Resonant interactions of drift vortex solitons in a convective motion of a plasma

Masayoshi Tajiri and Hiroshi Maesono
Department of Mathematical Sciences, College of Engineering, University of Osaka Prefecture, Sakai 593, Japan
(Received 1 April 1996)

The three dimensionality of drift vortex solitons in a convective motion is investigated. The propagation of vortex solitons is described by the Kadomtsev-Petviashvili equation with negative dispersion. It is pointed out that under a certain condition the vortex soliton resonance is possible. [S1063-651X(97)05102-7]

PACS number(s): 52.35.Kt, 52.35.Ra

I. INTRODUCTION

The pseudo-three-dimensional dynamics of a magnetized (with magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$) and inhomogeneous [with density $n_0(\mathbf{x})$] plasma with $T_e \gg T_i$, by taking account of the motion of electrons along the magnetic field, can be described by the Hasegawa and Mima equation [1,2]

$$\frac{\partial}{\partial t}(\nabla_\perp^2 \phi - \phi) - [(\boldsymbol{\nabla}_\perp \phi \times \hat{\mathbf{z}}) \cdot \boldsymbol{\nabla}_\perp](\nabla_\perp^2 \phi - \ln n_0) = 0, \quad (1)$$

where $\hat{\mathbf{z}}$ is the unit vector of the z axis; the time and space coordinates are normalized by $\omega_{\mathrm{ci}}^{-1}$ and $\rho_s = (T_e/m_i)^{1/2}/\omega_{\mathrm{ci}}$ (ω_{ci} is the ion cyclotron frequency, T_e is the electron temperature), the electric potential ϕ by T_e/e, and the subscript $\perp$ indicates the components perpendicular to $\mathbf{B}_0$. When we take the y axis in the direction of the density gradient, Eq. (1) admits a linear wave whose dispersion relation is given by

$$\omega = -\frac{[(\mathbf{k} \times \hat{\mathbf{z}}) \cdot \boldsymbol{\nabla}_\perp \ln n_0(y)]}{1+k^2} = \frac{\kappa k_x}{1+k^2}, \quad (2)$$

where $\mathbf{k}$ is the wave vector in the direction perpendicular to $\hat{\mathbf{z}}$, k_x is the x component of $\mathbf{k}$, and $\kappa = |\boldsymbol{\nabla}_\perp \ln n_0(y)|$. The drift wave which exists in such a magnetized and nonuniform plasma has interesting properties. Hasegawa and Kodama [3] showed that the spectrum cascade by mode coupling in drift wave turbulence described by Eq. (1) occurs at longer and shorter wavelengths. In a region of large wave numbers, the energy spectrum cascade to smaller $|\mathbf{k}|$, and in a small wave number region the energy tends to decay to a lower frequency, hence to smaller k_x. Thus, the energy spectrum tends to condense at a critical value of $k_y = k_c$ and $k_x = 0$. It is well known that as a consequence of cascade, a periodic zonal flow in the x direction perpendicular to both directions of inhomogeneity and applied magnetic field appears in the plasma [4]. Nozaki *et al.* [5] have shown that vortices formed by the shear flow propagate along the neutral sheet of the zonal flows at the Korteweg-de Vries (KdV) solitons. They have obtained the KdV equation for the motion of vortices by applying the reductive perturbation method to the Hasegawa and Mima model equation and also to the two dimensional ion-fluid equations with the Boltzmann distribution for the electron density.

As the KdV solitary wave is a nonlinear wave, which by virtue of the one dimensionality is fully stable, vortex solitons described by the KdV equation may be stable. However, we cannot tell whether the vortex soliton is stable against bending distortion. Laedke and Spatschek [6] showed that two dimensional perturbations of single vortices in the plane perpendicular to the magnetic field do not grow in time. Recently, the three dimensional stability of monopolar drift vortices has been studied by Åkerstedt *et al.* [7]. They showed that vortices with a monotonic decreasing or increasing radial profile of the potential vorticity are stable for long transverse perturbation. When we consider the three dimensionality of drift vortices, the three dimensional interaction between vortices is an interesting problem as well as the three dimensional stability. However, almost all studies on the interactions use two-dimensional models.

When we consider the vortex motion in an inhomogeneous plasma extended to the direction of magnetic field, the three dimensionality of the vortex must be taken into account. In this paper, we investigate the propagation of drift vortices in a two dimensional periodic zonal flow that extends to the direction of the magnetic field uniformly. We consider the case that the motion of ions is nearly two dimensional but vortex lines are inclined to the z axis. In Sec. II, we derive the Kadomtsev-Petviashvili (K-P) equation from the fluid equation by using the reductive perturbation method, which describes the propagation of drift vortex solitons. In Sec. III, the interaction between two inclined vortex solitons is investigated by making use of the two-soliton solution of the K-P equation, and the existence of vortex soliton resonance is shown. Summary and discussion are given in the last section.

