

Collective modes and correlations in one-component plasmas

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(Received 10 June 1996)

The static and time-dependent potential and surface charge correlations in a plasma with a boundary are computed for different shapes of the boundary. The case of a spheroidal or spherical one-component plasma is studied in detail because experimental results are available for such systems. Also, since there is some knowledge, both experimental and theoretical, about the electrostatic collective modes of these plasmas, the time-dependent correlations are computed using a method involving these modes. [S1063-651X(97)15502-7]

PACS number(s): 52.25.Kn, 05.20.-y, 52.35.Fp, 52.25.Wz

I. INTRODUCTION

The static correlations between charged particles in classical plasmas at equilibrium have attracted some theoretical attention [1]. For a plasma with a boundary, the surface charge correlations are especially interesting, because they are universal, in that sense that they do not depend on the detail of the microscopic constitution of the plasma, for length scales large compared to the microscopic ones [2] (although these correlations do depend on the geometry of the boundary).

Experimentally [3,4], classical plasmas in thermal equilibrium, with a boundary, have been obtained by confining particles of one sign in a Penning or Paul trap. These particles may be electrons or ions. In a Penning trap, a magnetic field along the z axis confines the particles radially, while an electric field due to suitable electrodes provides the axial confinement. In a Paul trap, the confinement is provided by an electrostatic potential and a radio frequency field. In both cases, one obtains a system which behaves like a one-component plasma: a system of particles of one sign immersed in a neutralizing uniform background (in the experiments, the confining fields play the role of the background). The particles form a blob of spheroidal shape. In the case of a Penning trap, the blob performs a rigid rotation around the z axis, and it is in the corotating frame that it behaves like a static one-component plasma in a magnetic field; however, if the rotation frequency is just half the cyclotron frequency (the Brillouin regime), the system behaves like a one-component plasma without a magnetic field. Some electrostatic modes of such spheroidal plasmas have been excited and their frequencies experimentally measured [5–8], and a theoretical discussion about all the modes has been made [9–11].

These experimental findings are an incentive for applying the general theory [2] of surface charge correlations to spheroidal plasmas; this will be done in Sec. II. Furthermore, since there is some knowledge about the electrostatic collective modes, it is desirable to check in a variety of cases that the static charge correlations can be correctly obtained (on

large length scales) as the sum of the contributions from all the thermally excited collective modes (it was found some time ago that this approach does work in the very simple special case of a plane boundary without a magnetic field [12]); this will be done in Sec. III. This method for computing the static correlations also gives the time-dependent correlations.

II. STATIC CORRELATIONS IN A SPHEROIDAL PLASMA

It should be emphasized that the calculations of this section are based on macroscopic electrostatics, and give results which are valid only on macroscopic length scales. Therefore, these calculations cannot account for the shell or crystal structures which have been observed [13] for plasmas in a Penning trap and reproduced by computer simulations [14]. The present calculations apply to correlations smoothed on microscopic oscillations. The surface of the plasma has microscopic oscillations responsible of the creation of an excess—or a lack—of electric charge near the surface, which at a macroscopical level can be seen as a surface charge density. Therefore, the surface charge density σ which will be considered here should be understood as a microscopic volume charge density integrated along the normal to the surface, and smoothed in directions parallel to the surface.

As explained in Sec. I, a plasma confined in a Penning trap behaves, in a rotating frame, like a static spheroidal one-component plasma submitted to a uniform magnetic field along its axis. We assume that the Debye length is much smaller than the plasma dimensions: this condition ensures that the spheroid has a well-defined surface, and that the spheroid size is macroscopic.

It is well known that in classical statistical mechanics a magnetic field has no effect on the static quantities. Thus we shall compute the static (equal-time) correlations of a spheroidal one-component plasma in the absence of a magnetic field. The results will also be applicable to the case with a magnetic field.

We use the method of Ref. [2]. Surface charge correlations can be derived from electric field correlations by considering the discontinuity of the normal electric field across the surface of the plasma. The two-point electric potential correlations can be computed on a macroscopic scale by using linear response theory and macroscopic electrostatics.

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Let us put a test charge q at \mathbf{r} . At some point \mathbf{r}' , the potential change $\delta\Phi(\mathbf{r}')$ due only to the plasma is related by linear response to the potential correlation at thermal equilibrium,

$$\delta\Phi(\mathbf{r}') = -\beta q \langle \Phi(\mathbf{r})\Phi(\mathbf{r}') \rangle^T, \quad (2.1)$$

where $\beta = (k_B T)^{-1}$, with T the temperature and k_B the Boltzmann constant, and $\langle AB \rangle^T$ means the truncated average $\langle AB \rangle - \langle A \rangle \langle B \rangle$.

The potential change $\delta\Phi(\mathbf{r}')$ can be computed using screening properties of the plasma and macroscopic electrostatic arguments, provided that $|\mathbf{r} - \mathbf{r}'|$ is large compared to the screening length. From now on we shall assume that this condition is satisfied.

If the test charge is inside the plasma, it would be completely screened. Conversely, if the particle is outside, it will not be screened, but the plasma will change its configuration in a special way, as we will see below. We will be able to compute the potential change $\delta\Phi(\mathbf{r}')$, and therefore the potential and surface charge correlations. In a certain way the screening dictates the form of the correlations functions, in particular the surface charge correlation which, as stated above, is related to the surface oscillation. Incidentally, we will see that the surface charge correlation has a decay slower than the volume charge correlation inside the plasma; this means that the screening at the surface is not as strong as it is in the bulk of the plasma.

