Nonlinear reflectivity of an inhomogeneous plasma in the strongly damped regime

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The nonlinear reflectivity of an inhomogeneous plasma slab in the strongly damped regime is investigated, taking into account the spatial and temporal characteristics of the thermal noise emission of waves. In the linear approximation, the spectral width corresponding to the frequencies that are effectively amplified is always found to be less than the spectral width of the unstable frequency domain. By conjecturing that this frequency filtering process remains valid in the nonlinear regime, the effective noise term appearing in Tang's formula [J. Appl. Phys. **37**, 2945 (1966)] can be obtained analytically. The validity of this conjecture is numerically checked for different values of the inhomogeneity parameter. Conditions are given that must be satisfied for the validity of one-dimensional modeling of three-dimensional scattering. [S1063-651X(97)07504-7]

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I. INTRODUCTION

The parametric instabilities induced by laser plasma interaction remain a domain of intense research in the context of inertial fusion. Parametric instabilities such as stimulated Brillouin scattering (SBS) and stimulated Raman scattering (SRS) can induce several effects deleterious for the good efficiency of the laser-plasma coupling. The linear behavior of these two instabilities is controlled mainly by inhomogeneity effects, namely, the density gradients in the case of SRS and the flow inhomogeneity in the case of SBS, as well as by multidimensional effects [1]. A good indicator of the potential importance of these instabilities is the value of their reflectivity R in the limit where the only saturation mechanism is the pump depletion (i.e., corresponding to the standard three-wave model). Since this simple saturation mechanism yields an upper bound to the actual reflectivity, one can compute reasonable estimates of the maximum energy transferred to the scattered wave and to nonthermal plasma particles in the case of a reflectivity very small compared to unity. In the opposite case of a large reflectivity predicted by the three-wave model, one must carefully investigate all the possible nonlinear saturation mechanisms of parametric growth: pump depletion with multidimensional effects [1], nonlinearity of one of the daughter waves [2], coupling of the daughter waves to other types of waves [3], wave-particle interaction, wave breaking and subsequent particle heating [4], and interplay between instabilities [5]. It is therefore crucial to be able to accurately compute the reflectivity of an inhomogeneous plasma in the simple limit of three-wave coupling.

In the convective regime where the low-frequency daughter wave is strongly damped, the value of the reflectivity is determined not only by the incident laser intensity, but also by the plasma noise level corresponding to the spontaneous emission of the waves (bremsstrahlung and Čerenkov emission). Subsequently, a proper description of the instability in this regime requires solving the problem of the parametric growth including stochastic source terms for each wave into play (along with consistent fluctuating boundary conditions for these waves). Unfortunately, such a solution cannot be carried out analytically in the nonlinear saturated regime. To overcome this difficulty Tang [6] considered the simplified problem in which the effect of the noise sources is modeled by a deterministic boundary value. Considering the most unstable wave triplet and restricting himself to the case of a homogeneous plasma, he obtained the relation

$$R(1+B-R)=B\exp[(1-R)G],$$

which can be solved to give the plasma reflectivity R as a function of the gain factor G and of a *fixed* boundary value for the backscattered wave B. This boundary value plays the role of an effective noise term that is supposed to account for the averaged effects of the actual plasma noise. It is important to notice that the value of B cannot be obtained from a deterministic theory such as that of Tang. Here the quantity B is a *free* parameter that must be chosen in such a way that the theoretical results fit the experimental (and/or numerical) data.

This paper is devoted to the computation of the quantity B in the case of an inhomogeneous plasma, taking into account the role played by the space and time nature of the thermal noise emission. In Sec. II we introduce our theoretical model and give the statistical properties of the source terms that must be added to the usual coupled-mode equations in order to account for the thermal noise emission of waves. In Sec. III we consider the linear stage of the instability. We show that taking into account explicitly the spatial and temporal characteristics of the thermal noise emission leads to the result of the spectral width corresponding to the effectively amplified frequencies always being less than the spectral width of the unstable frequency domain. Conjecturing that this result still holds in the nonlinear regime, we show in Sec. IV A that the proper value of B can be obtained

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as the solution to an implicit equation that links B to both the plasma noise parameters and the effectively amplified spectral width. Reliability of this result is checked numerically in Sec. IV B. An alternative derivation of the nonlinear reflectivity in the strongly inhomogeneous plasma limit is given in Sec. IV C. In Sec. V the conditions are given that must be satisfied for the validity of a one-dimensional (1D) modeling of 3D scattering. Numerical applications are made in the case of SBS and SRS.

II. STANDARD THREE-WAVE COUPLING

A. Equations

In the following we restrict ourselves to parametric instabilities in the limit of the envelope approximation. We consider a 1D static inhomogeneous plasma slab model in which $x=L$ and $x=0$ denote the points where the laser light, propagating from right to left, enters and exits, respectively. Such a model is correct only when the diffraction effects do not modify the value of the gain factor, as compared with the usual 1D prediction. The conditions that must be satisfied in order to be able to neglect the diffraction effects when computing the gain coefficients have been derived in [1,7], and are recalled in Sec. V. Stochastic functions in space and time are added to the usual coupled-mode equations in order to account for the thermal noise emission of waves.

In the standard decay regime, the coupled-mode equations for an inhomogeneous plasma take the form

$$(\partial_t+V_0\partial_x+\nu_0)a_0+\gamma_0 a_1 a_2 e^{-i\kappa' x^2/2}=S_0, \tag{1a}$$

$$(\partial_t+V_1\partial_x+\nu_1)a_1-\gamma_0 a_0 a_2^* e^{i\kappa' x^2/2}=S_1, \tag{1b}$$

$$(\partial_t+V_2\partial_x+\nu_2)a_2-\gamma_0 a_0 a_1^* e^{i\kappa' x^2/2}=S_2, \tag{1c}$$

where a_0, a_1, and a_2 stand for the amplitudes of the incoming laser light and the transverse and the longitudinal decay waves, respectively. These amplitudes have been normalized in such a way that one simply has $a_0(L)=1$ at the incoming point $x=L$; γ_0 is the linear homogeneous growth rate of the parametric instability. The inhomogeneity is taken into account in the WKB approximation by $\kappa' \equiv (d/dx)[K_0(x)-K_1(x)-K_2(x)]_{x=0}$, where K_α is the local wave vector of wave α associated with the resonance condition that is assumed to be fulfilled at $x=0$, so that $[K_0(x)-K_1(x)-K_2(x)]_{x=0}=0$. The quantities V_α and ν_α denote the group velocity and the linear damping of wave α, respectively. In the following we restrict ourselves to backscattering instabilities only so that one has $V_0<0$, $V_1>0$, and $V_2<0$. The source terms S_α on the right-hand side of Eqs. (1) account for the thermal noise emission of wave α and are therefore stochastic functions in space and time. The statistical properties of the processes S_α are detailed in the next subsection.

B. Statistical properties of the source terms

The source terms S_α have to be chosen so as to reproduce the equilibrium statistical properties of the fields a_α properly within the spectral domain of interest for the instability [8]. These stochastic processes are taken to be white noises in space and time with the statistical properties

$$\langle S_\alpha(x,t)\rangle=0, \tag{2a}$$

$$\langle S_\alpha(x,t)S_\alpha^*(x',t')\rangle=(2\pi)^2\Sigma_\alpha^{1D}\delta(x-x')\delta(t-t'), \tag{2b}$$

where $\langle\ \rangle$ denotes the statistical average. The space-time variables appearing in Eqs. (2) are the slow variables corresponding to the envelope approximation. The 1D spectral density Σ_α^{1D} is determined from the condition that the equilibrium fluctuations corresponding to Eqs. (1) in the limit $\gamma_0=0$ be identical to the thermal equilibrium values. One obtains

$$\Sigma_\alpha^{1D}=\frac{\nu_\alpha T_\alpha^{\mathrm{eff}}}{(2\pi)^4\omega_\alpha N_0}K_\alpha^2\Delta\Omega_\alpha, \tag{3}$$

where $\mathbf{K}_\alpha$ is the wave vector of wave α associated with the usual three-wave resonance conditions and $\Delta\Omega_\alpha$ is the solid angle in $\mathbf{K}_\alpha$ space characterizing the 3D plasma volume of the thermal noise effectively amplified in the process being considered. The expressions of $\Delta\Omega_\alpha$ are given in Sec. V. The effective temperature T_α^{eff} is given for the transverse wave (TW) and the electron plasma wave (EPW) by $T_{\mathrm{TW}}^{\mathrm{eff}}=T_{\mathrm{EPW}}^{\mathrm{eff}}=T_e$ and for the ion sound wave (ISW) by $T_{\mathrm{ISW}}^{\mathrm{eff}}=T_e(1+ZQ)/[1+(ZT_e/T_i)Q]$ with $Q=(v_{\mathrm{the}}/v_{\mathrm{thi}})\times\exp[-(ZT_e+3T_i)/2T_i]$, where T_s and $v_{\mathrm{ths}}=(T_s/m_s)^{1/2}$ denote the temperature (in units of energy) and the thermal velocity of species s, respectively. In the case of SBS, the angular frequency ω_α is given by $\omega_1\approx\omega_0$ and $\omega_2\approx 2\omega_0(c_S/c)(1-n/n_c)^{1/2}$, where n, n_c, and $c_S=[(ZT_e+3T_i)/m_i]^{1/2}$ are, respectively, the electron density, the critical density, and the ion sound velocity. In the case of SRS, ω_α is given by $\omega_1\approx\omega_0-\omega_{pe}$ and $\omega_2\approx\omega_{pe}$, where ω_{pe} is the local electron plasma frequency. The normalizing constant N_0 is the density of quanta for the pump wave