II. DERIVATION OF THE K-P EQUATION FROM A FLUID MODEL

The three dimensionality of drift vortices in the zonal flow in the direction perpendicular to both directions of inhomogeneity and magnetric field is investigated. We assume that the ion temperature is much smaller than the electron temperature, and use a cold ion approximation. The fluid equations for cold ions take the forms

$$\frac{\partial n}{\partial t} + v_x \frac{\partial n}{\partial x} + v_y \frac{\partial n}{\partial y} + v_z \frac{\partial n}{\partial z} + n\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) = 0, \quad (3)$$

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} = -\frac{\partial \phi}{\partial x} + v_y, \quad (4)$$

$$\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}=-\frac{\partial\phi}{\partial y}-v_x, \tag{5}$$

$$\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}=-\frac{\partial\phi}{\partial z}, \tag{6}$$

where we take the z axis in the direction of the magnetic field, the time and space coordinates are normalized by $\omega_{\rm ci}^{-1}$ and $\rho_s=(T_e/m_i)^{1/2}/\omega_{\rm ci}$, respectively, and the electric potential ϕ by T_e/e.

As the ion density in drift waves is equal to the electron density to a high accuracy, the quasineutrality condition relates n to the electron density which is given by the Boltzmann distribution

$$n=\exp(\phi). \tag{7}$$

In this section, we extend the reductive perturbation [5] for the one-dimensional propagation of drift vortices in the two dimensional space to the quasi one-dimensional propagation in the three-dimensional space. We assume that an electrostatic drift wave is propagated in the x direction while it varies slowly in the z direction, so that by means of a parameter ϵ, ordering a smallness of the amplitude stretched variables may be expressed as

$$\begin{aligned}\xi&=\epsilon^{1/2}(x-\epsilon\lambda t),\\ \eta&=\epsilon^{1/2+a}z,\\ \tau&=\epsilon^{5/2}t,\end{aligned} \tag{8}$$

where λ is the phase velocity in the x direction of the drift wave in the long wave limit and so that ϕ is expanded as

$$\phi=\epsilon\phi^{(1)}+\epsilon^2\phi^{(2)}+\epsilon^3\phi^{(3)}\cdots, \tag{9}$$

and also

$$v_x=\epsilon v_x^{(1)}+\epsilon^2 v_x^{(2)}+\epsilon^3 v_x^{(3)}+\epsilon^4 v_x^{(4)}+\cdots, \tag{10}$$

$$v_y=\epsilon^{1/2}(\epsilon v_y^{(1)}+\epsilon^2 v_y^{(2)}+\epsilon^3 v_y^{(3)}+\epsilon^4 v_y^{(4)}+\cdots), \tag{11}$$

$$v_z=\epsilon^{1/2+b}(\epsilon v_z^{(1)}+\epsilon^2 v_z^{(2)}+\epsilon^3 v_z^{(3)}+\epsilon^4 v_z^{(4)}+\cdots), \tag{12}$$

where a (>0) and b (>0) are parameters to be determined and $\phi^{(1)}$ and $(v_x^{(1)},v_y^{(1)})$ are the electrostatic potential and drift velocity related to the zonal flow. When we take the y axis in the direction of the density gradient, $\phi^{(1)}$, $v_x^{(1)}$, and $v_y^{(1)}$ are regarded as functions of y only. We note here that the quasineutrality condition does not break down because the vorticity is not strong. Introducing the transformations [Eq. (8)] and the expansions [Eqs. (9)–(12)] into Eqs. (3)–(7), we have $a=3/2$ and $b=1$ from the consistency of the ordering and the requirement that the reduced equation must contain the η derivatives. In the lowest order, $O(\epsilon)$ and $O(\epsilon^{3/2})$, and in the second order $O(\epsilon^2)$ and $O(\epsilon^{5/2})$, we have the guiding-center drift,

$$v_x^{(1)}=-\frac{d\phi^{(1)}}{dy}=-\Phi(y),\quad v_y^{(1)}=0, \tag{13}$$

$$v_x^{(2)}=-\frac{\partial\phi^{(2)}}{\partial y},\quad v_y^{(2)}=\frac{\partial\phi^{(2)}}{\partial\xi}, \tag{14}$$

respectively. In the third order, $O(\epsilon^3)$ and $O(\epsilon^{7/2})$, we have the polarization drift

$$v_x^{(3)}=-\frac{\partial\phi^{(3)}}{\partial y}, \tag{15}$$

$$v_y^{(3)}=(\lambda+\Phi)\frac{\partial^2\phi^{(2)}}{\partial\xi\partial y}-\frac{\partial\phi^{(2)}}{\partial\xi}\Phi'+\frac{\partial\phi^{(3)}}{\partial\xi}, \tag{16}$$

and the equation for $\phi^{(2)}$

$$\left\{(\lambda+\Phi)\frac{\partial^2}{\partial y^2}-(\lambda+\Phi'')\right\}\frac{\partial\phi^{(2)}}{\partial\xi}=0, \tag{17}$$

which are in agreement with the results of Ref. [5] to this point. We assume that $\phi^{(2)}$ is separable

$$\phi^{(2)}=X^{(2)}(\xi,\eta,\tau)Y^{(2)}(y). \tag{18}$$

Substituting Eq. (18) into Eq. (17), we have

$$W(\lambda)Y^{(2)}(y)=\left(\frac{d^2}{dy^2}-\frac{\lambda+\Phi''}{\lambda+\Phi}\right)Y^{(2)}(y)=0. \tag{19}$$

