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Beam coupling impedances of obstacles protruding into a beam pipe

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The beam coupling impedances of small obstacles protruding inside the vacuum chamber of an accelerator are calculated at frequencies for which the wavelength is large compared to a typical size of the obstacle. Formulas for a few important particular cases including both essentially three-dimensional objects like a post or a mask and axisymmetric irises, are presented. These results allow simple practical estimates of the broad-band impedance contributions from such discontinuities. [S1063-651X(97)00903-3]

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I. INTRODUCTION

Due to high currents in modern accelerators and colliders, even contributions from small discontinuities of the vacuum chamber to the impedance budget of the machine have to be accounted for. Numerous pumping holes—a few hundred per meter—in the vacuum screen of the Large Hadron Collider (LHC) [1] can serve as an example. Their total contribution to the machine impedance in the initial design was calculated [2,3] and found to be dangerously large, close to the beam instability threshold, but after the proposed design changes it was reduced by more than an order of magnitude [4]. This analytical calculation was based on the Bethe theory of diffraction of electromagnetic waves by a small hole in a metal plane [5]. The method's basic idea is that the hole in the frequency range where the wavelength is large compared to the typical hole size can be replaced by two induced dipoles, an electric and a magnetic one. Since essentially the same idea works for any small obstacle, the method was extended for arbitrarily small discontinuities on the pipe of an arbitrary-shaped cross section, see [6] for the summary and references therein. The problem of calculating the impedance contribution from a given small discontinuity was therefore reduced to finding its electric and magnetic polarizabilities. Useful analytical results in this direction have been obtained for various axisymmetric obstacles [7], as well as for holes and slots: circular [5] and elliptic [8] hole in a zero-thickness wall, circular [9] and elliptic [10] hole in a thick wall, various slots (some results are compiled in [11]), and a ring-shaped cut [12].

In the present paper we utilize the method to calculate the coupling impedances of a few types of obstacles protruding inside the beam pipe, such as a narrow post or a mask intercepting synchrotron radiation. Formulas are derived which make practical estimates very simple. Numerical simulations required to obtain similar results are necessarily three-dimensional (3D) ones, and therefore are rather involved. This statement is generally applicable for any small discontinuities, but especially for those protruding into the vacuum chamber.

II. GENERAL SOLUTION

For brevity we restrict ourselves to the longitudinal coupling impedance of a small discontinuity on the wall of a circular beam pipe of radius R, which is [2]

$$Z(k) = -iZ_0 k \frac{\alpha_e + \alpha_m}{4\pi^2 R^2}, \tag{1}$$

when the wavelength $2\pi/k$ is large compared to the obstacle size. Here $Z_0 = 120\pi\ \Omega$ is the impedance of free space, $k = \omega/c$ is the wave number, and α_e, α_m are the electric and magnetic polarizabilities of the discontinuity. One should note that the transverse impedance is proportional to the same combination of polarizabilities, $\alpha_e + \alpha_m$, and the real part of the impedance is small at such frequencies (see [6,4] for detail, as well as for other chamber cross sections). Let the obstacle shape be a half ellipsoid with semiaxis a in the longitudinal direction (along the chamber axis), b in the radial direction, and c in the azimuthal one, with $a,b,c \ll R$, which means that the obstacle is small, and the Bethe approach can be applied. To find the polarizabilities, one needs to calculate the induced electric dipole moment P of the obstacle illuminated by a homogeneous radial electric field E_0, and the magnetic dipole moment M when it is illuminated by an azimuthal magnetic field H_0. This problem, however, is essentially the same as that for an ellipsoid immersed in a homogeneous field. Using the known solution of the last problem, e.g., [8], and adding obvious symmetry considerations, we get

$$\alpha_e = \frac{P}{2\varepsilon_0 E_0} = \frac{2\pi abc}{3I_b} \tag{2}$$

and

$$\alpha_m = \frac{M}{2H_0} = \frac{2\pi abc}{3(I_c - 1)}, \tag{3}$$

where

$$I_b = \frac{abc}{2}\int_0^\infty \frac{ds}{(s+b^2)^{3/2}(s+a^2)^{1/2}(s+c^2)^{1/2}}, \tag{4}$$

and I_c is given by Eq. (4) with b and c interchanged. Although Eqs. (2)–(4) solve the problem in general, important practical results can be obtained by considering particular cases.

