Light diffraction at mixed phase and absorption gratings in anisotropic media for arbitrary geometries

G. Montemezzani and M. Zgonik
Nonlinear Optics Laboratory, HPF E17, Institute of Quantum Electronics, Swiss Federal Institute of Technology, CH-8093 Zürich, Switzerland
(Received 3 June 1996)

The coupled wave theory of Kogelnik [H. Kogelnik, Bell Syst. Tech. J. **48**, 2909 (1969)] is extended to the case of moderately absorbing thick anisotropic materials with grating vector and medium boundaries arbitrarily oriented with respect to the main axes of the optical indicatrix. Dielectric and absorption modulation with common grating vector and of arbitrary relative phase shift is considered. Solutions for the wave amplitudes, diffraction efficiencies, and angular mismatch sensitivities are given in transmission and reflection geometries. The main difference of the new results with respect to the expressions valid for isotropic media arise due to the walk-off between the wave-front and energy propagation directions. The difference is particularly important in materials with large birefringence, such as organic crystals, ordered polymers, and liquid crystalline cells. The special case of Bragg diffraction and two-beam coupling at holograms recorded in optically inactive photorefractive crystals is analyzed in detail. It is found that the two-beam coupling gain is influenced substantially by an absorption anisotropy. [S1063-651X(96)10212-9]

PACS number(s): 42.40.Pa, 42.65.Hw, 42.25.Fx, 42.25.Lc

I. INTRODUCTION

Scattering of light in thick holographic media has been the subject of investigation for a long time in the fields of acousto-optics and holographic recording by absorption and photorefractive gratings. The theoretical efforts to understand light diffraction in thick media have culminated in the coupled wave theory of Kogelnik [1], which applies to isotropic materials. Despite the fact that a large fraction of the materials used for volume holography is optically anisotropic, only limited effort has been made to theoretically analyze the diffraction of light in this kind of media [2–6]. Kojima [2] analyzed the problem of diffraction of light at phase gratings in absorptionless anisotropic materials finding solutions in the Raman-Nath diffraction regime using a phase function method and in the Bragg diffraction regime using the Born approximation in the undepleted pump limit. Rokushima and Yamakita [3] developed a matrix formalism to solve the same kind of problems and Johnson and Tanguay [4] analyzed phase gratings using a numerical beam propagation method. Glytsis and Gaylord [5] presented a three-dimensional coupled wave diffraction theory for the study of cascaded anisotropic gratings and waveguide geometries. Vachss and Hesselink [6] considered the case of optically active anisotropic photorefractive media. They found solutions for the Bragg diffraction efficiency in the undepleted pump limit. Dielectric and absorption gratings with a common phase and some special crystal cuts were assumed.

The advent of materials with strong birefringence, such as liquid crystals, ordered polymers, or organic crystals [7–10] in the field of volume holography asks for a novel consideration of the anisotropy effects. In these materials, not only anisotropic [11], but also isotropic Bragg diffraction is strongly affected by the optical anisotropy. The main reason lies in the difference between the energy propagation direction and the wave-front normal. Many materials also show an anisotropic absorption constant, that is, absorption depends strongly on the direction of light polarization. A complete analysis of dielectric and absorption gratings in anisotropic materials should include this effect also.

In this paper we develop a coupled wave theory valid for moderately absorbing nonoptically active anisotropic thick media. The phase and absorption gratings in these media may have an arbitrary relative phase shift. The model is valid for every direction of the grating wave-vector in three dimensions. The entrance and exit surfaces of the medium are parallel to each other and may have an arbitrary orientation with respect to the main axis of the optical indicatrix. We treat the cases of transmission and reflection gratings, the former being characterized by a diffracted beam exiting the medium through the same surface as the transmitted incident beam, the latter being characterized by a diffracted beam back reflected through the incidence surface. The coupled wave equations are solved for both grating types to give the diffraction efficiency and the angle-mismatch sensitivity. The special case of photorefractive phase gratings is discussed in a separate section, where we also discuss the correct expression for the light modulation index that has to be used while considering two-wave mixing processes induced by self-generated gratings.

Section II brings the basic equations and the derivation of the two coupled wave equations valid, in general, in anisotropic media. In Sec. III the general solution of the coupled wave equations for diffraction at transmission gratings is derived. Section IV is devoted to reflection gratings, while Sec. V presents the special cases of Bragg diffraction and two-beam coupling at photorefractive phase gratings. The linear propagation of waves is treated in Appendix A, where we derive the relationship between the complex dielectric tensor and the effective absorption constant that describe the propagation of a wave submitted to boundary conditions at the entrance surface of the medium.

II. BASIC EQUATIONS

We consider a medium containing a phase (refractive index) and/or an absorption grating. Our analysis treats the case of thick holograms only. An exact definition of a thick grating has been given by Gaylord and Moharam [12] and the conditions to be fulfilled are $Q=K^2\lambda d/(2\pi n)>1$ and $\rho=\lambda^2/(\Lambda^2 n\sigma)\geqslant 10$, where $\sigma=\Delta n$ for dielectric gratings and $\sigma=\Delta\alpha\lambda/2\pi$ for absorption gratings. In our case of anisotropic materials the refractive index change Δn and the absorption modulation $\Delta\alpha$ are defined later in connection with Eqs. (41) and (44), respectively. The other quantities in the two above conditions are the medium thickness d, the vacuum wavelength λ, the average refractive index n, the grating spacing Λ, and the grating wave vector $K=2\pi/\Lambda$. We notice that if the two above conditions are not strictly fulfilled the diffraction may be described by a mixture of Bragg and Raman-Nath regime. In such an intermediate regime the theory presented in this work gives only approximate results and the diffraction would be calculated more precisely by a rigorous coupled wave analysis similar to the one presented earlier for the isotropic case [13].

As shown by Kogelnik [1] for thick gratings it is sufficient to consider the propagation of only two plane waves p and s. Since we consider the general case of anisotropic materials the waves p and s should represent eigenwaves of the medium. The total electric field amplitude is given by

$$\vec{\mathcal{E}}(\vec{r},t)=[\vec{\mathcal{E}}_s(\vec{r})e^{i\vec{k}_s\cdot\vec{r}}+\vec{\mathcal{E}}_p(\vec{r})e^{i\vec{k}_p\cdot\vec{r}}]e^{-i\omega t}+\text{c.c.}, \tag{1}$$

where $\vec{\mathcal{E}}_s$ and $\vec{\mathcal{E}}_p$ are complex amplitudes cleaned of the absorption contribution. This means that they are always constant in the absence of nonlinear effects, as explained later. In absorbing crystals the wave vectors $\vec{k}_s$ and $\vec{k}_p$ are complex with the imaginary part, which possibly has a different direction than the real part [14]

$$\vec{k}_s=\vec{k}_{s,r}+i\vec{k}_{s,i}\,,\quad \vec{k}_p=\vec{k}_{p,r}+i\vec{k}_{p,i}\,. \tag{2}$$

The real part, as usual, is related to the wave-front propagation direction for an eigenpolarization in the crystal, while the imaginary part is related to the linear absorption experienced by the waves and is calculated as derived in Appendix A. The wave of Eq. (1) has to fulfill the time-independent vector wave equation