It follows that λ is one of the eigenvalues of Eq. (19), under the periodic boundary condition for $Y^{(2)}$,

$$\begin{aligned}Y^{(2)}(0)&=Y^{(2)}(l),\\ Y^{(2)\prime}(0)&=Y^{(2)\prime}(l).\end{aligned} \tag{20}$$

In the fourth order $O(\epsilon^4)$ and $O(\epsilon^{9/2})$, we obtain $v_x^{(4)}$, $v_y^{(4)}$, $v_z^{(1)}$ and the equation for $\phi^{(3)}$ as follows:

$$v_x^{(4)}=(\lambda+\Phi)\frac{\partial^2\phi^{(2)}}{\partial\xi^2}-\frac{\partial\phi^{(4)}}{\partial y}, \tag{21}$$

$$\begin{aligned}v_y^{(4)}=&-\frac{\partial^2\phi^{(2)}}{\partial\tau\partial y}+(\lambda+\Phi)\frac{\partial^2\phi^{(3)}}{\partial\xi\partial y}+\frac{\partial\phi^{(2)}}{\partial y}\frac{\partial^2\phi^{(2)}}{\partial\xi\partial y}\\ &-\frac{\partial\phi^{(2)}}{\partial\xi}\frac{\partial^2\phi^{(2)}}{\partial y^2}-\Phi'\left\{(\lambda+\Phi)\frac{\partial^2\phi^{(2)}}{\partial\xi\partial y}\right.\\ &\left.-\Phi'\frac{\partial\phi^{(2)}}{\partial\xi}+\frac{\partial\phi^{(3)}}{\partial\xi}\right\}+\frac{\partial\phi^{(4)}}{\partial\xi},\end{aligned} \tag{22}$$

$$(\lambda+\Phi)\frac{\partial v_z^{(1)}}{\partial\xi}=\frac{\partial\phi^{(2)}}{\partial\eta}, \tag{23}$$

and

$$\left\{(\lambda+\Phi)\frac{\partial^2}{\partial y^2}-(\lambda+\Phi'')\right\}\frac{\partial\phi^{(3)}}{\partial\xi}+\frac{\partial}{\partial\tau}\left(1-\frac{\partial^2}{\partial y^2}\right)\phi^{(2)}$$
$$+\left(\frac{\partial^3\phi^{(2)}}{\partial\xi\partial y^2}\frac{\partial\phi^{(2)}}{\partial y}-\frac{\partial\phi^{(2)}}{\partial\xi}\frac{\partial^3\phi^{(2)}}{\partial y^3}\right)+(\lambda+\Phi)\frac{\partial^3\phi^{(2)}}{\partial\xi^3}$$
$$-\frac{\partial\phi^{(2)}}{\partial\xi}(\lambda\phi^{(1)}+\Phi\Phi'-2\Phi'\Phi''+\phi^{(1)}\Phi'')$$
$$+\frac{\partial^2\phi^{(2)}}{\partial\xi\partial y}\{(\lambda+\Phi)\Phi-(\lambda+\Phi)\Phi''\}$$
$$+\frac{\partial^3\phi^{(2)}}{\partial\xi\partial y^2}\{(\lambda+\Phi)(\phi^{(1)}-\Phi')\}+\frac{\partial v_z^{(1)}}{\partial\eta}=0. \qquad (24)$$

Differentiating Eq. (24) with respect to ξ, substituting Eq. (23) into the equation and taking account of Eq. (18), we have

$$W(\lambda)\frac{\partial^2\phi^{(3)}}{\partial\xi^2}-\frac{1}{\lambda+\Phi}\left(\frac{d^2Y^{(2)}}{dy^2}-Y^{(2)}\right)\frac{\partial^2X^{(2)}}{\partial\tau\partial\xi}$$
$$-\frac{1}{\lambda+\Phi}\left(Y^{(2)}\frac{d^3Y^{(2)}}{dy^3}-\frac{dY^{(2)}}{dy}\frac{d^2Y^{(2)}}{dy^2}\right)$$
$$\times\frac{\partial}{\partial\xi}\left(X^{(2)}\frac{\partial X^{(2)}}{\partial\xi}\right)+\left\{\Phi'\left(-1+\frac{\Phi''}{\lambda+\Phi}\right)Y^{(2)}\right.$$
$$\left.+(\Phi-\Phi'')\frac{dY^{(2)}}{dy}\right\}\frac{\partial^2X^{(2)}}{\partial\xi^2}+Y^{(2)}\frac{\partial^4X^{(2)}}{\partial\xi^4}$$
$$+\frac{1}{(\lambda+\Phi)^2}Y^{(2)}\frac{\partial^2X^{(2)}}{\partial\eta^2}=0. \qquad (25)$$

