves were taken as given by the shooting method, but for the pulse shapes, the VA analytical expressions (10) and (11) were plugged in. The aim in choosing the initial conditions in this mixed form was twofold: first, it is much easier to insert the initial conditions into the numerical code when they are known in an analytical form; second, a small deviation of the initial conditions from the (practically) exact solitary-wave shape generated by the shooting seeds a small perturbation which is necessary to observe the dynamics.

In all the cases in which the asymmetric stationary solitons were found by the shooting method, the PDE simulations have demonstrated their stability. A typical example is displayed, for $Q=0.1$, in Fig. 3. In this figure, two well separated solitons are seen. The first one is the large u_1 component of the stationary soliton, while the second soliton is

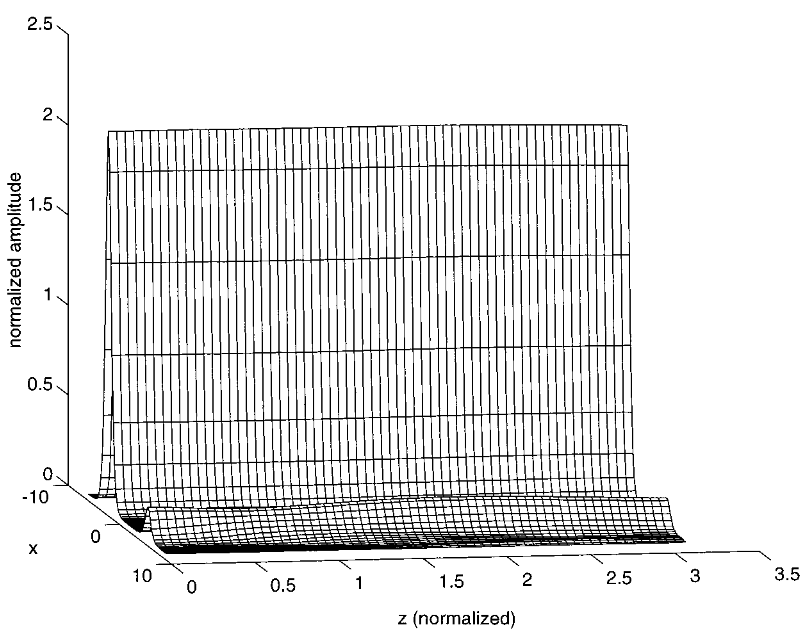

FIG. 3. An example of the evolution of a slightly perturbed asymmetric soliton at $Q=0.1$. Shown are the fundamental harmonics in both waveguides.

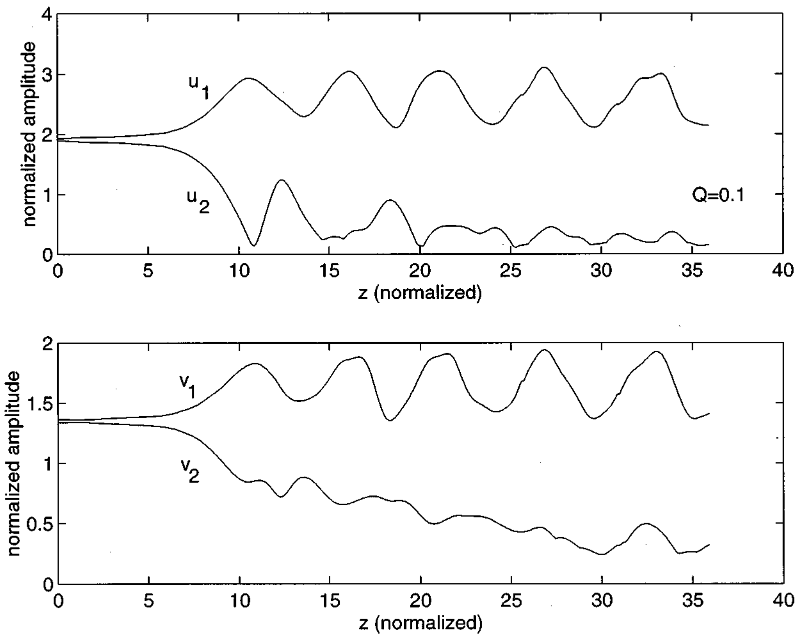

FIG. 4. Evolution of the peak values of the components $u_{1,2}$ and $v_{1,2}$ illustrating the instability of the symmetric soliton at $Q=0.1$.

represented by the small u_2 component of essentially the same solution (recall that at $Q=0.1$ the solution is strongly asymmetric, see Fig. 2). The peak values of the components of the perturbed soliton undergo minor fluctuations (within 1%). The fluctuations show no sign of decay, but they are not growing either (to check this, we made some runs much longer). Thus we conclude that all the stationary asymmetric solitons are, in effect, neutrally stable.