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{\mathcal{E}})-k_0^2\overleftrightarrow{\epsilon}\cdot\vec{\mathcal{E}}=0, \tag{3}$$

where $\overleftrightarrow{\epsilon}=\overleftrightarrow{\epsilon}_r+i\overleftrightarrow{\epsilon}_i$ is the complex second-rank dielectric tensor that includes the effects of the material refractive index and absorption [15], and $k_0=\omega/c$ is the free-space wave number. From now on the explicit time dependence $\exp(-i\omega t)$ will always be dropped. We consider a medium containing a phase and/or an amplitude plane holographic grating. The complex dielectric tensor $\overleftrightarrow{\epsilon}$ can then be expressed as

$$\begin{aligned}\overleftrightarrow{\epsilon}&=[\overleftrightarrow{\epsilon}{}_r^0+\overleftrightarrow{\epsilon}{}_r^1\cos(\vec{K}\cdot\vec{r})]+i[\overleftrightarrow{\epsilon}{}_i^0+\overleftrightarrow{\epsilon}{}_i^1\cos(\vec{K}\cdot\vec{r}+\phi)]\\&=[\overleftrightarrow{\epsilon}{}_r^0+\tfrac{1}{2}\overleftrightarrow{\epsilon}{}_r^1(e^{i\vec{K}\cdot\vec{r}}+e^{-i\vec{K}\cdot\vec{r}})]\\&\quad+i[\overleftrightarrow{\epsilon}{}_i^0+\tfrac{1}{2}\overleftrightarrow{\epsilon}{}_i^1(e^{i(\vec{K}\cdot\vec{r}+\phi)}+e^{-i(\vec{K}\cdot\vec{r}+\phi)}),\end{aligned} \tag{4}$$

where the superscripts 0 and 1 denote the constant and the modulated component, respectively. The grating vector $\vec{K}$ in Eq. (4) has an arbitrary direction with respect to the geometrical or crystallographic axis of the anisotropic medium. The absorption grating [modulated term in the imaginary part of Eq. (4)] may be phase shifted by a phase ϕ with respect to the refractive index grating. We may choose our coordinate system to coincide with the main axes of the optical indicatrix so that the tensor $\overleftrightarrow{\epsilon}{}_r^0$ contains only diagonal elements. In contrast, the modulated part $\overleftrightarrow{\epsilon}{}_r^1$ of the real dielectric tensor is generally nondiagonal. That is

$$\overleftrightarrow{\epsilon}{}_r^0=\begin{pmatrix}\epsilon_{r,11}^0&0&0\\0&\epsilon_{r,22}^0&0\\0&0&\epsilon_{r,33}^0\end{pmatrix},\quad \overleftrightarrow{\epsilon}{}_r^1=\begin{pmatrix}\epsilon_{r,11}^1&\epsilon_{r,12}^1&\epsilon_{r,13}^1\\\epsilon_{r,12}^1&\epsilon_{r,22}^1&\epsilon_{r,23}^1\\\epsilon_{r,13}^1&\epsilon_{r,23}^1&\epsilon_{r,33}^1\end{pmatrix}, \tag{5}$$

e.g., nondiagonal elements can be produced by shear acoustic waves and by space-charge induced electro-optic effects [16]. For crystalline materials with orthorhombic or higher symmetry the main axes of the imaginary dielectric tensor coincide with those of the real one [15]. For these materials also $\overleftrightarrow{\epsilon}{}_i^0$ and $\overleftrightarrow{\epsilon}{}_i^1$ are diagonal tensors

$$\overleftrightarrow{\epsilon}{}_i^0=\begin{pmatrix}\epsilon_{i,11}^0&0&0\\0&\epsilon_{i,22}^0&0\\0&0&\epsilon_{i,33}^0\end{pmatrix},\quad \overleftrightarrow{\epsilon}{}_i^1=\begin{pmatrix}\epsilon_{i,11}^1&0&0\\0&\epsilon_{i,22}^1&0\\0&0&\epsilon_{i,33}^1\end{pmatrix}. \tag{6}$$

For crystals with lower symmetry the main axes of the absorption ellipsoid may differ from those of the refractive index ellipsoid [17] and the tensors $\overleftrightarrow{\epsilon}{}_i^0$ and $\overleftrightarrow{\epsilon}{}_i^1$ may contain also nondiagonal elements in our coordinate system. We want to consider only materials with positive absorption (no gain). This property has to be fulfilled for any wave polarization and any position in the crystal, thus giving some constraints on the elements of the tensors $\overleftrightarrow{\epsilon}{}_i^0$ and $\overleftrightarrow{\epsilon}{}_i^1$,

$$\epsilon_{i,kl}^0\geqslant\epsilon_{i,kl}^1\geqslant 0. \tag{7}$$

We proceed by analyzing the coupled wave equations and we insert Eqs. (4) and (1) into the wave equation (3). We notice that the first term of Eq. (3) can be represented in the following form:

$$\begin{aligned}\vec{\nabla}\times(\vec{\nabla}\times\vec{\mathcal{E}})&\\&=e^{i\vec{k}_s\cdot\vec{r}}\{\vec{\nabla}\times\vec{\nabla}\times\vec{\mathcal{E}}_s-i[(\vec{\nabla}\times\vec{\mathcal{E}}_s)\times\vec{k}_s+\vec{\nabla}\times(\vec{\mathcal{E}}_s\times\vec{k}_s)]\\&\quad-(\vec{\mathcal{E}}_s\times\vec{k}_s)\times\vec{k}_s\}+e^{i\vec{k}_p\cdot\vec{r}}\{\cdots\},\end{aligned} \tag{8}$$

where we have listed only the terms proportional to $\exp[i\vec{k}_s\cdot\vec{r}]$ and the second set of curly brackets contains analogous terms in $\vec{\mathcal{E}}_p$ and $\vec{k}_p$. The first term on the right-hand side of Eq. (8) contains only second-order derivatives

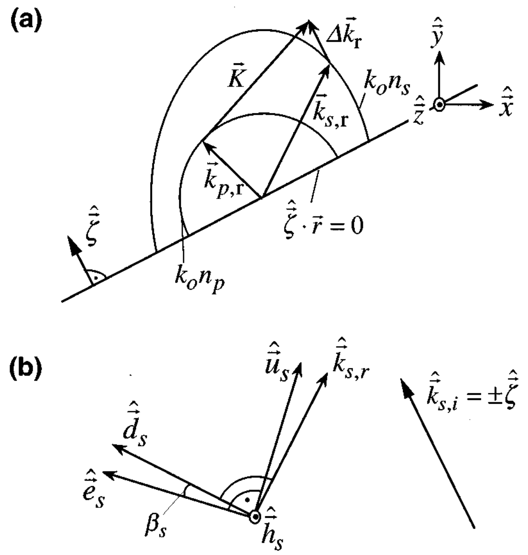

FIG. 1. (a) Projection of the wave-vector diagram for the holographic interaction. The coordinate axes are parallel to the main axes of the optical indicatrix. The input surface plane $\hat{\vec{\zeta}}\cdot\vec{r}=0$ does not necessarily contain the axis $\hat{\vec{z}}$. The vectors $\vec{k}_{p,r}$, $\vec{k}_{s,r}$, $\vec{K}$, and $\Delta\vec{k}_r$ do not need to be all coplanar. (b) Unit vectors in direction of the electric field ($\hat{\vec{e}}_s$), the dielectric displacement ($\hat{\vec{d}}_s$), the magnetic field ($\hat{\vec{h}}_s$), the energy propagation ($\hat{\vec{u}}_s$), the real and imaginary component of the propagation vector ($\hat{\vec{k}}_{s,r}$,$\hat{\vec{k}}_{s,i}$) for the wave s, and the input surface normal ($\hat{\vec{\zeta}}$). It holds $\hat{\vec{e}}_s\perp\hat{\vec{u}}_s\perp\hat{\vec{h}}_s$, $\hat{\vec{d}}_s\perp\hat{\vec{k}}_{s,r}\perp\hat{\vec{h}}_s$, and $\hat{\vec{e}}_s\cdot\hat{\vec{d}}_s=\hat{\vec{u}}_s\cdot\hat{\vec{k}}_{s,r}=\cos\beta_s$.

of the wave amplitude and can be neglected applying the slowly varying amplitude approximation. The last term together with the second term of Eq. (3) that contains the contribution of the nonmodulated dielectric tensors describe the linear propagation of the wave as discussed in Appendix A. For the s wave it is

$$-[(\vec{\mathcal{E}}_s\times\vec{k}_s)\times\vec{k}_s]e^{i\vec{k}_s\cdot\vec{r}}=k_0^2[\overleftrightarrow{\epsilon}_r^{\,0}+i\overleftrightarrow{\epsilon}_i^{\,0}]\cdot\vec{\mathcal{E}}_s e^{i\vec{k}_s\cdot\vec{r}}, \qquad (9)$$

and an analogous expression holds for the p wave.