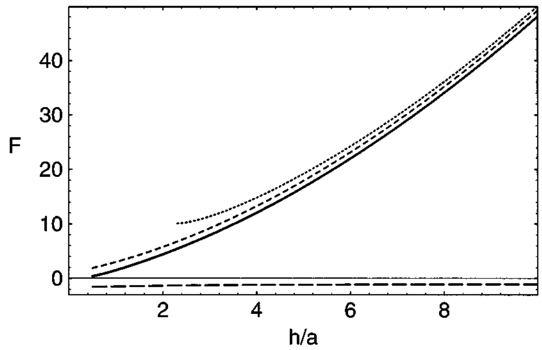

FIG. 1. Function $F\equiv(\alpha_e+\alpha_m)/V$ versus aspect ratio h/a for a pinlike obstacle (solid line). The electric contribution is short dashed, the magnetic one is long dashed, and the dotted line shows the asymptotic form used in Eq. (7).

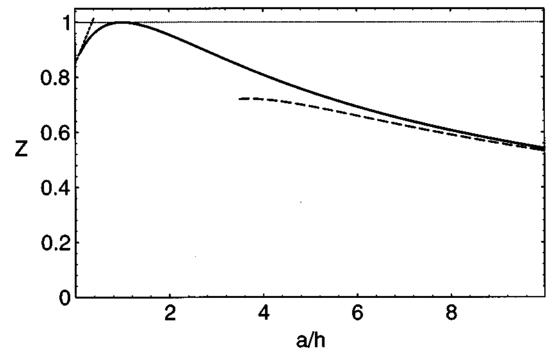

FIG. 2. Impedance Z of a mask [in units of that for a semisphere with the same depth, Eq. (8) with $a=h$] versus its length. The narrow-mask approximation, Eq. (9), is short dashed, and the long-mask one, Eq. (10), is long dashed.

III. POST AND MASK

In the case $a=c$, $b=h$ we have an ellipsoid of revolution, and the integral in Eq. (4) can be expressed in terms of the hypergeometric function ${}_2F_1$ to yield

$$\alpha_e=\frac{2\pi a^2 h}{{}_2F_1(1,1;5/2;1-h^2/a^2)} \tag{5}$$

and

$$\alpha_m=\frac{2\pi a^2 h}{3[{}_2F_1(1,1;5/2;1-a^2/h^2)-1]}. \tag{6}$$

In the limit $a\ll h$, corresponding to a pinlike obstacle, we get

$$\alpha_e\simeq\frac{2\pi h^3}{3\,[\ln(2h/a)-1]},$$

that is much larger than $\alpha_m\simeq-2\pi a^2h/3$. Note that in this limit $\alpha_m\simeq-V$, where $V=2\pi a^2h/3$ is the volume occupied by the obstacle (and subtracted from that occupied by the beam magnetic field), similarly to the axisymmetric case [7]. These results give us a simple expression for the inductive impedance of a narrow pin (post) of height h and radius a, $a\ll h$, protruding radially into the beam pipe:

$$Z(k)\simeq-ikZ_0\frac{h^3}{6\pi R^2[\ln(2h/a)-1]}. \tag{7}$$

[One could use the known result for the induced electric dipole of a narrow cylinder parallel to the electric field [13]. It will only change $\ln(2h/a)-1$ in Eq. (7) to $\ln(4h/a)-7/3$.] The factor $F\equiv(\alpha_e+\alpha_m)/V$ is plotted in Fig. 1 versus the ratio h/a. The figure also shows comparison with the asymptotic approximation given by Eq. (7).

One more particular case of interest here is $h=a$, i.e., a semispherical obstacle of radius a. From Eqs. (5) and (6) the impedance of such a discontinuity is

$$Z(k)=-ikZ_0\frac{a^3}{4\pi R^2}, \tag{8}$$

which is $3\pi/2$ times that for a circular hole of the same radius in a thin wall [2].

Another practical result that can be derived from the general solution, Eqs. (2)–(4), is the impedance of a mask intended to intercept synchrotron radiation. We set $b=c=h$, so that our model mask has the semicircular shape with radius h in its largest transverse cross section. Then the integral in Eq. (4) is reduced to

$$I_b=I_c=\frac{1}{3}{}_2F_1\left(1,\frac{1}{2};\frac{5}{2};1-\frac{h^2}{a^2}\right),$$

and we can further simplify the result for two particular cases.

