Laser-driven acceleration with Bessel beams

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The possibility of enhancing the energy gain in laser-driven accelerators by using Bessel laser beams is examined. A formalism based on Huygens' principle is developed to describe the diffraction of finite power (bounded) Bessel beams. An analytical expression for the maximum propagation distance is derived and found to be in excellent agreement with numerical calculations. Scaling laws are derived for the propagation length, acceleration gradient, and energy gain in various accelerators. Assuming that the energy gain is limited only by diffraction (i.e., in the absence of phase velocity slippage), a comparison is made between Gaussian and Bessel beam drivers. For equal beam powers, the energy gain can be increased by a factor of $N^{1/2}$ by utilizing a Bessel beam with N lobes, provided that the acceleration gradient is linearly proportional to the laser field. This is the case in the inverse free electron laser and the inverse Cherenkov accelerators. If the acceleration gradient is proportional to the square of the laser field (e.g., the laser wakefield, plasma beat wave, and vacuum beat wave accelerators), the energy gain is comparable with either beam profile. [S1063-651X(97)13503-6]

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I. INTRODUCTION

Laser-driven accelerators rely on the large intensities that can be achieved when laser beams are focused down to spot sizes on the order of several wavelengths [1–17]. The associated gradients are typically much larger than the ~100 MV/m in proposed next-generation X-band linacs. However, a shortcoming of many of these schemes is that the interaction length over which the high intensity can be sustained is relatively short due to transverse spreading (diffraction). Radiation from a laser cavity is usually in the form of the fundamental and higher-order Gaussian modes. For such a beam the Rayleigh length, i.e., the free-space scale length for diffraction, is given by

$$Z_{RG}=kr_0^2/2, \tag{1}$$

where r_0 is the minimum spot size of the beam at the focal point and $\lambda=2\pi/k$ is the free-space wavelength [18]. In vacuum or in a gas [4–13] acceleration can be achieved by direct interaction of the axial component of the laser field E_z with the particles, where z is the propagation direction. Using $\boldsymbol{\nabla}\cdot\mathbf{E}=0$, the axial electric field is related to the (dominant) transverse field $\mathbf{E}_\perp$ by $\partial E_z/\partial z=-\boldsymbol{\nabla}_\perp\cdot\mathbf{E}_\perp$. For a Gaussian beam, $E_z=O(E_0/kr_0)$, where E_0 is the transverse field amplitude. The product E_zZ_{RG} provides an estimate of the energy gain, assuming that the interaction is synchronous, i.e., neglecting phase velocity slippage.

This paper addresses the scaling of and the maximization of the energy gain in various accelerators driven by lasers with two different transverse mode profiles. In particular, laser accelerators driven by Gaussian beams will be compared to those driven by Bessel beams [19–22]. The diffraction of Bessel beams is examined and an analytical expression for the maximum propagation distance is derived and compared to numerical calculations. It is shown that a Bessel beam can enhance the energy gain by a factor of $N^{1/2}$ compared to a Gaussian beam of the same power, provided that (i) the acceleration gradient is linearly proportional to the laser field, and (ii) the acceleration distance is limited by diffraction and not by phase detuning (or some other mechanism), where N is the number of transverse rings (lobes) in the Bessel beam. The specific example of the inverse Cherenkov accelerator is examined in detail.

The mode structure of a laser beam can be altered using common optical elements, including holographically generated zone plates [23] and axicons [11,12,24–29]. Notable examples of such beams are the Bessel beam of order n, $J_n(k_\perp r)$, where $k_\perp$ is the transverse wave number and r is the radial coordinate. The J_0 beam has been the subject of much theoretical and experimental analysis as a paradigm of what are referred to as "diffraction-free" beams [19]. In reality any beam with finite transverse extent is subject to spreading and the designation diffraction free is a misnomer. Indeed, careful comparison of a Bessel beam with a Gaussian beam reveals that the latter has a better energy transfer capability [20–22].

However, since Bessel beams are sharply peaked and have a large depth of field they may be more useful than the familiar Gaussian beams in certain applications. For example, direct laser acceleration relies on the interaction of a particle with the axial electric field of the laser. The fundamental Gaussian and the J_0 beams are not efficient for direct acceleration since there is no on-axis electric field associated with either. However, $E_z(r=0)\neq 0$ for higher-order Gaussian and Bessel beams. Bessel beams have been created using axicon lenses and, in particular, a zeroth-order Bessel beam was used in channel guiding experiments [29] and a radially polarized, first-order Bessel beam has been used in experiments on the inverse Cherenkov accelerator [12].

