Collective-induced computation

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Many natural systems, such as social insects, perform complex computations collectively. In these groups, large numbers of individuals communicate in a local way and send information to its nearest neighbors. Interestingly, a general observation of these societies reveals that the cognitive capabilities of individuals are fairly limited, suggesting that the complex dynamics observed inside the collective is induced by the interactions among elements and is not defined at the individual level. In this paper we use globally coupled maps, as a generic theoretical model of a distributed system, and Crutchfield's statistical complexity, as our theoretical definition of complexity, to study the relation between the complexity the collective is able to induce on the individual and the complexity of the latter. It is conjectured that the observed patterns could be a generic property of complex dynamical nonlinear networks. [S1063-651X(97)00203-1]

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I. INTRODUCTION

The topic this paper addresses is easy to state: *The more complex a society, the more simple the individual* [1]. This sentence, of course, concerns social insects, among which we will take ants as a main example. It is a well-known fact that all living species of ants are eusocial (i.e., all species have the following properties: cooperation in caring for the young, overlap of at least two generations capable of contributing to colony labor, and reproductive division of labor [2]); nevertheless, there exist large differences among species with respect to the number of ants that compose the colony, their collective capabilities, and the cognitive skills of individuals. A specific example is that of recruitement strategies: There is a clear correlation between the size of the colony and the behavioral sophistication of individual members [3]. In one extreme we find the more advanced evolutionary grade: mass communication (information that can be transmitted only from one group of individuals to another group of individuals, according to [2], p. 271). Mass communication is the recruitement strategy used by army ants (e.g., *Eciton burchelli*), whose colonies are composed of a huge number of individuals, who are, nevertheless, almost blind and extremely simple in behavior when isolated. The other extreme is occupied by those ants using individual foraging strategies (e.g., the desert ant *Cataglyphis bicolor*), who displays very complex solitary behavior.

Our interest here is not as much to study this remarkable feature of eusocial insects as to see if this could be a general trait of collectives of agents. That is, is there a trade-off between individual complexity and collective behavior in such a way that complex emergent properties cannot appear if individuals are too much complex?

In order to continue with our work, let us start considering the concept of emergence. According to Haken [4], the emergent properties of a system can be studied with the notion of *order parameter* and its associated slaving principle. As we can see in Fig. 1, we can look for an answer in two directions: from the individual to the collective and vice versa. Immediately we can discard the former because the simplest individuals are those who display collectively the most complex behavior. So we can ask now a more concrete question: What kind of behavior does the collective induce on the otherwise simple individual to attain emergent functional capabilities? Of course we can answer it from an evolutionary point of view, arguing that adaptation to the environment is the ultimate reason of those diverse features of ant colonies. This is not the unique answer we can provide [5] because we can also look for relations between the order parameter and the individuals in such a way that, perhaps, complex solitary behavior imposes severe constraints on the behavior that a collective would induce on individuals. This would be a structural solution of our problem and it will be the answer we are seeking.

Although we will not provide a complete solution, we will make an initial move towards a theoretical account of the problem. First of all, we review in Secs. II and III the theoretical framework we use: Kaneko's globally coupled maps (GCMs) [6] and Crutchfield's statistical complexity and ϵ-machine reconstruction [7]. Furthermore, in Sec. II we characterize the phase space of GCMs with information-theoretic measures. In Sec. IV we detail our work with

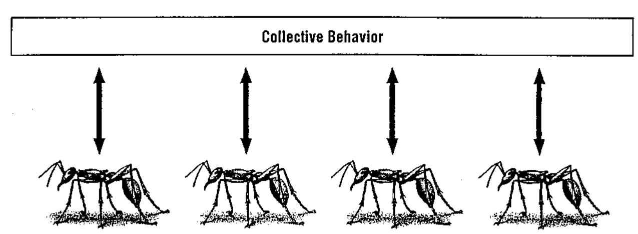

FIG. 1. Emergence in collective behavior. Individual ants interact either by physical contact or by laying pheromones. Coordinated collective actions emerge from these patterns of interaction, which in turn affects individual behavior. This causal circularity pervades complex systems.