The first one is the thin mask, $a\ll h$, in which case $\alpha_e\simeq 8h^3/3$, and again it dominates the magnetic term, $\alpha_m\simeq-V=-2\pi ah^2/3$. The coupling impedance for such an obstacle—a half disk of radius h and thickness $2a$, $a\ll h$, transverse to the chamber axis—is therefore

$$Z(k)=-ikZ_0\frac{2h^3}{3\pi^2R^2}\left[1+\left(\frac{4}{\pi}-\frac{\pi}{4}\right)\frac{a}{h}+\cdots\right], \tag{9}$$

where the next-to-leading term is shown explicitly.

In the opposite limit, $h\ll a$, which corresponds to a long (along the beam) mask, the leading terms $\alpha_e\simeq-\alpha_m\simeq 4\pi ah^2/3$ cancel each other. As a result, the impedance of a long mask with length $l=2a$ and height h, $h\ll l$, is

$$Z(k)\simeq-ikZ_0\frac{4h^4}{3\pi R^2 l}\left(\ln\frac{l}{h}-1\right), \tag{10}$$

which is relatively small due to the ''aerodynamic'' shape of this obstacle, in complete analogy with results for long elliptic slots [2,3,11].

Figure 2 shows the impedance of a mask with a given semicircular transverse cross section of radius h versus its normalized half length, a/h. The comparison with the asymptotic approximations Eqs. (9) and (10) is also shown. One can see that the asymptotic behavior (10) starts to work well only for very long masks, namely, when $l=2a\geqslant 10h$. Figure 2 demonstrates that the mask impedance depends rather weakly on the length. Even a very thin mask ($a\ll h$) contributes as much as $8/(3\pi)\simeq 0.85$ times the semisphere

($a=h$) impedance, Eq. (8), while for long masks the impedance decreases slowly: at $l/h=20$, it is still 0.54 of that for the semisphere.

In practice, however, the mask has usually an abrupt cut toward the incident synchrotron radiation, so that it is rather one-half of a long mask. From considerations above one can suggest as a reasonable impedance estimate for such a discontinuity the half sum of the impedances given by Eqs. (9) and (10). This estimate is corroborated by 3D numerical simulations using the MAFIA code [14], at least, for the masks which are not too long. In fact, the low-frequency impedances of a semisphere and a half semisphere of the same depth—which can be considered as a relatively short realistic mask—were found numerically to be almost equal (within the errors), and close to that for a longer half mask. From these results one can conclude that a good estimate for the mask impedance is given simply by Eq. (8). The simulations mentioned are rather involved, and a detailed comparison, as well as numerical results for other types of discontinuities, will be reported elsewhere.

IV. AXISYMMETRIC IRIS

Following a similar procedure one can also easily obtain the results for axisymmetric irises having a semielliptic profile in the longitudinal chamber cross section, with depth $b=h$ and length $2a$ along the beam. For that purpose, one should consider limit $c\to\infty$ in Eq. (4) to calculate polarizabilities $\tilde{\alpha}_e$ and $\tilde{\alpha}_m$ per unit length of the circumference of the chamber transverse cross section. The broadband impedances of axisymmetric discontinuities have been studied in [7], and the longitudinal coupling impedance is given by

$$Z(k)=-iZ_0k\frac{\tilde{\alpha}_e+\tilde{\alpha}_m}{2\pi R}, \tag{11}$$

quite similar to Eq. (1). As $c\to\infty$, the integral $I_c\to 0$, and I_b is expressed in elementary functions as

$$I_b=\frac{1}{2}{}_2F_1\left(1,\frac{1}{2};2;1-\frac{h^2}{a^2}\right)=\frac{a}{a+h}.$$

It gives us immediately

$$\tilde{\alpha}_e=\frac{\pi}{2}h(h+a), \quad \tilde{\alpha}_m=-\frac{\pi}{2}ah, \tag{12}$$

and the resulting impedance of the iris of depth h with the semielliptic profile is simply

$$Z(k)=-ikZ_0\frac{h^2}{4R}, \tag{13}$$

which proves to be independent of the iris thickness a. The same result has been recently obtained by Gluckstern and Kurennoy using another method [15]. One should emphasize that $\tilde{\alpha}_m$ in Eq. (12) is just an iris cross-section area (with negative sign), which is correct for any small axisymmetric discontinuity, as was pointed out in [7]. However, calculating $\tilde{\alpha}_e$ is not so easy: a conformal mapping was used for that purpose in [7] for irises (as well as for chamber enlargements) having a trapezoid (or rectangular, or triangular) profile. An interesting fact is that the leading behavior for thin irises of all shapes is exactly the same as Eq. (13).