$$N_0=\frac{n_c}{4}\frac{m_e c^2}{\omega_0}\left(\frac{v_{\mathrm{osc}}}{c}\right)^2,$$

where $v_{\mathrm{osc}}=q_e E_0/m_e\omega_0$ is the electron quiver velocity, E_0 denoting the amplitude of the pump wave electric field at its incoming point $x=L$. In the case of 1D particle in cell simulations, the whole factor $T_\alpha^{\mathrm{eff}}K_\alpha^2\Delta\Omega_\alpha$ should be determined so as to match the equilibrium characteristics corresponding to the numerical scheme properly.

III. REFLECTIVITY IN THE LINEAR APPROXIMATION

In the strongly damped regime, defined by $\nu_2\gg 2\gamma_0|V_2/V_1|^{1/2}$, the damping ν_2 is sufficiently strong for wave 2 to be locally enslaved to wave 1 (except in a negligibly thin boundary layer at the vicinity of $x=L$). In this limit, the instability quickly reaches a steady state of spatial amplification. The fluctuation $\langle|a_1|^2\rangle$ does not depend on time and one has

$$\langle|a_1|^2\rangle=\int_{-\infty}^{+\infty}n_1(\omega,x)d\omega, \tag{4}$$

where $n_1(\omega,x)$ denotes the spectral density of wave 1. In the linear regime where the pump depletion is negligible (i.e., $a_0 \equiv 1$), the equation for $n_1(\omega,x)$ can readily be derived from Eqs. (1b), (1c), (2a), and (2b). As shown in Appendix A, one obtains

$$\left[\partial_x + \frac{2}{V_1}\left(\nu_1 - \frac{\gamma_0^2 \nu_2}{\nu_2^2 + (\omega + \varepsilon \nu_2 x/x_c)^2}\right)\right] n_1(\omega,x) = \frac{2\pi}{V_1^2}\left[\Sigma_1^{1D} + \frac{\gamma_0^2}{\nu_2^2 + (\omega + \varepsilon \nu_2 x/x_c)^2}\Sigma_2^{1D}\right], \tag{5}$$

with $\varepsilon = \mathrm{sgn}(V_2 \kappa')$ and where $x_c = \nu_2 / |V_2 \kappa'|$ is the inhomogeneous amplification length in the strongly damped limit [9]. In the following we restrict ourselves to the case where the inverse bremsstrahlung absorption can be neglected in the interacting plasma ($\nu_1 = 0$). In this limit and assuming, for the sake of simplicity, that the spectral density at the boundary $x=0$ is given by the equilibrium value $n_1(\omega,0) = \pi \Sigma_1^{1D}/V_1 \nu_1$, one finds that the solution to Eq. (5) reduces to (cf. Appendix A)

$$n_1(\omega,x) = \pi \Sigma_1^{1D}/V_1 \nu_1 + \frac{\pi}{V_1}\left(\frac{\Sigma_1^{1D}}{\nu_1} + \frac{\Sigma_2^{1D}}{\nu_2}\right)[\exp G(\omega,x) - 1], \tag{6}$$

where the gain factor $G(\omega,x)$ is given by

$$G(\omega,x) = \frac{2\gamma_0^2}{V_1 V_2 \kappa'}\left[\tan^{-1}\left(\frac{\omega}{\nu_2} + \varepsilon \frac{x}{x_c}\right) - \tan^{-1}\left(\frac{\omega}{\nu_2}\right)\right].$$

It is worth mentioning that in the limit of a strongly inhomogeneous plasma defined by $x_c \ll x$, our results reduce to those of Berger *et al.* [10], as they should.

Before proceeding further into the calculations, it is interesting to *a priori* estimate the order of magnitude of the contribution of each term on the right-hand side of Eq. (6) when substituted into expression (4) for the fluctuation $\langle |a_1|^2 \rangle$. The order of magnitude of the first term, denoted in what follows as $\langle |a_1|^2 \rangle_{\mathrm{th}}$, is given by

$$\langle |a_1|^2 \rangle_{\mathrm{th}} \approx \frac{\pi}{V_1}\frac{\Sigma_1^{1D}}{\nu_1}\Delta\omega_{\mathrm{obs}}, \tag{7}$$

where $\Delta\omega_{\mathrm{obs}}$ is the total spectral width corresponding to the domain of observation for wave 1. Regarding now the following terms, denoted as $\langle |a_1|^2 \rangle_{\mathrm{coupl}}$, one expects their order of magnitude to be given by

$$\langle |a_1|^2 \rangle_{\mathrm{coupl}} \approx \frac{\pi}{V_1}\left(\frac{\Sigma_1^{1D}}{\nu_1} + \frac{\Sigma_2^{1D}}{\nu_2}\right)\Delta\omega_{\mathrm{unst}}[\exp(G_{\max}) - 1], \tag{8}$$

where the maximum gain factor $G_{\max} \equiv \max_\omega[G(\omega,x)]$ is given by

$$G_{\max} = (2/\pi) G_{\mathrm{Ros}} \tan^{-1}(x/2x_c), \tag{9}$$

G_{Ros} being the so-called Rosenbluth gain factor $G_{\mathrm{Ros}} \equiv 2\pi\gamma_0^2/|V_1 V_2 \kappa'|$. The quantity $\Delta\omega_{\mathrm{unst}}$ [the expression of which is given in Eq. (11)] denotes the characteristic spectral width of the unstable frequency domain for which the gain factor $G(\omega,x)$ is of the same order as its maximum value $G_{\max}$. In the large-gain-factor limit the *a priori* estimate (8) can be obtained easily by substituting $\Delta\omega_{\mathrm{unst}}$ for $\Delta\omega_{\mathrm{obs}}$ into Eq. (7) and then multiplying the result by the amplification factor $\exp(G_{\max})$ corresponding to the maximum gain factor. The expression of $\Delta\omega_{\mathrm{unst}}$ can be readily obtained from the small-gain-factor limit of Eq. (8), which has to reduce in this limit to the expression for Thomson scattering reflectivity

$$\langle |a_1|^2 \rangle_{\mathrm{Thoms}} = (2\pi)^2\left(\frac{\Sigma_1^{1D}}{\nu_1} + \frac{\Sigma_2^{1D}}{\nu_2}\right)\frac{\gamma_0^2 L}{2V_1^2}. \tag{10}$$

Identifying Eq. (8), in the limit $G_{\max} \ll 1$, with expression (10), one obtains

$$\Delta\omega_{\mathrm{unst}} \equiv \frac{1}{\max_\omega[G(\omega,L)]}\int_{-\infty}^{+\infty} G(\omega,L)\, d\omega = \pi \nu_2 \frac{(L/2x_c)}{\tan^{-1}(L/2x_c)}. \tag{11}$$

The remainder of this section is devoted to checking the validity of the estimate (8) with $\Delta\omega_{\mathrm{unst}}$ given by Eq. (11). Namely, we will show that the estimate (8) is correct to within a nontrivial numerical factor only, which leads to replace the spectral width $\Delta\omega_{\mathrm{unst}}$ by an effective spectral width $\Delta\omega_{\mathrm{eff}}$. As this effective width will be found to satisfy the inequality $\Delta\omega_{\mathrm{eff}} \leq \Delta\omega_{\mathrm{unst}}$, this replacement can be interpreted as corresponding to filtering in the domain of effectively amplified frequencies.