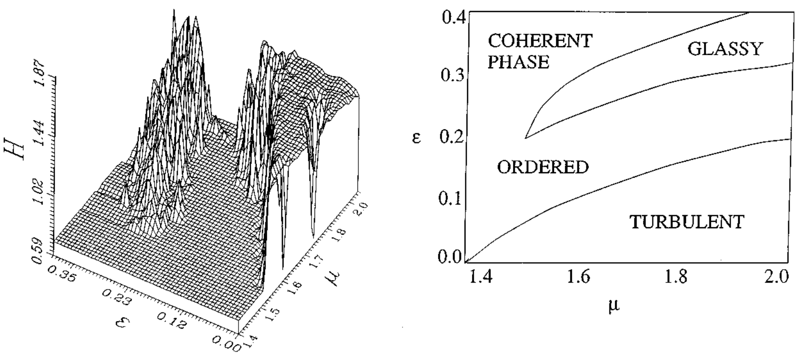

FIG. 2. Information-theoretic measures are able to discriminate among the different phases of GCM dynamical behavior. Right: joint entropy for $0 \leqslant \epsilon \leqslant 0.4$, $1.4 \leqslant \mu \leqslant 2.0$, and $N=100$. Left: GCM phase space (after [6]). The joint entropy is largest at the turbulent phase when all the binary pairs are equally explored. It is ln(2) for the ordered and coherent phase and it takes intermediate values at the glassy phase. ϵ and μ are dimensionless parameters of the GCM.

ϵ-machine reconstruction and we see how the complexity of a theoretical individual can be an obstacle to the collective in order to modify its behavior. Finally, we discuss our results and their possible implications in Sec. V.

We think we should say what this paper is not about. This paper does *not* analyze completely GCMs using statistical complexity. Of course this research deserves to be done, but the objectives of this paper are far more modest. We simply use those theoretical constructs to show a theoretical property that resembles a natural one.

II. GCM: PHASES AND INFORMATION

Globally coupled maps are usually defined by a set of nonlinear discrete equations

$$x_{n+1}(i)=(1-\epsilon)f_\mu(x_n(i))+\frac{\epsilon}{N}\sum_{j=1}^{N} f_\mu(x_n(j)) \qquad (1)$$

where n is a discrete time step and $i=1,\ldots,N$. The function $f_\mu(x)$ is assumed to have a bifurcation scenario leading to chaos. Here we use the logistic map

$$f_\mu(x)=1-\mu x^2, \qquad (2)$$

which is known to have a period-doubling route to chaos. GCMs are in fact the simplest approach to a wide class of nonlinear networks, from neural networks to the immune system [6]. They have been shown to have remarkably rich behavior, partly similar to the mean-field model for the spin glass by Sherrington and Kirkpatrick [6]. Their behavior in phase space is very rich, showing clustering among maps. These clusters are formed by sets of elements with the same phase.

The phase space of GCMs exhibits several transitions among coherent, ordered, intermittent, and turbulent phases. These phases are well characterized in terms of the so-called cluster distribution function $Q(k)$ [6] and can also be well characterized, as shown in this section, by means of information-theoretic measures [8].

In each phase, a given number of clusters N_r involving r maps will be observed. Specifically, a cluster is defined by the set of maps such that $x_n(i)=x_n(j)$ for all maps belonging to the cluster. We can calculate the number of clusters of size r, and for a given phase we have a set $\{N_1,N_2,\ldots,N_k\}$ of integer numbers. Then the $Q(k)$ function is defined as the fraction of initial conditions that collapse into a given k-cluster attractor (i.e., the volume of the attraction basin). An additional useful measure will be the mean number of clusters R_μ defined as $R_\mu=\Sigma_k kQ(k)$.

Here we also consider an information-based characterization of the different phases by means of the Markov partition

$$\Pi=\{x_n\in[-1,0)\Rightarrow S_i^j=0,\, x_n\in[0,1]\Rightarrow S_i^j=1\}, \qquad (3)$$

where $S_1^i S_2^i S_3^i \cdots$ will be the sequence of bits $S_i^j \in \Sigma \equiv \{0,1\}$ generated through the dynamics of the ith map, under the partition Π. We can compute the Boltzmann entropy for each map

$$H^i(\Sigma)=-\sum_{S_i^j=0,1} P(S_i^j)\log_2 P(S_i^j)$$

and the joint entropy for each pair of maps

$$H^{il}(\Sigma)=-\sum_{S_i^j=0,1}\sum_{S_l^r=0,1} P(S_i^j,S_l^r)\log_2 P(S_i^j,S_l^r).$$

From the previous quantities, we can compute the information transfer between two given units. It will be given by

$$M^{il}(\Sigma)=H^i(\Sigma)+H^l(\Sigma)-H^{il}(\Sigma).$$

These quantities have been widely used in the characterization of macroscopic properties of complex systems modeled by cellular automata and fluid neural networks [9,10]. As a way of quantifying complexity, it has been shown that information transfer is an appropiate measure of correlations [11] and in this context it is maximum near critical points [12]. Because our interest is in the computational structure behind the observed dynamics, we expect to have some well-defined relations between computational complexity and information transfer. Using these measures (see Fig. 2), the four basic phases exhibited by GCMs are the following.