We also ran simulations with large initial perturbations of the asymmetric solitons. Without displaying ponderous figures, we can formulate an inference that strongly perturbed solitons demonstrate persistent internal vibrations, without being destroyed by the perturbations, but also without emitting conspicuous amounts of radiation. From a number of numerical simulations, it is known that stable solitons in the *single* SHG nonlinear waveguide demonstrate similar properties [1,26].

We also checked numerically the stability of the exact symmetric solutions (6). First of all, one should expect that, for $Q<Q_{\mathrm{cr}}$, the symmetric soliton must be destabilized by the bifurcation producing the stable asymmetric solitons. As is illustrated by Fig. 4, this is indeed the case. Moreover, the instability evolution illustrated by Fig. 4 demonstrates a trend to rearrange the unstable symmetric soliton into a stable asymmetric one existing at the same value of Q. This process is, though, quite slow, because it gives rise to strong internal vibrations of the solitary wave, for which, in accord with a rather general property of the SHG systems mentioned above, the damping is very weak.

At $Q>Q_{\mathrm{cr}}$, the symmetric solitons are stable. An ex-

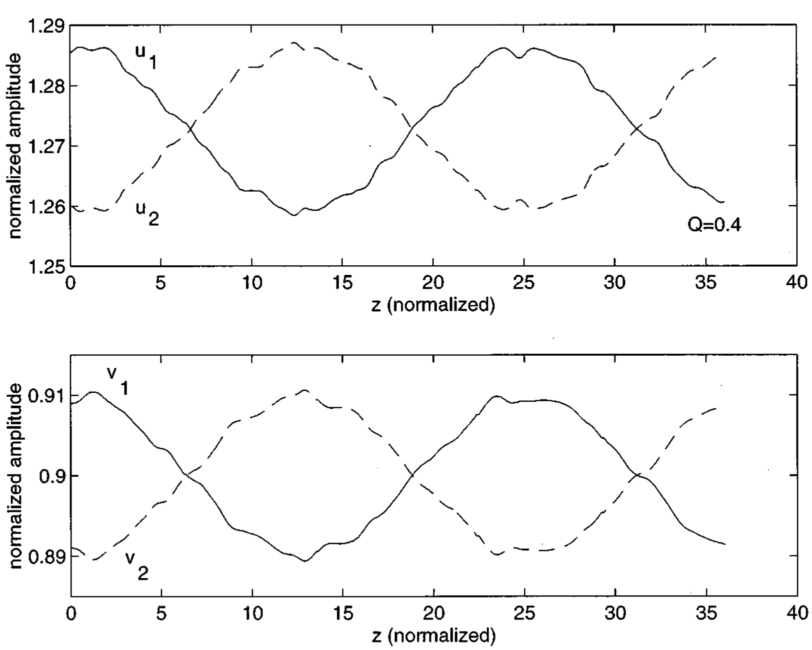

FIG. 5. The same as in Fig. 4 for $Q=0.4$. In this case, the symmetric soliton is (neutrally) stable.