The second and third terms on the right-hand side of Eq. (8), which are left, describe the coupling of the waves due to $\overleftrightarrow{\epsilon}_r^1$ and $\overleftrightarrow{\epsilon}_i^1$. The problem that we are analyzing is interesting for perfect phase matching and for small phase mismatch [Fig. 1(a)]. In this case we write the momentum conservation equation as

$$\vec{k}_p+\vec{K}=\vec{k}_s+\Delta\vec{k},$$

$$\Delta\vec{k}\equiv\Delta\vec{k}_r+i\Delta\vec{k}_i=(\vec{k}_{p,r}-\vec{k}_{s,r}+\vec{K})+i(\vec{k}_{p,i}-\vec{k}_{s,i}). \quad (10)$$

Using the above arguments Eq. (3) transforms in the two coupled wave equations

$$e^{i\vec{k}_s\cdot\vec{r}}[(\vec{\nabla}\times\vec{\mathcal{E}}_s)\times\vec{k}_s+\vec{\nabla}\times(\vec{\mathcal{E}}_s\times\vec{k}_s)]$$

$$=\frac{k_0^2}{2}[i\overleftrightarrow{\epsilon}_r^1\cdot\vec{\mathcal{E}}_p-\overleftrightarrow{\epsilon}_i^1\cdot\vec{\mathcal{E}}_p e^{i\phi}]e^{i\vec{k}_s\cdot\vec{r}}e^{i\Delta\vec{k}\cdot\vec{r}}, \qquad (11)$$

$$e^{i\vec{k}_p\cdot\vec{r}}[(\vec{\nabla}\times\vec{\mathcal{E}}_p)\times\vec{k}_p+\vec{\nabla}\times(\vec{\mathcal{E}}_p\times\vec{k}_p)]$$

$$=\frac{k_0^2}{2}[i\overleftrightarrow{\epsilon}_r^1\cdot\vec{\mathcal{E}}_s-\overleftrightarrow{\epsilon}_i^1\cdot\vec{\mathcal{E}}_s e^{-i\phi}]e^{i\vec{k}_p\cdot\vec{r}}e^{-i\Delta\vec{k}\cdot\vec{r}}. \qquad (12)$$

Using some vector algebra the terms on the left-hand side of Eq. (11) can be rewritten as

$$(\vec{\nabla}\times\vec{\mathcal{E}}_s)\times\vec{k}_s=|\vec{k}_{s,r}|\left\{\left(\hat{\vec{k}}_{s,r}\cdot\frac{\partial E_s}{\partial\vec{r}}\right)\hat{\vec{e}}_s-(\hat{\vec{e}}_s\cdot\hat{\vec{k}}_{s,r})\frac{\partial E_s}{\partial\vec{r}}\right\}$$

$$+i|\vec{k}_{s,i}|\left\{\left(\hat{\vec{k}}_{s,i}\cdot\frac{\partial E_s}{\partial\vec{r}}\right)\hat{\vec{e}}_s-(\hat{\vec{e}}_s\cdot\hat{\vec{k}}_{s,i})\frac{\partial E_s}{\partial\vec{r}}\right\}, \qquad (13)$$

and

$$\vec{\nabla}\times(\vec{\mathcal{E}}_s\times\vec{k}_s)=|\vec{k}_{s,r}|\left\{\left(\hat{\vec{k}}_{s,r}\cdot\frac{\partial E_s}{\partial\vec{r}}\right)\hat{\vec{e}}_s-\left(\hat{\vec{e}}_s\cdot\frac{\partial E_s}{\partial\vec{r}}\right)\hat{\vec{k}}_{s,r}\right\}$$

$$+i|\vec{k}_{s,i}|\left\{\left(\hat{\vec{k}}_{s,i}\cdot\frac{\partial E_s}{\partial\vec{r}}\right)\hat{\vec{e}}_s-\left(\hat{\vec{e}}_s\cdot\frac{\partial E_s}{\partial\vec{r}}\right)\hat{\vec{k}}_{s,i}\right\}, \qquad (14)$$

where $\vec{\mathcal{E}}_s=E_s\hat{\vec{e}}_s$ and $\hat{\vec{e}}_s$, $\hat{\vec{k}}_{s,r}$, and $\hat{\vec{k}}_{s,i}$ are real unit vectors along the electric-field vector and the real and imaginary wave vectors of the wave s, as shown in Fig. 1(b). $\partial E_s/\partial\vec{r}\equiv\vec{\nabla}E_s$ is the gradient of the scalar complex wave amplitude E_s. Similar expressions to Eqs. (13) and (14) hold for the wave p and the left-hand side of Eq. (12).

In this paper we consider only waves that are sufficiently far from the absorption resonance of the medium. In this limit one has only moderate absorption, that is $|\vec{k}_{s,i}|\ll|\vec{k}_{s,r}|$ and $|\vec{k}_{p,i}|\ll|\vec{k}_{p,r}|$. We can therefore neglect the terms involving $|\vec{k}_{s,i}|$ in Eqs. (13) and (14). All relationships derived in this work are valid in this limit. Summing Eqs. (13) and (14) and multiplying both sides of Eq. (11) with the unit vector $\hat{\vec{e}}_s$ one obtains

$$2|\vec{k}_{s,r}|\left\{\frac{\partial E_s}{\partial\vec{r}}\cdot[\hat{\vec{k}}_{s,r}-\hat{\vec{e}}_s(\hat{\vec{e}}_s\cdot\hat{\vec{k}}_{s,r})]\right\}$$

$$=\frac{k_0^2}{2}[i\hat{\vec{e}}_s\cdot\overleftrightarrow{\epsilon}_r^1\cdot\hat{\vec{e}}_p-\hat{\vec{e}}_s\cdot\overleftrightarrow{\epsilon}_i^1\cdot\hat{\vec{e}}_p e^{i\phi}]E_p e^{i\Delta\vec{k}\cdot\vec{r}}, \qquad (15)$$

where in the terms of the kind $\hat{\vec{e}}_s^T\cdot\overleftrightarrow{\epsilon}_r^1\cdot\hat{\vec{e}}_p$ we omit the transpose sign in the first vector in order to simplify the notation. The left-hand side vector expression in the square brackets gives a vector that is parallel to the energy propagation direction (Poynting vector) of the wave s [15]. One can write

$$\hat{\vec{k}}_{s,r}-\hat{\vec{e}}_s(\hat{\vec{e}}_s\cdot\hat{\vec{k}}_{s,r})=g_s\hat{\vec{u}}_s \qquad (16)$$

with $\hat{\vec{u}}_s$ being the unit vector along the Poynting vector [Fig. 1(b)]. Using $\hat{\vec{k}}_{s,r}\cdot\hat{\vec{u}}_s=\hat{\vec{e}}_s\cdot\hat{\vec{d}}_s=\cos\beta_s$ and $\hat{\vec{k}}_{s,r}\cdot\hat{\vec{d}}_s=\hat{\vec{e}}_s\cdot\hat{\vec{u}}_s=0$ we get $g_s=\hat{\vec{e}}_s\cdot\hat{\vec{d}}_s=\cos\beta_s$. The unit vector $\hat{d}_s$ points in the direction of the electric displacement vector for the wave s. Introducing the unperturbed refractive indices n_s and n_p seen

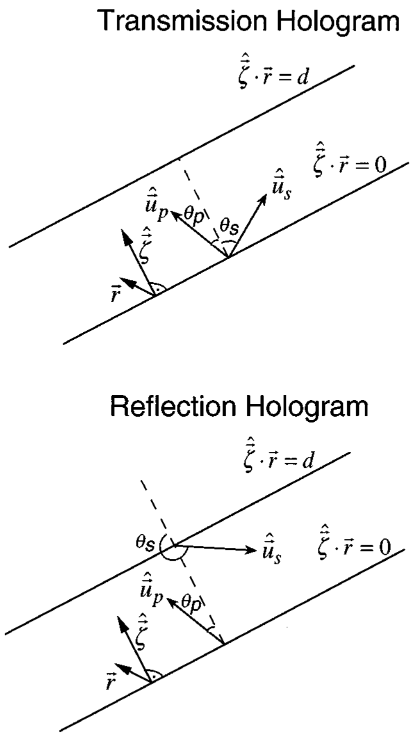

FIG. 2. Beam propagation directions for transmission holograms and reflection holograms.