(i) *Coherent phase*. The system is totally synchronous, i.e., $x(i)=x(j)$ for all i,j. The motion is then described by a single map $x_{n+1}=f_\mu(x_n)$ and the stability of this single attractor can be analytically characterized [6]. If λ_0 is the Lyapunov exponent for the single map, the Jacobi matrix is simply given by

$$\mathbf{J}_\mu=\partial f_\mu/\partial x_n\left[(1-\epsilon)\mathbf{I}+\frac{\epsilon}{N}\mathbf{D}\right],$$

where **I** and **D** are the identity matrix and a matrix of ones, respectively. From the Jacobi matrix we can get the stability condition

$$\lambda_0 + \ln(1-\epsilon) < 0.$$

Here almost all basins of attraction are occupied by the coherent attractor and $Q(1)=1$, so we have $R_\mu = 1$.

In terms of information transfer under the Markov partition, we will have $H^i(\Sigma)=H^l(\Sigma)$ (both maps are visiting the same points) and $P(S_i^j, S_l^r)=\delta_{jr}/2$, so it is easy to see that in this phase we have $H^{il}(\Sigma)=H^i(\Sigma)$ and the mutual information is given by $M^{il}=H^i$. The information is totally defined by the entropy of the single maps as long as the correlations are trivial.

(ii) *Turbulent phase*. This corresponds to the other extreme in the dynamical phases of GCMs. Here we have that the number of clusters is such that $R_\mu \approx N$. A first look at the dynamics of single maps seems to suggest that they behave independently. Under this hypothesis, the entropies can be easily estimated. If the maps are independent, then we have again $H^i(\Sigma)=H^l(\Sigma)$, but the joint probabilities will be such that $P(S_i^j, S_l^r)=P(S_i^j)P(S_l^r)$, and so we have $H^{il}(\Sigma)=2H^i(\Sigma)$ and as a consequence the mutual information will be zero. A close inspection of the numerical values for the mutual information shows, however, that $1 \gg M^{il} > 0$, so some amount of correlation is still present. Specifically, we found that typically $10^{-6} < M^{il} < 10^{-3}$. This result was obtained by Kaneko [13] in a remarkable work where it was shown that GCMs violate the law of large numbers (LLNs). This hidden order is shown to exist by means of the analysis of the local fields, defined as $h_n \equiv N^{-1}\Sigma_j f_\mu(x_n(j))$. The study of the mean-square deviation of this quantity, which is expected to decay as $O(1/N)$ if the units are really independent, was shown to saturate for a given $N \geq N_c(\mu)$ [13]. The analysis of the density distribution for two maps gives a pair of continuous functions $P_i(x)$ and $P_j(y)$ [i.e., $\int P_i(s)ds=1$] and a joint distribution $P_{i,j}(x,y)$ [with $\int\int P_{i,j}(x,y)dx\,dy=1$], which makes it possible to define a continuous mutual information

$$M_{i,j} = -\int\int \log_2\left[\frac{P_{i,j}(x,y)}{P_i(x)P_j(y)}\right]dx\,dy$$

and, after averaging over space and time, it also shows a saturation when N gets large. Numerical experiments gave $M_{i,j}(N\to\infty)=O(10^{-3})$, consistently with our bounds for the binary partition. Such remaining finite correlation is the origin of the breakdown of the LLNs and will be relevant in our discussion about computation in GCMs.

(iii) *Ordered phase*. Here we have a small number of clusters with many units. Specifically, we have $Q(k)=0$ for $k>k_c$ (where k_c does not depend on N) and

$$Q_L(k) \equiv \sum_{k>N/2} Q(k) = 0$$

and $Q(1)\neq 1$. We also get $R_\mu = b \ll N$. Again, a large number of elements will share the same state and we can easily estimate the entropies and information transfer. Given two maps, they could belong to the same cluster or two different clusters. In the first case, we get the same result as in the coherent phase and the same occurs if they belong to clusters that are in phase. If the maps belong to two clusters that are not in phase, we have $H^i(\Sigma)=H^l(\Sigma)=\ln(2)$ and now $P(S_i^j, S_l^r)=(1-\delta_{jr})/2$ so again we get $M^{il}=H^i$, as in the coherent phase (see Fig. 2).