II. BESSEL AND GAUSSIAN LASER BEAMS

A. Ideal Bessel beams

In vacuum, the Cartesian components of the laser electric field, E_i $(i=x,y,z)$, satisfy the scalar wave equation

$$(\nabla^2+k^2)E_i=0, \tag{2}$$

where $\omega=ck=2\pi c/\lambda$ is the laser frequency, c is the speed of light, and $E_i\sim\exp(-i\omega t)$ has been assumed. Letting $E_i=\mathrm{Re}\hat{E}_i\exp(ikz-i\omega t)$, the laser field envelope $\hat{E}_i$ satisfies the paraxial wave equation [18]

$$\left(\nabla_\perp^2+2ik\frac{\partial}{\partial z}\right)\hat{E}_i=0, \tag{3}$$

where $|\partial\hat{E}_i/\partial z|\ll|k\hat{E}_i|$ has been assumed. An exact solution of the paraxial wave equation is the fundamental Bessel mode [19–22]

$$\hat{E}_x=E_0J_0(k_\perp r)\exp(-ik_\perp^2z/2k), \tag{4}$$

where J_0 is the zeroth-order Bessel function, E_0 is the peak field amplitude, and $k_\perp\ll k$ is the transverse wave number. The fundamental Bessel mode is peaked along the z axis and the radius of the central lobe is given by $r_a=p_{01}/k_\perp$, where p_{01} is the first zero of J_0. Associated with the transverse field is an axial field component which satisfies $\nabla\cdot\mathbf{E}=0$. For $E_x\sim J_0$, however, E_z is zero along $r=0$.

For laser acceleration of particles in vacuum or in gas [4–13], a more useful laser field is a radially polarized, first-order Bessel mode of the form

$$\hat{E}_r=E_0J_1(k_\perp r)\exp(-ik_\perp^2z/2k). \tag{5}$$

In terms of its Cartesian components,

$$\hat{E}_x=E_0J_1(k_\perp r)\exp(-ik_\perp^2z/2k)\cos\theta, \tag{6a}$$

$$\hat{E}_y=E_0J_1(k_\perp r)\exp(-ik_\perp^2z/2k)\sin\theta. \tag{6b}$$

The associated axial field component is

$$\hat{E}_z=\frac{ik_\perp E_0}{(k-k_\perp^2/2k)}J_0(k_\perp r)\exp(-ik_\perp^2z/2k). \tag{7}$$

Here, $\hat{E}_x$, $\hat{E}_y$, and $\hat{E}_z$ are exact solutions to the paraxial wave equation, Eq. (3). For the first-order Bessel beam, E_z is maximum along $r=0$ whereas E_r is zero. Hence, the first-order Bessel beam described by Eqs. (5)–(7) is well suited for acceleration of particles along the z axis. Notice that the axial wave number for the above Bessel beams is $k_z\simeq k-k_\perp^2/2k$. This implies an axial phase velocity $v_p=\omega/k_z$ given by

$$v_p/c=1+k_\perp^2/2k^2. \tag{8}$$

That is, the phase velocity exceeds c and particle slippage prevents acceleration to high energies in a single stage.

Although the ideal Bessel beam solutions given by Eqs. (4)–(7) do not diffract, they have infinite power. This is due to the fact that $J_{0,1}\sim r^{-1/2}$ for large r. The asymptotic form ($k_\perp r\gg1$) for the Bessel functions is given by

$$J_n(k_\perp r)\sim(2/\pi k_\perp r)^{1/2}\cos[k_\perp r-(2n+1)\pi/4]. \tag{9}$$

An ideal Bessel beam consists of an infinite number of rings (lobes) each having a radial width of $r_b\simeq\pi/k_\perp$. Since the asymptotic width of each ring is the same, the power contained each ring $P_b\simeq(c/4)E_0^2/k_\perp^2$ is approximately equal. In essence, the ideal Bessel beams are the cylindrical equivalents of plane waves.

B. Gaussian beams

It is useful to compare the Bessel beam solutions to the well-known Gaussian beam solutions [18]. The fundamental Gaussian beam is given by

$$\hat{E}_z=E_0\frac{r_0}{r_s}\exp\left[-(1-i\alpha)\frac{r^2}{r_s^2}-i\tan^{-1}\alpha\right], \tag{10}$$

where $r_s=r_0(1+\alpha^2)^{1/2}$ is the spot size, $\alpha=(z-z_0)/Z_{RG}$ is proportional to the wave-front curvature, r_0 is the minimum spot size at the focal point $z=z_0$, and $Z_{RG}=kr_0^2/2$ is the Rayleigh length. Equation (10) is an exact solution to the paraxial wave equation, Eq. (3). The radially polarized, first-order Gaussian mode is given by

$$\hat{E}_r=E_0\frac{rr_0}{r_s^2}\exp\left[-(1-i\alpha)\frac{r^2}{r_s^2}-2i\tan^{-1}\alpha\right], \tag{11}$$

and the axial field component is

$$\hat{E}_z\simeq\frac{2ir_0E_0}{kr_s^2}\left[1-(1-i\alpha)\frac{r^2}{r_s^2}\right]\times\exp\left[-(1-i\alpha)\frac{r^2}{r_s^2}-2i\tan^{-1}\alpha\right]. \tag{12}$$

Near the focal point ($\alpha=0$) and along the z axis, the axial wave number associated with this field is given by $k_z\simeq k-2/Z_{RG}$. This corresponds to an axial phase velocity

$$v_p/c=1+2/kZ_{RG}. \tag{13}$$

In vacuum $v_p>c$ and particle slippage prevents acceleration to high energies, as is the case for a Bessel beam. The scale length over which the Gaussian beams diffract is the Rayleigh length Z_{RG}. The total power associated with the Gaussian beams is

$$P_G=\frac{c}{4}\int_0^\infty dr\, r|\hat{E}_\perp|^2=\frac{c}{16}E_0^2r_0^2f_g, \tag{14}$$

where $f_g=1$ for the fundamental and $f_g=1/2$ for the first-order Gaussian beam.