by the signal and pump wave, respectively, and with $|\vec{k}_{s,r}| = k_0 n_s$ and $|\vec{k}_{p,r}| = k_0 n_p$, the coupled wave equations (11) and (12) are rewritten as

$$\frac{\partial E_s}{\partial \vec{r}} \cdot \hat{\vec{u}}_s = \frac{k_0}{4 n_s g_s} [i \hat{\vec{e}}_s \cdot \overleftrightarrow{\epsilon}^1_r \cdot \hat{\vec{e}}_p - \hat{\vec{e}}_s \cdot \overleftrightarrow{\epsilon}^1_i \cdot \hat{\vec{e}}_p e^{i\phi}] E_p e^{i\Delta\vec{k}\cdot\vec{r}}, \quad (17)$$

$$\frac{\partial E_p}{\partial \vec{r}} \cdot \hat{\vec{u}}_p = \frac{k_0}{4 n_p g_p} [i \hat{\vec{e}}_p \cdot \overleftrightarrow{\epsilon}^1_r \cdot \hat{\vec{e}}_s - \hat{\vec{e}}_p \cdot \overleftrightarrow{\epsilon}^1_i \cdot \hat{\vec{e}}_s e^{-i\phi}] E_s e^{-i\Delta\vec{k}\cdot\vec{r}}, \quad (18)$$

where $g_p = \hat{\vec{e}}_p \cdot \hat{\vec{d}}_p$. Equations (17) and (18) describe the coupling of two plane waves in any general geometry in anisotropic media containing phase and/or absorption gratings. It is important to notice that the coupling terms describe the projection of the amplitude gradients along the Poynting vector direction of the corresponding wave and not along the wave-vector direction as in the theory of Kogelnik [1] that applies to isotropic materials only.

III. TRANSMISSION GRATINGS

A. Mixed transmission gratings

We consider first transmission gratings, that is both beams s and p leave the material through the same surface. More precisely, this geometry is characterized by the condition $(\hat{\vec{u}}_p \cdot \hat{\vec{\zeta}})(\hat{\vec{u}}_s \cdot \hat{\vec{\zeta}}) = \cos\theta_p \cos\theta_s > 0$, where $\hat{\vec{\zeta}}$ is the unit vector in the direction of the normal to the entrance surface of the wave P in the holographic medium (Fig. 2). We assume the medium to be infinite in the directions normal to $\hat{\vec{\zeta}}$. We look for a general expression for the holographic diffraction efficiency under pump depletion conditions. To find the spatial evolution of the signal wave S we extract E_p from Eq. (17) and insert it into Eq. (18) to get the second-order differential equation

$$\left(\frac{\partial^2 E_s}{\partial \vec{r}^2} \cdot \hat{\vec{u}}_s\right) \cdot \hat{\vec{u}}_p - i \left(\frac{\partial E_s}{\partial \vec{r}} \cdot \hat{\vec{u}}_s\right) (\Delta\vec{k} \cdot \hat{\vec{u}}_p) + \frac{k_0^2}{16 n_s n_p g_s g_p} E_s [A_r^2 - A_i^2 + 2 i A_r A_i \cos\phi] = 0, \quad (19)$$

where $\partial^2 / \partial \vec{r}^2 \equiv \vec{\nabla} \otimes \vec{\nabla}$ and $\otimes$ indicates outer product. The coupling constants A_r and A_i are defined as

$$A_r = \hat{\vec{e}}_s \cdot \overleftrightarrow{\epsilon}^1_r \cdot \hat{\vec{e}}_p = \hat{\vec{e}}_p \cdot \overleftrightarrow{\epsilon}^1_r \cdot \hat{\vec{e}}_s, \quad (20)$$

$$A_i = \hat{\vec{e}}_s \cdot \overleftrightarrow{\epsilon}^1_i \cdot \hat{\vec{e}}_p = \hat{\vec{e}}_p \cdot \overleftrightarrow{\epsilon}^1_i \cdot \hat{\vec{e}}_s, \quad (21)$$

where the second equalities are valid because the tensors $\overleftrightarrow{\epsilon}^1_r$ and $\overleftrightarrow{\epsilon}^1_i$ are symmetric. The boundary conditions for diffraction from a transmission grating are

$$E_s(\hat{\vec{\zeta}} \cdot \vec{r} = 0) = 0 \quad (22)$$

and

$$\frac{\partial E_s}{\partial \vec{r}} \cdot \hat{\vec{u}}_s(\hat{\vec{\zeta}} \cdot \vec{r} = 0) = \frac{k_0}{4 n_s g_s} [i A_r - A_i e^{i\phi}] E_{p0} e^{i\Delta\vec{k}\cdot\vec{r}}, \quad (23)$$

where $E_{p0} = E_p(\hat{\vec{\zeta}} \cdot \vec{r} = 0)$ is the pump wave amplitude at the entrance face of the anisotropic holographic medium. The general solution of the differential equation (19) has the form

$$E_s = E_{s1} \exp(\vec{\gamma}_1 \cdot \vec{r}) + E_{s2} \exp(\vec{\gamma}_2 \cdot \vec{r}), \quad (24)$$

where E_{s1} and E_{s2} are complex constants. The direction of the vectors $\vec{\gamma}_1$ and $\vec{\gamma}_2$ is not strictly defined because inserting Eq. (24) into Eq. (19) one obtains constraints only on the scalar products $\vec{\gamma} \cdot \hat{\vec{u}}_s$ and $\vec{\gamma} \cdot \hat{\vec{u}}_p$. In view of the boundary conditions given by Eqs. (22) and (23) it is useful to choose $\vec{\gamma}_1$ and $\vec{\gamma}_2$ parallel to the surface normal $\hat{\vec{\zeta}}$, which gives

$$\vec{\gamma}_{1,2} = \left(i \frac{\Delta\vec{k} \cdot \hat{\vec{u}}_p}{2 \cos\theta_p} \pm i W \right) \hat{\vec{\zeta}}, \quad (25)$$

where $W = \sqrt{W^2}$ is a complex quantity with

$$W^2 = \left(\frac{\Delta\vec{k} \cdot \hat{\vec{u}}_p}{2 \cos\theta_p} \right)^2 + \frac{k_0^2}{16 n_s n_p g_s g_p \cos\theta_s \cos\theta_p} \times (A_r^2 - A_i^2 + 2 i A_r A_i \cos\phi) \quad (26)$$

and

$$\cos\theta_s = \hat{\vec{\zeta}} \cdot \hat{\vec{u}}_s, \quad \cos\theta_p = \hat{\vec{\zeta}} \cdot \hat{\vec{u}}_p. \quad (27)$$

Note that all projection cosines in Eq. (26) are taken with respect to the Poynting vector direction and not with respect to the wave-vector direction. The constants E_{s1} and E_{s2} are obtained by using Eqs. (22) and (23) and one finds

$$E_{s1}=-E_{s2}=\frac{k_0}{8n_s g_s \cos\theta_s}\, e^{i\Delta\vec{k}\cdot\vec{r}_\parallel}\,\frac{A_r+iA_i e^{i\phi}}{W}\,E_{p0}, \tag{28}$$

where $\vec{r}_\parallel$ is a position vector on the entrance surface defined by $\hat{\vec{\zeta}}\cdot\vec{r}=0$. The constant E_{s1} is $\vec{r}$ independent and Eq. (24) fulfills Eq. (19) only if $\Delta\vec{k}\cdot\vec{r}_\parallel=0$ for all $\vec{r}_\parallel$, thus constraining the real and imaginary part of the vector $\Delta\vec{k}$ to be parallel to the normal to the surface $\hat{\vec{\zeta}}$, as shown in Fig. 1(a). This property is a direct consequence of the fact that waves and gratings have infinite extent in the transversal directions. The wave-front propagation direction $\vec{k}_s$ of the wave s is now well defined and is obtained with Eq. (10). Inserting the complex amplitudes (28) and the complex gain constants (25) into Eq. (24) one finds the general solution for the evolution of the signal wave amplitude

$$E_s(\vec{r})=\frac{k_0}{8n_s g_s\cos\theta_s}\,\frac{A_r+iA_i e^{i\phi}}{W}\,e^{i[(\Delta k_r+i\Delta k_i)/2]\hat{\vec{\zeta}}\cdot\vec{r}} \times[e^{iW(\hat{\vec{\zeta}}\cdot\vec{r})}-e^{-iW(\hat{\vec{\zeta}}\cdot\vec{r})}]E_{p0}, \tag{29}$$

where we have defined the real scalar mismatch quantities Δk_r and Δk_i by $\Delta\vec{k}_r=\Delta k_r\hat{\vec{\zeta}}$ and $\Delta\vec{k}_i=\Delta k_i\hat{\vec{\zeta}}$. In analogy, one can also find the wave amplitude of the transmitted pump wave, which is

$$E_p(\vec{r})=e^{-i[(\Delta k_r+i\Delta k_i)/2]\hat{\vec{\zeta}}\cdot\vec{r}}\left[\frac{2W+(\Delta k_r+i\Delta k_i)}{4W}\,e^{iW(\hat{\vec{\zeta}}\cdot\vec{r})} +\frac{2W-(\Delta k_r+i\Delta k_i)}{4W}\,e^{-iW(\hat{\vec{\zeta}}\cdot\vec{r})}\right]E_{p0}. \tag{30}$$