It is easy to check the result in Eq. (12) for the particular case $h=a$, when the iris has a semicircular profile of radius a. The required conformal mapping for this case is very simple, $w=(z/a+a/z)/2$. The ratio of the coefficients of the second and first terms in this expression is $\tilde{\alpha}_e/\pi$, cf. [7], which gives us $\tilde{\alpha}_e=\pi a^2$, in agreement with Eq. (12).

The more general conformal mapping from the upper half plane w into z with the boundary including the iris having a semielliptic profile is given by

$$z=aw+h\sqrt{w^2-1}.$$

We need an inverse mapping, but, fortunately, it is enough to find its asymptotic behavior at large z and w [7], which is

$$w=\frac{z}{a+h}+\frac{h}{2}\frac{1}{z}+\cdots.$$

Comparison of the second and first terms leads us exactly to the result for $\tilde{\alpha}_e$ in Eq. (12).

In fact, one can readily obtain an answer also for irises having the profile shaped as a circle segment with the chord of length s along the chamber wall in the longitudinal direction, and opening angle 2φ, where $0\leqslant\varphi\leqslant\pi$. The conformal mapping for this case is

$$w=\frac{s\beta}{1-(1-s/z)^{\beta}},$$

where $\beta=\pi/(\pi-\varphi)$. Considering its asymptotic behavior at $z\to\infty$, and comparing terms z and $1/z$, one gets $\tilde{\alpha}_e=\pi s^2(\beta^2-1)/12$, which can be used in Eq. (11) to derive the impedance. Just for reference, we present the impedance of such an exotic iris, expressed in terms of its height $h=s(1-\cos\varphi)/(2\sin\varphi)$:

$$Z(k)=-ikZ_0\frac{h^2}{2R(1-\cos\varphi)^2}\times\left[\frac{\varphi(2\pi-\varphi)}{3(\pi-\varphi)^2}\sin^2\varphi-\frac{2\varphi-\sin 2\varphi}{2\pi}\right]. \tag{14}$$

Again, the impedance is proportional to h^2, but the coefficient now depends (in fact, rather weakly) on φ.

V. SUMMARY

Combining above the general method for calculating impedances of small discontinuities with the well-known solution of the problem of an ellipsoid in a homogeneous field, we obtained a few analytical results for both 3D and axisymmetric obstacles protruding inside the beam pipe. These results can greatly simplify calculations of the broadband contributions to the coupling impedances from such discontinuities, especially in the 3D case. One should mention that the present approach does not work for enlargements of the vacuum chamber. However, the existing results for holes and slots [2–4,6] and for axisymmetric enlargements [7] cover this case quite well.

It is worthwhile to mention that the above results for the polarizabilities can also be used to obtain the real part of the longitudinal impedance, which is proportional to $(\alpha_e^2+\alpha_m^2)$, as a function of frequency, and then to calculate the related loss factors for the considered discontinuities, see [6] for detail.

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[1] CERN Report No. CERN/AC/95-05 (LHC), 1995 (unpublished).
[2] S.S. Kurennoy, Part. Accel. **39**, 1 (1992).
[3] R.L. Gluckstern, Phys. Rev. A **46**, 1106 (1992); **46**, 1110 (1992).
[4] S.S. Kurennoy, Part. Accel. **50**, 167 (1995).
[5] H.A. Bethe, Phys. Rev. **66**, 163 (1944).
[6] S.S. Kurennoy, R.L. Gluckstern, and G.V. Stupakov, Phys. Rev. E **52**, 4354 (1995).
[7] S.S. Kurennoy and G.V. Stupakov, Part. Accel. **45**, 95 (1994).
[8] R.E. Collin, *Field Theory of Guided Waves* (IEEE Press, New York, 1991).
[9] R.L. Gluckstern and J.A. Diamond, IEEE Trans. Microwave Theory Tech. **39**, 274 (1991).
[10] B. Radak and R.L. Gluckstern, IEEE Trans. Microwave Theory Tech. **43**, 194 (1995).
[11] S.S. Kurennoy, SSC Lab Report No. SSCL-636, 1993 (unpublished).
[12] S.S. Kurennoy, IEEE Trans. Microwave Theory Tech. **44**, 1109 (1996).
[13] L.D. Landau and I.M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon Press, Oxford, 1960), Sec. 3.
[14] M. Bartsch *et al.*, Comput. Phys. Commun. **72**, 22 (1992)
[15] R.L. Gluckstern and S.S. Kurennoy, following paper, Phys. Rev. E **55**, 3533 (1997).