(iv) *Glassy phase*. Also called intermittent phase, in this domain of parameter space we have many clusters, but they have a wide distribution of sizes. We have $\Sigma_{k>N/2}Q(k)>0$ and also $\Sigma_{k<N/2}Q(k)>0$. So $R_\mu = rN$ with $r<1$. Here the competition of some attractors with different cluster size leads to frustration [6]. Following our previous arguments, it is not difficult to show that $0<M^{il}(\Sigma)<\ln(2)$. So in this phase the joint entropy has a finite (but not large) value, as expected from the existence of a decaying distribution of cluster sizes.

We have shown that the use of information-based measures involving the previously defined Markov partition provides an accurate characterization of the GCM phases. As we can see, some phases have a high information transfer, while others have a nearly zero correlation among units. The basic qualitative observation of this phase space is that the greater the nonlinearity (the parameter μ), the more widespread the disorder, and the greater the averaging effect (parametrized by ϵ), the more the overall coherence. So each unit in the GCM is subject to two competing forces: the individual tendency to chaos and the tendency to conformity arising from the averaging effect of the system as a whole.

This conflict between order and disorder changes suddenly at the boundaries between the different phases. In recent studies, it has been suggested that such phase transitions can be very important in sustaining higher computational capabilities [7,9,10]. Usually the transition is defined as involving the maximum information transfer (and the higher correlations), information being lower in both phases. Here, however, each phase is roughly characterized by rather constant entropies and information so we could ask whether or not intrinsic computation will reach higher values at the transitions. In the next section we explore this problem by means of the ϵ-machine reconstruction algorithm.

III. STATISTICAL COMPLEXITY

Statistical complexity is a recent measure of complexity based on a computational view of what an orbit of a dynamical system is [7]. Chaotic dynamical systems (with a period doubling or a quasiperiodic route to chaos) [7,14,16] and one-dimensional spin systems [21] have been adequately characterized using statistical complexity. There is the ϵ-machine reconstruction algorithm (ϵ-MRA [20]) associated with it. This algorithm has been the basis of much work relating dynamical systems and computation. It has been successfully applied to characterize computationally the above-mentioned onset of chaos, to the characterization of cellular automata in terms of domains, attractors and basins of attraction [17], and to finding out the mechanisms by which an evolved cellular automata can compute (particle-based computation [18]).

Here we will use the ϵ-MRA to ascertain the *intrinsic computation* [16] of the individual logistic maps in the GCM. In general, in order to apply ϵ-MRA, we need to know the orbit of a dynamical system $\ldots, x_{t-2}, x_{t-1}, x_t, x_{t+1}, x_{t+2}, \ldots$ (which we assume to be stationary) and we also need to specify an instrument to observe the above-mentioned orbit. This instrument will have some resolution ϵ. The instrument used in this paper is precisely the Markov partition Π (3).

Π will define a generating partition (i.e., where there is a finite to one correspondence between infinite bit strings and initial conditions) for the logistic map (2), so it is clearly the best choice for a logistic map in a GCM. Applying the instrument to the orbit will provide us with a bit string, which, in practice, will have finite length M, that will be used to construct a deterministic finite automaton (DFA) (see [19]) with probabilistic labels. This automaton, if found, will be a minimal model describing the intrinsic computation of the observed process (dynamical system plus instrument). The ϵ-MRA proceeds, very briefly, as

$$\{\ldots 011101010 \ldots\} \Rightarrow \text{tree}(L) \Rightarrow \epsilon\text{-machine}(D),$$

where L will be used to scan the entire bit string extracting bit strings of length L to build a parse tree and D (the "morph depth," in practice $\lfloor L/2 \rfloor$) will be used to construct the states of the ϵ-machine. Here we will not go into the details of the ϵ-MRA (see [7,14,15] and [17], Chap. 5). We just say that if the stationarity assumption is violated, the ϵ-MRA will fail in reconstructing any DFA. This will be the case when we have GCMs with supertransients or when a high-dimensional attractor is reached. Once we have the ϵ-machine, the statistical complexity will be defined as the logarithm of the number of (recurrent) ϵ-machine states [16].

Measuring the intrinsic computation provides us with an upper bound to usable computation [16]. Of course it does not make much sense to talk about the usable computation of a logistic map, but, in real systems, it would be quite interesting to have a good description of their intrinsic computation in order to be compared with the intrinsic computation of the dynamical systems modeling them. Furthermore, if we could find the intrinsic computation of, say, a real ant, we could know what the maximum computational capability of that ant would be. This would allow a deeper understanding of the problem stated in the Introduction.