Now let us consider the particular case of an insulated spheroidal plasma. We consider a ellipsoid of revolution around the z axis. Let $2b$ be its axial length and $2a$ its diameter. Let $d^2 = b^2 - a^2$. Then $|d|$ is the distance between the foci of the spheroid. If $d^2 > 0$, then the spheroid is prolate; otherwise it is oblate. We use spheroidal coordinates (ξ, η, ϕ) defined by

$$\begin{aligned} x &= [(\xi^2 - d^2)(1 - \eta^2)]^{1/2} \cos \phi, \\ y &= [(\xi^2 - d^2)(1 - \eta^2)]^{1/2} \sin \phi, \\ z &= \xi \eta. \end{aligned} \quad (2.2)$$

The boundary between the plasma and the vacuum is then the spheroidal surface defined by

$$\xi = b. \quad (2.3)$$

Let us consider first the case where \mathbf{r} and \mathbf{r}' are inside the plasma. Due to the plasma's screening properties the charge q at \mathbf{r} will be surrounded by a polarization cloud of microscopic dimensions carrying a charge $-q$, giving to $\delta\Phi(\mathbf{r}')$ a contribution $-q/|\mathbf{r} - \mathbf{r}'|$. Since the plasma is insulated a charge $+q$ spreads on the surface of the spheroid $\xi = b$. This surface charge gives to $\delta\Phi(\mathbf{r}')$ another contribution equal to q/C , with the capacitance C given by [15]

$$C = \frac{d}{Q_0(b/d)}, \quad (2.4)$$

where Q_0 is a Legendre function of second kind. Using the total $\delta\Phi(\mathbf{r}')$ in Eq. (2.1) gives, for \mathbf{r} and \mathbf{r}' inside the spheroid,

$$\beta \langle \Phi(\mathbf{r})\Phi(\mathbf{r}') \rangle^T = \frac{1}{|\mathbf{r} - \mathbf{r}'|} - \frac{1}{d} Q_0(b/d). \quad (2.5)$$

If \mathbf{r}' is outside but \mathbf{r} is still inside the plasma, the potential change at \mathbf{r}' created by the surface charge q is equal to $qQ_0(\xi'/d)/d$. So in that case we have

$$\beta \langle \Phi(\mathbf{r})\Phi(\mathbf{r}') \rangle^T = \frac{1}{|\mathbf{r} - \mathbf{r}'|} - \frac{1}{d} Q_0(\xi'/d). \quad (2.6)$$

If \mathbf{r} and \mathbf{r}' are both outside the plasma the potential change at \mathbf{r}' is $q[G(\mathbf{r}, \mathbf{r}') - |\mathbf{r} - \mathbf{r}'|^{-1}]$, where $G(\mathbf{r}, \mathbf{r}')$ is the potential at \mathbf{r}' when a unit charge is put at \mathbf{r} outside the insulated spheroidal conductor. G is given by [15]

$$\begin{aligned} G(\mathbf{r}, \mathbf{r}') &= \frac{1}{d} \sum_{n=0}^{\infty} (2n+1) \sum_{m=0}^n \epsilon_m (-1)^m \left[\frac{(n-m)!}{(n+m)!} \right]^2 \\ &\quad \times \cos[m(\phi - \phi')] P_n^m(\eta) P_n^m(\eta') \\ &\quad \times \left[-Q_n^m(\xi/d) Q_n^m(\xi'/d) \frac{P_n^m(b/d)}{Q_n^m(b/d)} \right. \\ &\quad \left. + \begin{cases} P_n^m(\xi/d) Q_n^m(\xi'/d) & \text{if } \xi < \xi' \\ P_n^m(\xi'/d) Q_n^m(\xi/d) & \text{if } \xi' < \xi \end{cases} \right] \\ &\quad + \frac{Q_0(\xi/d) Q_0(\xi'/d)}{Q_0(b/d)}, \end{aligned} \quad (2.7)$$

where P_n^m and Q_n^m are associated Legendre functions of the first and second kinds, respectively, and $\epsilon_m = 2 - \delta_{m0}$ is the Neumann factor. The $P_n^m(\xi/d) Q_n^m(\xi'/d)$ or $P_n^m(\xi'/d) Q_n^m(\xi/d)$ terms in Eq. 2.7 come from an expansion of $|\mathbf{r} - \mathbf{r}'|^{-1}$, and finally the electric potential correlation is given by

$$\begin{aligned} \beta \langle \Phi(\mathbf{r})\Phi(\mathbf{r}') \rangle^T &= \frac{1}{d} \sum_{n=1}^{\infty} (2n+1) \sum_{m=0}^n \epsilon_m (-1)^m \left[\frac{(n-m)!}{(n+m)!} \right]^2 \\ &\quad \times \cos[m(\phi - \phi')] P_n^m(\eta) P_n^m(\eta') \\ &\quad \times Q_n^m(\xi/d) Q_n^m(\xi'/d) \frac{P_n^m(b/d)}{Q_n^m(b/d)}. \end{aligned} \quad (2.8)$$

The surface charge correlation is

$$\begin{aligned} \langle \sigma(\mathbf{r})\sigma(\mathbf{r}') \rangle^T &= \frac{1}{(4\pi)^2} \langle (E_n^{\text{out}}(\mathbf{r}) - E_n^{\text{in}}(\mathbf{r})) \\ &\quad \times (E_n^{\text{out}}(\mathbf{r}') - E_n^{\text{in}}(\mathbf{r}'))^T, \end{aligned} \quad (2.9)$$

where $E_n^{\text{in(out)}}(\mathbf{r})$ denotes the limit of the normal component of the electric field when \mathbf{r} approaches the surface from the inside (outside). Using expressions (2.5), (2.6), and (2.8) for the electric potential correlations, we find

$$\beta\langle\sigma(\mathbf{r})\sigma(\mathbf{r}')\rangle^T = -\frac{1}{(4\pi)^2}\frac{b^2-d^2}{\sqrt{(b^2-d^2\eta^2)(b^2-d^2\eta'^2)}} \times \left. \frac{\partial^2 G(\mathbf{r},\mathbf{r}')}{\partial\xi\partial\xi'} \right|_{\mathbf{r},\mathbf{r}' \in \text{surface}}, \quad (2.10)$$

which finally gives

$$\begin{aligned} \beta\langle\sigma(\mathbf{r})\sigma(\mathbf{r}')\rangle^T &= -\frac{1}{(4\pi)^2 d \sqrt{(b^2-d^2\eta^2)(b^2-d^2\eta'^2)}} \\ &\times \sum_{n=1}^{\infty} \sum_{m=0}^n (2n+1) \epsilon_m \frac{(n-m)!}{(n+m)!} P_n^m(\eta) P_n^m(\eta') \\ &\times \frac{Q_n^{m'}(b/d)}{Q_n^m(b/d)} \cos[m(\phi-\phi')]. \end{aligned} \quad (2.11)$$

In the case of a globally neutral spheroid, $\langle\sigma(\mathbf{r})\rangle=0$, and the truncation sign T may be omitted.

