Soliton and electromagnetic wave propagation in a ferromagnetic medium

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We find that when an electromagnetic wave propagates in an isotropic ferromagnetic medium, an effective field, equivalent to the anisotropic and constant external field, is created. The magnetic induction lies in a plane normal to the direction of propagation. Also, the excitation of magnetization of the ferromagnetic medium is restricted to the normal plane at the lowest order of perturbation, and goes out of plane at higher orders. The excitations of the magnetization and magnetic induction, and hence also the magnetic field, are governed by soliton modes. [S1063-651X(97)10103-9]

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I. INTRODUCTION

The soliton, a concept introduced by Zabusky and Kruskal [1] three decades ago has acquired a unique place in the realm of nonlinear physics. Over the years, the soliton has been identified in many branches of physics including plasma, fluids, molecular systems, condensed matter, etc. [2]. Very recently, the soliton has started playing an important role in technological applications as well. Notable among these is the lossless propagation of the optical soliton through a fiber (dielectric) medium [3]. In a different context, one-dimensional classical continuum Heisenberg ferromagnetic spin systems with different kinds of magnetic interactions act as interesting nonlinear dynamical models exhibiting magnetic solitons to represent the elementary spin excitations. The above models are constituted by the inherent magnetic interactions of the materials, such as exchange, anisotropy, and weak interaction, and also interaction with constant external magnetic fields [4–11]. At the present time, the concept of magneto-optical recording has become technologically very important for the purpose of higher storage and fast reading [12]. In this context, in the present paper, we investigate the effect of propagation of electromagnetic waves (EMW) in an isotropic ferromagnetic medium and the nature of the excitations of the magnetization. In Sec. II, we study the effect of EMW propagation on the dynamics of magnetization of the medium. The effect of interaction between the magnetic induction and the magnetization of the medium is investigated in Sec. III. The results are concluded in Sec. IV.

II. MAGNETIC SOLITON DUE TO ELECTROMAGNETIC WAVE PROPAGATION

In the absence of static and moving electric charges, the Maxwell's equations of electromagnetics [13] can be written as

$$\nabla\times\mathbf{H}=\varepsilon_0\,\frac{\partial\mathbf{E}}{\partial t}, \tag{1a}$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}, \tag{1b}$$

$$\nabla\cdot\mathbf{B}=0, \tag{1c}$$

$$\nabla\cdot\mathbf{E}=0. \tag{1d}$$

Here the fields $\mathbf{H}=(H^x,H^y,H^z)$, $\mathbf{E}=(E^x,E^y,E^z)$, and $\mathbf{B}=(B^x,B^y,B^z)$ have the usual meaning of magnetic fields, electric fields, and magnetic induction, respectively. ε_0 is the dielectric constant of the medium.

The change in orientation of the magnetization $\mathbf{M}(\mathbf{r},t)$ in an isotropic ferromagnet in the classical continuum limit in the presence of an external magnetic field $\mathbf{H}(\mathbf{r},t)$ can be expressed in terms of the Landau-Lifshitz equation [14],

$$\frac{\partial\mathbf{M}}{\partial t}(\mathbf{r},t)=\mathbf{M}\times[\nabla^2\mathbf{M}+2A\mathbf{H}],\qquad \mathbf{M}=(M^x,M^y,M^z),\quad \mathbf{M}^2=1, \tag{2}$$

where $2A=g\mu_B$, g is the gyromagnetic ratio and μ_B is the Bohr magneton. The set of coupled Eqs. (1) and (2) describe the propagation of EMW in an isotropic ferromagnetic medium in which the dynamics of magnetization is governed by Eq. (2). Now taking curl on Eq. (1a), and using Eq. (1b), and with little algebra we can write

$$\nabla(\nabla\cdot\mathbf{H})-\nabla^2\mathbf{H}=-\varepsilon_0\,\frac{\partial^2\mathbf{B}}{\partial t^2}. \tag{3}$$

In the case of untreated ferromagnetic materials, the magnetic induction, the magnetic field, and the magnetization are connected by the linear relation [13]

$$\mathbf{H}=\frac{\mathbf{B}}{\mu_0}-\mathbf{M}, \tag{4}$$

where μ_0 is the permeability of the medium. Substituting Eq. (4) into Eq. (3) and using Eq. (1c), we obtain

$$c^2\nabla^2\mathbf{B}-\frac{\partial^2\mathbf{B}}{\partial t^2}=\frac{1}{\varepsilon_0}\,[\nabla^2\mathbf{M}-\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{M})], \tag{5}$$

where $c=1/(\mu_0\varepsilon_0)^{1/2}$ is the velocity of the EMW. Assuming the direction of propagation of the EMW along the z direction, Eq. (5) can be written as

$$c^2\,\frac{\partial^2\mathbf{B}}{\partial z^2}-\frac{\partial^2\mathbf{B}}{\partial t^2}=\frac{1}{\varepsilon_0}\left[\frac{\partial^2\mathbf{M}}{\partial z^2}-\frac{\partial^2 M^z}{\partial z}\,\mathbf{k}\right],\quad \mathbf{k}=(0,0,1). \tag{6}$$