IV. COLLECTIVE-INDUCED COMPUTATION

In this paper the collective system we are working on is a globally coupled map, i.e., N logistic maps (2) interacting as has been described in Sec. II, and our individual will be a randomly chosen logistic map of the system. This approach has a clear advantage: The statistical complexity of the logistic map is well known [7,14], so our individuals have a well-defined intrinsic computation. Our purpose is to see how the collective is (or is not) able to induce more complex behavior than the individual is able to show.

A. Complex individuals

Given a logistic map (our individual) a high statistical complexity is observed for μ close to μ_∞, i.e., the onset of chaos. There we need a large number of states to model the high periodicity of the orbits. We have chosen $\mu = 1.4$, whose statistical complexity is $C_{1.4} \simeq 4$. As we can see in Fig. 3(a), this automaton has a large number of states.

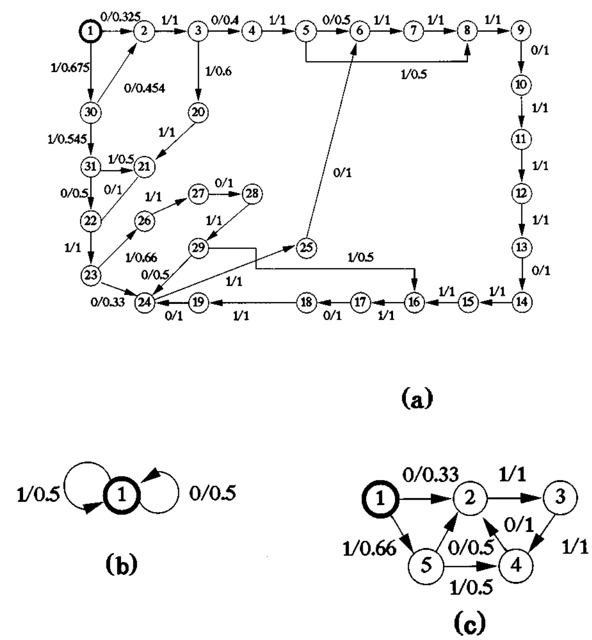

FIG. 3. DFA with probabilistic labelings resulting from the ϵ-MRA applied to (a) logistic map (2) with $\mu = 1.4$, (b) logistic map (2) with $\mu = 2$, and (c) logistic map (2) with $\mu = 1.75$. These are the individuals over which we will check if the collective can induce more complex behavior. As is obvious from the automata, (a) is much more complex than (b) and (c) (see the text). In all cases the ϵ-MRA parameters are $M = 10^7$, $L = 32$, and $D = 16$. In (a)–(c) state 1 is the initial state, all other states are accepting states. μ is the dimensionless parameter of the logistic map.

The next step is to define a GCM, such as that of Eq. (1), with $\mu = 1.4$, and look at the statistical complexity of an individual (all are, in principle, equal) chosen at random, say, i, as the degree of interaction increases, i.e., we examine $C^i_{1.4}$ as the parameter ϵ goes from 0 to 0.4.

The result is simply that there are no changes (as can be seen in Fig. 4). The intrinsic computation of the individual remains the same, $C^i_{1.4} \simeq 4$, no matter how large the interaction is with the rest of the system. So the collective has not been able to induce any kind of added complexity to the individual. In this case there is no emergent behavior. The collective behavior can be reduced to that of the individuals.

B. Simple individuals

If we take $\mu = 2$ the logistic map has completely chaotic dynamics. It is, in statistical complexity terms, the same as a fair coin toss. So its automaton has $C_2 = 0$ with just one state [Fig. 3(b)]. Now, we can apply the ϵ-MRA to the symbolic dynamics (i.e., the bit string of length M) of an individual chosen at random among the N that compose the GCM. The ϵ-MRA failed to reconstruct any automaton in the turbulent phase (either for $\mu = 2$ or for $\mu = 1.75$, in Sec. IV C). This could be because of high-dimensional chaos and the existence of supertransients [22]. In any case, it seems that the stationarity assumption was not fulfilled, causing the nonconvergence of the ϵ-MRA (see [17], Chap. 5). There are also some values of ϵ in the ordered and the glassy phase where no finite automaton was obtained. The reason here is the fine structure of those phases [22].