From this last expression we can recover the charge correlation for some particular geometries. For example, if d goes to zero, we have the case of a spherical plasma. In that limit b becomes the radius of the sphere and $Q_n^{m'}(b/d)/Q_n^m(b/d) \rightarrow -(n+1)d/b$; then Eq. (2.11) becomes

$$\begin{aligned} \beta\langle\sigma(\mathbf{r})\sigma(\mathbf{r}')\rangle^T &= \frac{1}{(4\pi)^2 b^3} \sum_{n=1}^{\infty} \sum_{m=0}^n (2n+1)(n+1) \\ &\times \epsilon_m \frac{(n-m)!}{(n+m)!} P_n^m(\eta) P_n^m(\eta') \\ &\times \cos[m(\phi-\phi')]. \end{aligned} \quad (2.12)$$

The sum can be performed to give the already known result [2,16]

$$\beta\langle\sigma(\mathbf{r})\sigma(\mathbf{r}')\rangle^T = -\frac{1}{8\pi^2 b^3} \left[\frac{1}{\left(2\sin\frac{\alpha}{2}\right)^3} + \frac{1}{2} \right], \quad (2.13)$$

where α is the angle between \mathbf{r} and \mathbf{r}' .

Another special case is the cylindrical geometry obtained taking the limit $b \rightarrow \infty$; then a is the radius of the cylinder. In that case it is interesting to define $k=n/b$. The sum over n times b^{-1} becomes an integral over k , $b/d \sim 1+(a^2/2b^2)$, $\eta \sim z/b$, and using the asymptotic expansions

$$Q_n^m(b/d) \sim (in)^m K_m(ka), \quad (2.14a)$$

$$P_n^m(\eta) \sim \left(\frac{2}{n\pi}\right)^{1/2} n^m \cos\left[(n-m)\frac{\pi}{2} - kz\right], \quad (2.14b)$$

where the K_m are modified Bessel function of the third kind, we find

$$\begin{aligned} \beta\langle\sigma(\mathbf{r})\sigma(\mathbf{r}')\rangle^T &= -\frac{1}{8\pi^3} \sum_{m=0}^{\infty} \epsilon_m \cos[m(\phi-\phi')] \\ &\times \int_0^{+\infty} k \frac{K_m'(ka)}{a K_m(ka)} \cos[k(z-z')] dk. \end{aligned} \quad (2.15)$$

III. COLLECTIVE MODES AND CORRELATIONS

When the microscopic detail is disregarded, the thermal fluctuations are expected to be correctly described by the set of collective modes. If each collective mode n is considered as a harmonic oscillator of frequency ω_n , the electric potential associated with this mode is of the form

$$[\Phi_n(\mathbf{r})e^{-i\omega_n t} + \overline{\Phi_n(\mathbf{r})}e^{+i\omega_n t}]/\sqrt{2}, \quad (3.1)$$

with an amplitude of Φ_n such that the corresponding average energy is $k_B T$, at temperature T . Then the time-displaced potential correlation will be

$$\langle\Phi(\mathbf{r},t)\Phi(\mathbf{r}',t')\rangle^T = \text{Re} \sum_n \langle\overline{\Phi_n(\mathbf{r})}\Phi_n(\mathbf{r}')\rangle e^{-i\omega(t-t')}, \quad (3.2)$$

and the correlation functions of the other functions can be deduced from Eq. (3.2).

In the general case $t \neq t'$, expression (3.2) is expected to depend on the magnetic field applied to the plasma. However, the static limit $t=t'$ should be magnetic field independent.

The explicit calculation of Eq. (3.2) for a spheroidal plasma in a magnetic field would involve complicated expressions which are not very illuminating. Therefore, only the special case (A) of a spherical plasma without a magnetic field is considered here. However, as exercises, we consider simpler models, with magnetic field, on which it can be explicitly checked that the static limit $t=t'$ is field independent. These models are (B), a plasma along a plane boundary in a magnetic field normal to the boundary, and (C) a two-dimensional plasma (with two-dimensional logarithmic Coulomb interactions) in a disk with a magnetic field normal to the plasma plane.

A. Spherical plasma without magnetic field

In this section we consider a spherical one-component plasma of radius R composed of particles of mass m and charge q in a uniform charged background with charge density $-qn_0$, without a magnetic field. Experimentally this could be achieved in a Penning trap in the Brillouin regime (in the rotating frame the plasma behaves as an unmagnetized plasma) or in a Paul trap (as stated above, the confining fields play the role of the uniform neutralizing background).

We use spherical coordinates (r, θ, ϕ) . We compute the time-dependent correlations which can be seen as the sum of contributions from different collective modes, each one oscillating at its own frequency. It should be noted that the time-dependent correlations can also be computed by a generalization of the linear response method explained in Sec. II, now using the dynamical linear response theory.

The linearized equations of motion for the electric potential Φ , the volume charge density ρ , and the current density \mathbf{j} inside the plasma are

$$\Delta\Phi = -4\pi\rho, \quad (3.3a)$$

$$\frac{\partial\mathbf{j}}{\partial t} = -\frac{\omega_p^2}{4\pi}\nabla\Phi, \quad (3.3b)$$

$$\frac{\partial\rho}{\partial t} = -\nabla\cdot\mathbf{j}, \quad (3.3c)$$

where $\omega_p = (4\pi q^2 n_0/m)^{1/2}$ is the plasma frequency. We look for a mode of frequency ω . Manipulating equations (3.3), we find for Φ , inside the plasma, the equation

$$\epsilon\Delta\Phi = 0, \quad (3.4)$$

where $\epsilon = 1 - \omega_p^2/\omega^2$. The equation for Φ outside the plasma is the usual Laplace equation

$$\Delta\Phi = 0. \quad (3.5)$$

The problem has been reduced to the electrostatic problem of a dielectric filling the sphere of radius R . Equations (3.4) and (3.5) must be supplemented with the boundary conditions

$$\Phi \rightarrow 0 \quad \text{when } r \rightarrow +\infty, \quad (3.6a)$$

$$\lim_{r \rightarrow R^+} \Phi(\mathbf{r}) = \lim_{r \rightarrow R^-} \Phi(\mathbf{r}), \quad (3.6b)$$

$$\epsilon \partial_r \Phi(R^-, \theta, \phi) = \partial_r \Phi(R^+, \theta, \phi). \quad (3.6c)$$