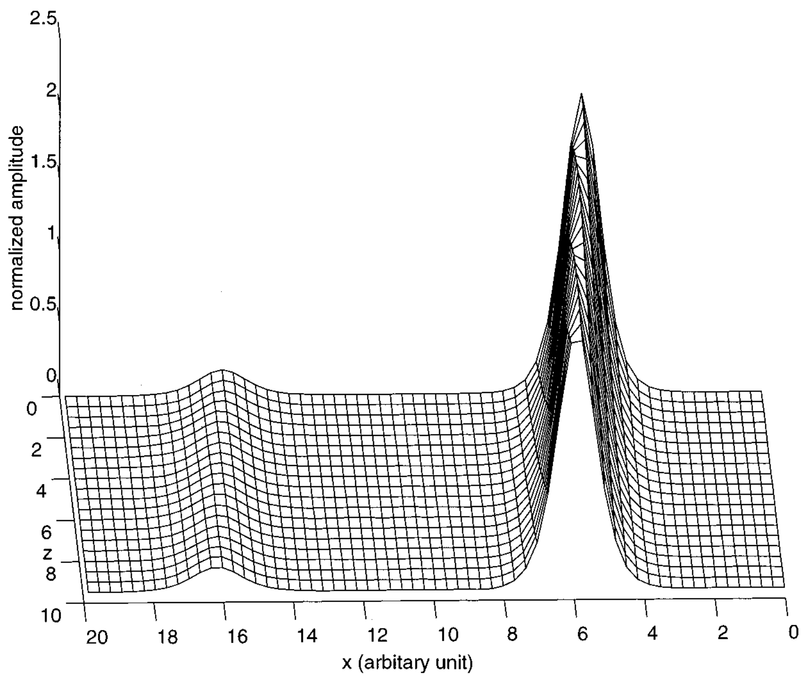

FIG. 6. Evolution of the asymmetric soliton at $Q = 0.1$ under the action of the walkoff terms with $\delta = 0.1$. The FH components are shown. The SH components are of the same shape, and the only difference from the FH components is just the amplitudes; thus they are not shown. Comparing this figure with Fig. 3, there is no essential difference, except a change in propagation direction.

ample, shown in Fig. 5 for $Q = 0.4$, shows that the initially introduced perturbations trigger internal vibrations of the solitary wave around the stationary symmetric solution. Very little damping can be seen, but the vibrations are not growing either. Thus we conclude that the symmetric solitons here are also, effectively, neutrally stable, as the asymmetric solitons that exist beyond the bifurcation point.

As was said above, the asymmetric solitons had not been found for $Q < -0.3$. In this region (detailed simulations were performed, e.g., at $Q = -0.9$), the symmetric solitons always demonstrate an *oscillatory* instability. At sufficiently large negative Q, simulation shows that the unstable symmetric solitons quite quickly decay into dispersive radiation.

In all the above consideration, we considered only the no-walkoff case, $\delta = 0$ in Eqs. (1)–(4). Because a spatial walkoff will always be present in an experiment, it is crucially important to test the robustness of the asymmetric solitons against adding walkoff into the model.

The investigation of this walkoff effect is now being undertaken, using direct PDE simulations. Preliminary results showed that both asymmetric and symmetric solitons, if they were stable in the absence of walkoff, remained stable, provided δ was not too large. For example, as shown in Fig. 6, the solitons are still stable when $\delta = 0.1$, which corresponds to a misalignment of about 10°. At large δ, say, 0.6, the solitons have been shown, from numerical simulations, to be destroyed, for all values of Q. A detailed account of the walkoff effects will be presented elsewhere, when the investigation is completed.

IV. CONCLUSION

We have formulated and analyzed a model describing two linearly coupled quadratically nonlinear waveguides. The model includes two equations for the fundamental harmonics, and two equations for the second harmonics. We have considered in detail the most important special case of no walkoff and fully matched harmonics, when the only control parameter is the coupling constant, the same for both harmonics. It was demonstrated that, alongside the obvious symmetric solitons, the model supports asymmetric solitary waves. A bifurcation point at which the asymmetric solutions appear was found exactly. A full description of these solutions in an analytical form was based on a simple variational approximation. Comparison with numerical results obtained by the shooting method has demonstrated that this approximation provides a fairly good accuracy in a part of the range where existence of the stationary asymmetric solitons was predicted, while in another part of this range, asymmetric solitary-wave solutions were not found, although periodic solutions can be easily obtained. Direct simulations of the full PDE's have shown that the asymmetric solitons, whenever they exist, are always neutrally stable. On the contrary, the symmetric solitons are stable (effectively, also neutrally) only to the right of the bifurcation point, where the asymmetric solitons do not exist. To the left of the bifurcation point, the symmetric soliton is found to be unstable, demonstrating a trend to rearrange itself into the stable asymmetric soliton that exists at the same value of the coupling constant Q . For negative values of Q, where the asymmetric solitons do not exist, the symmetric ones demonstrate a very different oscillatory instability mode, sometimes quickly decaying into radiation. Finally, some preliminary results, also by means of direct PDE simulations, demonstrate that the asymmetric solitons survive and remain stable after adding walkoff to the model, provided that the new terms are not too large.

Thus the results obtained in this work point to the existence of novel stable soliton states in the parallel-coupled second-harmonic-generating waveguides. As the next step, it is necessary to consider effects of a mismatch between the harmonics, and to analyze, in more detail, influence of walkoff between the beams in the two waveguides. In any case, the above results strongly suggest that the asymmetric solitons are robust. Moreover, the strong dependence of the stable asymmetric solutions upon the effective coupling parameter, or, in physical units, upon the energy of the beam, may open ways to use these states for optical switching.