C. Nonideal Bessel beams

Finite power, nonideal, Bessel beams can be created by clipping the ideal beam with an aperture of radius $r=a$ [19–22]. A nonideal Bessel beam consists of $N\simeq a/r_b=ak_\perp/\pi$ rings, with a total power given by N times the power in a single ring, $P_B\simeq NP_b$, i.e.,

$$P_B\simeq(c/4)NE_0^2/k_\perp^2. \tag{15}$$

Roughly speaking, a nonideal Bessel beam consisting of N rings diffracts away sequentially starting with the outermost ring [20]. The outermost ring diffracts after a distance $\sim\pi r_b^2/\lambda$, the next ring diffracts after a distance $2\pi r_b^2/\lambda$, and

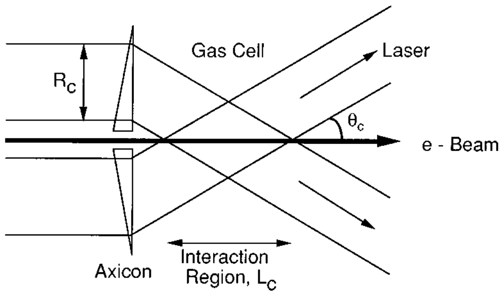

FIG. 1. Schematic of the axicon focusing geometry used to create a line focus. At the midplane of the focus, the transverse profile of the laser field is a nonideal Bessel beam. The axicon focal parameters (R_c, L_c, and $\theta_c \ll 1$) are related to the Bessel beam parameters ($k_\perp = \pi/r_b$, $L_{\max}=2NZ_{RB}/\pi$, and $N=a/r_b$) by $R_c \simeq L_c\theta_c \simeq 2a$, $k_\perp \simeq k\theta_c$, $L_c \simeq 2L_{\max}$, and $N \simeq (R_c/\lambda)\theta_c$.

so on until the innermost ring diffracts away after a distance $\sim N\pi r_b^2/\lambda$. Hence, the maximum propagation distance of a nonideal Bessel beam consisting of N rings is [19,20]

$$L_{\max} \simeq NZ_{RB}, \tag{16}$$

where $Z_{RB}=kr_b^2/2=(\pi^2/2)k/k_\perp^2$ is the Rayleigh length associated with the asymptotic width of an individual ring. More accurately, the analysis presented in Sec. III gives $L_{\max}=(2/\pi)NZ_{RB}=ak/k_\perp$.

Axicon lenses can be used to create nonideal Bessel beams [12,29]. A schematic for creating a radially polarized, first-order Bessel beam is shown in Fig. 1 [11,12,27,28]. Here a radially polarized beam is focused by an axicon lens of radius R_c, such that it crosses the z axis at an angle θ_c and forms a focal region of length L_c. A circularly symmetric interference pattern develops along the focal region with a radial field component given by $\hat{E}_r=E_c(z)J_1(k_\perp r)$, where the field amplitude $E_c(z)$ depends on the laser intensity at the surface of the axicon. Assuming the field components have the form given by Eqs. (5)–(7), the axicon focal parameters (R_c, L_c, and $\theta_c \ll 1$) are related to the Bessel beam parameters ($k_\perp=\pi/r_b$, $L_{\max}=2NZ_{RB}/\pi$, and $N=a/r_b$) by $R_c \simeq L_c\theta_c \simeq 2a$, $k_\perp \simeq k\theta_c$, $L_c \simeq 2L_{\max}$, and $N \simeq (R_c/\lambda)\theta_c$.

III. BESSEL BEAM DIFFRACTION

The scale length for diffraction of a nonideal Bessel beam can be determined analytically using the scalar diffraction theory based on Huygens' principle [18]. In this method the transverse beam profile is specified in the plane of an aperture and the beam is propagated forward using an integral formulation with an approximate form for the Green's function. The solution to the wave equation, Eq. (3), for a laser beam of frequency $\omega=ck$ is given by the Kirchhoff integral [18]

$$E_i(r)=\frac{k}{2\pi i}\int_A dS'\left(1+\frac{i}{kR}\right)\frac{\hat{\mathbf{n}}\cdot\mathbf{R}}{R^2}\exp(ikR)E_i(\mathbf{r}'), \tag{17}$$