Equations (3.4), (3.5), and (3.6) have two types of solutions:

(1) Surface modes: for $\epsilon \neq 0$, Φ satisfies the Laplace equation inside and outside the plasma. One finds modes (3.1) with

$$\Phi_{nm}(\mathbf{r}) = \begin{cases} A_n r^l Y_n^m(\theta, \phi) & \text{if } r < R \\ A_n R^{2l+1} r^{-l-1} Y_n^m(\theta, \phi) & \text{if } r > R \end{cases} \quad (3.7)$$

and

$$\omega^2 = \omega_n^2 = \frac{n}{2n+1} \omega_p^2, \quad (3.8)$$

where n and m are integers ($n > 0$ and $|m| \leq n$), and Y_n^m are the spherical harmonics.

Equating the average potential energy of this mode to $k_B T/2$ gives the average squared amplitude

$$\beta \langle |A_n|^2 \rangle = \frac{4\pi}{(2n+1)R^{2n+1}}. \quad (3.9)$$

Using the time-displaced analog of Eq. (2.9), one finds that the surface modes contribute to the time-dependent surface charge correlation $\langle \sigma(\mathbf{r}, t) \sigma(\mathbf{r}', t') \rangle^T$ a term

$$\sum_{nm} \frac{(2n+1)^2}{(4\pi)^2} R^{2n-2} \langle |A_n|^2 \rangle Y_n^m(\theta, \phi) \overline{Y_n^m(\theta', \phi')} \cos \omega_n(t-t') = k_B T \sum_{nm} \frac{2n+1}{4\pi R^3} Y_n^m(\theta, \phi) \overline{Y_n^m(\theta', \phi')} \cos \omega_n(t-t'). \quad (3.10)$$

(2) Volume modes: when $\epsilon = 0$, then $\omega^2 = \omega_p^2$. There is now an infinite number of modes for each (n, m) : $\Phi^{\text{out}} = 0$ and any $\Phi^{\text{in}} = f(r) Y_n^m(\theta, \phi)$ with $f(R) = 0$ is acceptable. However, a complete basis for Φ^{in} can be chosen as the eigenfunctions of the Laplacian with Dirichlet boundary conditions:

$$\Phi^{\text{in}}(\mathbf{r}) = \sum_{\gamma} a_{\gamma} f_{\gamma}(\mathbf{r}), \quad (3.11)$$

with $\Delta f_{\gamma} = \lambda_{\gamma} f_{\gamma}$, $f_{\gamma}(R, \theta, \phi) = 0$, and $\int |f_{\gamma}(\mathbf{r})|^2 d\mathbf{r} = 1$.

Equating the average potential energy of each mode to $k_B T/2$, we find $\beta \langle |a_{\gamma}|^2 \rangle = -4\pi/\lambda_{\gamma}$. So the contribution of the volume modes to $\langle \Phi(\mathbf{r}, t) \Phi(\mathbf{r}', t') \rangle^T$ is

$$\begin{aligned} & \sum_{\gamma} \langle |a_{\gamma}|^2 \rangle f_{\gamma}(\mathbf{r}) \overline{f_{\gamma}(\mathbf{r}')} \cos \omega_p(t-t') \\ &= -4\pi k_B T \sum_{\gamma} \lambda_{\gamma}^{-1} f_{\gamma}(\mathbf{r}) \overline{f_{\gamma}(\mathbf{r}')} \cos \omega_p(t-t') \\ &= -4\pi k_B T G_D(\mathbf{r}, \mathbf{r}') \cos \omega_p(t-t'), \end{aligned} \quad (3.12)$$

where G_D is the Green function of the Laplacian with Dirichlet boundary conditions on the sphere,

$$G_D(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi} \left[\frac{r}{R} \mathbf{r}' - \frac{R}{r} \mathbf{r} \right]^{-1} - |\mathbf{r} - \mathbf{r}'|^{-1}. \quad (3.13)$$

Finally the contribution of the volume modes to $\langle \sigma(\mathbf{r}, t) \sigma(\mathbf{r}', t') \rangle^T$ is found to be

$$\frac{k_B T}{8\pi^2} \frac{1}{(2R \sin(\alpha/2))^3} \cos \omega_p(t-t'), \quad (3.14)$$

where α is the angle between \mathbf{r} and \mathbf{r}' . Putting Eqs. (3.10) and (3.14) together

$$\begin{aligned} \beta \langle \sigma(\mathbf{r}, t) \sigma(\mathbf{r}', t') \rangle^T &= \sum_{n,m} \frac{2n+1}{4\pi R^3} Y_n^m(\theta, \phi) \\ &\times \overline{Y_n^m(\theta', \phi')} \cos \omega_n(t-t') \\ &+ \frac{1}{8\pi^2} \frac{1}{(2R \sin(\alpha/2))^3} \cos \omega_p(t-t'). \end{aligned} \quad (3.15)$$

For $t=t'$ the sum in Eq. (3.15) can be performed and we recover the static result (2.13).

B. Plasma in a half space with a magnetic field

Let us consider now a one-component plasma filling the half space $z < 0$ with a uniform magnetic field in the z direction, $\mathbf{B} = B\hat{\mathbf{z}}$. Sum rules for the time-dependent correlations have been obtained for this case using the dynamical linear response theory [17]. Here we use the collective mode method to find expressions for correlations that are valid macroscopically. This collective mode method has been used in the case $\mathbf{B} = \mathbf{0}$ in [12]; here we extend it to the case $\mathbf{B} \neq \mathbf{0}$. With the same notation as in Sec. III A, the linearized equations of motion now are

$$\Delta \Phi = -4\pi\rho, \quad (3.16a)$$

$$\frac{\partial \mathbf{j}}{\partial t} = -\frac{\omega_p^2}{4\pi} \nabla \Phi + \Omega \mathbf{j} \times \hat{\mathbf{z}}, \quad (3.16b)$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{j}, \quad (3.16c)$$

where $\Omega = qB/m$ is the cyclotron frequency.