Treating Eq. (6) as an inhomogeneous linear wave equation in $\mathbf{B}$, it can be solved to give

$$\mathbf{B}(z,t)=-\frac{1}{2c\varepsilon_0}\int dt\left[\frac{\partial\mathbf{M}}{\partial z}-\frac{\partial M^z}{\partial z}\,\mathbf{k}+f(t)\right], \tag{7}$$

where $f(t)$ is an arbitrary function of time.

Treating the external magnetic field $\mathbf{H}(z,t)$ in Eq. (2) as the magnetic-field component $\mathbf{H}(z,t)$ of the EMW, and upon using Eqs. (4) and (7), Eq. (2) can be written (in one dimension) as

$$\frac{\partial\mathbf{M}}{\partial t}\,(z,t)=\mathbf{M}\times\left[\frac{\partial^2\mathbf{M}}{\partial z^2}+Ac\mathbf{B}\right], \tag{8a}$$

$$\frac{\partial\mathbf{B}}{\partial t}\,(z,t)=\frac{\partial M^z}{\partial t}\,\mathbf{k}-\frac{\partial\mathbf{M}}{\partial z}+f(t). \tag{8b}$$

Introducing the wave variable $\xi=z-ct$ in Eq. (8b), integrating once, and substituting the resultant expression for $\mathbf{B}$ into Eq. (8a), we obtain

$$\frac{\partial\mathbf{M}}{\partial t}\,(z,t)=\mathbf{M}\times\left[\frac{\partial^2\mathbf{M}}{\partial z^2}+AM^z\mathbf{k}+\mathbf{G}\right],\quad \mathbf{M}^2=1, \tag{9}$$

where $\mathbf{G}=(G^x,G^y,G^z)$ is a constant vector. It is interesting to note that the second and third terms in the right-hand side of Eq. (9) represent effective fields similar to fields due to uniaxial single ion anisotropy along the z direction and Zeeman energy due to a constant external magnetic field $\mathbf{G}$, respectively. The corresponding energy density will be $-A(M^z)^2-\mathbf{M}\cdot\mathbf{G}$. Thus we find that when EMW propagates along the z direction in an isotropic ferromagnetic medium, effective fields similar to the anisotropic field due to the presence of an easy axis of magnetization along the z direction and a constant magnetic field have been created.

Equation (9) is an important nonlinear dynamical model system exhibiting an interesting class of nonlinear excitations of the magnetization. For example in the isotropic limit ($A=0$), Eq. (9) is an integrable model possessing N-soliton solutions [4,15]. When $\mathbf{G}$ is aligned parallel to the direction of propagation ($\mathbf{k}$), Eq. (9) takes the form

$$\frac{\partial\mathbf{M}}{\partial t}\,(z,t)=\mathbf{M}\times\left[\frac{\partial^2\mathbf{M}}{\partial z^2}-AM^z\mathbf{k}+G^z\mathbf{k}\right]. \tag{10}$$

In Eq. (10), the constant field term can be transformed away by a simple transformation $M^{\pm}=M^{\pm}\exp(iG^zt)$, where $M^{\pm}=M^x\pm iM^y$, and we obtain

$$\frac{\partial\mathbf{M}}{\partial t}\,(z,t)=\mathbf{M}\times\left[\frac{\partial^2\mathbf{M}}{\partial z^2}-AM^z\mathbf{k}\right]. \tag{11}$$

Equation (11) is also a completely integrable nonlinear dynamical model possessing soliton solution [5]. Further, Eq. (11) can be made gauge equivalent to the soliton possessing nonlinear Schrödinger equation [16]. Thus the soliton excitations of magnetization of the isotropic ferromagnetic medium when an EMW propagates along the direction of the magnetic chain can be expressed again in terms of solitons. On the other hand, when the constant vector $\mathbf{G}$ points in a direction normal to the direction of propagation, say along the x direction, then the magnetic excitations of the ferromagnet is governed by chaotic structures [17].