Multiplying Eq. (25) by the eigenfunction $Y^{(2)}$ for λ, and then integrating over y from 0 to l, we obtain the K-P equation for $X^{(2)}$ as a compatibility condition,

$$X^{(2)}_{\tau\xi}+\alpha(X^{(2)}X^{(2)}_{\xi})_{\xi}+\beta X^{(2)}_{\xi\xi\xi\xi}+\gamma X^{(2)}_{\eta\eta}+\delta X^{(2)}_{\xi\xi}=0. \qquad (26)$$

where the coefficients α, β, γ and δ are given by

$$\alpha=-\frac{1}{N}\int_0^l(Y^{(2)})^3\frac{1}{\lambda+\Phi}\frac{d}{dy}\left(\frac{\lambda+\Phi''}{\lambda+\Phi}\right)dy, \qquad (27)$$

$$\beta=\frac{1}{N}\int_0^l(Y^{(2)})^2dy, \qquad (28)$$

$$\gamma=\frac{1}{N}\int_0^l(Y^{(2)})^2\frac{1}{(\lambda+\Phi)^2}dy, \qquad (29)$$

$$\delta=-\frac{1}{N}\int_0^l\left\{(Y^{(2)})^2\Phi'\left(1-\frac{\Phi''}{\Phi+\lambda}\right)\right.$$
$$\left.-Y^{(2)}Y^{(2)\prime}(\Phi-\Phi'')\right\}dy, \qquad (30)$$

$$N=\int_0^l(Y^{(2)})^2\frac{\Phi-\Phi''}{(\lambda+\Phi)^2}dy. \qquad (31)$$

The last term of Eq. (26) can be eliminated by the Galilei transformation. The coefficients α and β are in agreement with the coefficients of a nonlinear term and a dispersion term of the KdV equation, which is derived in Ref. [5], respectively. Following Ref. [5], we assume a periodic electric-field with small variation about the constant value ϵc_0 in order to get the finite coefficient α of the nonlinear term of Eq. (26),

$$-\Phi=c_0+fV(y), \qquad (32)$$

where V is given by

$$V(y)=\sum_{m=1}a_m\sin k_my+\sum_{m=1}b_m\cos k_my,$$

$$k_m=2\pi m/l, \qquad (33)$$

where f is a small parameter but is much greater than ϵ. Expanding W, λ and $Y^{(2)}$ in power of f,

$$W=W_0+fW_1+\cdots,$$
$$\lambda=\lambda_0+f\lambda_1+\cdots,$$
$$Y^{(2)}=Y^{(2)}_0+fY^{(2)}_1+\cdots, \qquad (34)$$

and following the same calculation plan to Ref. [5], we have

$$\lambda_0=\frac{c_0k_m^2}{1+k_m^2}, \qquad (35)$$

$$Y^{(2)}_{0,m}=\frac{\sqrt{2/l}}{\sqrt{A_m^2+B_m^2}}\{A_m\sin k_my+B_m\cos k_my\}, \qquad (36)$$

where A_m and B_m are arbitrary constants. Let the mth eigenmode be excited. Substituting Eqs. (35) and (36) into Eqs. (27)–(31), we have

$$\alpha_m=-\frac{f}{\sqrt{8l}}\frac{k_{3m}}{c_0}(k_m^2-k_{3m}^2)\left\{a_{3m}\frac{B_m}{\sqrt{A_m^2+B_m^2}}\right.$$
$$\left.\times\left(1-\frac{4A_m^2}{A_m^2+B_m^2}\right)+b_{3m}\frac{A_m}{\sqrt{A_m^2+B_m^2}}\left(1-\frac{4B_m^2}{A_m^2+B_m^2}\right)\right\}$$
$$+O(f^2), \qquad (37)$$

$$\beta_m=-\frac{c_0}{(1+k_m^2)^2}+O(f), \qquad (38)$$

$$\gamma_m=-\frac{1}{c_0}+O(f), \qquad (39)$$

and

$$\delta_m = \frac{c_0}{(1+k_m^2)^2}\{k_{2m}+k_m(1+k_{2m}^2)\}\left(\frac{1}{2}\frac{A_m^2-B_m^2}{A_m^2+B_m^2}a_{2m} + \frac{A_mB_m}{A_m^2+B_m^2}b_{2m}\right)f+O(f^2). \tag{40}$$

It should be noted that $\beta_m\gamma_m = 1/(1+k_m^2)^2 > 0$. Thus the propagation of the three dimensional drift vortex solitons is described by the K-P equation with negative dispersion. As being pointed out by Nozaki *et al.* [5], the vortex soliton is produced by means of the coupling between the mth mode and the $3m$ harmonic component in the zonal flow. The shift of the soliton-velocity δ_m is produced by the coupling between the mth mode and the $2m$ harmonic component in the zonal flow.