In order to establish this result one has to evaluate the right-hand side of Eq. (4) more carefully. Considering first the large-gain-factor limit $G_{\max} \gg 1$, the leading term of the asymptotic expansion of the integral (4) is found to be given by

$$\langle |a_1|^2 \rangle_{\mathrm{coupl}}(x) \sim \frac{\pi}{V_1}\left(\frac{\Sigma_1^{1D}}{\nu_1} + \frac{\Sigma_2^{1D}}{\nu_2}\right)\mathfrak{I}\exp(G_{\max}),$$

where $\mathfrak{I}$ is written in terms of the error function $\mathrm{erf}(z) \equiv 2/\sqrt{\pi}\int_0^z \exp(-u^2)du$ as

$$\mathfrak{I} = \Delta\omega_{\mathrm{unst}}\frac{\mathrm{erf}[\sqrt{\pi^3/4(\nu_2/\Delta\omega_{\mathrm{unst}}) f(x/2x_c) G_{\max}}]}{\sqrt{\pi^2(\nu_2/\Delta\omega_{\mathrm{unst}}) f(x/2x_c) G_{\max}}},$$

where the function $f(x)$ is defined by

$$f(x) \equiv [x/(\sqrt{1+x^2}\tan^{-1}x)]^4.$$

[It can be useful to have in mind the behavior of $f(x)$ in the limits of small and large values of its argument. One has $f(x) \approx 1$ for $x \ll 1$ and $f(x) \approx (2/\pi)^4$ for $x \gg 1$.] Substituting for the error function the simple algebraic fit $\mathrm{erf}(\sqrt{x}) \sim (4x/\pi)^{1/2}(1+4x/\pi)^{-1/2}$, one obtains for $G_{\max} \gg 1$

$$\langle |a_1|^2\rangle_{\rm coupl}(x)\sim \frac{\pi}{V_1}\left(\frac{\Sigma_1^{\rm 1D}}{\nu_1}+\frac{\Sigma_2^{\rm 1D}}{\nu_2}\right) \times\frac{\Delta\omega_{\rm unst}\exp(G_{\max})}{\sqrt{1+\pi^2(\nu_2/\Delta\omega_{\rm unst})f(x/2x_c)G_{\max}}}. \tag{12}$$

In the opposite limit of small gain factor, the integral on the right-hand side of Eq. (4) can be carried out perturbatively and written as a power series of $G_{\max}$. At the lowest order, one obtains for $G_{\max}\ll 1$

$$\langle |a_1|^2\rangle_{\rm coupl}(x)=\frac{\pi}{V_1}\left(\frac{\Sigma_1^{\rm 1D}}{\nu_1}+\frac{\Sigma_2^{\rm 1D}}{\nu_2}\right)\Delta\omega_{\rm unst}G_{\max}, \tag{13}$$

which is simply the Thomson scattering reflectivity (10), as it should be.

It is now possible to recast the two equations (12) and (13) into a single uniform expression, i.e., an expression that is correct for both small and large values of the gain factor. This expression reads

$$\langle |a_1|^2\rangle_{\rm coupl}(x)=\frac{\pi}{V_1}\left(\frac{\Sigma_1^{\rm 1D}}{\nu_1}+\frac{\Sigma_2^{\rm 1D}}{\nu_2}\right)\Delta\omega_{\rm eff}(G_{\max})[e^{G_{\max}}-1], \tag{14}$$

where $\Delta\omega_{\rm eff}(G_{\max})$ denotes the spectral width associated with the effectively amplified frequencies, namely,

$$\Delta\omega_{\rm eff}(G_{\max})=\frac{\Delta\omega_{\rm unst}}{\sqrt{1+\pi^2(\nu_2/\Delta\omega_{\rm unst})f(x/2x_c)G_{\max}}}. \tag{15}$$

Equation (14) can be quite simply obtained from the simple estimate (8) by substituting $\Delta\omega_{\rm eff}(G_{\max})$ for $\Delta\omega_{\rm unst}$ into the latter equation. As said before, this substitution represents a reduction in the frequency domain of the Fourier components that are effectively amplified, as compared to the simple estimate (11), because one has $\Delta\omega_{\rm eff}\leqslant\Delta\omega_{\rm unst}$. As one has $\pi\nu_2/\Delta\omega_{\rm unst}\leqslant 1$ and $f(x/2x_c)\leqslant 1$, $\Delta\omega_{\rm eff}$ can be significantly smaller than $\Delta\omega_{\rm unst}$ in the regime of large gain factor only, which corresponds to the well-known effect of frequency filtering induced by gain narrowing. More precisely, comparing the exact result (14) with the estimate (8) in the regime of large-gain-factor limit, one may quantify this gain narrowing effect as follows: (i) in the limit corresponding to a quasihomogeneous plasma, namely, for $x/x_c<1<G_{\max}$, the simple expression (8) overestimates the correct result by the factor $(\pi G_{\max})^{1/2}>1$ with $G_{\max}\approx 2\gamma_0^2 x/V_1\nu_2$; (ii) in the limit of moderately inhomogeneous plasmas defined by the inequality $1<x/x_c<G_{\max}$, one has $G_{\max}\approx G_{\rm Ros}$ and Eq. (8) overestimates the correct result by the factor $(4/\pi)(x_cG_{\rm Ros}/x)^{1/2}>1$; (iii) it is only in the strongly inhomogeneous limit $1<G_{\max}<x/x_c$ that one has $\Delta\omega_{\rm eff}(G_{\max})=\Delta\omega_{\rm unst}$ and the simple estimate (8) gives the correct result (14).

The final expression (14) can also be written in the convenient form

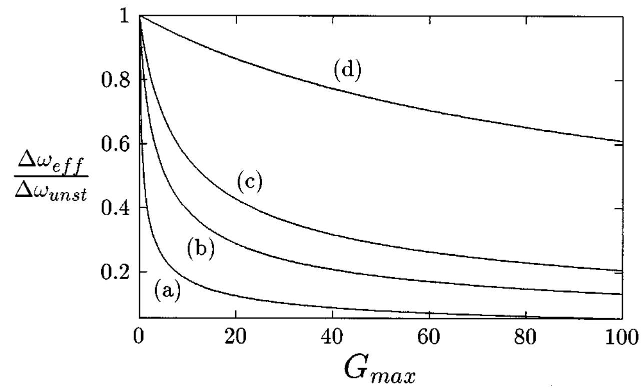

FIG. 1. Ratio $\Delta\omega_{\rm eff}/\Delta\omega_{\rm unst}$ as a function of $G_{\max}$ for different values of $L/2x_c$: (a) $L/2x_c=0$, (b) $L/2x_c=2.5$, (c) $L/2x_c=5$, and (d) $L/2x_c=50$.

$$\langle |a_1|^2\rangle_{\rm coupl}=\langle |a_1|^2\rangle_{\rm Thoms}\frac{[\exp(G_{\max})-1]}{G_{\max}}\frac{\Delta\omega_{\rm eff}}{\Delta\omega_{\rm unst}}, \tag{16}$$

which can be regarded as a mere generalization of the Thomson scattering expression consisting, on the one hand, in the replacement of the gain factor $G_{\max}$ by the quantity $[\exp(G_{\max})-1]$ and, on the other hand, in taking into account the gain narrowing by the ratio $\Delta\omega_{\rm eff}/\Delta\omega_{\rm unst}$.

Figure 1 shows the ratio $\Delta\omega_{\rm eff}/\Delta\omega_{\rm unst}$ as a function of $G_{\max}$ for different values of $x/2x_c$. It can be seen that the frequency filtering that amounts to replace $\Delta\omega_{\rm unst}$ by $\Delta\omega_{\rm eff}$ strongly modifies the reflectivity in the homogeneous limit $x/x_c\ll 1$. In this limit and for a gain factor as small as 10, one has $\Delta\omega_{\rm eff}/\Delta\omega_{\rm unst}\approx 0.17$, which corresponds to a reduction of almost one order of magnitude between the correct result (14) and the estimate (8). In the opposite limit of an inhomogeneous plasma $x/x_c=100$ and for a gain factor as large as 100, one obtains $\Delta\omega_{\rm eff}/\Delta\omega_{\rm unst}\approx 0.61$, which means that the simple estimate (8) gives the correct order of magnitude for the reflectivity.

All the results obtained in this section assume that the spectral density $n_1(\omega,0)$ at the boundary $x=0$ is nearly constant over a frequency range of width $\Delta\omega_{\rm eff}(G_{\max})$ around the most unstable frequency $\omega_{\max}=-\varepsilon\nu_2x/2x_c$. If this is not the case, e.g., due to departure from thermal equilibrium, a similar treatment can be carried out, which leads either to the modification of $\Delta\omega_{\rm eff}$ [in the case of a smooth, nonthermal function $n_1(\omega,0)$] or the modification of both $\Delta\omega_{\rm eff}$ and $G_{\max}$ [in the case of a peaked function $n_1(\omega,0)$ corresponding, for instance, to the existence of particle beams].

IV. NONLINEAR REFLECTIVITY

A. Analytical results

The validity of the linear approximation is strongly constrained by the inequality $\langle |a_1|^2\rangle\ll 1$, which is practically never fulfilled in the very-large-gain-factor limit. A proper description of the nonlinear saturation of the quantity $\langle |a_1|^2\rangle$ would require the solution of the whole system (1); unfortunately, it is not possible to carry out such a solution analytically. To overcome this difficulty one has to seek a more simple system of equations that could easily be solved analytically and give a good description of the nonlinear saturation of the reflectivity.