III. SOLITON AND MODULATION OF ELECTROMAGNETIC WAVES IN A FERROMAGNETIC MEDIUM

Having analyzed the effect of EMW propagation on the dynamics of the magnetization of the ferromagnetic medium, we now investigate the nature of excitations of the magnetic induction and hence the modulation that takes place in the magnetic field component of the EMW due to the interaction between the magnetic induction and magnetization of the medium. For this, we stretch the wave variable ξ and the time t by introducing the variables

$$\xi=\varepsilon\xi\equiv\varepsilon(z-ct),\quad \tau=\varepsilon^3 t, \tag{12}$$

where ε is a very small parameter.

In order to understand the effect, we make a perturbation [18–20] by expanding the magnetization and magnetic induction vectors as

$$\mathbf{M}=\mathbf{M}_0+\varepsilon\mathbf{M}_1+\varepsilon^2\mathbf{M}_2+\cdots, \tag{13a}$$

$$\mathbf{B}=\mathbf{B}_0+\varepsilon\mathbf{B}_1+\varepsilon^2\mathbf{B}_2+\cdots. \tag{13b}$$

We substitute Eqs. (12) and (13) into Eqs. (8a) and (8b) in component forms, and collect the coefficients of different powers of ε, and try to solve the resultant equations at different orders of ε. For example, on solving the equations at the order of ε^0, we obtain

$$M_0^x=cB_0^x, \tag{14a}$$

$$M_0^y=cB_0^y, \tag{14b}$$

$$M_0^z=0, \tag{14c}$$

$$B_0^z=0. \tag{14d}$$

Similarly on solving the equations at $0(\varepsilon)$ and using Eq. (14), we obtain

$$M_1^x=cB_1^x, \tag{15a}$$

$$M_1^y=cB_1^y, \tag{15b}$$

$$B_1^z=0, \tag{15c}$$

and

$$\frac{\partial M_0^x}{\partial \xi} = A B_0^y M_1^z, \tag{16a}$$

$$\frac{\partial M_0^y}{\partial \xi} = -A B_0^x M_1^z. \tag{16b}$$

Finally at $0(\varepsilon^2)$, after using Eqs. (14), we obtain the following set of equations:

$$\frac{\partial}{\partial \xi}\left[B_2^x - \frac{1}{c} M_2^x\right] = -\frac{1}{c^2}\frac{\partial M_0^x}{\partial \tau}, \tag{17a}$$

$$\frac{\partial}{\partial \xi}\left[B_2^y - \frac{1}{c} M_2^y\right] = -\frac{1}{c^2}\frac{\partial M_0^y}{\partial \tau}, \tag{17b}$$

$$B_2^z = 0 \tag{17c}$$

and

$$-c\frac{\partial M_1^x}{\partial \xi} = M_0^y \frac{\partial^2 M_0^z}{\partial \xi^2} - M_0^z \frac{\partial^2 M_0^y}{\partial \xi^2} + Ac\{M_0^y B_2^z + M_1^y B_1^z + M_2^y B_0^z - M_0^z B_2^y - M_1^z B_1^y - M_2^z B_0^y\}, \tag{18a}$$

$$-c\frac{\partial M_1^y}{\partial \xi} = M_0^z \frac{\partial^2 M_0^x}{\partial \xi^2} - M_0^x \frac{\partial^2 M_0^z}{\partial \xi^2} + Ac\{M_0^z B_2^x + M_1^z B_1^x + M_2^z B_0^x - M_0^x B_2^z - M_1^x B_1^z - M_2^x B_0^z\}, \tag{18b}$$

$$-c\frac{\partial M_1^z}{\partial \xi} = M_0^x \frac{\partial^2 M_0^y}{\partial \xi^2} - M_0^y \frac{\partial^2 M_0^x}{\partial \xi^2} + Ac\{M_0^x B_2^y + M_1^x B_1^y + M_2^x B_0^y - M_0^y B_2^x - M_1^y B_1^x - M_2^y B_0^x\}. \tag{18c}$$

Using Eqs. (15a), (15b), (17a), and (17b) in Eq. (18c), we obtain

$$-c\frac{\partial M_1^z}{\partial \xi} = M_0^x \frac{\partial^2 M_0^y}{\partial \xi^2} - M_0^y \frac{\partial^2 M_0^x}{\partial \xi^2} - \frac{A}{c}\left\{M_0^y - \frac{\partial}{\partial \tau}\int M_0^x d\xi + M_0^x \frac{\partial}{\partial \tau}\int M_0^y d\xi\right\}. \tag{19}$$