ACKNOWLEDGMENTS

We are indebted to Yuri Kivshar for useful discussions. One of the authors (B.A.M.) appreciates support from the School of Electrical Engineering at the University of New South Wales (Sydney, Australia).

[1] R. DeSalvo, D. Hagan, M. Sheik-Bahae, G. Stegeman, and E. Van Stryland, Opt. Lett. **17**, 28 (1992).
[2] G.I. Stegeman, M. Sheik-Bahae, E. Van Stryland, and G. Assanto, Opt. Lett. **18**, 13 (1993).
[3] R. Schiek, J. Opt. Soc. Am. B **10**, 1848 (1993).
[4] M.J. Werner and P.D. Drummond, J. Opt. Soc. Am. B **10**, 2390 (1993).
[5] A.V. Buryak and Y.S. Kivshar, Opt. Lett. **19**, 1612 (1994); Phys. Lett. A **197**, 407 (1995).
[6] L. Torner, C.M. Menyuk, and G.I. Stegeman, Opt. Lett. **19**, 1615 (1994); J. Opt. Soc. Am. B **12**, 889 (1995).
[7] W.E. Torruellas, G. Krijen, D.Y. Kim, R. Schiek, G.I. Stegeman, P. Vidakovich, and J. Zyss, Opt. Commun. **112**, 122 (1994).
[8] W.E. Torruellas, Z. Wang, D.J. Hagan, E.W. Van Stryland, G.I. Stegeman, L. Torner, and C.R. Menyuk, Phys. Rev. Lett. **74**, 5036 (1995).
[9] D.E. Pelinovsky, A.V. Buryak, and Y.S. Kivshar, Phys. Rev. Lett. **75**, 591 (1995).
[10] C. Etrich, U. Peschel, F. Lederer, and B.A. Malomed, Phys. Rev. A **52**, R3444 (1995).
[11] R. Schiek, Y. Baek, and G.I. Stegeman, Phys. Rev. E **53**, 1138 (1996).
[12] H. He, M.J. Werner, and P.D. Drummond, Phys. Rev. E **54**, 896 (1996).
[13] G. Assanto, G. Stegeman, M. Sheik-Bahae, and E. Van Stryland, Appl. Phys. Lett. **62**, 1323 (1993); G. Assanto, A. Laureti-Palma, C. Sibilia, and M. Bertolotti, Opt. Comm. **110**, 599 (1994).
[14] M.A. Karpierz, in *Nonlinear Guided Waves and Their Applications*, OSA 1996 Technical Digest Series Vol. 15 (Optical Society of America, Washington, DC, 1996), p. 70.
[15] V. Steblina, Y.S. Kivshar, M. Lisak, and B.A. Malomed, Opt. Comm. **118**, 345 (1995).
[16] Y.N Karamzin and A.P. Sukhorukov, Zh. Éksp. Teor. Fiz. **68**, 834 (1975) [Sov. Phys. JETP **41**, 414 (1976)].
[17] N. Akhmediev and J.M. Soto-Crespo, Phys. Rev. E **49**, 4519 (1994).
[18] B.A. Malomed, I.M. Skinner, P.L. Chu, and G.D. Peng, Phys. Rev. E **53**, 4084 (1996).
[19] A. Ankiewicz, N. Akhmediev, and J.M. Sotocrespo, Opt. Commun. **116**, 411 (1996).
[20] P.L. Chu, B.A. Malomed, and G.D. Peng, J. Opt. Soc. Am. B **10**, 1379 (1993).
[21] A. Ankiewicz, N. Akhmediev, and G.D. Peng, Opt. Quantum Electron. **27**, 193 (1995).
[22] J.M. Sotocrespo and N. Akhmediev, Phys. Rev. E **48**, 4710 (1993).
[23] N. Akhmediev, A. Ankiewicz, and H.T. Tran, J. Opt. Soc. Am. B **10**, 230 (1993).
[24] G. Cohen, Phys. Rev. E **52**, 5565 (1995).
[25] G.R. Hadley, IEEE J. Quantum Electron **QE-28**, 363 (1992).
[26] C. Etrich, T. Peschel, F. Lederer, B.A. Malomed, and Y.S. Kivshar, Phys. Rev. E **54**, 4321 (1996).