where $\mathbf{R}=\mathbf{x}-\mathbf{x}'$ is the position vector from the element of surface integration dS' at $\mathbf{x}'$ to the point of observation $\mathbf{x}$, $\hat{\mathbf{n}}$ is a unit vector that is normal to the plane of the aperture and directed towards the observation point, A denotes the surface area of the aperture, and $E_i \sim \exp(-i\omega t)$ has been assumed. The key dependence in Eq. (17) is the phase factor $\exp(ikR)$. In a cylindrical coordinate system and in the paraxial limit, the binomial expansion

$$R \simeq z\left[1+\frac{r^2+r'^2}{2z^2}-\frac{rr'}{z^2}\cos(\theta-\theta')\right] \tag{18}$$

can be used in the exponent, where $(r^2+r'^2)/z^2 \ll 1$ has been assumed. Here, the aperture is taken to be planar and at $z'=0$, the unit vector $\hat{\mathbf{n}}$ points along the positive z axis, which is the direction of propagation, and θ and θ' denote the azimuthal angle at the observation point and at the aperture, respectively. Substituting this expansion into Eq. (17), the field at the observation point to leading order is given by

$$\hat{E}_i(\mathbf{r}) \simeq \frac{k}{2\pi iz}\exp\left(\frac{ikr^2}{2z}\right)\int_0^a dr'r'\int_0^{2\pi}d\theta' \times\exp\left\{\frac{ik}{2z^2}[r'^2-2rr'\cos(\theta-\theta')]\right\}\hat{E}_i(r',\theta'), \tag{19}$$

where $E_i=\mathrm{Re}\hat{E}_i\exp(ikz-i\omega t)$ and a circular aperture of radius a has been assumed. In the limit of a cylindrically symmetric field at the aperture surface, $\hat{E}_i(r',\theta')=\hat{E}_i(r')$, the θ' integral can be evaluated and Eq. (19) reduces to

$$\hat{E}_i(r,z) \simeq \frac{k}{iz}\exp\left(\frac{ikr^2}{2z}\right)\int_0^a dr'r' \times\exp\left(\frac{ikr'^2}{2z}\right)J_0\left(\frac{krr'}{z}\right)\hat{E}_i(r'). \tag{20}$$

Equation (20) will be used to described the diffraction of a Bessel beam that is incident on a circular aperture of radius a. Specifically, the propagation distance of the Bessel beam will be determined by evaluating Eq. (20) along the z axis ($r=0$).

Consider the diffraction of a nonideal fundamental Bessel beam, in which the field has been truncated by a circular aperture of radius a. At the aperture, $\hat{E}_x(r')=E_0J_0(k_\perp r')$ for $r \leq a$. Inserting this into Eq. (20) gives

$$\hat{E}_x(r,z) \simeq \frac{kE_0}{iz}\exp\left(\frac{ikr^2}{2z}\right)\int_0^a dr'r' \times\exp\left(\frac{ikr'^2}{2z}\right)J_0\left(\frac{krr'}{z}\right)J_0(k_\perp r'). \tag{21}$$

Note that in the limit $a \to \infty$,

$$\hat{E}_x(a\to\infty) \simeq E_0J_0(k_\perp r)\exp\left(-\frac{ik_\perp^2 z}{2k}\right), \tag{22}$$

which is the exact solution to the paraxial wave equation, Eq. (3), describing the ideal fundamental Bessel beam, Eq. (4).

Suppose now that the radius of the aperture is large but finite. Along the z axis, Eq. (21) can be written as

$$\hat{E}_x(r=0,z) \simeq \frac{kE_0}{iz}\left(\int_0^\infty dr' r' - \int_a^\infty dr' r'\right) \times \exp\left(\frac{ikr'^2}{2z}\right) J_0(k_\perp r'). \quad (23)$$

The first r' integral gives the ideal Bessel beam result, Eq. (22), evaluated at $r=0$. To evaluate the second integral, the asymptotic form for the Bessel function can be used, Eq. (9), assuming $k_\perp a \gg 1$. Equation (23) can be written as

$$\hat{E}_x(r=0,z) = E_0 \exp\left(-\frac{ik_\perp^2 z}{2k}\right) - \frac{kE_0 F(z)}{iz(2\pi k_\perp)^{1/2}}, \quad (24)$$

where

$$F(z) = \int_a^\infty dr' \sqrt{r'}[\exp(i\phi_+) + \exp(i\phi_-)], \quad (25a)$$

and

$$\phi_\pm(r',z) = \frac{kr'^2}{2z} \pm \left(k_\perp r' - \frac{\pi}{4}\right). \quad (25b)$$