As one expects the behavior of $\langle|a_1|^2\rangle_{\rm coupl}(x=L)$ not to be very different from that of the most unstable component $|a_1(\omega^L_{\rm max},L)|^2$, with $\omega^L_{\rm max}=-\varepsilon\nu_2 L/2x_c$, we will consider the coupled equations for $|a_0|^2$ and $|a_1(\omega^L_{\rm max})|^2$. These equations can be readily derived from the system (1) written in the strongly damped $(\nu_2 \gg 2\gamma_0|V_2/V_1|^{1/2})$ deterministic $(S_\alpha=0)$ limits and for the most unstable triplet $a_0(t,x)=a_0(x)$, $a_1(t,x)=a_1(\omega^L_{\rm max},x)\exp(-i\omega^L_{\rm max}t)$, and $a_2(t,x)=a_2(-\omega^L_{\rm max},x)\exp(i\omega^L_{\rm max}t)$. One obtains

$$\partial_x|a_0|^2+\frac{2\gamma_0^2|a_0|^2|a_1(\omega^L_{\rm max})|^2}{V_0\nu_2[1+(x-L/2)^2/x_c^2]}=0, \tag{17a}$$

$$\partial_x|a_1(\omega^L_{\rm max})|^2-\frac{2\gamma_0^2|a_0|^2|a_1(\omega^L_{\rm max})|^2}{V_1\nu_2[1+(x-L/2)^2/x_c^2]}=0. \tag{17b}$$

The solution of this system for the boundary condition $|a_1(\omega^L_{\rm max},0)|^2=B$ yields the inhomogeneous plasma version of the Tang formula, namely

$$R\left[1+\frac{V_1}{|V_0|}(B-R)\right]=B\exp\left[\left(1-\frac{V_1}{|V_0|}R\right)G_{\rm max}\right], \tag{18}$$

where R and $G_{\rm max}$ denote, respectively, the quantity $|a_1(\omega^L_{\rm max},L)|^2$ and the maximum inhomogeneous gain factor (9) at the incoming point $x=L$. Note that Eq. (18) can be rewritten into the usual form of the Tang formula $R_v(1+B_v-R_v)=B_v\exp[(1-R_v)G_{\rm max}]$, where $B_v \equiv V_1B/|V_0|$ and $R_v\equiv V_1R/|V_0|$ denote, respectively, the incoming and outgoing intensity $|a_1|^2$ in vacuum. In this classical approach of the nonlinear calculation of the reflectivity, it is the fixed boundary condition $|a_1(\omega^L_{\rm max},0)|^2=B$ that is supposed to account for the averaged effects of the stochastic functions S_α. The problem is now to determine the effective value of B so that the reflectivity R as given by Eq. (18) reproduces the correct reflectivity $\langle|a_1|^2\rangle_{\rm coupl}(x=L)$ properly.

In the linear regime, corresponding to the limit $B\ll R\ll 1$, Eq. (18) is found to reduce to $R=B\exp(G_{\rm max})$, which coincides with the large-gain-factor limit of Eq. (14) provided B is taken to be equal to $(\pi/V_1)(\Sigma_1^{\rm 1D}/\nu_1+\Sigma_2^{\rm 1D}/\nu_2)\Delta\omega_{\rm eff}(G_{\rm max})$.

In the nonlinear regime, one expects the nonlinear saturation of the reflectivity to limit the decreasing of $\Delta\omega_{\rm eff}$ with increasing $G_{\rm max}$. The condition $R<1$ and the linear expression $R=B\exp(G_{\rm max})$ yield the inequality $G_{\rm max}<\ln(1/B)$, which gives a lowest-order estimate of the maximum value of $G_{\rm max}$ to be taken as an argument of $\Delta\omega_{\rm eff}$ in the nonlinear regime.

One is thus led to conjecture that the generalized Tang formula (18) reproduces the actual reflectivity properly if B is given by the implicit equation

$$B=\frac{\pi}{V_1}\left(\frac{\Sigma_1^{\rm 1D}}{\nu_1}+\frac{\Sigma_2^{\rm 1D}}{\nu_2}\right)\Delta\omega_{\rm eff}\{\min[G_{\rm max},\ln(1/B)]\}, \tag{19}$$

where the quantity $\Delta\omega_{\rm eff}$ is given by Eq. (15).This equation can also be written under the convenient form

$$B=\langle|a_1|^2\rangle_{\rm Thoms}\frac{1}{G_{\rm max}}\frac{\Delta\omega_{\rm eff}\{\min[G_{\rm max},\ln(1/B)]\}}{\Delta\omega_{\rm unst}}. \tag{20}$$

Expressions (19) and (20) show that the proper value of B depends not only on the expected plasma noise parameters, but also, through the effectively amplified spectral width $\Delta\omega_{\rm eff}$, on more unexpected quantities such as the plasma length L, the inhomogeneous amplification length x_c, and the gain factor $G_{\rm max}(L)$. In particular, it is interesting to notice that in the limit of inhomogeneous plasmas $(L/x_c \gg 1)$ one has $\Delta\omega_{\rm eff}\approx\Delta\omega_{\rm unst}\approx\nu_2 L/x_c$, so that B is proportional to the interaction length L (besides some logarithmic corrections). The expression for B as given by Eq. (19) or (20) is the main result of our paper; the numerical applications to SBS and SRS are given in Sec. V.

B. Numerical results

We have checked the validity of Eq. (19) by numerically integrating the set of equations (1). In our simulations, the stochastic source terms $S_\alpha(x,t)$ are computed such as to reproduce the characteristics of a colored noise, with the statistical properties

$$\langle S_\alpha(x,t)\rangle=0, \tag{21a}$$

$$\langle S_\alpha(x,t)S^*_\alpha(x',t')\rangle=\frac{\pi^2\Sigma_\alpha^{\rm 1D}}{l_c\tau_c}e^{-|x-x'|/l_c}e^{-|t-t'|/\tau_c}. \tag{21b}$$

In the limit where the correlation length l_c and the correlation time τ_c tend to zero, these source terms reduce to a white noise that satisfies the statistical properties (2). To generate the quantities S_α we have used two coupled Langevin equations

$$\partial_x F_1+l_c^{-1}F_1=F_0, \tag{22a}$$

$$\partial_t S+\tau_c^{-1}S=F_1, \tag{22b}$$

where the stochastic function F_0 is a white noise in space and time. In the discretized equations this means that, at each time step, the function F_0 takes random values uniformly distributed between two bounds determined such as to reproduce the statistical properties (21). The space and time steps are chosen so as to fulfill the conditions

$$\Delta x \ll l_c \ll L_{\rm min}, \tag{23a}$$

$$\Delta t \ll \tau_c \ll \tau_{\rm min}, \tag{23b}$$

where the quantities $L_{\rm min}$ and $\tau_{\rm min}$ denote the smallest characteristic scales for the evolution of the waves into play. It should be noted that these conditions can be very constraining for large inhomogeneous plasmas (see Appendix B).

The reflectivity obtained by solving Eqs. (1) numerically is a stochastic function of time. Before it can be compared with the analytical results given by Eqs. (18) and (19) it must be smoothed out by averaging it over a sufficiently long time

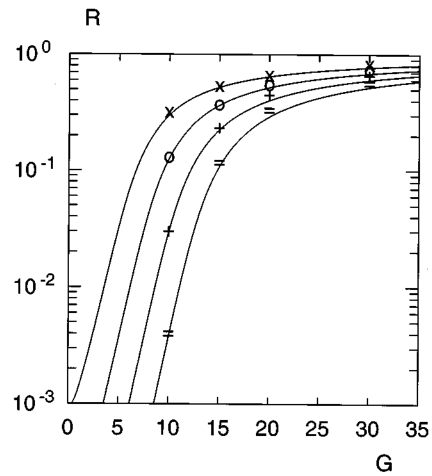

FIG. 2. Reflectivity of a quasihomogeneous plasma ($L/2x_c = 0.01$) as a function of $G_{\max}$ for different values of the noise parameter b: $b=10^{-6}$ (=), $b=10^{-5}$ (+), $b=10^{-4}$ (○), and $b=10^{-3}$ (×). The solid lines correspond to the analytical results (18) and (19).

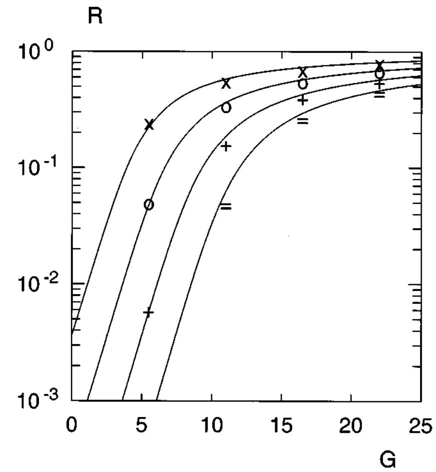

FIG. 4. Reflectivity of a strongly inhomogeneous plasma ($L/2x_c = 5$) as a function of $G_{\max}$ for different values of the noise parameter b: $b=10^{-6}$ (=), $b=10^{-5}$ (+), $b=10^{-4}$ (○), and $b=10^{-3}$ (×). The solid lines correspond to the analytical results (18) and (19).