For the one-soliton solution of Eq. (26), $\phi_m^{(2)}$ takes the form

$$\phi_m^{(2)} = 3\frac{\beta_m}{\alpha_m}K^2\mathrm{sech}^2\frac{1}{2}\left\{K\left(\xi+\frac{L}{K}\eta\right)-\Omega\tau\right\}\sqrt{\frac{2}{l}} \times\sin\left(k_my+\frac{\pi}{2}\right)+O(f), \tag{41}$$

where K and L are the x and y components of soliton wave number, respectively, and Ω is given by

$$\Omega_m = \beta_mK^3+\gamma_m\frac{L^2}{K}, \tag{42}$$

and $\sin^{-1}(A_m/\sqrt{A_m^2+B_m^2})$ is taken to be $\pi/2$.

Equation (41) shows that the potential-well and potential-hump solitons are lined up in the y direction alternately. The electric field is directed to the center of the soliton in the potential-well and it is directed outward from the center in the potential-hump soliton. It may be noted that the direction of rotation due to the $\mathbf{E}\times\mathbf{B}$ drift between the neighboring vortex is opposite. One of the vortex solitons is drawn in Fig. 1. The figure shows the contour $\phi_m^{(2)}=$const for Eq. (41) depicted in the frame moving with soliton.

Finally, we note that if some of the coefficients α, β, and γ in Eq. (26) become extremely small or large for a given configuration, others stretching from Eqs. (8)–(12) must be introduced. In that case, we shall obtain another equation instead of Eq. (26).

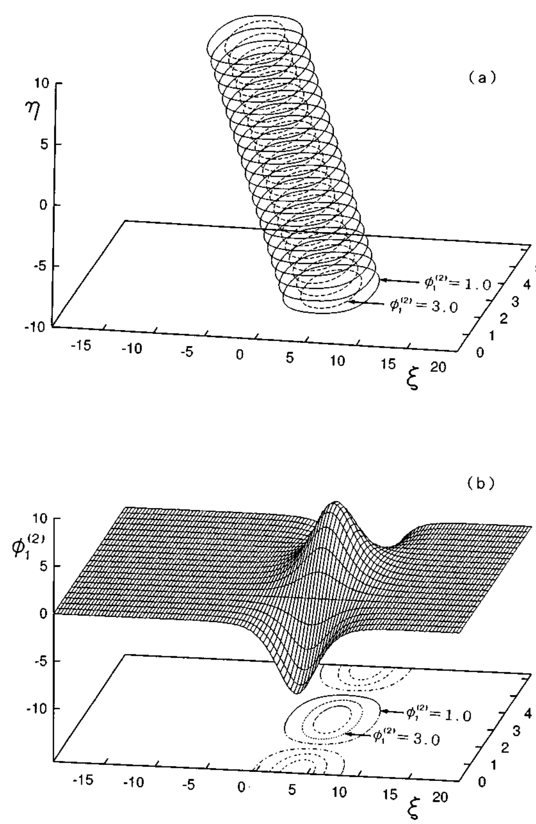

FIG. 1. Vortex soliton. (a) The perturbed electric potential contour lines, $\phi_1^{(2)}=1.0$ and 3.0, are depicted at different values of η in the frame moving with the soliton. The parameters are set as follows: $m=1$, $a_m=b_m=1$, $a_{3m}=b_{3m}=0.1$, $f=0.1$, and $K=1.52$, and $L=0.6$, where y, $\phi_1^{(2)}$, ξ, and η are normalized by ρ_s, $\epsilon^2T_ee^{-1}$, $\epsilon^{-1/2}\rho_s$, and $\epsilon^{-2}\rho_s$, respectively. (b) The perturbed electric potential $\phi_1^{(2)}$ [Eq. (41)] at $\eta=-10$ and the contour map of equipotential.

III. RESONANT INTERACTION OF VORTEX SOLITONS

We have obtained the K-P equation for the motion of vortices in zonal flow by applying the reductive perturbation method to be three dimensional cold ion-fluid equations with the Boltzmann distribution for electron density. The K-P equation was first introduced in order to discuss the stability of the line soliton against long transverse perturbation by Kadomtsev and Petviashvili [8]. The results were obtained that the line soliton of the KdV equation is unstable in the case of positive dispersion and is stable for negative dispersion. The equation corresponds to the case of negative and positive dispersion when $\beta\gamma>0$ and $\beta\gamma<0$, respectively. As the equation that is derived in previous section has $\beta_m\gamma_m=1/(1+k_m^2)^2>0$, the motion of vortex solitons is described by the K-P equation with negative dispersion. This means that vortex line soliton is stable against a transverse perturbation. It is interesting to study the interaction between two obliquely moving vortex solitons. The study of the interaction of two obliquely moving line solitons has been made by Miles [9]. He has shown that, when relative inclination between wave normals is at a certain small critical angle, two solitons interact strongly in the case of negative dispersion to make a resonant soliton from a point at which the two incident solitons meet together. On the other hand, in the case of positive dispersion, line solitons never satisfy the resonant condition. From the procedure to construct the multisoliton solutions of the K-P equation [10], the two-soliton solution is given by