The integral $F(z)$, Eq. (25), may be evaluated by observing that for sufficiently large a the ϕ_+ term phase mixes away and that the leading contribution comes from the ϕ_- term in the vicinity of the stationary phase point. The stationary point $r' = r_f$ where $\partial\phi_-/\partial r' = 0$ is given by $r_f = k_\perp z/k$. If r_f lies outside the integration region, $r_f < a$, the integrand is highly oscillatory and the integral tends to phase mix to zero. If r_f lies within the integration region, $r_f > a$, the integral can be evaluated by expanding the phase ϕ_- about $r' = r_f$. The leading order contribution to the integral $F(z)$ is given by

$$F(z) \simeq \left(\frac{2\pi r_f}{\phi''(r_f)}\right)^{1/2} \exp\left[i\phi(r_f) + i\frac{\pi}{4}\right], \quad (26)$$

for $r_f > a$ and $I \simeq 0$ otherwise, where the double prime denotes $\partial^2/\partial r^2$. Hence, to leading order, the amplitude of the laser field along the axis of propagation is given by

$$\hat{E}_x(r=0,z) \simeq E_0 \exp\left(-\frac{ik_\perp^2 z}{2k}\right)\left[1 - H\left(z - \frac{ka}{k_\perp}\right)\right], \quad (27)$$

where $H(z - ka/k_\perp)$ is the Heaviside step function. This demonstrates that the maximum propagation distance for a Bessel beam passing through an aperture of radius $r=a$ is

$$L_{\max} = ka/k_\perp = (2/\pi)NZ_{RB}, \quad (28)$$

where $r_b = \pi/k_\perp$ is the asymptotic width of a Bessel ring, $N = a/r_b$ is the total number of rings, and $Z_{RB} = kr_b^2/2$ is the Rayleigh length associated with an individual Bessel ring. This value for $L_{\max}$ is in agreement with previous estimates [19,20].

To verify the analytical results based on the method of stationary phase, the field intensity $|E_x(z)|^2$ is plotted along $r=0$ as a function of $z/L_{\max}$ in Fig. 2 for (a) $k_\perp a = 20$ and (b) $k_\perp a = 40$. These plots were obtained by numerically integrating the expression for the field given by Eq. (21). The plots show that the field intensity falls off dramatically at $z = L_{\max}$.

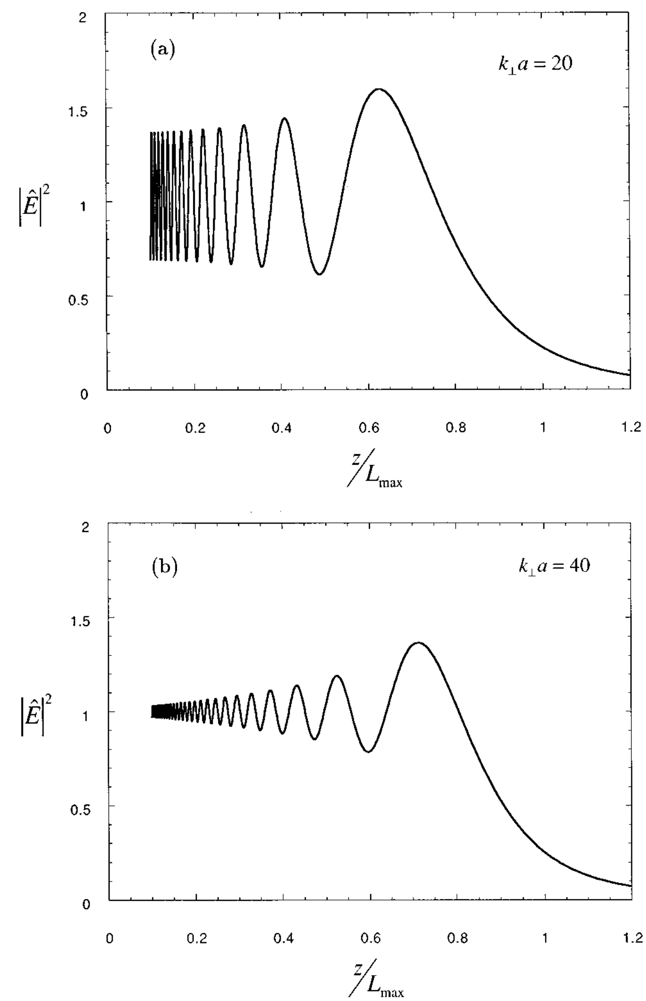

FIG. 2. Normalized field intensity $|E_x(z)|^2$ for a zero-order Bessel beam plotted along $r=0$ as a function of $z/L_{\max}$ for (a) $k_\perp a = 20$ and (b) $k_\perp a = 40$, as obtained from a numerical integration of Eq. (21).

As the number of Bessel lobes increases, the amplitude of the axial oscillations in the intensity tends to decrease and the intensity terminates more sharply at $z = L_{\max}$.

Although this calculation has assumed a fundamental Bessel beam with $E_x \sim J_0(k_\perp r)$, an identical result is obtained for the radially polarized first-order Bessel beam with $E_r \sim J_1(k_\perp r)$, as given by Eqs. (5)–(7). This is readily seen by considering the diffraction of the axial component of the first-order Bessel field, $E_z \sim J_0(k_\perp r)$, Eq. (7). The above results Eqs. (21)–(28) directly apply to $E_z \sim J_0(k_\perp r)$.