(after the initial transient stage has disappeared). Figures 2–4 show the reflectivity as a function of the maximum gain factor $G_{\max}$ for different values of the noise parameter b, with

$$b \equiv \frac{\pi^2 \nu_2}{V_1}\left(\frac{\Sigma_1^{1D}}{\nu_1} + \frac{\Sigma_2^{1D}}{\nu_2}\right).$$

Three different cases have been considered: quasihomogeneous plasma ($L/2x_c = 0.01$), moderately inhomogeneous plasma ($L/2x_c = 1$), and strongly inhomogeneous plasma ($L/2x_c = 5$), corresponding to Figs. 2, 3, and 4, respectively. The solid line curves correspond to the analytical results (18) and (19). The points correspond to the smoothed numerical reflectivity for $b=10^{-3}$, 10^{-4}, 10^{-5}, and 10^{-6},

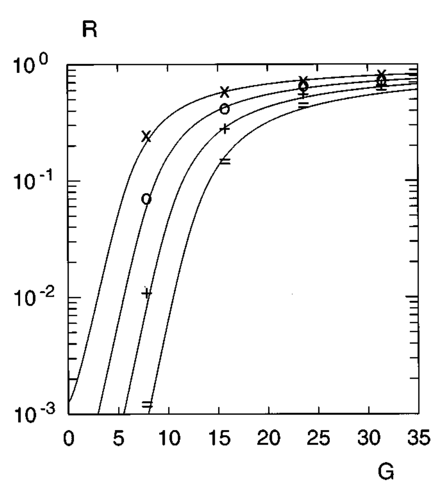

FIG. 3. Reflectivity of a moderately inhomogeneous plasma ($L/2x_c = 1$) as a function of $G_{\max}$ for different values of the noise parameter b: $b=10^{-6}$ (=), $b=10^{-5}$ (+), $b=10^{-4}$ (○), and $b=10^{-3}$ (×). The solid lines correspond to the analytical results (18) and (19).

respectively. It can be seen that the analytical and numerical results are in excellent agreement.

C. An alternative derivation of the nonlinear reflectivity in the strongly inhomogeneous plasma limit

To obtain the generalized Tang formula (18) we have assumed that the pump depletion could be described as being due to the amplification of only one unstable component $a_1(\omega_{\max}^L, L)$, with $\omega_{\max}^L = -\varepsilon \nu_2 L/2x_c$, the influence of all the other unstable components on the pump depletion being accounted for by the quantity $\Delta\omega_{\text{eff}}$ in the expression (19) of the effective noise. Thus, according to this description, in the strongly inhomogeneous plasma limit, all the reflectivity and all the pump depletion are modeled as if they were coming from the narrow resonance region of the most unstable component, the latter being assumed to be growing from this strongly enhanced effective noise. As seen in Sec. IV B, this description yields very good results as far as one is concerned with the expression of the plasma reflectivity (cf. Fig. 4). For the sake of completeness, we present in the remainder of this section an alternative derivation of the nonlinear reflectivity in the case of a strongly inhomogeneous plasma. This alternative description is found, on the one hand, to give an expression for the plasma reflectivity that is in good agreement with the generalized Tang formula (18) and therefore with the numerical results displayed before; on the other hand, it provides a better description of the spatial profile of the pump intensity within the plasma.

In the limit of a large gain factor and a strongly inhomogeneous plasma, Eq. (6) can be rewritten as

$$n_1(\omega,x) \approx \frac{\pi}{V_1}\left(\frac{\Sigma_1^{1D}}{\nu_1} + \frac{\Sigma_2^{1D}}{\nu_2}\right) H\left(x - \frac{\varepsilon\omega x_c}{\nu_2}\right) \times \exp\left[G_{\text{Ros}}|a_0|^2\left(\frac{\varepsilon\omega x_c}{\nu_2}\right)\right], \tag{24}$$

where $H(x)$ is the Heaviside step function and G_{Ros} denotes the Rosenbluth gain factor *without* pump depletion (i.e., with

$|a_0|^2=1)$. Inserting this expression into the flux conservation equation, one obtains the equation for the pump intensity [11,7]

$$\frac{d|a_0|^2(x)}{dx}=\frac{d}{dx}\int_{-\infty}^{+\infty}n_1(\omega,x)d\omega$$
$$=\frac{\nu_2\pi}{V_1x_c}\left(\frac{\Sigma_1^{1D}}{\nu_1}+\frac{\Sigma_2^{1D}}{\nu_2}\right)\exp[G_{\rm Ros}|a_0|^2(x)], \tag{25}$$

where we have taken $|V_0|=V_1$ for simplicity. Integrating this equation between $x=0$ and $x=L$, one obtains

$$\frac{1-\exp(-G_{\rm Ros}R)}{G_{\rm Ros}}=\frac{2b}{\pi}\left(\frac{L}{2x_c}\right)\exp[G_{\rm Ros}(1-R)], \tag{26}$$

where $R=1-|a_0|^2(0)$ is the reflectivity and b is the noise parameter defined at the end of Sec. IV B. This expression is an alternative expression for the plasma reflectivity, which has to be compared with the generalized Tang formula (18), which takes the form

$$R(1-R)=\frac{2b}{\pi}\left(\frac{L}{2x_c}\right)\exp[G_{\rm Ros}(1-R)] \tag{27}$$

in the strongly inhomogeneous plasma limit considered here $(\Delta\omega_{\rm eff}=\Delta\omega_{\rm unst}=\nu_2L/x_c)$. It can be seen from Eq. (25) that each unstable component contributes to the pump depletion by an infinitesimal amount, and the overall pump depletion as given by Eq. (26) is caused by the superposition of all these infinitesimal contributions. In the strongly inhomogeneous regime considered in this subsection, the derivation underlying (26) is therefore better than the generalized Tang description [in which one unstable component gives the whole (macroscopic) contribution to the pump depletion], whenever one is concerned with the spatial profile of the waves *within* the interaction region. As said before, concerning the plasma reflectivity itself, the numerical results displayed in Sec. IV B show that the generalized Tang expression gives a very good estimate of the plasma reflectivity.

In order to compare the results of Eqs. (26) and (27) we have plotted in Fig. 5 the reflectivity as a function of the Rosenbluth gain factor in the same case as in Fig. 4 $(L/2x_c=5)$. The solid lines correspond to the generalized Tang formula (27) (these are the same curves as in Fig. 4) and the dashed lines correspond to the strongly inhomogeneous plasma limit formula (26). It can be seen that the discrepancy is always much less than one order of magnitude. (In this particular case, the largest relative deviation is $\Delta R/R=23\%$ with $R\approx0.23$, which corresponds to $b=10^{-6}$ and $G_{\rm Ros}\approx13$.) This result confirms that, in the strongly inhomogeneous plasma limit, Eqs. (18) and (19) give a reliable estimate of the reflectivity even though they correspond to a rough description of the pump depletion within the interaction region.

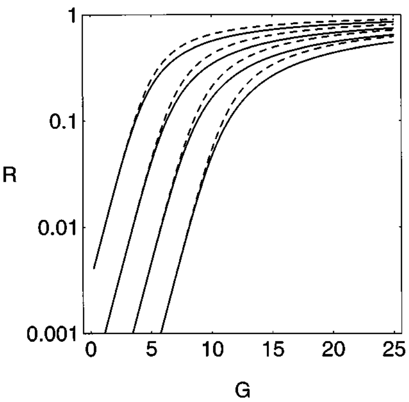

FIG. 5. Reflectivity as a function of the gain factor in the same case as in Fig. 4. The solid lines correspond to Tang's formula (27) (i.e., same curves as in Fig. 4) and the dashed lines correspond to the strongly inhomogeneous plasma limit formula (26).

V. VALIDITY CONDITIONS FOR 1D MODELING AND NUMERICAL APPLICATIONS TO SBS AND SRS

Before our results can be applied to practical computation of SBS or SRS reflectivity, it is necessary to determine the plasma parameters appearing in the expressions of the gain factor (9) and the noise term (19). These plasma parameters are to be obtained either from experimental results (temperature diagnostics, etc.) or from numerical simulations of the hydrodynamical evolution of the irradiated plasma (using hydrodynamic codes such as LASNEX [12], FILM [13], and MULTI [14]). It is only after these data are known that one can then compute the interaction parameters (gain factor and noise term) that appear in the generalized Tang formula (18). It follows in particular that, due to the intensity dependence of the plasma parameters, the actual dependence of the interaction parameters on the laser intensity is not the simple explicit dependence. It is also worth noting that, since the characteristic time scale for the hydrodynamical evolution of the plasma is much longer than the characteristic saturation time of the instability (typically the light transit time across the resonant region), the generalized Tang formula (18) gives the instantaneous reflectivity (on the hydrodynamical time scale), the time dependence of which is given by the time dependence of the plasma parameters.