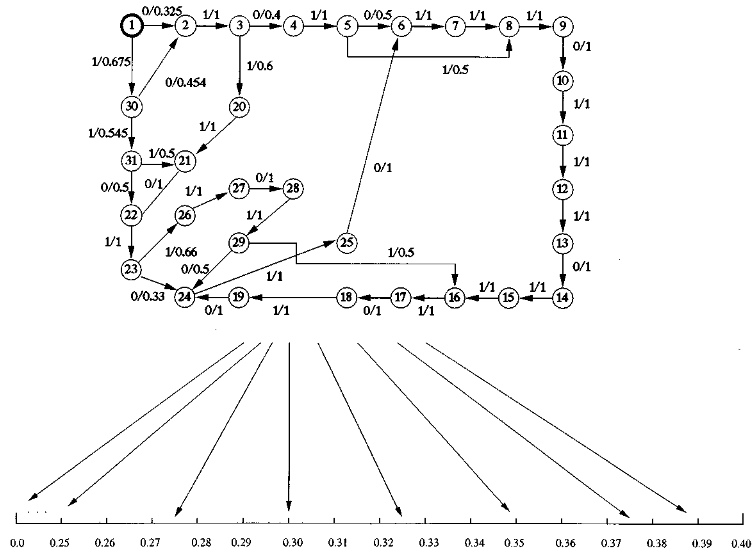

FIG. 4. If we have a complex individual, no matter how much interaction it receives, its behavior will not change. The collective cannot induce on the individual any kind of added behavior. In the figure, the individual possesses the same statistical complexity for all ϵ. The parameters of the ϵ-MRA are $M=10^7$, $L=32$, and $D=16$. All the automata have 1 as the initial state and all other states are accepting states. ϵ is a dimensionless parameter of the GCM (see the text). $N=500$.

Our result is somewhat surprising (Fig. 5). If we exclude the automaton at $\epsilon=0.26$ and the gaps at $\epsilon=0.27$ and $\epsilon=0.28$ (which indicates some kind of irregular behavior in the regions, although according to the phase space of Fig. 2 we should have ordered behavior) our individual reaches high complexity, $C_2^i \simeq 3$, near the boundary of the turbulent phase. Beyond this point we find the same automaton around $\epsilon \simeq 0.295$, perhaps pointing out another boundary (that of the above-mentioned irregular behavior). After that the complexity decreases with ϵ while going deeply into the ordered phase: First $C_2^i=2$ at $\epsilon=0.31$ and then it goes down to $C_2^i=1$ at $\epsilon=0.32$, $\epsilon=0.325$, and $\epsilon=0.33$, to end up in $C_2^i=0$ at $\epsilon=0.34$ and $\epsilon=0.35$. The complexity increases slightly again at the glassy phase: $C_2^i \simeq 1.585$ at $\epsilon=0.375$ and $\epsilon=0.39$. The more complex behavior is displayed near phase boundaries, as has been observed also in other systems [12].

If we compare this case with the previous one, we see that simple individual behavior allows the interaction to create more sophisticated behavior in the individual, inducing a certain amount of statistical complexity that was not present at the individual level. So a coordinated behavior, which the individual is unable to show, emerges from the collective through interactions.

C. Intermediate individuals

Here we have $\mu=1.75$ with an individual of complexity $C_{1.75} \simeq 1.585$ [Fig. 3(c)] and we take a logistic map randomly from a GCM with the same μ value. In this case, as in the previous one, we find a maximum intrinsic computation at the boundary between the turbulent phase and the ordered