IV. LASER ACCELERATION OF PARTICLES

For a fixed total laser power, a Bessel laser beam can lead to enhanced energy gain in a laser-driven accelerator provided that (i) the accelerating gradient $E_{\rm acc}$ is linearly proportional to the laser field amplitude and (ii) the acceleration distance is limited by laser diffraction and not phase detuning (or some other mechanism). When these two conditions are met, the energy gain can be enhanced by a factor of $N^{1/2}$, where $N = a/r_b$ is the number of rings in the Bessel beam.

A. Ponderomotive accelerators

In many laser-driven accelerators, such as the laser wakefield accelerator, the plasma beat wave accelerator, and the vacuum beat wave accelerator, particle acceleration is the result of the ponderomotive force of the laser fields, i.e., the accelerating gradient is proportional to the square of the laser field amplitude, $E_{\rm acc}\sim E_0^2$ [1–5,8,10]. In this case, there is no enhancement in the energy gain when a Bessel beam is used, as is evident by the following scaling laws. For a Gaussian beam undergoing vacuum diffraction, the acceleration distance is on the order of a Rayleigh length, $L\sim Z_{RG}$. In terms of the total power $P_G\sim E_0^2 r_0^2$, the energy gain scales as $W_G\sim LE_{\rm acc}\sim P_G$. For a J_0 Bessel beam, the total propagation length is $L\sim NZ_{RB}$ and the total beam power is $P_B\simeq NP_b\sim NE_0^2 r_b^2$. Hence, $W_B\sim NZ_{RB}E_0^2\sim P_B$. The ratios of the energy gains scales as

$$W_B/W_G\sim P_B/P_G, \tag{29}$$

where the subscripts B and G refer to the Bessel and Gaussian beams, respectively. For equal beam powers, there is no enhancement in the energy gain, assuming that the acceleration distance is limited by diffraction of the drive laser beams and not by phase velocity slippage.

B. Inverse free electron laser

On the other hand, consider the case of an accelerating gradient that is linearly proportional to the amplitude of the transverse laser field, $E_{\rm acc}\sim E_0$. This is the case in the inverse free electron laser [14–17], in which acceleration results from the ponderomotive force of the beat wave of the laser field with the wiggler magnetic field, i.e., $E_{\rm acc}\sim(a_w/\gamma)E_0$, where a_w is the normalized wiggler magnetic field and γ is the relativistic factor of the electron. For a Gaussian driver, $W_G\sim E_0 Z_{RG}\sim r_0 P_G^{1/2}$. For a J_0 Bessel driver, $W_B\sim E_0 NZ_{RB}\sim r_b(NP_B)^{1/2}$. Hence,

$$W_B/W_G\sim(r_b/r_0)(NP_B/P_G)^{1/2}. \tag{30}$$

Uniform acceleration of the electron beam requires both r_0 and r_b to be greater than the electron beam radius. Assuming $r_b\simeq r_0$ and $P_G\simeq P_B$, the energy gain can be enhanced by the use of a Bessel beam by a factor of $N^{1/2}$. This assumes that the acceleration distance is limited by the diffraction of the drive laser beams and not by phase velocity slippage.

C. Direct acceleration in vacuum

The above arguments assume that the acceleration length is limited only by the propagation distance of the laser beam and that phase synchronism is maintained by matching the phase velocity of the accelerating wave to the velocity of the electron beam. For direct acceleration in vacuum [4–9], in which the particles are accelerated by the axial component of the laser field ($E_{\rm acc}\simeq E_z$), phase detuning will limit the acceleration length. The detuning length is defined as the distance required for a particle to phase slip by one-half period with respect to the axial electric field, i.e., $L_d(v_e^{-1}-v_p^{-1})\simeq\pi/\omega$, where v_e is the axial particle velocity. For a Gaussian beam, the phase velocity is given by Eq. (13) and the detuning length for a highly relativistic electron ($v_e\simeq c$) is $L_d\simeq(\pi/2)Z_{RG}$. For a Bessel beam, v_p is given by Eq. (8) and $L_d\simeq(4/\pi)Z_{RB}$. Hence, in vacuum the effective acceleration length will be limited to $L_{\rm acc}=L_d$, even if the beam propagates a distance longer than L_d. For a Gaussian beam $E_z\simeq 2E_0/kr_0$ and $W_G\sim(2E_0/kr_0)Z_{RG}\sim P_G^{1/2}$. For a J_1 Bessel beam $E_z\simeq k_\perp E_0/k$ and $W_B\sim(kE_0/k_\perp)Z_{RB}\sim(P_B/N)^{1/2}$. Thus

$$W_B/W_G\sim(P_B/NP_G)^{1/2}, \tag{31}$$

and the energy gain for direct acceleration in vacuum will be reduced by a factor of $N^{1/2}$ for the Bessel beam, assuming equal beam powers.