One can now calculate the diffraction efficiency defined as the ratio of the output-signal intensity to the incident-pump intensity

$$\eta=\frac{I_s(\hat{\vec{\zeta}}\cdot\vec{r}=d)}{I_p(\hat{\vec{\zeta}}\cdot\vec{r}=0)}=\frac{E_sE_s^*n_sg_s}{E_{p0}E_{p0}^*n_pg_p}\,\frac{\cos\theta_s}{\cos\theta_p}\,e^{-2\vec{k}_{s,i}\cdot\vec{r}}, \tag{31}$$

where we recall that g_s and g_p are projection cosines between $\hat{\vec{e}}$ and $\hat{\vec{d}}$ [Eq. (16)]. The factor $\cos\theta_s/\cos\theta_p$ is an obliquity term that assures consistent results in a general case when we are interested in the optical energy flow through the input and output surfaces of the medium. The term n_sg_s/n_pg_p has been often overlooked in the literature. Neglecting this term is allowed only in isotropic materials or in anisotropic materials in the case of a configuration fully symmetric with respect to the axis $\hat{\vec{\zeta}}$ and the optical indicatrix. Using the expression for the diffraction efficiency (31) and the solution for the signal wave amplitude $E_s(\vec{r})$ (29) one obtains the general expression

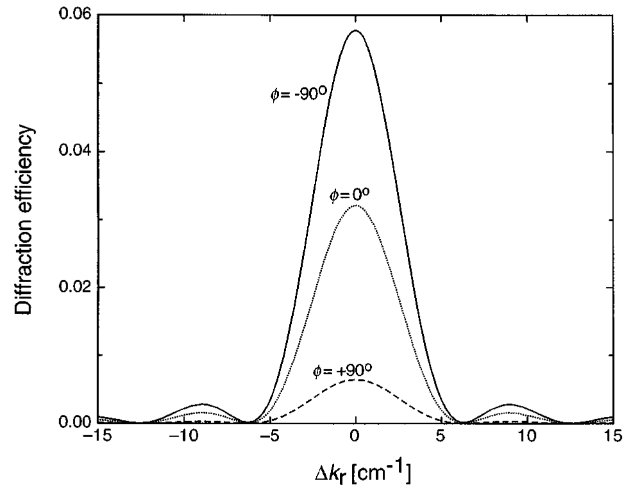

FIG. 3. Mixed transmission grating. Diffraction efficiency vs the real grating mismatch parameter Δk_r for three values of the phase-shift angle ϕ between phase and absorption grating. Parameters: $A_r=2\times10^{-5}$, $A_i=1\times10^{-5}$, $\lambda=633$ nm, $d=1$ cm, $\alpha_s=0.4$ cm^{-1}, $\alpha_p=0.7$ cm^{-1}, $n_s=2.2$, $n_p=2.0$, $g_s=1.0$, $g_p=0.95$, $\theta_s=10°$, and $\theta_p=-40°$.

$$\eta(\hat{\vec{\zeta}}\cdot\vec{r}=d) =\frac{k_0^2}{16n_sn_pg_sg_p\cos\theta_s\cos\theta_p}\,\frac{A_r^2+A_i^2-2A_rA_i\sin\phi}{|W^2|} \times\{\sin^2(\mathrm{Re}[W]d)+\sinh^2(\mathrm{Im}[W]d)\}e^{-(\alpha_s+\alpha_p)d}. \tag{32}$$

The quantities $\alpha_s=|\vec{k}_{s,i}|$ and $\alpha_p=|\vec{k}_{p,i}|$ are the effective amplitude absorption constants experienced by the signal wave s and pump wave p in direction $\hat{\vec{\zeta}}$, respectively, as derived in Appendix A [Eq. (A9)], that is

$$\alpha_s=\frac{k_0(\hat{\vec{e}}_s\cdot\overleftrightarrow{\epsilon}_i^{\,0}\cdot\hat{\vec{e}}_s)}{2n_sg_s|\cos\theta_s|},\quad \alpha_p=\frac{k_0(\hat{\vec{e}}_p\cdot\overleftrightarrow{\epsilon}_i^{\,0}\cdot\hat{\vec{e}}_p)}{2n_pg_p|\cos\theta_p|}. \tag{33}$$

It should be noted that the effective absorption constants for the waves s and p differ even in a fully isotropic situation if their directions of propagation are not symmetric with respect to the surface normal $\hat{\vec{\zeta}}$, as is expected due to a different propagation distance of the two waves inside the absorbing medium.

Equation (32) describes completely the diffraction at a mixed phase and absorption transmission grating in anisotropic media. As an example, Fig. 3 shows that the total diffraction efficiency strongly depends on the phase shift ϕ between phase and absorption grating, which is in agreement with an analysis of mixed phase and absorption gratings in isotropic media by Guibelalde [18]. This behavior is easily explained by the interference of the waves scattered off the phase and absorption grating, respectively.

B. Transmission gratings with refractive index modulation only

We consider here the case where the grating consists only of a refractive index modulation. In absence of absorption modulation we have $A_i=0$ and the quantity W^2 can be simplified and rewritten as

$$W^2=\frac{1}{d^2}\,(\nu^2+\xi^2+i\chi^2), \tag{34}$$

where we have defined the real quantities

$$\nu^2=\frac{k_0^2A_r^2}{16n_sn_pg_sg_p\cos\theta_s\cos\theta_p}\,d^2, \tag{35}$$

$$\xi^2=\frac{\Delta k_r^2-\Delta k_i^2}{4}\,d^2=\left[\frac{\Delta k_r^2}{4}-\frac{(\alpha_p-\alpha_s)^2}{4}\right]d^2, \tag{36}$$

and

$$\chi^2=\frac{\Delta\vec{k}_r\cdot\Delta\vec{k}_i}{2}\,d^2=\left[\frac{\Delta k_r(\alpha_p-\alpha_s)}{2}\right]d^2. \tag{37}$$

The diffraction efficiency of Eq. (32) reads then

$$\eta(d)=\frac{\nu^2}{\sqrt{(\nu^2+\xi^2)^2+\chi^4}}\times\left\{\sin^2\left(\frac{(\nu^2+\xi^2)+\sqrt{(\nu^2+\xi^2)^2+\chi^4}}{2}\right)^{1/2}+\sinh^2\left(\frac{-(\nu^2+\xi^2)+\sqrt{(\nu^2+\xi^2)^2+\chi^4}}{2}\right)^{1/2}\right\}\times e^{-(\alpha_s+\alpha_p)d}. \tag{38}$$

Note that the arguments of the $\sin^2$ and $\sinh^2$ functions are always real although ξ^2 and χ^2 can be negative numbers. We notice also that even in absence of absorption modulation there is still a term proportional to $\sinh^2$. This term takes accurately into account the effect on the diffraction efficiency of a different absorption constant for the pump and signal waves. It vanishes if the effective absorption constant seen by the two waves is the same ($\alpha_s=\alpha_p=\alpha$, $\chi^2=0$), in which case Eq. (38) simplifies further to

$$\eta(d)=\frac{\sin^2\sqrt{\nu^2+\xi^2}}{(1+\xi^2/\nu^2)}\,e^{-2\alpha d}. \tag{39}$$

Equation (39) has exactly the same form as the one given in Ref. [1]. However the quantities ν^2, ξ^2, and α are defined differently. The quantity ξ^2 in this case reduces to

$$\xi^2=\frac{\Delta k_r^2}{4}\,d^2, \tag{40}$$

ν^2 is redefined according to Eq. (35) with the projection cosines given by Eq. (27), and the effective amplitude absorption constant α is given by Eq. (A9).