$$X = 12\frac{\beta}{\alpha}(\log f)_{\xi\xi}, \tag{43}$$

$$f = 1+\exp\eta_1+\exp\eta_2+a(1,2)\exp(\eta_1+\eta_2), \tag{44}$$

$$\eta_j = K_j\xi+L_j\eta-\Omega_j\tau-\eta_j^{\,0}, \tag{45}$$

$$K_j\Omega_j = \beta K_j^4+\gamma L_j^2 (j=1,2), \tag{46}$$

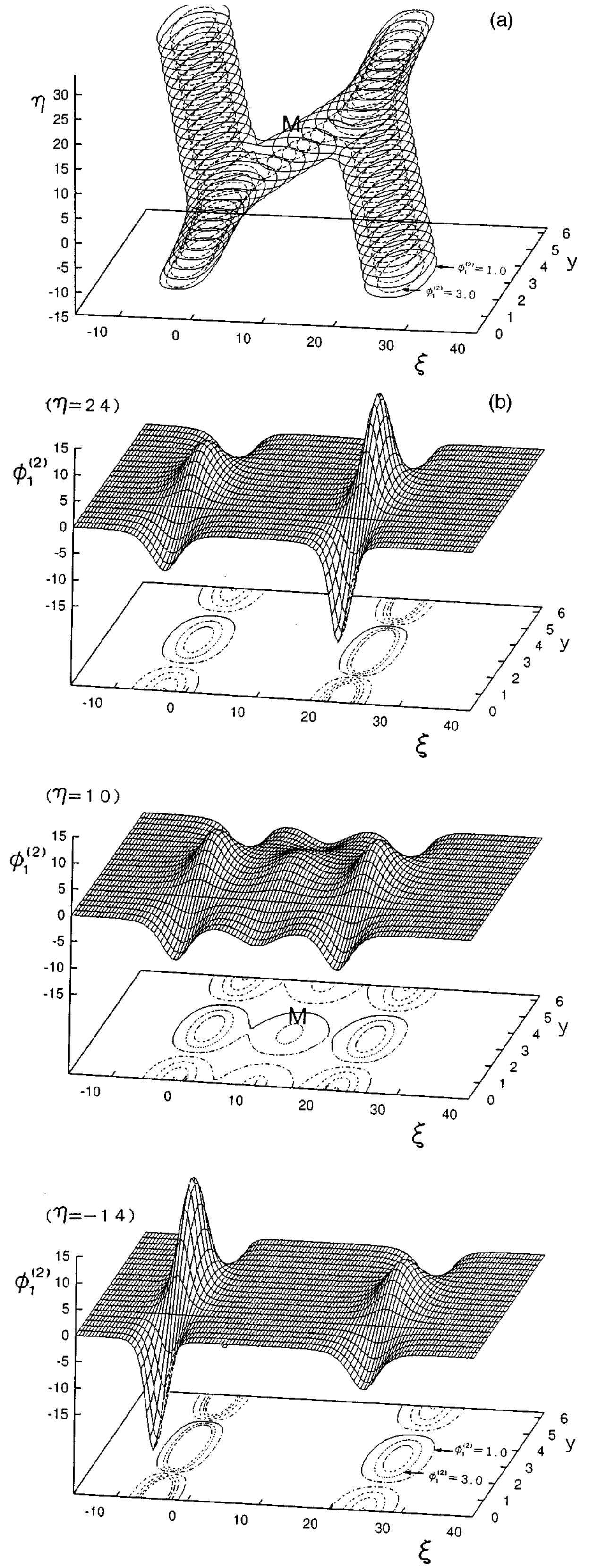

FIG. 2. The repulsive interaction between two vortex solitons $(0<a(1,2)<1)$. (a) The perturbed electric potential contour lines, $\phi_1^{(2)}=1.0$ and 3.0, are depicted at different values of η. The parameters are set as follows: $m=1$, $a_m=b_m=1$, $a_{3m}=b_{3m}=0.1$, $f=0.1$, and $K_1=0.76$, $L_1=0.3$, $K_2=1.3$, and $L_2=-0.44$. (b) The perturbed electric potentials $\phi_1^{(2)}$ and the contour maps of equipotential at $\eta=24$, $\eta=10$, and $\eta=-14$.