D. Inverse Cherenkov accelerator

In the inverse Cherenkov accelerator (ICA) [4,11–13], a background of neutral gas is introduced to control the phase velocity of the laser field. Acceleration is the result of the axial component of a radially polarized first-order laser field. In a gas the dispersion relation is $n\omega=ck$, where $n>1$ is the index of refraction. Typically, $\Delta n=n-1$ is much less than unity and proportional to the gas density. Hence, the phase velocity can be tuned by adjusting the gas density and phase synchronism can be achieved. For example, the axial phase velocity of the Bessel beam of Eqs. (5)–(7) is given by

$$v_p/c=1-\Delta n+k_\perp^2/2k^2. \tag{32}$$

For a highly relativistic electron, phase matching $v_p=c$ requires $2\Delta n=k_\perp^2/k^2$.

1. Gaussian beam ICA

For a Gaussian beam of the form given by Eqs. (11) and (12), the energy gain in the ICA can be calculated by integrating the axial electric field, Eq. (12), over $-\infty<z<\infty$. A particle with relativistic factor γ moving at nearly constant velocity $v_e\simeq c(1-1/2\gamma^2)$ ($\gamma\gg 1$) along the z axis is subject to an electric field given by

$$E_z(t=z/v_e)=iE_{z0}\frac{(1-i\alpha)^2}{(1+\alpha^2)^2}\exp[i(\Delta kz+\phi_0)], \tag{33}$$

where $E_{z0}=2E_0/kr_0$ is the peak axial field amplitude, $\Delta k=k(\Delta n-1/2\gamma^2)$, $\alpha=z/Z_{RG}$, $z=0$ is the focal point, and ϕ_0 is the phase of the electron at the focal point as determined by the initial conditions. The energy gain is given by

$$\begin{aligned} W_G&=-q\int_{-\infty}^{\infty}dz\; E_z(t=z/v_e),\\ &=qE_{z0}\sin t\phi_0\int_{-\infty}^{\infty}\frac{dz}{(1+\alpha^2)^2}[(1-\alpha^2)\cos\Delta kz\\ &\quad+2\alpha\;\sin\Delta kz],\\ &=2\pi qE_{z0}\Delta kZ_{RG}^2\exp(-\Delta kZ_{RG})\sin\phi_0, \end{aligned} \tag{34}$$

for $\Delta k\geqslant 0$ and $W_G=0$ for $\Delta k<0$, where q is the electron charge. The energy gain is maximum when $\sin\phi_0=1$ and when $\Delta k Z_{RG}=1$, i.e., $\Delta n = 1/kZ_{RG} + 1/2\gamma^2$, which minimizes the effects of phase detuning. In this case, the maximum energy gain is given by

$$W_G = (2\pi q/e) E_{z0} Z_{RG} = (2\pi q/e) E_0 r_0 . \tag{35}$$

In terms of the total power, Eq. (35) can be written as

$$W_G[\mathrm{MeV}] \simeq 2.3 P_G^{1/2}[\mathrm{GW}]. \tag{36}$$

To enhance the energy gain it is necessary to increase the laser propagation distance. A Gaussian laser beam can be self-guided in a gas by a proper balancing of diffraction, nonlinear self-focusing, and plasma defocusing [4,30].

2. Bessel beam ICA

For a first-order Bessel beam of the form given by Eqs. (5)–(7), the amplitude and phase of the electric field is approximately constant over a total propagation length of $2L_{\max}$, where $L_{\max}$ is given by Eq. (28). Hence, the maximum energy gain can be estimated by the product of the peak axial field $E_{z0} = k_\perp E_0/k$, Eq. (7), times the total propagation distance, $(4/\pi) N Z_{RB}$, i.e.,

$$W_B = 2qE_{z0} L_{\max} = 2qaE_0 . \tag{37}$$

Here it has been assumed that the electron remains phase matched to the laser field, i.e., $2\Delta n = k_\perp^2/k^2 + 1/\gamma^2$, as implied by Eq. (32). In terms of the total Bessel beam power, $P_B \simeq caE_0^2/4\pi k_\perp$, the maximum energy gain is given by

$$W_B \simeq 4\pi q (NP_B/c)^{1/2}. \tag{38}$$

In practical units,

$$W_B[\mathrm{MeV}] \simeq 2.2 N^{1/2} P_B^{1/2}[\mathrm{GW}]. \tag{39}$$

This is in agreement with previous estimates [11]. Hence, by using a Bessel beam with N rings, the energy gain can be enhanced by a factor of $N^{1/2}$ compared to a Gaussian beam with the same total power,

$$W_B/W_G \simeq (NP_B/P_G)^{1/2}. \tag{40}$$

It is also of interest to compare the effective acceleration lengths $L_{G,B}$ and peak accelerating gradients $E_{zB,G}$, where $(W = qE_z L)_{B,G}$, $L_G = (2\pi/e) Z_{RG}$, and $L_B = (4/\pi) Z_{RB}$. In particular,

$$L_B/L_G \simeq 0.55 N (r_b/r_0)^2, \tag{41a}$$

$$E_{zB}/E_{zG} \simeq 1.7 (r_0/r_b)^2 (P_B/NP_G)^{1/2}. \tag{41b}$$

Clearly, these ratios depend on the transverse dimensions of the beams. For example, if equal powers and equal apertures are assumed, $P_G = P_B$ and $r_0 = a_0 = Nr_b$, then $L_B/L_G \simeq 0.55 N^{-1}$ and $E_{zB}/E_{zG} \simeq 1.7 N^{3/2}$.