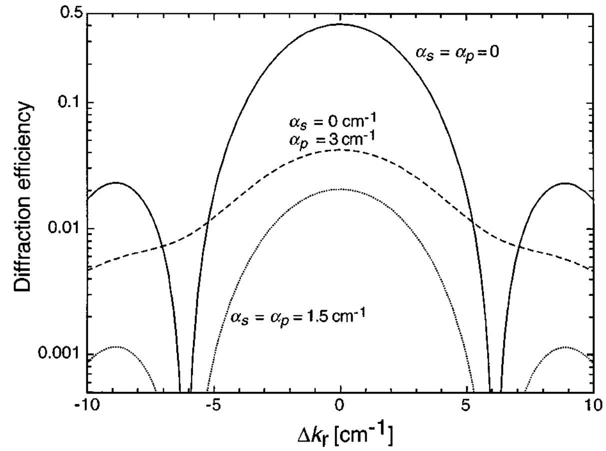

FIG. 4. Effect of absorption on diffraction efficiency and Bragg-angle selectivity. Parameters: $A_r=5\times10^{-5}$, $A_i=0$, $\lambda=633$ nm, $d=1$ cm, $n_s=2.2$, $n_p=2.0$, $g_s=1.0$, $g_p=0.95$, $\theta_s=10°$, and $\theta_p=-40°$.

A further simplification is obtained in the case of perfect Bragg matching ($\Delta\vec{k}_r=\vec{0}$, $\xi^2=0$). In this case Eq. (39) becomes

$$\eta(d)=\sin^2\left(\frac{\pi A_rd}{2\lambda(n_sn_pg_sg_p\cos\theta_s\cos\theta_p)^{1/2}}\right)e^{-2\alpha d}, \tag{41}$$

where λ is the vacuum wavelength. The argument of the sin function is of the form $(\pi\Delta nd/\lambda\cos\theta)$ in analogy with Ref. [1], with $\Delta n=A_r/[2(n_sn_pg_sg_p)^{1/2}]$ and $\cos\theta=(\cos\theta_s\cos\theta_p)^{1/2}$. In nonabsorbing materials the maximum possible diffraction efficiency is exactly 100% for phase-only gratings, regardless of the fact whether isotropic or anisotropic diffraction processes are considered.

The effect of the background absorption α_s and α_p on the Bragg-angle selectivity of a phase-only grating is shown in Fig. 4 [Eq. (38)]. The main effect of absorption is to reduce the maximum diffraction efficiency. In addition, a broadening of the Bragg selectivity curve is observed if signal and pump are absorbed differently ($\alpha_s\neq\alpha_p$). For a given total absorption ($\alpha_s+\alpha_p$) the more favorable diffraction efficiency is found when the absorption difference between signal and pump is maximum. A strong difference in the effective absorption for the two waves may be observed in a number of crystals under anisotropic Bragg diffraction geometries. It should be noted that, despite the fact that the mismatch term ξ^2 in Eq. (36) contains the term $(\alpha_p-\alpha_s)$, there is no shift in the Bragg angle for $(\alpha_s\neq\alpha_p)$, i.e., the maximum diffraction efficiency is still obtained for $\Delta k_r=0$. The absorption characteristics can introduce a significant shift in the direction for which one observes the maximum diffraction efficiency only when the grating strength ν exceeds $\pi/2$. However, for absorbing materials it is usually convenient to reduce the thickness of the material and avoid this regime.

To visualize the essential features brought about by the material anisotropy we compare in a concrete example our theory for anisotropic media with Kogelnik's coupled wave theory for isotropic materials. We choose the example of the organic material 4-*N*,*N*-dimethylamino-4′-*N*-methyl-

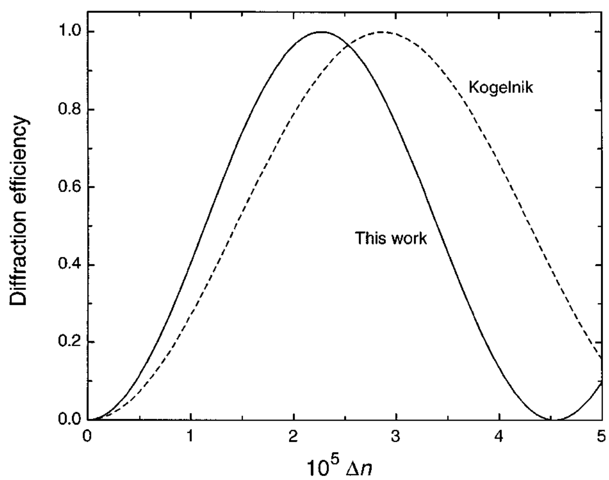

FIG. 5. Diffraction efficiency vs refractive index change Δn using our coupled wave theory for anisotropic materials (solid line) and the coupled wave theory of Kogelnik (dashed line). The diffraction is modeled for the organic crystal 4-N,N-dimethylamino-4-N-methyl-stilbazolium tosylate (DAST) with symmetric p-polarized signal and pump wave propagating in the 1,3 plane ($\hat{\vec{\zeta}}=\hat{\vec{x}}_3$) and the grating wave vector parallel to the 1 axis. Parameters: $A_i=0$, $\lambda=860$ nm, $d=1$ cm, $\alpha_s=\alpha_p=0$, and $\angle(\hat{\vec{k}}_s,\hat{\vec{x}}_3)=-\angle(\hat{\vec{k}}_p,\hat{\vec{x}}_3)=25°$, which gives $n_s=n_p=2.119$, $g_s=g_p=0.945$, and $\theta_s=-\theta_p=44.2°$.

stilbazolium tosylate (DAST) [19] which has a very strong birefringence. At $\lambda=860$ nm, $n_1=2.315$, $n_2=1.660$, and $n_3=1.604$ [20]. For a crystal cut along the dielectric principal axes (x_1,x_2,x_3) and pump and scattered signal beams with $\vec{k}$ vectors in the 1,3 plane and directed at $\pm 25°$ to the x_3 axis one obtains $n_p=n_s=2.119$. The energy propagation vectors $\hat{\vec{u}}_p$ and $\hat{\vec{u}}_s$ are then directed at $\pm 44.2°$ to the x_3 axis, giving a big walk-off angle of the order of 20° and $g_s=g_p=0.945$. Figure 5 compares the dependence of the diffraction efficiency on refractive index change Δn as obtained from our new results given by Eq. (41) and from Eq. (47) in Ref. [1]. It becomes clear that in such highly birefringent materials the use of Kogelnik's expressions leads to large errors even in such fully symmetric beam geometries.

C. Transmission gratings with absorption modulation only

In this case one has $A_r=0$. The expression for the diffraction efficiency differs from Eq. (38) only by a $(-)$ sign

$$\eta(d)=\frac{-\nu^2}{\sqrt{(\nu^2+\xi^2)^2+\chi^4}}\times\left\{\sin^2\left(\frac{(\nu^2+\xi^2)+\sqrt{(\nu^2+\xi^2)^2+\chi^4}}{2}\right)^{1/2}+\sinh^2\left(\frac{-(\nu^2+\xi^2)+\sqrt{(\nu^2+\xi^2)^2+\chi^4}}{2}\right)^{1/2}\right\}\times e^{-(\alpha_s+\alpha_p)d}. \tag{42}$$

Here

$$\nu^2=\frac{-k_0^2A_i^2}{16n_sn_pg_sg_p\cos\theta_s\cos\theta_p}\,d^2 \tag{43}$$

is a negative number, while ξ^2 and χ^2 are still given by Eqs. (36) and (37), respectively. In the limit of Bragg condition fulfillment and no absorption difference between the two waves ($\xi^2=0$, $\chi^2=0$), Eq. (42) reduces to

$$\eta(d)=\sinh^2\left(\frac{\pi A_i d}{2\lambda(n_sn_pg_sg_p\cos\theta_s\cos\theta_p)^{1/2}}\right)e^{-2\alpha d}. \tag{44}$$

In analogy with Ref. [1] the argument of the sinh function is of the form $(\Delta\alpha d/2\cos\theta)$ with $\Delta\alpha=\pi A_i/\lambda(n_sn_pg_sg_p)^{1/2}$ and $\cos\theta=(\cos\theta_s\cos\theta_p)^{1/2}$.