As above we look for a mode with frequency ω . From Eqs. (3.16), we find, for Φ inside the plasma,

$$\nabla \cdot \epsilon \nabla \Phi = 0, \quad (3.17)$$

where ϵ is now the plasma dielectric tensor defined in Cartesian coordinates by

$$\epsilon = \begin{pmatrix} \epsilon_1 & -i\epsilon_2 & 0 \\ i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}, \quad (3.18)$$

with $\epsilon_1 = 1 - \omega_p^2/(\omega^2 - \Omega^2)$, $\epsilon_2 = \Omega \omega_p^2/[\omega(\omega^2 - \Omega^2)]$, and $\epsilon_3 = 1 - \omega_p^2/\omega^2$. As to Φ outside the plasma ($z > 0$), it obeys

$$\Delta \Phi = 0. \quad (3.19)$$

We now have the problem of an anisotropic dielectric filling the $z < 0$ half space. The boundary conditions are

$$\Phi \rightarrow 0 \quad \text{when } z \rightarrow +\infty, \quad (3.20a)$$

$$\Phi(x, y, 0^-) = \Phi(x, y, 0^+), \quad (3.20b)$$

$$\epsilon_3 \partial_z \Phi(x, y, 0^-) = \partial_z \Phi(x, y, 0^+). \quad (3.20c)$$

Let us first consider the case $\epsilon_1/\epsilon_3 < 0$ which give modes with a frequency in the ranges $0 < |\omega| < \min(\omega_p, \Omega)$ called magnetized plasma modes and $\max(\omega_p, \Omega) < |\omega| < \Omega_u = (\omega_p^2 + \Omega^2)^{1/2}$ called upper hybrid modes.

We look for a mode of the form

$$\Phi_{\mathbf{k}}(\mathbf{r}) = \begin{cases} (A e^{i\mathbf{k}_{\parallel} z} + B e^{-i\mathbf{k}_{\parallel} z}) e^{i\mathbf{k}_{\perp} \cdot \mathbf{r}_{\perp}} & \text{for } z < 0 \\ (A + B) e^{-k_{\perp} z} e^{i\mathbf{k}_{\perp} \cdot \mathbf{r}_{\perp}} & \text{for } z > 0, \end{cases} \quad (3.21)$$

where $\mathbf{k} = \mathbf{k}_{\perp} + k_{\parallel} \hat{\mathbf{z}}$, with \mathbf{k}_{\perp} in the xy plane, $k_{\perp} = |\mathbf{k}_{\perp}|$ and $\mathbf{r}_{\perp} = (x, y, 0)$. The Laplace equation (3.19) is satisfied for $z > 0$, and Eq. (3.17) gives the dispersion relation

$$\left(1 - \frac{\omega_p^2}{\omega^2}\right) k_{\parallel}^2 + \left(1 - \frac{\omega_p^2}{\omega^2 - \Omega^2}\right) k_{\perp}^2 = 0. \quad (3.22)$$

Solving this equation we find two modes: one upper hybrid mode with frequency ω_+ , and one magnetized plasma mode with frequency ω_- :

$$\omega_{\pm}^2 = \frac{1}{2} \left[\omega_p^2 + \Omega^2 \pm \left((\omega_p^2 + \Omega^2)^2 - 4\omega_p^2 \Omega^2 \frac{k_{\parallel}^2}{k_{\perp}^2 + k_{\parallel}^2} \right)^{1/2} \right]. \quad (3.23)$$

Equation (3.20c) gives a relation between the incident and reflected amplitudes A and B ,

$$\frac{A}{B} = \frac{-1 - \epsilon_1 \epsilon_3 + 2i\epsilon_3 \sqrt{-\epsilon_1/\epsilon_3}}{1 - \epsilon_1 \epsilon_3}. \quad (3.24)$$

It should be noted that $|A/B| = 1$: there is total reflection on the surface $z = 0$.

The energy of this mode is a quadratic form of $\partial \Phi / \partial t$ and Φ . In the particular case $B = 0$ this quadratic form is diagonal [12], which means that the potential and kinetic energies have equal averages and by the energy equipartition theorem each average is $k_B T/2$ (this was the case in Sec. III A). If $B \neq 0$, the potential and kinetic average energies are different (take for example the simple case of a single charged particle with a circular trajectory in the plane normal to a magnetic field when the whole energy is kinetical). But we can still use the energy equipartition theorem and say that the total average energy of each mode is equal to $k_B T$.

The total average energy for a large volume V of plasma is found to be

$$\langle E \rangle = V \frac{\langle |A|^2 \rangle k_{\perp}^2 \Omega^2 \omega_p^2 (2\omega^2 - \Omega^2 - \omega_p^2)}{2\pi (\omega^2 - \Omega^2)^2 (\omega^2 - \omega_p^2)}. \quad (3.25)$$

Equating $\langle E \rangle$ to $k_B T$ gives the average squared amplitude $\langle |A|^2 \rangle$ of the mode.

Finally these modes will give a contribution to the electric potential time-dependent correlation $\langle \Phi(\mathbf{r}, t) \Phi(\mathbf{r}', t') \rangle^T$ equal to

$$V \int \frac{d^3 \mathbf{k}}{(2\pi)^3} [\langle \overline{\Phi_{\mathbf{k},+}(\mathbf{r}) \Phi_{\mathbf{k},+}(\mathbf{r}')} \rangle \cos \omega_+(t-t') + \langle \overline{\Phi_{\mathbf{k},-}(\mathbf{r}) \Phi_{\mathbf{k},-}(\mathbf{r}')} \rangle \cos \omega_-(t-t')], \quad (3.26)$$

where $\Phi_{\mathbf{k},+}$ is the electric potential for the upper hybrid mode, and $\Phi_{\mathbf{k},-}$ the potential for the magnetized mode.