In this section we first give the validity conditions for the 1D modeling developed in this article. We consider a Gaussian laser beam, characterized by a focal plane waist $a_0=f\lambda_0$ and a length along the direction of propagation $l_R=2\pi f^2\lambda_0$, where $f=F/D$ is the ratio of the focal length F of the focusing lens over the laser beam diameter D in vacuum. The size of the plasma along the direction of laser propagation will be denoted as L_p and L will represent, as before, the length of the 1D plasma slab modeling the 3D real plasma. The expression determining L as a function of the physical parameters L_p and l_R is given further on in Eq. (33). We restrict ourselves to the interesting regime of gain factors $G_{\rm max}$ greater than unity; we will then give the expressions of the quantities $G_{\rm max}$ and $(\Sigma_1^{1D}/\nu_1+\Sigma_2^{1D}/\nu_2)$ in practical units for SBS and SRS in an underdense plasma.

The validity conditions for a 1D model can be obtained from 3D calculations of SBS in a laser hot spot [1,7]. It is found that the diffraction does not modify the value of the gain factor $G_{\max}$ as compared with its 1D expression whenever the following inequality is satisfied:

$$l_R \geqslant (G_{\max}/2) l_{\text{ampl}}, \tag{28}$$

where l_{ampl} denotes the effective length for spatial amplification. The latter is defined through the relation $G_{\max}=(2\gamma_0^2/V_1\nu_2) l_{\text{ampl}}$, so that one has

$$l_{\text{ampl}} \equiv 2x_c \tan^{-1}(L_p/2x_c) \simeq \min(L_p, \pi x_c). \tag{29}$$

Whenever the inequality (28) is fulfilled, the diffraction effects are negligible for what concerns the value of $G_{\max}$. The quantity $(\Sigma_1^{\text{1D}}/\nu_1+\Sigma_2^{\text{1D}}/\nu_2)$ can then be expressed through Eq. (3) as a function of the physical parameters and of the solid angles $\Delta\Omega_1$ and $\Delta\Omega_2$; the latter can be determined precisely from the 3D calculation and are found to be given in terms of a solid angle, denoted as $\Delta\Omega_{\text{noise}}$, characterizing the plasma volume of the thermal noise effectively amplified in the process considered (SBS or SRS). Namely, one obtains, for $\alpha=1$ and 2, $K_\alpha^2\Delta\Omega_\alpha = K_1^2\Delta\Omega_{\text{noise}}$ and in the limit where the inequality (28) is satisfied, $\Delta\Omega_{\text{noise}}$ is related to the 3D characteristics of the backscattered light by the expression

$$\Delta\Omega_{\text{noise}} \equiv \left(\frac{r_{\text{eff}}}{2a_0}\right)^2 \Delta\Omega_{\text{scat}}, \tag{30}$$

where r_{eff} is the radius of the active backscattering region and $\Delta\Omega_{\text{scat}}$ denotes the solid angle within which the backscattered light is scattered in the far-field domain. The quantities r_{eff} and $\Delta\Omega_{\text{scat}}$ are found to be given by [7]

$$r_{\text{eff}} = 2a_0/\sqrt{G_{\max}}, \tag{31}$$

$$\Delta\Omega_{\text{scat}} = \left(\frac{12\pi}{G_{\max}}\right)\left(\frac{a_0}{l_{\text{ampl}}}\right)^2. \tag{32}$$

The quantity B as given by the expression (19) involves the quantity $(\Sigma_1^{\text{1D}}/\nu_1+\Sigma_2^{\text{1D}}/\nu_2)$, the expression of which is given further on for SBS and SRS, and $\Delta\omega_{\text{eff}}$. The expression for $\Delta\omega_{\text{eff}}$ follows from (15), in which the length L of the 1D plasma slab modeling the real plasma of length L_p enters. In order to properly determine L as a function of the physical parameters, we will restrict ourselves to the cases where either the plasma length L_p is larger than the Rayleigh length l_R or, in the opposite case, the center of the plasma is located in the focal plane. In these two realistic limits the dependence of the laser intensity upon the longitudinal coordinate x can easily be taken into account as follows. It can be seen in Eq. (25) that, in the strongly inhomogeneous plasma limit, the effect of the spatial variation of the laser intensity amounts to simply replace $|a_0|^2(x)$ by $|a_0|^2(x)/(1+x^2/l_R^2)$. Expanding $G_{\text{Ros}}|a_0|^2(x)/(1+x^2/l_R^2) \simeq G_{\text{Ros}}|a_0|^2(x) - G_{\text{Ros}}x^2/l_R^2$, valid for $|1-|a_0|^2(x)| \ll 1$, i.e., for not too large reflectivity ($R\leqslant 30\%$), one sees that the spatial dependence of the laser intensity results in replacing the coordinate x by x_{eff} defined by $dx_{\text{eff}} \equiv dx \exp - G_{\text{Ros}}x^2/l_R^2$; therefore, in the inhomogeneous plasma limit, the 1D plasma slab length L must be defined as

$$L \equiv l_R\sqrt{\pi/G_{\text{Ros}}}\,\text{erf}(L_p\sqrt{G_{\text{Ros}}}/2l_R) \simeq \min(L_p, l_R\sqrt{\pi/G_{\text{Ros}}}).$$

In the opposite limit of homogeneous plasma one has simply $L=L_p$, which can also be written as $L=\min(L_p, l_R\sqrt{\pi/G_{\max}})$ since Eqs. (28), (29), and the condition $G_{\max}\geqslant 1$ yield $L_p \ll l_R\sqrt{\pi/G_{\max}}$. It follows therefore that whatever the inhomogeneity of the plasma, the 1D plasma slab length L must be defined by

$$L \equiv l_R\sqrt{\pi/G_{\max}}\,\text{erf}(L_p\sqrt{G_{\max}}/2l_R) \simeq \min(L_p, l_R\sqrt{\pi/G_{\max}}). \tag{33}$$

We now give the expressions of $G_{\max}$ and $(\Sigma_1^{\text{1D}}/\nu_1+\Sigma_2^{\text{1D}}/\nu_2)$ in practical units for SBS and SRS in an underdense plasma. In the case of SBS, the gain factor $G_{\max}$ is given by

$$G_{\max} = 5.9\times 10^{-2}\frac{L_v}{\lambda_0}\frac{(n/n_c)I_{14}\lambda^2}{T_e(1+3T_i/ZT_e)}\tan^{-1}\left(\frac{L}{2\tilde{\nu}_2 L_v}\right), \tag{34}$$

where n, n_c, and T_s denote the plasma density, the critical density, and the temperature (in keV) of species s, respectively. The subscript 0 refers to the laser in vacuum, I_{14} is the laser intensity in units of 10^{14} W/cm^2, and λ is the laser wavelength in μm. In Eq. (34) we have introduced the characteristic length L_v defined as $L_v \equiv K_2 c_S/|V_2\kappa'|$, where c_S is the ion sound velocity. This length reduces to

$$L_v = c_S\left|\frac{\partial V_{\exp}}{\partial x}\right|^{-1}$$

in the case where the dominant term in the expression of κ' is the contribution due to the expansion velocity inhomogeneity. The quantity $\tilde{\nu}_2$ denotes the dimensionless damping coefficient of the acoustic waves $\tilde{\nu}_2 \equiv \nu_2/K_2 c_S$, which can be easily obtained from convenient empirical expressions such as those of Ref. [15]. The expression for the quantity $(\Sigma_1^{\text{1D}}/\nu_1+\Sigma_2^{\text{1D}}/\nu_2)$ reads

$$(\Sigma_1^{\text{1D}}/\nu_1+\Sigma_2^{\text{1D}}/\nu_2) = 2.72\frac{T_e(1-n/n_c)}{I_{14}\lambda^2}\Delta\Omega_{\text{SBS}}\left\{1+\frac{484\sqrt{A/Z}(1+ZQ)}{[T_e(1+3T_i/ZT_e)(1-n/n_c)]^{1/2}[1+(ZT_e/T_i)Q]}\right\}\frac{1}{n_c\lambda_0^2}, \tag{35}$$

where $Q=42.85\sqrt{AT_e/T_i}\exp[-(ZT_e+3T_i)/2T_i]$, A denoting the ion mass number. In Eq. (35), n_c must be expressed in $\mu\mathrm{m}^{-3}$ and $(\Sigma_1^{1D}/\nu_1+\Sigma_2^{1D}/\nu_2)$ is in μm; $\Delta\Omega_{\mathrm{SBS}}$ denotes the solid angle $\Delta\Omega_{\mathrm{noise}}$ in the case of SBS. In practical units, $\Delta\Omega_{\mathrm{SBS}}$, as given by Eqs. (30) and (31), reads

$$\Delta\Omega_{\mathrm{SBS}}=2.8\times10^3\left[\frac{T_e(1+3T_i/ZT_e)\tilde{\nu}_2}{(n/n_c)I_{14}\lambda^2}\right]^2 \times\frac{f^2\lambda_0^4}{[\tilde{\nu}_2L_v\tan^{-1}(L/2\tilde{\nu}_2L_v)]^4}. \tag{36}$$