Now, we write $\mathbf{M}_0$ in the polar coordinate representation as $\mathbf{M}=(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, and fix θ as $\pi/2$, i.e.,

$$M_0^x = \cos\phi, \quad M_0^y = \sin\phi. \tag{20}$$

Using Eq. (14b) and Eq. (20) in Eq. (16b), we obtain

$$M_1^z = -\frac{c}{A}\frac{\partial \phi}{\partial \xi}. \tag{21}$$

Substituting Eqs. (20) and (21) into Eq. (19), we obtain

$$\frac{c^2}{A}\frac{\partial^2 \phi}{\partial \xi^2} = \frac{A}{c}\left\{\sin\phi \frac{\partial}{\partial \tau}\int \cos\phi d\xi - \cos\phi \frac{\partial}{\partial \tau}\int \sin\phi d\xi\right\}. \tag{22}$$

Differentiating Eq. (22) with respect to ξ, we obtain

$$\mu\frac{\partial^3 \phi}{\partial \xi^3} + \frac{\partial \phi}{\partial \tau} = \left(\sin\phi \frac{\partial}{\partial \tau}\int \sin\phi d\xi + \cos\phi \frac{\partial}{\partial \tau}\int \cos\phi d\xi\right)\frac{\partial \phi}{\partial \xi}, \tag{23}$$

where $\mu = A^{-2}c^3$. After some lengthy calculations, Eq. (23) can be made equivalent to the modified Korteweg de Vries (MKdV) equation in the form

$$\frac{\partial f}{\partial \tau} + \frac{3}{2}\mu f^2 \frac{\partial f}{\partial \xi} + \mu\frac{\partial^3 f}{\partial \xi^3} = 0, \tag{24}$$

where $f = \partial\phi/\partial\xi$. The MKdV equation is known to be an integrable equation possessing N-soliton solutions [21]. Thus the excitation of magnetization, magnetic induction, and hence the magnetic-field component of the EMW, are governed by soliton modes. For instance, the one soliton solution of Eq. (24) can be written as

$$f = 2a \operatorname{sech} a\zeta, \tag{25}$$

where $\zeta = \xi - \lambda\tau$, $a^2 = \lambda/\mu$, and λ=const. Knowing f, ϕ can be obtained, and hence from Eqs. (20) and (21), we obtain the components of the magnetization as

$$M_0^x = 1 - 2\operatorname{sech}^2 a\zeta, \tag{26a}$$

$$M_0^y = 2\tanh a\zeta \operatorname{sech} a\zeta, \tag{26b}$$

$$M_1^z = -\frac{2ac}{A}\operatorname{sech} a\zeta. \tag{26c}$$

Using Eqs. (26) in Eqs. (14a) and (14b), we obtain the components of the magnetic induction as

$$B_0^x = \frac{1}{c}(1 - 2\operatorname{sech}^2 a\zeta), \tag{27a}$$

$$B_0^y = \frac{2}{c}\tanh a\zeta \operatorname{sech} a\zeta. \tag{27b}$$

Knowing **B** and **M**, the magnetic field **H** can be obtained using the relation (4). From Eqs. (26) and (27), we see that the excitation of the magnetization, the magnetic induction, and hence the magnetic field are spatially localized. To elucidate this we have plotted the components of magnetization [Eq. (26)] in Figs. (1a)–(1c) by choosing $a=\lambda=1$ and $A=2c$. We have already found that $M_0^z=0$ and $B_0^z=B_1^z=B_2^z=\cdots=0$. The above solutions indicate that the magnetic induction of the EMW is restricted to the plane (B^x-B^y) normal to the direction of propagation of the EMW. However, the magnetic field of the EMW is restricted to the normal (H^x-H^y) plane only at the leading order of perturba-