To this contribution we must add the one from the possible modes in the range $\min(\omega_p, \Omega) < |\omega| < \max(\omega_p, \Omega)$, called evanescent modes. In this case $\epsilon_1/\epsilon_3 > 0$, and we look for a solution of the form

$$\Phi_{\mathbf{k}_\perp}(\mathbf{r}) = \begin{cases} C e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp + \sqrt{\epsilon_1/\epsilon_3} k_\perp z} & \text{if } z < 0 \\ C e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp - k_\perp z} & \text{if } z > 0. \end{cases} \quad (3.27)$$

This form satisfies Eqs. (3.17), (3.19), (3.20a), and (3.20b). Equation (3.20c) implies $\epsilon_3 < 0$ and $\epsilon_1 \epsilon_3 = 1$. This means that we have an evanescent mode only if $\Omega < \omega_p$, with frequency given by $\omega^2 = \omega_e^2 = (\Omega^2 + \omega_p^2)/2$. To compute $\langle |C|^2 \rangle$ we proceed to compute the average total energy of the mode, and equate it with $k_B T$. In this case the energy is proportional to the surface S of the boundary between the plasma and the vacuum. We finally find

$$\beta \langle |C|^2 \rangle = \frac{2\pi}{S k_\perp} \frac{\omega_p^2 - \Omega^2}{\omega_p^2}. \quad (3.28)$$

This mode adds a contribution to $\langle \Phi(\mathbf{r}, t) \Phi(\mathbf{r}', t') \rangle^T$ equal to

$$S \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} \langle \overline{\Phi_{\mathbf{k}_\perp}(\mathbf{r}) \Phi_{\mathbf{k}_\perp}(\mathbf{r}')} \rangle \cos \omega_e(t-t'). \quad (3.29)$$

In Eq. (3.26) it is convenient to make a change of variable in the integral over k_\parallel and have an integral over ω . In this way we can express the electric potential correlation $\langle \Phi(\mathbf{r}, t) \Phi(\mathbf{r}', t') \rangle^T$ in terms of its Fourier transform $\widetilde{C}_{\Phi\Phi}(\mathbf{k}_\perp, z, z', \omega)$ with respect to time and the x and y coordinates:

$$\begin{aligned} & \langle \Phi(\mathbf{r}, t) \Phi(\mathbf{r}', t') \rangle^T \\ &= \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} \int_{-\infty}^{+\infty} d\omega \widetilde{C}_{\Phi\Phi}(\mathbf{k}_\perp, z, z', \omega) \\ & \times e^{-i\omega(t-t') + i\mathbf{k}_\perp \cdot (\mathbf{r}_\perp - \mathbf{r}'_\perp)}. \end{aligned} \quad (3.30)$$

$\widetilde{C}_{\Phi\Phi}(\mathbf{k}_\perp, z, z', \omega) = 0$ if $|\omega| \in]\min(\omega_p, \Omega), \max(\omega_p, \Omega)[\cup]\Omega_u, +\infty[$ [except maybe at $\omega = \pm \omega_e$ if $\Omega < \omega_p$]. If ω is not in that range, the following apply

(a) If $z > 0$ and $z' > 0$,

$$\beta \widetilde{C}_{\Phi\Phi}(\mathbf{k}_\perp, z, z', \omega) = -\frac{4\epsilon_3}{\omega k_\perp} \frac{(-\epsilon_1/\epsilon_3)^{1/2}}{1 - \epsilon_1 \epsilon_3} e^{-k_\perp(z+z')}. \quad (3.31a)$$

(b) If $z < 0$ and $z' < 0$,

$$\begin{aligned} & \beta \widetilde{C}_{\Phi\Phi}(\mathbf{k}_\perp, z, z', \omega) \\ &= -\frac{2}{\omega k_\perp \epsilon_3 (-\epsilon_1/\epsilon_3)^{1/2}} \left[\cos[k_\perp (-\epsilon_1/\epsilon_3)^{1/2} (z-z')] \right. \\ & \quad - \frac{1}{1 - \epsilon_1 \epsilon_3} ((1 + \epsilon_1 \epsilon_3) \cos[k_\perp (-\epsilon_1/\epsilon_3)^{1/2} (z+z')] \\ & \quad \left. + 2\epsilon_3 (-\epsilon_1/\epsilon_3)^{1/2} \sin[k_\perp (-\epsilon_1/\epsilon_3)^{1/2} (z+z')] \right]. \end{aligned} \quad (3.31b)$$

(c) If $z < 0$ and $z' > 0$,

$$\begin{aligned} \beta \widetilde{C}_{\Phi\Phi}(\mathbf{k}_\perp, z, z', \omega) &= -\frac{4}{\omega k_\perp} \frac{e^{-k_\perp z'}}{1 - \epsilon_1 \epsilon_3} \\ & \times (-\sin[(-\epsilon_1/\epsilon_3)^{1/2} k_\perp z]) \\ & + \epsilon_3 (-\epsilon_1/\epsilon_3)^{1/2} \\ & \times \cos[(-\epsilon_1/\epsilon_3)^{1/2} k_\perp z]. \end{aligned} \quad (3.31c)$$

In the case $\Omega < \omega_p$ we must add to $\beta \widetilde{C}_{\Phi\Phi}(\mathbf{k}_\perp, z, z', \omega)$ the term corresponding to the evanescent mode:

$$\begin{aligned} & \frac{\pi}{k_\perp} \frac{\omega_p^2 - \Omega^2}{\omega_p^2} (\delta(\omega - \omega_e) + \delta(\omega + \omega_e)) \\ & \times \begin{cases} \exp[-k_\perp(z+z')] & \text{if } z > 0 \text{ and } z' > 0 \\ \exp\left(k_\perp \frac{\omega_p^2 + \Omega^2}{\omega_p^2 - \Omega^2} (z+z')\right) & \text{if } z < 0 \text{ and } z' < 0 \\ \exp\left[k_\perp \left(\frac{\omega_p^2 + \Omega^2}{\omega_p^2 - \Omega^2} z - z'\right)\right] & \text{if } z < 0 \text{ and } z' > 0. \end{cases} \end{aligned} \quad (3.32)$$

From these expressions we can compute the surface charge correlation, using Eq. (2.9). The surface charge correlation Fourier transform is found to be

$$\begin{aligned} \beta \widetilde{C}_{\sigma\sigma}(\mathbf{k}_\perp, \omega) &= -\frac{4k_\perp(1 - \epsilon_3)^2}{\omega \epsilon_3 (-\epsilon_1/\epsilon_3)^{1/2} (1 - \epsilon_1 \epsilon_3)} \\ & + \frac{k_\perp}{4\pi} \frac{\omega_p^2}{\omega_p^2 - \Omega^2} (\delta(\omega - \omega_e) + \delta(\omega + \omega_e)). \end{aligned} \quad (3.33)$$

The last term is to be included only when $\Omega < \omega_p$.