In the case of SRS, the gain factor $G_{\max}$ is given by

$$G_{\max}=1.15\times10^{-4}\frac{L_n}{\lambda_0} \times\frac{[(1-n/n_c)^{1/2}+(1-2\sqrt{n/n_c})^{1/2}]^2I_{14}\lambda^2}{(1-2\sqrt{n/n_c})^{1/2}} \times\tan^{-1}\left(\frac{L}{2\tilde{\nu}_2L_n}\right). \tag{37}$$

Here the characteristic length L_n is defined as $L_n\equiv\omega_{pe}/|V_2\kappa'|$, where ω_{pe} is the local electron plasma frequency, which reduces to

$$L_n=2\left|\frac{1}{n}\frac{\partial n}{\partial x}\right|^{-1}$$

in the case where the dominant term in the expression of κ' is the contribution due to the density inhomogeneity. The quantity $\tilde{\nu}_2$ denotes the dimensionless damping coefficient of the plasma waves $\tilde{\nu}_2\equiv\nu_2/\omega_{pe}$. The expression for the quantity $(\Sigma_1^{1D}/\nu_1+\Sigma_2^{1D}/\nu_2)$ reads

$$(\Sigma_1^{1D}/\nu_1+\Sigma_2^{1D}/\nu_2)=2.72\frac{T_e(1-2\sqrt{n/n_c})}{I_{14}\lambda^2}\Delta\Omega_{\mathrm{SRS}} \times\left(\frac{1}{1-\sqrt{n/n_c}}+\sqrt{\frac{n_c}{n}}\right)\frac{1}{n_c\lambda_0^2}. \tag{38}$$

Here, as in Eq. (35), n_c must be expressed in $\mu\mathrm{m}^{-3}$ and $(\Sigma_1^{1D}/\nu_1+\Sigma_2^{1D}/\nu_2)$ is in μm; $\Delta\Omega_{\mathrm{SRS}}$ denotes the solid angle $\Delta\Omega_{\mathrm{noise}}$ in the case of SRS. In practical units, $\Delta\Omega_{\mathrm{SRS}}$, as given by Eqs. (30) and (31), reads

$$\Delta\Omega_{\mathrm{SRS}}=7.12\times10^8 \times\left\{\frac{(1-2\sqrt{n/n_c})^{1/2}\tilde{\nu}_2}{[(1-n/n_c)^{1/2}+(1-2\sqrt{n/n_c})^{1/2}]^2I_{14}\lambda^2}\right\}^2 \times\frac{f^2\lambda_0^4}{[\tilde{\nu}_2L_n\tan^{-1}(L/2\tilde{\nu}_2L_n)]^4}. \tag{39}$$

Finally, the relation between the experimentally measured reflectivity $R_{\exp}$ and the theoretical 1D reflectivity R obtained from Eqs. (18) and (19) is given by

$$R_{\exp}=\alpha\min(\beta,1)R, \tag{40}$$

with

$$\alpha=1,$$

$$\beta=\frac{1}{\sqrt{3\pi}}\left(\frac{\tilde{\nu}_2L_v}{f\lambda_0}\right)\tan^{-1}\left(\frac{L}{2\tilde{\nu}_2L_v}\right)\frac{\Delta\Omega_{\exp}}{\sqrt{\Delta\Omega_{\mathrm{SBS}}}},$$

in the case of SBS, where $\Delta\Omega_{\exp}$ is the solid angle corresponding to the collecting optics, and with

$$\alpha=\left[\frac{1-2(n/n_c)^{1/2}}{1-n/n_c}\right]^{1/2},$$

$$\beta=\frac{1}{\sqrt{3\pi}}\left(\frac{\tilde{\nu}_2L_n}{f\lambda_0}\right)\tan^{-1}\left(\frac{L}{2\tilde{\nu}_2L_n}\right)\frac{\Delta\Omega_{\exp}}{\sqrt{\Delta\Omega_{\mathrm{SRS}}}},$$

in the case of SRS. In these expressions, the parameter α represents the ratio $(\omega_1V_1)/(\omega_0|V_0|)$. The parameter β is simply the quantity $\Delta\Omega_{\exp}/\Delta\Omega_{\mathrm{scat}}$, in which $\Delta\Omega_{\mathrm{scat}}$ as given by Eq. (32) has been reexpressed in terms of $\Delta\Omega_{\mathrm{SBS}}$ and $\Delta\Omega_{\mathrm{SRS}}$ for the reader's convenience.

VI. CONCLUSION

In this paper we have investigated the reflectivity of an inhomogeneous plasma in the strongly damped convective regime. We have taken into account the stochastic characteristics (in space and time) of the thermal noise emission of waves. In the linear regime, we have found that the spectral width corresponding to the frequencies that are effectively amplified is always less than the spectral width of the unstable frequency domain. This spectral narrowing appears as a frequency filtering in the expression for the reflectivity. In the nonlinear regime, we first generalized the Tang formula to the case of an inhomogeneous plasma. Then, conjecturing that the linear frequency filtering process remains valid in the nonlinear regime, we obtained the effective noise term of the Tang formula analytically as the solution to a simple implicit equation. Checking the validity of this result numerically, we found excellent agreement between the analytical and numerical results. Finally, we gave the validity conditions for 1D modeling of 3D scattering in the case of an incident Gaussian laser beam.

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APPENDIX A: DERIVATION AND SOLUTION OF THE EQUATION FOR $n_1(\omega,x)$ IN THE LINEAR APPROXIMATION

Setting $a_2=\tilde{a}_2\exp(i\kappa' x^2/2)$ and $a_0\equiv1$ (linear approximation), Eqs. (1b) and (1c) can be written as

$$(\partial_t + V_1\partial_x + \nu_1)a_1 - \gamma_0\widetilde{a}_2^* = S_1, \tag{A1a}$$

$$(\partial_t + V_2\partial_x + iV_2\kappa' x + \nu_2)\widetilde{a}_2 - \gamma_0 a_1^* = \widetilde{S}_2, \tag{A1b}$$

with $\widetilde{S}_2 = S_2\exp(-i\kappa' x^2/2)$. In the strongly damped case defined by $\nu_2 \gg 2\gamma_0|V_2/V_1|^{1/2}$, one can neglect the operator $V_2\partial_x$ in Eq. (A1b): the damping ν_2 is sufficiently strong for wave 2 to be locally enslaved to wave 1, except in a negligibly thin boundary layer of width $|V_2|/\nu_2$ at the vicinity of $x=L$ (with $\nu_2 L/|V_2| \gg 1$). In this limit, the equation for $a_1(\omega,x)$ in the space domain $0<x<L$ reads

$$\left[\partial_x + \frac{1}{V_1}\left(\nu_1 - i\omega - \frac{\gamma_0^2}{\nu_2 - i(\omega+\varepsilon\nu_2 x/x_c)}\right)\right]a_1(\omega,x) = \frac{1}{V_1}\left[S_1(\omega,x) + \frac{\gamma_0(\widetilde{S}_2^*)(\omega,x)}{\nu_2 - i(\omega+\varepsilon\nu_2 x/x_c)}\right], \tag{A2}$$

which gives

$$a_1(\omega,x) = K_\omega^+(x,0)a_1(\omega,0) + \frac{1}{V_1}\int_0^x K_\omega^+(x,x') \times\left[S_1(\omega,x') + \frac{\gamma_0(\widetilde{S}_2^*)(\omega,x')}{\nu_2 - i(\omega+\varepsilon\nu_2 x'/x_c)}\right]dx', \tag{A3}$$

where the kernel $K_\omega^+(x,x')$ is defined by

$$K_\omega^+(x,x') \equiv \exp - \frac{1}{V_1} \times\int_{x'}^x \left(\nu_1 - i\omega - \frac{\gamma_0^2}{\nu_2 - i(\omega+\varepsilon\nu_2 u/x_c)}\right)du.$$