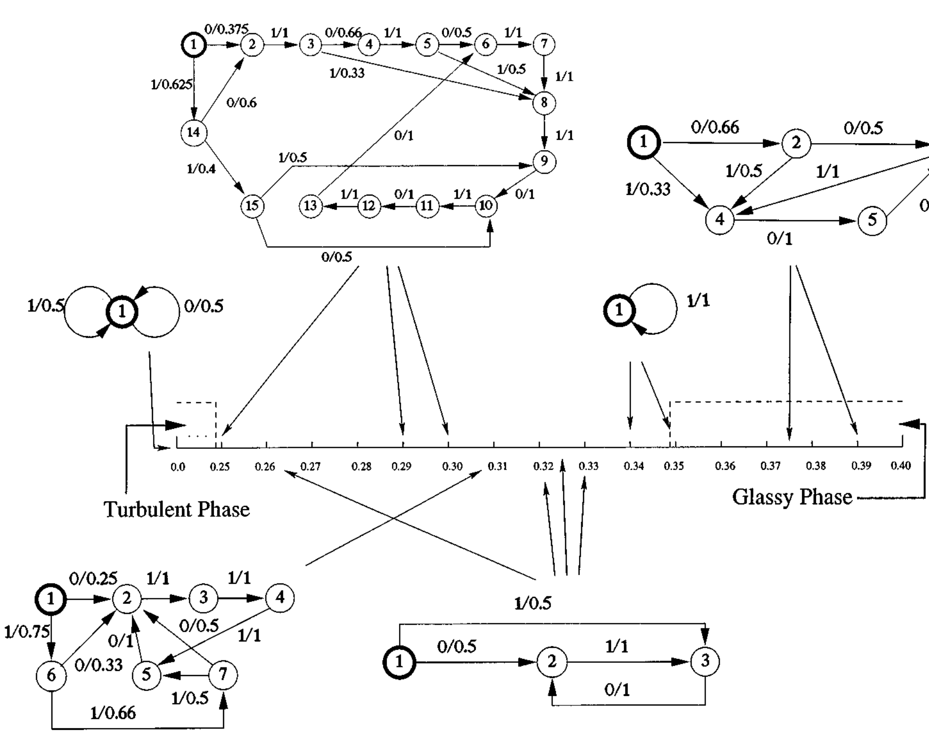

FIG. 5. With a simple individual like that of $\epsilon=0$ (in this figure), the collective is able to impose additional behavior on the individual. We have a decreasing complexity from the turbulent phase boundary onward with increasing ϵ, except in the region of 0.27 (see the text). We can observe also a slight increase in complexity at the glassy phase. The parameters of the ϵ-MRA are $M=10^7$, $L=32$, and $D=16$. All the automata have 1 as the initial state and all other states are accepting states. ϵ is a dimensionless parameter of the GCM. $N=500$.

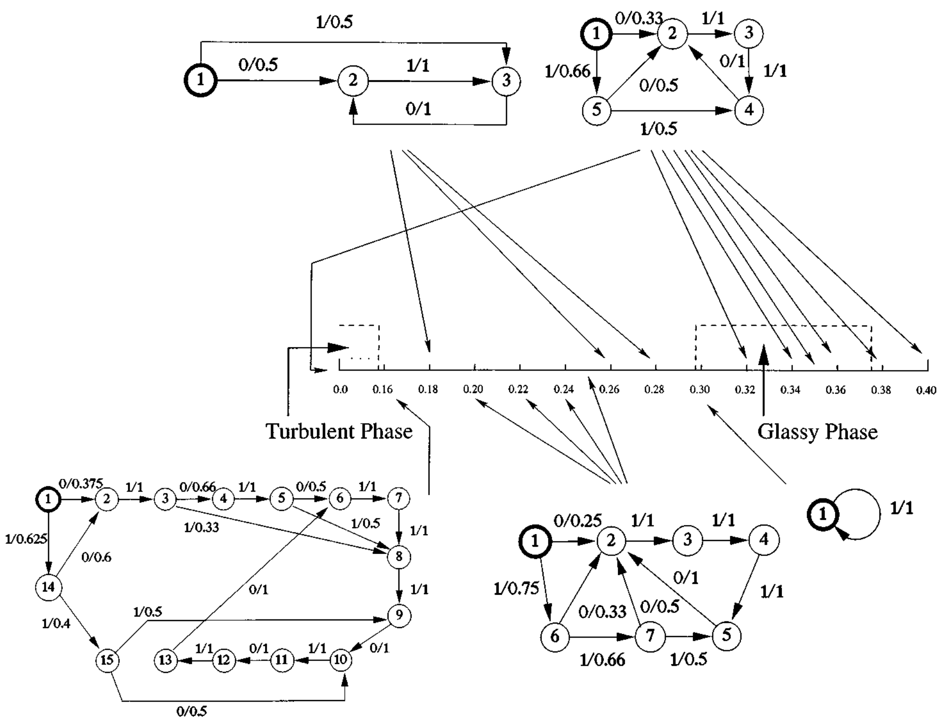

FIG. 6. Here we see intermediate behavior between the cases shown in Figs. 4 and 5. Just for $\epsilon=0.16$ we found much greater complexity than that of the individual. We also bear in mind that the individual statistical complexity is $C_{1.75}\simeq 1.585$, so that the increment is not as large as in the $\mu=2$ case (see the text). The parameters of the ϵ-MRA are $M=10^7$, $L=32$, and $D=16$. All the automata have 1 as the initial state and all other states are accepting states. ϵ and μ are the dimensionless parameters of the GCM. $N=500$.

phase. In fact, the automaton in this boundary is the same one we found at the same boundary for $\mu=2$. Although the individual is more complex than that of $\mu=2$, we can observe the same behavior of the automata with growing ϵ (Fig. 6): at $\epsilon=1.6$ we get a statistical complexity of $C^i_{1.75}\simeq 3$, at $\epsilon=0.2$, $\epsilon=0.22$, $\epsilon=0.24$, $\epsilon=0.25$, and $\epsilon=0.26$ statistical complexity decreases to $C^i_{1.75}\simeq 2$, then statistical complexity keeps decreasing down to a value of $C^i_{1.75}\simeq 1$ ($\epsilon=0.26$ and $\epsilon=0.28$), and finally it reaches the zero value at the boundary of the glassy phase. However, this picture fails at $\epsilon=1.8$, perhaps due to a small window located in the region of that ϵ. Again, at the glassy phase, there is a slight increase of complexity, i.e., $C^i_{1.75}\simeq 1.585$, which is precisely its individual value. The individual keeps this complexity value until $\epsilon=0.4$, although there is another boundary, separating glassy and coherent phases.