IV. REFLECTION GRATINGS

A. Mixed reflection gratings

Reflection gratings are characterized by the conditions $\hat{\vec{u}}_p\cdot\hat{\vec{\zeta}}=\cos\theta_p>0$ and $\hat{\vec{u}}_s\cdot\hat{\vec{\zeta}}=\cos\theta_s<0$. As shown in Fig. 2, we assume the medium to be a plane parallel plate of thickness d with surfaces oriented in arbitrary directions with respect to the optical main axes and of infinite lateral dimensions. Let the pump wave p enter the holographic medium from the face defined by $\hat{\vec{\zeta}}\cdot\vec{r}=0$. The boundary conditions valid for reflection gratings are then

$$E_s(\hat{\vec{\zeta}}\cdot\vec{r}=d)=0 \tag{45}$$

and

$$\frac{\partial E_s}{\partial\vec{r}}\cdot\hat{\vec{u}}_s(\hat{\vec{\zeta}}\cdot\vec{r}=0)=\frac{k_0}{4n_sg_s}[iA_r-A_ie^{i\phi}]E_{p0}e^{i\Delta\vec{k}\cdot\vec{r}}, \tag{46}$$

where $E_{p0}=E_p(\hat{\vec{\zeta}}\cdot\vec{r}=0)$. Proceeding in the same way as for transmission holograms we insert $E_s=E_{s1}\exp(\vec{\gamma}_1\cdot\vec{r})+E_{s2}\exp(\vec{\gamma}_2\cdot\vec{r})$ into the second-order differential equation (19) and use the boundary conditions (45) and (46) to obtain the general solution for the evolution of the signal wave amplitude

$$E_s(\vec{r})=\frac{k_0}{4n_sg_s\cos\theta_s}\times\frac{A_r+iA_ie^{i\phi}}{\left(\frac{\Delta k_r+i\Delta k_i}{2}\right)[e^{iWd}-e^{-iWd}]+W[e^{iWd}+e^{-iWd}]}\times e^{i[(\Delta k_r+i\Delta k_i)/2]\hat{\vec{\zeta}}\cdot\vec{r}}[e^{iW(\hat{\vec{\zeta}}\cdot\vec{r}-d)}-e^{-iW(\hat{\vec{\zeta}}\cdot\vec{r}-d)}]E_{p0}, \tag{47}$$

where we have made use again of the property that the vector $\Delta\vec{k}$ is constrained to be parallel to $\hat{\vec{\zeta}}$, so that $\Delta\vec{k}_r=\Delta k_r\hat{\vec{\zeta}}$ and $\Delta\vec{k}_i=\Delta k_i\hat{\vec{\zeta}}$. The quantity $W=\sqrt{W^2}$ is the same as given in Eq. (26), which for reflection gratings we can rewrite as

$$W^2=\left(\frac{\Delta k_r+i(\alpha_s+\alpha_p)}{2}\right)^2+\frac{k_0^2}{16n_sn_pg_sg_p\cos\theta_s\cos\theta_p}\times(A_r^2-A_i^2+2iA_rA_i\cos\phi), \quad (48)$$

where we have used Eqs. (2) and (10) and $\vec{k}_{s,i}=-\alpha_s\hat{\vec{\zeta}}$, $\vec{k}_{p,i}=+\alpha_p\hat{\vec{\zeta}}$. In analogy to Eq. (30), the evolution of the pump wave amplitude is obtained as

$$E_p(\vec{r})=\left[\frac{[2W+(\Delta k_r+i\Delta k_i)]e^{iW(\hat{\vec{\zeta}}\cdot\vec{r}-d)}+[2W-(\Delta k_r+i\Delta k_i)]e^{-iW(\hat{\vec{\zeta}}\cdot\vec{r}-d)}}{[2W+(\Delta k_r+i\Delta k_i)]e^{-iWd}+[2W-(\Delta k_r+i\Delta k_i)]e^{iWd}}\right]e^{-i[(\Delta k_r+i\Delta k_i)/2]\hat{\vec{\zeta}}\cdot\vec{r}}E_{p0}. \quad (49)$$

The diffraction efficiency of a reflection hologram is defined as

$$\eta=\frac{I_s(\hat{\vec{\zeta}}\cdot\vec{r}=0)}{I_p(\hat{\vec{\zeta}}\cdot\vec{r}=0)}=\frac{E_sE_s^*n_sg_s}{E_{p0}E_{p0}^*n_pg_p}\left|\frac{\cos\theta_s}{\cos\theta_p}\right|, \quad (50)$$

where again an obliquity factor is introduced in the definition. Inserting the complex signal wave amplitude (47) into Eq. (50) one obtains the general expression for the diffraction efficiency of a mixed phase and absorption reflection grating in anisotropic media with absorption anisotropy

$$\eta=\frac{-k_0^2(A_r^2+A_i^2-2A_rA_i\sin\phi)}{16n_sn_pg_sg_p\cos\theta_s\cos\theta_p}\frac{1}{R}\{\sin^2(\mathrm{Re}[W]d)+\sinh^2(\mathrm{Im}[W]d)\}, \quad (51)$$

where

$$R=\left[\frac{(\Delta k_r)^2}{4}+\frac{(\alpha_s+\alpha_p)^2}{4}\right]\{\sinh^2(\mathrm{Im}[W]d)+\sin^2(\mathrm{Re}[W]d)\}+|W^2|\{\cosh^2(\mathrm{Im}[W]d)-\sin^2(\mathrm{Re}[W]d)\}+\mathrm{Re}[W]\left[\frac{(\alpha_s+\alpha_p)}{2}\sin(2\,\mathrm{Re}[W]d)+\frac{\Delta k_r}{2}\sinh(2\,\mathrm{Im}[W]d)\right]+\mathrm{Im}[W]\times\left[\frac{(\alpha_s+\alpha_p)}{2}\sinh(2\,\mathrm{Im}[W]d)-\frac{\Delta k_r}{2}\sin(2\,\mathrm{Re}[W]d)\right]. \quad (52)$$

The overall diffraction efficiency of mixed reflection gratings depends again on the phase shift ϕ between phase and absorption grating components, as shown in Fig. 6 where η is plotted versus the material thickness. An example of angular mismatch characteristics for reflection gratings is plotted as an inset in the same figure.

B. Reflection gratings with refractive index modulation only

In analogy with Sec. III B we can write $W^2=(\nu^2+\xi^2+i\chi^2)/d^2$, with ν^2 given by Eq. (35) being now a negative real number,

$$\xi^2=\left[\frac{(\Delta k_r)^2-(\alpha_s+\alpha_p)^2}{4}\right]d^2, \quad (53)$$

and

$$\chi^2=-\left[\frac{\Delta k_r(\alpha_s+\alpha_p)}{2}\right]d^2. \quad (54)$$

The diffraction efficiency is found from Eq. (51) as

$$\eta=\frac{-\nu^2}{d^2R}\{\sin^2(\mathrm{Re}[W]d)+\sinh^2(\mathrm{Im}[W]d)\}, \quad (55)$$

where R is obtained from (52) and

$$\mathrm{Re}[W]=\pm\frac{1}{d}\left(\frac{(\nu^2+\xi^2)+\sqrt{(\nu^2+\xi^2)^2+\chi^4}}{2}\right)^{1/2}, \quad (56)$$

$$\mathrm{Im}[W]=\pm\frac{1}{d}\left(\frac{-(\nu^2+\xi^2)+\sqrt{(\nu^2+\xi^2)^2+\chi^4}}{2}\right)^{1/2}, \quad (57)$$