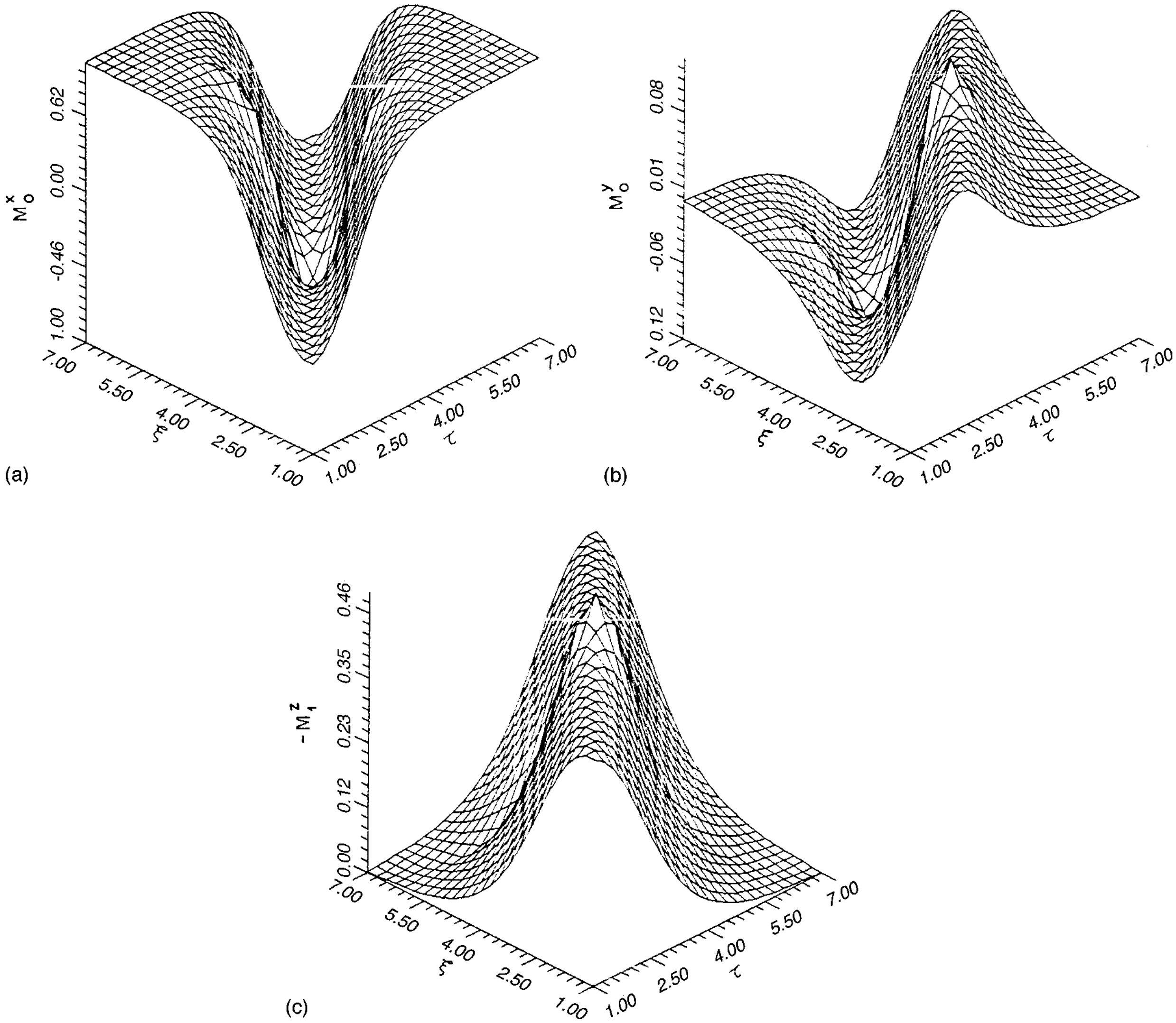

FIG. 1. (a) Evolution of magnetization component M_0^x. (b) Evolution of magnetization component M_0^y. (c) Evolution of magnetization component M_1^z.

tion, and comes out of it at higher orders. This is because, at the leading order of perturbation, the magnetization of the medium is restricted to the M^x-M^y plane, and comes out of it at higher orders.

IV. CONCLUSION

In this paper, we considered the propagation of an electromagnetic wave in an untreated ferromagnetic medium, and attempted to find answers to the following two questions. (i) What happens to the dynamics of the magnetization of the ferromagnet when an electromagnetic wave propagates through it? (ii) What is the nature of excitations due to the interaction between the magnetic induction of the EMW and the magnetization of the medium, and hence the nature of the modulation that takes place on the magnetic field component of the EMW? We find that when the electromagnetic wave propagates in an isotropic ferromagnetic medium in which the magnetization excitation is governed by soliton modes, the magnetic field component of the EMW introduces effective fields similar to the anisotropic field due to single ion uniaxial anisotropy with the easy axis of magnetization along the direction of propagation (i.e., the z axis), and a constant magnetic field which can have either solitonic or chaotic excitation of the magnetization depending on the orientation of the constant field. A perturbation analysis carried out to understand the nature of excitations due to interaction between the magnetization of the medium and the magnetic induction of the EMW (treated as more dominant) reveals that the magnetic induction is modulated, and the oscillations are restricted to a plane normal to the direction of propagation. In addition, the excitation of the magnetization of the medium and the magnetic-field component of the EMW are now restricted to the normal plane only at the lowest order of perturbation and come out of it at higher orders. Further, the above excitations take place in the form of solitons.

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