It is clear that now the individual is complex enough to have nonzero statistical complexity and it is simple enough to let the collective induce some amount of complexity. Of course the complexity growth is not as large as in the previous case because here the maximum complexity reached at the boundaries is the same that was reached with individuals of zero complexity. Furthermore, we have not detected any similar growth of complexity for any other ϵ value. To sum up, what has been observed is an intermediate behavior between the two cases previously studied. There is an induced complexity, although smaller than the $\mu=2$ case because of the difference between the individual complexity and the induced complexity and because complexity is not high except at the boundary between turbulent and ordered phases.

V. DISCUSSION

In this paper we have analyzed some computational properties of GCMs. Our interest was to explore the existence of collective-induced computation in some natural systems (such as ant colonies) where the single units behave very simply in isolation and in a complex way when forming part of the entire system. More precisely, we should ask how ant colonies formed by rather simple individuals (when isolated) are able to induce them to perform complex computation, as observed.

The information-theoretic characterization of the phase space has shown that the Markov partition defined on the logistic map provides an adequate characterization. Information transfer, in particular, shows three different types of behavior: It is high at the coherent and ordered phases, close to zero at the turbulent regime, and takes intermediate values for glassy dynamics.

These quantities change rather sharply at the boundaries between different phases. This makes some difference in relation to previous studies, where information transfer becomes maximum at the phase transition (where correlations diverge) [12]. GCMs do not show this type of maximum because of the globally coupled nature of the interactions. But for the same reason we expect to find some generic, common properties (in terms of both computation and dynamical properties) at each phase.

The ϵ-machine reconstruction of single maps close to the onset of chaos gives us a finite automaton with many states (here 31). So at this point we have a complex object in terms of computation. Interestingly, the coupling with other units via GCMs does not modify this complexity. So entities that are computationally complex in isolation do not change in the presence of coupling: Nothing new is induced by the collective. This observation matches the behavior of weakly evolved, primitive ants, where individuals are complex enough to work in isolation and the interactions among them are irrelevant.

However, if we start with random, computationally trivial maps and then couple them, the situation ends up being very different. At $\mu=2.0$ a fully chaotic map is obtained. The

Markov partition of this chaotic (nonfractal) attractor defines a Bernouilli sequence and so we have a $C=0$ complexity. Starting from low couplings, at the turbulent domain, the reconstruction algorithm does not converge, as expected given the disordered, high-dimensional nature of the attractors. In spite of the remaining coherence (as discussed in Sec. II) no finite machines are obtained.

But as we reach the boundary between the turbulent and the ordered phases, the situation changes radically. Now the coherent motion and the spontaneous emergence of clustering also gives rise to well-defined ϵ-machines. Suddenly, the coupling starts to control the dynamics of individuals and they behave in a computationally complex way. Nothing except the coupling has been introduced, but it is enough to generate complexity. As in the real ant colonies discussed in the Introduction, simple isolated individuals can behave in a complex way inside the collective. This is precisely what we have observed. A very important suggestion emerging from this result is that in insect societies complex behavior is only defined at the level of individuals *inside* the colony and not as isolated entities. In this sense, the observed behavior is the result of an emergent property. An interesting observation is that the ϵ-machine reconstruction captures the fine scale implicit in each phase (these phases have internal, fine-scale structure).

Several extensions of this work can be made. One observation in our study was that there is some dependence on the system's size. For some parameter combinations, we found that the automata reconstructed were different as a function of N. This is also interesting insofar as it is well known that social insect colonies use different ways of communicating as a function of the number of individuals engaged. Our preliminary results suggest that these transitions could be also present in the GCM models. Another extension is the finer-scale analysis of the transition points in terms of statistical complexity: Is there a systematic trend? A third extension could be the effects of noise in the reconstruction. As far as noise is an intrinsic part of real systems, we should ask how noise can modify the present results. Finally, one of the remarkable results of Kaneko's study was the presence of coding by means of attractors. The present results immediately suggest a possible connection between such coding mechanism and the underlying finite automaton.

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