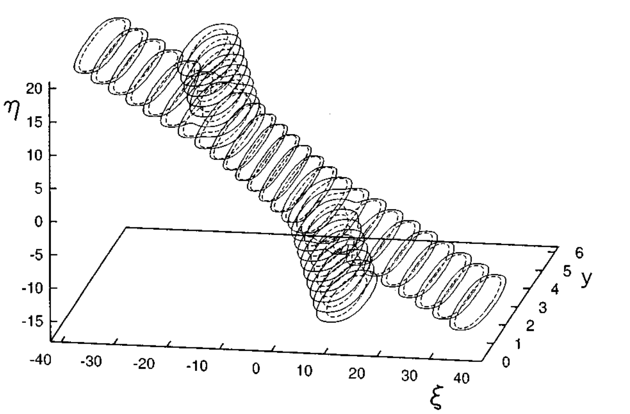

FIG. 3. The attractive interaction between two vortex solitons $[a(1,2)>1]$: The perturbed electric potential contour lines $\phi_1^{(2)}=1.0$ and 3.0 are depicted at different values of η ($K_1=0.76$, $L_1=0.3$, $K_2=1.3$, $L_2=2.3$).

where η_j^0 is the real constant and $a(1,2)$ relates to the phase shift ϕ of the two interacting solitons,

$$\exp(2\phi)$$
$$=a(1,2)$$
$$=-\frac{(K_1-K_2)(\Omega_1-\Omega_2)-\gamma(L_1-L_2)^2-\beta(K_1-K_2)^4}{(K_1+K_2)(\Omega_1+\Omega_2)-\gamma(L_1+L_2)^2-\beta(K_1+K_2)^4}. \tag{47}$$

For the case of $a(1,2)>0$, the solution [Eq. (43)] represents regular interaction of two solitons. According to $a(1,2)>1$ or $a(1,2)<1$, the interaction is attractive or repulsive in the x-direction. In Figs. 2 and 3, typical patterns of solution are shown for the two cases. Figures 2 and 3 correspond to repulsive and attractive cases, respectively. In the limit $a(1,2)\rightarrow\infty$, the phase shift becomes infinite. This is thought to be resonance between two inclined vortex solitons, whose condition is given by setting the dominator of $a(1,2)$ to zero

$$\tan\theta_1-\tan\theta_2=\pm\frac{\sqrt{3}c_0}{1+k_m^2}(K_1+K_2), \tag{48}$$

where $\theta_j=\arctan(L_j/K_j)$. Figure 4 is the snapshot of the resonant interaction between two inclined vortex solitons with parameters near the resonant condition in Eq. (48). Two vortex solitons interact strongly to make a resonant vortex soliton from a point at which the two incident solitons meet together.

The condition of the other limit $a(1,2)\rightarrow 0$, in which the phase shift becomes minus infinity, is given by equating the numerator of $a(1,2)$ to zero, which is expressed as follows:

$$\tan\theta_1-\tan\theta_2=\pm\frac{\sqrt{3}c_0}{1+k_m^2}(K_1-K_2). \tag{49}$$

Figure 2 is the snapshot of the interaction between two inclined vortex solitons with parameters near the condition in Eq. (49). In this case, two vortex solitons cannot approach each other closely. They interact through the messenger soliton denoted by M in Fig. 2. The solution of the messenger

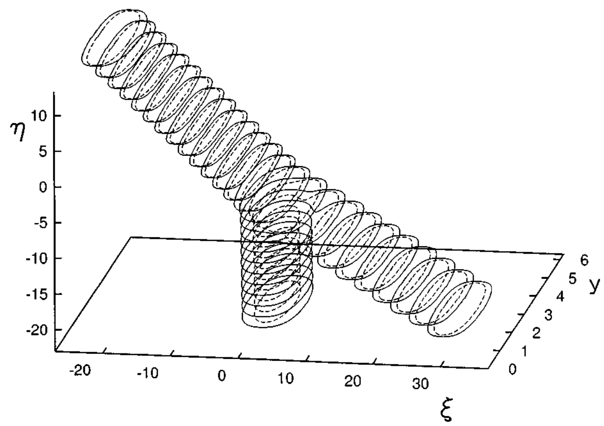

FIG. 4. The resonant interaction between two vortex solitons with parameters near the resonant condition ($K_1=0.76$, $L_1=0.01$, $K_2=1.3$, $L_2=2.3$).

soliton is given by taking the limit $\eta_1 \to \infty$, $\eta_2 + \log a(1,2) \to -\infty$ but $\eta_1 - \eta_2 \sim O(1)$ as follows,

$$\phi_M^{(2)} = 3\frac{\beta_m}{\alpha_m}(K_1-K_2)^2 \times \text{sech}^2 \frac{1}{2}\left\{(K_1-K_2)\left(\xi + \frac{L_1-L_2}{K_1-K_2}\eta\right) - \Omega_M \tau\right\} \times \sqrt{\frac{2}{l}}\sin\left(k_m y + \frac{\pi}{2}\right) + O(f\ ), \tag{50}$$

where

$$\Omega_M = \beta_m (K_1-K_2)^3 + \gamma_m \frac{(L_1-L_2)^2}{K_1-K_2}. \tag{51}$$

It should be noted that the messenger soliton and the second soliton satisfy the resonant condition. Thus, the messenger soliton and the second soliton interact resonantly to make the first soliton which is shifted by $\log a(1,2)$.