Now, let us briefly show how these results can be obtained using dynamical linear response. If we set an oscillating charge density $\delta(z-z') \exp[i(\mathbf{k}_\perp \cdot \mathbf{r}_\perp - \omega t)]$ at $z = z'$, the electric potential change at z is $\chi(\mathbf{k}_\perp, z, z', \omega) \exp[i(\mathbf{k}_\perp \cdot \mathbf{r}_\perp - \omega t)]$. The response function χ is related to the Fourier transform $\widetilde{C}_{\Phi\Phi}(\mathbf{k}_\perp, z, z', \omega)$ of the time-dependent correlation $\langle \Phi(\mathbf{r}, t) \Phi(\mathbf{r}', t') \rangle^T$ in the non-perturbed system by the fluctuation-dissipation theorem:

$$\beta \widetilde{C}_{\Phi\Phi}(\mathbf{k}_\perp, z, z', \omega) = -\frac{1}{\pi\omega} \text{Im}\chi(\mathbf{k}_\perp, z, z', \omega), \quad (3.34)$$

and $\chi(\mathbf{k}_\perp, z, z', \omega) = \Psi(\mathbf{k}_\perp, z, z', \omega) - 2\pi \exp[-k_\perp |z - z'|] / k_\perp$, where Ψ is the total electric potential solution, due to the plasma and the external charge, of

$$\begin{aligned} \nabla \cdot \epsilon \nabla [\Psi(\mathbf{k}_\perp, z, z', \omega) e^{i(\mathbf{k}_\perp \cdot \mathbf{r}_\perp)}] \\ = -4\pi \delta(z - z') \exp[i(\mathbf{k}_\perp \cdot \mathbf{r}_\perp)], \end{aligned} \quad (3.35)$$

where ϵ is the dielectric tensor given by Eq. (3.18) if $z < 0$, or equal to 1 if $z > 0$, and the boundary conditions (3.20).

Solving equation (3.35) gives the following results for χ

(a) If $z < 0$ and $z' < 0$,

$$\begin{aligned} \chi(\mathbf{k}_\perp, z, z', \omega) = \frac{2\pi}{k_\perp} \left[\frac{1}{\epsilon_3} \left(\frac{\epsilon_3}{\epsilon_1} \right)^{1/2} \exp \left[-k_\perp \left(\frac{\epsilon_1}{\epsilon_3} \right)^{1/2} |z - z'| \right] \right. \\ \left. + \frac{1 - \epsilon_3^{-1} \sqrt{\epsilon_3/\epsilon_1}}{1 + \epsilon_3 \sqrt{\epsilon_1/\epsilon_3}} \exp \left[-k_\perp \left(\frac{\epsilon_1}{\epsilon_3} \right)^{1/2} (z + z') \right] \right] \exp[-k_\perp |z - z'|]. \end{aligned} \quad (3.36a)$$

(b) If $z > 0$ and $z' < 0$,

$$\begin{aligned} \chi(\mathbf{k}_\perp, z, z', \omega) = \frac{2\pi}{k_\perp} \left[\frac{2}{1 + \epsilon_3 \sqrt{\epsilon_1/\epsilon_3}} \right. \\ \left. \times \exp \left[-k_\perp z + \left(\frac{\epsilon_1}{\epsilon_3} \right)^{1/2} k_\perp z' \right] \right. \\ \left. - \exp[-k_\perp |z - z'|] \right]. \end{aligned} \quad (3.36b)$$

(c) If $z > 0$ and $z' > 0$,

$$\chi(\mathbf{k}_\perp, z, z', \omega) = \frac{2\pi}{k_\perp} \frac{1 - \epsilon_3 \sqrt{\epsilon_1/\epsilon_3}}{1 + \epsilon_3 \sqrt{\epsilon_1/\epsilon_3}} \exp[-k_\perp (z + z')]. \quad (3.36c)$$

From these expressions it is easy to verify that Eq. (3.34) leads to Eqs. (3.31) and (3.32), and therefore both methods give the same results. Furthermore, for $t = t'$, Eq. (3.34) and the Kramers-Kronig relation

$$\pi \text{Re}\chi(\mathbf{k}_\perp, z, z', 0) = \text{P} \int_{-\infty}^{+\infty} \frac{\text{Im}\chi(\mathbf{k}_\perp, z, z', \omega)}{\omega} d\omega \quad (3.37)$$

give the well-known static correlation [2]

$$\begin{aligned} \beta \langle \Phi(\mathbf{r}) \Phi(\mathbf{r}') \rangle^T \\ = \begin{cases} |\mathbf{r} - \mathbf{r}'|^{-1} & \text{if } z < 0 \text{ or } z' < 0 \\ [|\mathbf{r}_\perp - \mathbf{r}'_\perp|^2 + (z + z')^2]^{-1/2} & \text{if } z > 0 \text{ and } z' > 0, \end{cases} \end{aligned} \quad (3.38)$$

which is, as expected, independent of the presence of the magnetic field.