A more relevant constraint for the ICA is to assume that the peak intensities in the two beams are equal. If phase matching between the electron and the laser field is to be maintained in the ICA, ionization should be avoided, which places a limit on the maximum intensity. For the first-order Gaussian beam given by Eq. (11), the field reaches at maximum of $|E_x|_{\max} = E_{0G}/\sqrt{2e}$ at $r = r_0/\sqrt{2}$. For the first-order Bessel beam given by Eq. (5), the field reaches at maximum of $|E_r|_{\max} = (0.58) E_{0B}$ at $r = 1.8/k_\perp$. Assuming equal peak intensities $|E_x|_{\max} = |E_r|_{\max}$ implies

$$r_b^2/r_0^2 \simeq 2.3 P_B/NP_G . \tag{42}$$

Inserting this into Eqs. (41a) and (41b) gives

$$L_B/L_G \simeq 1.2 P_B/P_G , \tag{43a}$$

$$E_{zB}/E_{zG} \simeq 0.77 (NP_G/P_B)^{1/2}. \tag{43b}$$

Hence, for equal powers and equal peak intensities, the propagation lengths are approximately equal for the two beams and the peak accelerating gradient for the Bessel beam is approximately $N^{1/2}$ times larger than that for the Gaussian beam. Furthermore, note that in the highly relativistic limit, the phase matching condition is $\Delta n \simeq k_\perp^2/2k^2 = \pi^2/2k^2 r_b^2$ for the Bessel beam and $\Delta n \simeq 1/kZ_{RG} = 2/k^2 r_0^2$ for the Gaussian beam. For equal powers and intensities, Eq. (42) implies $r_b^2/r_0^2 \simeq 2.3/N$. For large N this implies that phase matching with a Bessel beam requires much higher neutral gas densities than is required with a Gaussian beam, which may impose technological difficulties.

V. DISCUSSION

The first-order limitation in any laser-driven accelerator configuration is diffraction of the drive laser beams. The possibility of using Bessel laser beams to extend the acceleration distance and enhance the energy gain has been examined. A formalism based on Huygens' principle has been developed to describe the diffraction of nonideal Bessel beams, i.e., a Bessel beam profile which has been truncated by an aperture of radius a. Such a beam has a finite power and can be generated in the laboratory. An analytical expression for the maximum propagation distance was derived, $L_{\max} = (2/\pi) N Z_{RB}$, where $N = a/r_b$ is the number of transverse lobes in the Bessel beam, r_b is the asymptotic width of an individual lobe, and $Z_{RB} = kr_b^2/2$ is the Rayleigh length of an individual lode. This analytical expression was verified by numerical calculations.

Comparisons were made between accelerators driven by Gaussian laser beams and those driven by Bessel laser beams. For equal beam powers, it was shown that the energy gain using a Bessel beam is approximately $N^{1/2}$ times that using a Gaussian beam provide that (i) the accelerating gradient is linearly proportional to the laser field and (ii) the acceleration distance is limited by diffraction and not by phase detuning (or some other mechanism). This is the case for the inverse Cherenkov accelerator (phased-matched acceleration using a J_1 beam in a gas) and for the inverse free electron laser (phase-matched acceleration resulting from the ponderomotive interaction of a J_0 beam and a wiggler magnetic field). The inverse Cherenkov accelerator was analyzed in detail. It was shown that for equal powers and equal peak intensities, the acceleration length, peak accelerating field, and energy gain scale as $L_B/L_G \sim 1$, $E_{zB}/E_{zG} \sim N^{1/2}$, and $W_B/W_G \sim N^{1/2}$, respectively.

Scaling laws have been derived for other configurations as well. Direct acceleration by the E_z field in vacuum is limited by phase detuning, hence, there is no advantage in using a Bessel beam (in fact, the energy gain is reduced by $N^{1/2}$, assuming equal beam powers). For cases in which the accelerating gradient is proportional to the square of the laser field (e.g., the laser wakefield accelerator, the plasma beat wave accelerator, and the vacuum beat wave accelerator), the energy gain for both Gaussian and Bessel beam drivers scales as $W \sim P$ (independent of N), assuming an acceleration distance limited by diffraction. For very intense lasers, however, there exists a highly nonlinear regime of the laser wakefield accelerator for which $E_{\rm acc}$ is linearly proportional to the laser field [3,4], hence, there may be an advantage to using a Bessel beam if the propagation distance is limited by diffraction. In addition, for all laser-driven plasma-based accelerators, it appears possible to guide a laser beam large distances (many Rayleigh lengths) using a plasma density channel [3,4,29,31,32], thus enhancing acceleration length and the energy gain.

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