IV. SUMMARY

We have investigated the propagation of three-dimensional drift vortices in a two-dimensional periodic zonal flow which extends to the direction of the applied magnetic field. The propagation of vortices is described by the *K-P* equation with negative dispersion. Therefore, vortices propagate as the *K-P* solitons in the direction normal to the static electric field with inhomogeneity. The solutions show that the potential-hump and potential-well solitons are lined up in the direction of the static electric field alternatively and propagate with the same velocity. These hump and well parts rotate clockwise and counterclockwise in the sense of the **E**×**B** drift, respectively.

As the propagation of the vortex solitons is described by the *K-P* equation with negative dispersion, the vortex may be stable against a transverse perturbation. The interactions between two obliquely moving vortex solitons are also investigated by using the two-soliton solution of the *K-P* equation. It is shown that the drift vortex soliton resonance is possible under a certain condition.

When the angle of intersection between two solitons has the critical value given by Eq. (48), the resonance occurs to form the triad soliton as shown in Fig. 4. However, it should be noted that when the angle of intersection of the two solitons is between the critical values, the two-soliton solution [Eq. (43)] becomes singular. As pointed out by Miles [9], the resonance solution is on the borderline between regular and singular regimes in the parameter space. Although the narrowness of the resonance region may cast doubt on the existence of the soliton resonance in a real system, it is possible to produce a virtual resonant soliton in the region close to the exact resonance state. In fact, Folkes *et al.* [11] and Nishida and Nagasawa [12] verfied the resonance conditions by the observation of such a virtual state of plane ion-acoustic soli--tons. The resonant interaction of ion-acoustic solitons has been studied both theoretically and experimentally by many authors [13–17]. The importance of soliton resonance in nonlinear development of the two dimensional wave system is clear. Now the possibility of drift vortex soliton resonance has been shown even in the special case. We believe that the vortex soliton resonance is also important in understanding the time evolution of the vortex soliton systems.

[1] A. Hasegawa and K. Mima, Phys. Rev. Lett. **39**, 205 (1977).
[2] A. Hasegawa and K. Mima, Phys. Fluids **21**, 87 (1978).
[3] A. Hasegawa and Y. Kodama, Phys. Rev. Lett. **41**, 1470 (1978).
[4] A. Hasegawa, C. G. Maclennan, and Y. Kodama, Phys. Fluids **22**, 2122 (1979).
[5] K. Nozaki, T. Taniuti, and K. Watanabe, J. Phys. Soc. Jpn. **46**, 983 (1979).
[6] E. W. Laedke and K. H. Spatschek, Phys. Lett. A **113**, 259 (1985).
[7] H. O. Åkerstedt, J. Nycander, and V. P. Pavlenko, Phys. Plasmas **3**, 160 (1996).
[8] B. B. Kadomtsev and V. I. Petviashvili, Sov. Phys. Dokl. **15**, 539 (1970).
[9] J. W. Miles, J. Fluid Mech. **79**, 157 (1977); **79**, 171 (1977).
[10] J. Satsuma, J. Phys. Soc. Jpn. **40**, 286 (1976).
[11] P. A. Folkes, H. Ikezi, and R. Davis, Phys. Rev. Lett. **45**, 902 (1980).
[12] Y. Nishida and T. Nagasawa, Phys. Rev. Lett. **45**, 1626 (1980).
[13] F. Kako and N. Yajima, J. Phys. Soc. Jpn. **49**, 2063 (1980); F. Kako and N. Yajima, J. Phys. Soc. Jpn. **51**, 311 (1982).
[14] Y. Nishida, T. Nagasawa, and S. Kawamata, Phys. Rev. Lett. **42**, 379 (1979); T. Nagasawa, M. Shimizu, and Y. Nishida, Phys. Lett. A **87**, 37 (1981); T. Nagasawa and Nishida, Phys. Rev. A **28**, 3043 (1983).

[15] F. Ze, N. Hershkowitz, C. Chan, and K. E. Lonngren, Phys. Rev. Lett. **42**, 1747 (1979); F. Ze, N. Hershkowitz, and K. E. Lonngren, Phys. Fluids **23**, 1155 (1980).

[16] I. Tsukabayashi and Y. Nakamura, Phys. Lett. A **85**, 151 (1981).

[17] K. E. Lonngren, E. Gabl, J. M. Bulson, and M. Khazei, Physica D **9**, 372 (1983); M. Khazei, J. M. Bulson, and K. E. Lonngren, Phys. Fluids **25**, 759 (1982).