C. Plasma in a disk with a magnetic field

Here we consider the model of a two-dimensional one-component plasma in a disk of radius R with a magnetic field normal to the plane where the disk lies. The particles interact

through the two-dimensional Coulomb potential $-\ln r$. We look for modes with frequency ω . The dielectric formalism is also valid in this case. Only the definition of the plasma frequency is slightly changed: $\omega_p = (2\pi q^2 n_0/m)^{1/2}$. The equation for the potential Φ inside the disk is

$$\epsilon_1 \Delta \Phi = 0, \quad (3.39)$$

while outside the disk it is the usual Laplace equation. In polar coordinates (r, θ) the boundary conditions are

$$\Phi \rightarrow 0 \quad \text{when } r \rightarrow +\infty, \quad (3.40a)$$

$$\lim_{r \rightarrow R^+} \Phi(\mathbf{r}) = \lim_{r \rightarrow R^-} \Phi(\mathbf{r}), \quad (3.40b)$$

$$\epsilon_1 \frac{\partial \Phi(R^-, \theta)}{\partial r} - i\epsilon_2 \frac{\partial \Phi(R^-, \theta)}{r \partial \theta} = \frac{\partial \Phi(R^+, \theta)}{\partial r}. \quad (3.40c)$$

As in Sec. III A, there are two types of solutions:

(1) If $\epsilon_1 \neq 0$, Φ satisfies the Laplace equation inside and outside the disk,

$$\Phi(\mathbf{r}) = \begin{cases} A_m r^{|m|} e^{im\theta} & \text{if } r < R \\ A_m R^{2|m|} r^{-|m|} e^{im\theta} & \text{if } r > R, \end{cases} \quad (3.41)$$

and there are two possible frequencies for each integer m ($m \neq 0$), $\omega = \text{sgn}(m) \omega_\pm$, where $\text{sgn}(m)$ denotes the sign of m , and

$$\omega_\pm = (-\Omega \pm \sqrt{\Omega^2 + 2\omega_p^2})/2. \quad (3.42)$$

Equating the total average energy of the mode to $k_B T$ gives

$$\beta \langle |A_m|^2 \rangle = \frac{1}{|m| R^{2|m|}} \frac{\omega'}{\omega' - \omega}, \quad (3.43)$$

where ω is the frequency of the mode, and ω' the other root of Eq. (3.42). The contribution from these modes to $\langle \sigma(\mathbf{r}, t) \sigma(\mathbf{r}', t') \rangle^T$ is

$$\begin{aligned} k_B T \sum_m \frac{|m| e^{im(\theta - \theta')}}{(\pi R)^2 (\omega_+ - \omega_-)} \\ \times [\omega_+ e^{-i \text{sgn}(m) \omega_-(t-t')} - \omega_- e^{-i \text{sgn}(m) \omega_+(t-t')}]. \end{aligned} \quad (3.44)$$

(2) If $\epsilon_1 = 0$, $\omega^2 = \omega_p^2 + \Omega^2$, and any Φ satisfies Eq. (3.39) inside the disk. However, writing Φ as a Fourier series in $e^{im\theta}$, boundary conditions (3.40) and Laplace equation for Φ outside the disk implies that $\Phi = 0$ outside the disk. The situation is similar to the one in Sec. III A 2. Φ can be written in the base of the eigenfunctions of the Laplacian, with Dirichlet conditions on the boundary. Then equating the average total energy of each mode to $k_B T$ and following the calculations from Sec. III A 2, we find the contribution from these modes to $\langle \Phi(\mathbf{r}, t) \Phi(\mathbf{r}', t') \rangle^T$:

$$-2\pi k_B T \frac{\omega_p^2}{\Omega^2 + \omega_p^2} G_D(\mathbf{r}, \mathbf{r}') \cos[(\omega_p^2 + \Omega^2)^{1/2} (t - t')], \quad (3.45)$$

where G_D now is the Green function of the Laplacian with Dirichlet boundary conditions on the disk:

$$G_D(\mathbf{r}, \mathbf{r}') = \frac{1}{2\pi} [\ln|\mathbf{r} - \mathbf{r}'| - \ln|\mathbf{r} - (R/r')^2 \mathbf{r}'|]. \quad (3.46)$$

In two dimensions there is another mode corresponding to $\omega=0$. The dielectric formalism does not apply here (ϵ_2 diverges when $\omega=0$). Dealing directly with the equations of motion, we can show that $\Phi=0$ outside the disk. Then the contribution of this mode can be computed in the same way as in the case $\epsilon_1=0$. The contribution to $\langle \Phi(\mathbf{r}, t) \Phi(\mathbf{r}', t') \rangle^T$ is found to be

$$-2\pi k_B T \frac{\Omega^2}{\Omega^2 + \omega_p^2} G_D(\mathbf{r}, \mathbf{r}'). \quad (3.47)$$

Putting together all contributions gives

$$\begin{aligned} \beta \langle \sigma(\mathbf{r}, t) \sigma(\mathbf{r}', t') \rangle^T &= \sum_m \frac{|m| e^{im(\theta - \theta')}}{(\pi R)^2 (\omega_+ - \omega_-)} \\ &\times [\omega_+ e^{-i \operatorname{sgn}(m) \omega_- (t-t')} \\ &- \omega_- e^{-i \operatorname{sgn}(m) \omega_+ (t-t')}] \\ &+ \frac{\Omega^2 + \omega_p^2 \cos[(\omega_p^2 + \Omega^2)^{1/2} (t-t')]}{\Omega^2 + \omega_p^2} \\ &\times \frac{1}{8 \left(\pi R \sin \frac{\theta}{2} \right)^2}. \end{aligned} \quad (3.48)$$

For $t=t'$, the sum in Eq. (3.48) can be performed and the static result [2], independent of the magnetic field \mathbf{B} is recovered,

$$\beta \langle \sigma(\mathbf{r}) \sigma(\mathbf{r}') \rangle^T = -\frac{1}{8 \left(\pi R \sin \frac{\theta}{2} \right)^2}. \quad (3.49)$$

IV. CONCLUSION

Since one-component plasmas have been obtained experimentally, it would be interesting if the static or dynamical correlations could be measured and compared to the expressions obtained here. It should be noted that our calculations give only the dominant term of the expansion of the correlations in powers of the distance r between the points. It has been shown that there are other terms in the asymptotic expansion [18] of the time-dependent correlations (behaving like r^{-6} for the potential correlations in an infinite plasma). These algebraic corrections vanish in the static case.

ACKNOWLEDGMENT

The author wishes to thank B. Jancovici for useful discussions, and for reading the manuscript.

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