From Eqs. (A2) and (A3) and their counterparts for $(a_1^*)(\omega,x) \equiv [a_1(-\omega^*,x)]^*$, one can easily derive the equation for the correlation function $C(\omega,\omega',x) \equiv \langle a_1(\omega,x)(a_1^*)(\omega',x)\rangle$. One obtains

$$\left[\partial_x + \frac{1}{V_1}\left(2\nu_1 - i(\omega+\omega') - \frac{\gamma_0^2[2\nu_2 - i(\omega+\omega')]}{[\nu_2 - i(\omega+\varepsilon\nu_2 x/x_c)][\nu_2 - i(\omega'-\varepsilon\nu_2 x/x_c)]}\right)\right] \times C(\omega,\omega',x) = \frac{1}{V_1^2}[V_1\Gamma_1(\omega,\omega',x) + \Gamma_2(\omega,\omega',x) + \Gamma_3(\omega,\omega',x)], \tag{A4}$$

where the quantities $\Gamma_\alpha(\omega,\omega',x)$ are given by

$$\Gamma_1(\omega,\omega',x) = K_{\omega'}^-(x,0)\langle(a_1^*)(\omega',0)S_1(\omega,x)\rangle + K_\omega^+(x,0)\langle a_1(\omega,0)(S_1^*)(\omega',x)\rangle + \frac{K_{\omega'}^-(x,0)\langle(a_1^*)(\omega',0)(\widetilde{S}_2^*)(\omega,x)\rangle}{\nu_2 - i(\omega+\varepsilon\nu_2 x/x_c)} + \frac{K_\omega^+(x,0)\langle a_1(\omega,0)\widetilde{S}_2(\omega',x)\rangle}{\nu_2 - i(\omega'-\varepsilon\nu_2 x/x_c)},$$

$$\Gamma_2(\omega,\omega',x) = \int_0^x [K_{\omega'}^-(x,x')\langle S_1(\omega,x)(S_1^*)(\omega',x')\rangle + K_\omega^+(x,x')\langle S_1(\omega,x')(S_1^*)(\omega',x)\rangle]dx' + \gamma_0^2 \times\int_0^x \frac{K_{\omega'}^-(x,x')\langle\widetilde{S}_2(\omega',x')(\widetilde{S}_2^*)(\omega,x)\rangle}{[\nu_2 - i(\omega'-\varepsilon\nu_2 x'/x_c)][\nu_2 - i(\omega+\varepsilon\nu_2 x/x_c)]}dx' + \gamma_0^2 \times\int_0^x \frac{K_\omega^+(x,x')\langle\widetilde{S}_2(\omega',x)(\widetilde{S}_2^*)(\omega,x')\rangle}{[\nu_2 - i(\omega'-\varepsilon\nu_2 x/x_c)][\nu_2 - i(\omega+\varepsilon\nu_2 x'/x_c)]}dx',$$

and

$$\Gamma_3(\omega,\omega',x) = \gamma_0\int_0^x K_{\omega'}^-(x,x')\left[\frac{\langle\widetilde{S}_2(\omega',x')S_1(\omega,x)\rangle}{\nu_2 - i(\omega'-\varepsilon\nu_2 x'/x_c)} + \frac{\langle(S_1^*)(\omega',x')(\widetilde{S}_2^*)(\omega,x)\rangle}{\nu_2 - i(\omega+\varepsilon\nu_2 x/x_c)}\right]dx' + \gamma_0\int_0^x K_\omega^+(x,x')\left[\frac{\langle\widetilde{S}_2(\omega',x)S_1(\omega,x')\rangle}{\nu_2 - i(\omega'-\varepsilon\nu_2 x/x_c)} + \frac{\langle(S_1^*)(\omega',x)(\widetilde{S}_2^*)(\omega,x')\rangle}{\nu_2 - i(\omega+\varepsilon\nu_2 x'/x_c)}\right]dx'.$$

Writing Eq. (2b) as

$$\langle S_\alpha(\omega,x)(S_\alpha^*)(\omega',x')\rangle = 2\pi\Sigma_\alpha^{1D}\delta(x-x')\,\delta(\omega+\omega')$$

and using the relations $\langle S_1S_2\rangle = \langle S_1S_2^*\rangle = 0$ and $\langle a_1(0)(S_1^*)(x>0)\rangle = \langle a_1(0)S_2(x>0)\rangle = 0$, one finds that these intricate expressions for Γ_α reduce to

$$\Gamma_1(\omega,\omega',x) = \Gamma_3(\omega,\omega',x) = 0, \tag{A5a}$$

$$\Gamma_2(\omega,\omega',x) = 2\pi\left[\Sigma_1^{1D} + \Sigma_2^{1D}\frac{\gamma_0^2}{\nu_2^2 + (\omega+\varepsilon\nu_2 x/x_c)^2}\right] \times \delta(\omega+\omega'), \tag{A5b}$$

where we have used the prescription $\int^x \delta(x-x')dx' = 1/2$ [note that this prescription is in agreement with Eq. (21b) in the limit $l_c \to 0$]. Inserting Eqs. (A5) into Eq. (A4), using the stationarity condition $C(\omega,\omega',x) = n_1(\omega,x)\delta(\omega+\omega')$, and integrating over ω', one obtains the equation for the spectral density $n_1(\omega,x)$ as

$$\left[\partial_x + \frac{2}{V_1}\left(\nu_1 - \frac{\gamma_0^2\nu_2}{\nu_2^2 + (\omega+\varepsilon\nu_2 x/x_c)^2}\right)\right] n_1(\omega,x) = \frac{2\pi}{V_1^2}\left[\Sigma_1^{1D} + \frac{\gamma_0^2}{\nu_2^2 + (\omega+\varepsilon\nu_2 x/x_c)^2}\Sigma_2^{1D}\right]. \tag{A6}$$

The solution of Eq. (A6) is straightforward and leads to

$$n_1(\omega,x) = n_{1\mathrm{nat}}(\omega,x) + \left[n_1(\omega,0) - \frac{\pi}{V_1\nu_1}\Sigma_1^{1D}\right] \times [R_\omega(x,0) - e^{-2\nu_1 x/V_1}] + \frac{2\pi}{V_1^2}\gamma_0^2\nu_2 \times \left(\frac{\Sigma_1^{1D}}{\nu_1} + \frac{\Sigma_2^{1D}}{\nu_2}\right)\int_0^x \frac{R_\omega(x,x')}{\nu_2^2 + (\omega+\varepsilon\nu_2 x'/x_c)^2}dx', \tag{A7}$$

where the kernel $R_\omega(x,x')$ is given by

$$R_\omega(x,x') = \exp\left[G_\omega(x,x') - \frac{2\nu_1}{V_1}(x-x')\right],$$

with

$$G_\omega(x,x') = \frac{2\gamma_0^2}{V_1V_2\kappa'}\left[\tan^{-1}\left(\frac{\omega}{\nu_2} + \varepsilon\frac{x}{x_c}\right) - \tan^{-1}\left(\frac{\omega}{\nu_2} + \varepsilon\frac{x'}{x_c}\right)\right],$$

and where the quantity $n_{1\mathrm{nat}}(\omega,x)$ corresponds to the natural plasma emissivity [i.e., the solution to Eq. (A6) in the limit $\gamma_0 = 0$], namely,

$$n_{1\mathrm{nat}}(\omega,x) = n_1(\omega,0)e^{-2\nu_1 x/V_1} + \frac{\pi\Sigma_1^{1D}}{V_1\nu_1}(1 - e^{-2\nu_1 x/V_1}).$$

In the case where the inverse bremsstrahlung absorption can be neglected ($\nu_1 = 0$), it is possible to carry out the integral on the right-hand side of Eq. (A7) so that the expression for $n_1(\omega,x)$ finally reduces to

$$n_1(\omega,x) = n_1(\omega,0) + \left[n_1(\omega,0) + \frac{\pi}{V_1\nu_2}\Sigma_2^{1D}\right] \times [\lim_{\nu_1\to 0} R_\omega(x,0) - 1]. \tag{A8}$$

APPENDIX B: NUMERICAL CONSTRAINTS ON THE SPACE STEP Δx

In a large inhomogeneous plasma the smallest scale length is the phase-mismatch scale length given by

$$L_{\min} \sim \frac{1}{|\kappa'|L} = \frac{|V_2|}{\nu_2 L}x_c. \tag{B1}$$

For typical parameters ($|V_1| \approx c \approx 300\ \mu$m ps^{-1}, $\gamma_0 \sim 1$ ps^{-1}, $L \geqslant 10\ \mu$m, and $|V_2/V_1|^{1/2} \sim 10^{-2}$), the strong damping condition $\nu_2 \gg 2\gamma_0|V_2/V_1|^{1/2}$ yields

$$\frac{|V_2|}{\nu_2 L} \ll \frac{|V_1|}{2\gamma_0 L}\left|\frac{V_2}{V_1}\right|^{1/2} \sim 10^{-1}. \tag{B2}$$

Thus, from Eqs. (B1) and (B2) one finds that, in the case of a strongly inhomogeneous plasma ($x_c/L \leqslant 0.1$), one has the ordering

$$L_{\min}/L \ll 10^{-2}. \tag{B3}$$

Writing then the conditions (23a) as $\Delta x \sim 10^{-1}l_c \sim 10^{-2}L_{\min}$, one finds from Eq. (B3) that the order of magnitude of $\Delta x/L$ is given by

$$\Delta x/L \ll 10^{-4}. \tag{B4}$$

Such a small space step leads to very-time-consuming simulations in the strongly inhomogeneous plasma limit.

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