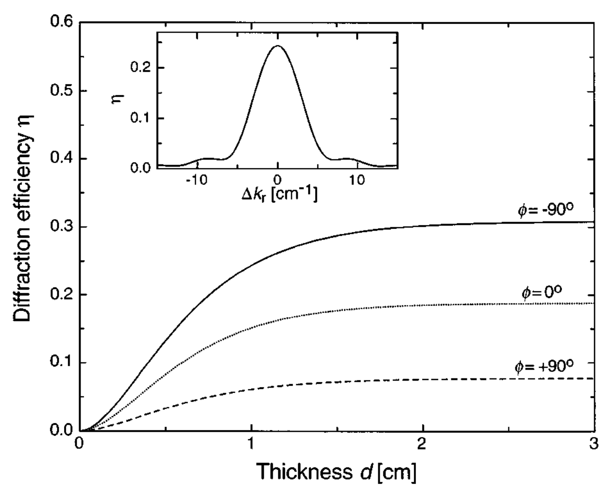

FIG. 6. Mixed reflection grating. Diffraction efficiency vs thickness d for three values of the phase-shift angle ϕ between phase and absorption grating. Parameters: $A_r=6\times10^{-5}$, $A_i=2\times10^{-5}$, $\lambda=633$ nm, $\Delta k_r=0$, $\alpha_s=0.7$ cm^{-1}, $\alpha_p=1.0$ cm^{-1}, $n_s=2.2$, $n_p=2.0$, $g_s=1.0$, $g_p=0.95$, $\theta_s=-170°$, and $\theta_p=-40°$. The inset shows the dependence of the diffraction efficiency on the phase mismatch Δk_r for $d=1$ cm, $\phi=-90°$, and the other parameters stayed the same.

where the signs (+) or (−) have to be selected in order to be consistent with the quadrant of the complex quantity W^2. For the practically most interesting case of perfect phase matching and no loss, Eq. (55) reduces to

$$\eta=\tanh^2\sqrt{-\nu^2}=\tanh^2\left(\frac{\pi A_r d}{2\lambda(n_s n_p g_s g_p|\cos\theta_s|\cos\theta_p)^{1/2}}\right). \quad (58)$$

C. Reflection gratings with absorption modulation only

For $\overleftrightarrow{\epsilon}_r^1=\overleftrightarrow{0}$ the quantity ν^2 is given by Eq. (43) and is now a positive real number. The diffraction efficiency is then expressed as

$$\eta=\frac{\nu^2}{d^2 R}\{\sin^2(\mathrm{Re}[W]d)+\sinh^2(\mathrm{Im}[W]d)\}, \quad (59)$$

with R, ξ^2, χ^2, Re$[W]$, and Im$[W]$ obtained from Eqs. (52), (53), (54), (56), and (57), respectively. Under Bragg incidence one has $\Delta\vec{k}_r=\vec{0}$ and $\chi^2=0$. It holds further that $\Delta\alpha<\alpha_s$, which implies $\nu^2+\xi^2<0$ in this case and Eq. (59) transforms for Bragg incidence into

$$\eta=\frac{\nu^2}{-\xi^2+\sqrt{-(\nu^2+\xi^2)}(\alpha_s+\alpha_p)d\ \coth\sqrt{-(\nu^2+\xi^2)}-(\nu^2+\xi^2)\coth^2\sqrt{-(\nu^2+\xi^2)}}. \quad (60)$$

V. PHOTOREFRACTIVE PHASE GRATINGS

In this section we use the expressions derived above and apply them as an example to a special kind of phase grating that has found much attention in recent years. The photorefractive effect [16] produces a phase grating as a result of a photoinduced internal space-charge electric field and the linear electro-optic effect. In crystalline materials the refractive index change is usually described in terms of the change in the real part of the inverse dielectric tensor that can be expressed as a function of the scalar amplitude of the sinusoidal space-charge electric field E_{sc} as

$$\Delta\overleftrightarrow{\epsilon}_r^{-1}=\overleftrightarrow{r}^{\,\mathrm{eff}}E_{\mathrm{sc}}. \quad (61)$$

The second-rank tensor $\overleftrightarrow{r}^{\,\mathrm{eff}}$ is a function of the direction $\vec{K}$ of the photorefractive grating and takes into account the effect of mechanical deformations of the materials due to the presence of the periodic field. It can be expressed explicitly in its elements r_{ij}^{eff} as [21]

$$r_{ij}^{\mathrm{eff}}=r_{ijk}^{S}\hat{\vec{K}}_k+p_{ijkl}^{\prime E}\hat{\vec{K}}_l A_{km}^{-1}B_m, \quad (62)$$

where $\hat{\vec{K}}=\vec{K}/|\vec{K}|$ is the unit vector along the grating vector, r_{ijk}^{S} is the clamped third-rank electro-optic tensor, $p_{ijkl}^{\prime E}$ is the modified elasto-optic tensor [22], and A_{km}^{-1} and B_m are defined as

$$A_{ik}=C_{ijkl}^{E}\hat{\vec{K}}_j\hat{\vec{K}}_l \quad \text{and} \quad B_i=e_{kij}\hat{\vec{K}}_k\hat{\vec{K}}_j, \quad (63)$$

with C_{ijkl}^{E} being the elastic stiffness tensor at constant electric field, and e_{ijk} being the piezoelectric stress tensor.

To be able to relate our expressions of Secs. III and IV to photorefractive gratings we need to express the tensor $\overleftrightarrow{\epsilon}_r^1=\overleftrightarrow{\epsilon}_r-\overleftrightarrow{\epsilon}_r^0=\Delta\overleftrightarrow{\epsilon}_r$ as a function of the tensor $\Delta\overleftrightarrow{\epsilon}_r^{-1}$ contained in Eq. (61). To do this we start from $\overleftrightarrow{\epsilon}_r\cdot\overleftrightarrow{\epsilon}_r^{-1}=\overleftrightarrow{1}$ and differentiate with respect to the electric field to obtain

$$\Delta(\overleftrightarrow{\epsilon}_r\cdot\overleftrightarrow{\epsilon}_r^{-1})=\overleftrightarrow{0}=\Delta\overleftrightarrow{\epsilon}_r\cdot\overleftrightarrow{\epsilon}_r^{-1}+\overleftrightarrow{\epsilon}_r\cdot\Delta\overleftrightarrow{\epsilon}_r^{-1}. \quad (64)$$

Multiplying each term on the right-hand side with $\overleftrightarrow{\epsilon}_r$ we obtain

$$\Delta\overleftrightarrow{\epsilon}_r=-(\overleftrightarrow{\epsilon}_r\cdot\Delta\overleftrightarrow{\epsilon}_r^{-1}\cdot\overleftrightarrow{\epsilon}_r)\cong-(\overleftrightarrow{\epsilon}_r^{\,0}\cdot\Delta\overleftrightarrow{\epsilon}_r^{-1}\cdot\overleftrightarrow{\epsilon}_r^{\,0}), \quad (65)$$

where the second equality is valid because we assumed $\|\overleftrightarrow{\epsilon}_r^1\|\ll\|\overleftrightarrow{\epsilon}_r^0\|$, which is usually fulfilled in photorefractive experiments. Using Eqs. (65), (61), and (20) one can now determine the form of the coupling constant A_r for a photorefractive grating

$$A_r=-\hat{\vec{e}}_s\cdot\overleftrightarrow{\epsilon}_r^{\,0}\cdot\overleftrightarrow{r}^{\,\mathrm{eff}}\cdot\overleftrightarrow{\epsilon}_r^{\,0}\cdot\hat{\vec{e}}_p E_{\mathrm{sc}}=-n_s^2 n_p^2 g_s g_p r_{\mathrm{eff}}E_{\mathrm{sc}}, \quad (66)$$

where $g_s=\hat{\vec{e}}_s\cdot\hat{\vec{d}}_s$, $g_p=\hat{\vec{e}}_p\cdot\hat{\vec{d}}_p$, n_s and n_p are the refractive indices seen by the signal and pump wave, respectively, and r_{eff} represents now a scalar effective electro-optic coefficient defined as

$$r_{\mathrm{eff}}=\hat{\vec{d}}_s\cdot\overleftrightarrow{r}^{\,\mathrm{eff}}\cdot\hat{\vec{d}}_p. \quad (67)$$

The scalar electro-optic coefficient is therefore related directly to the $\hat{\vec{d}}$-vector directions (polarization) of the two waves, and not to the electric-field-vector directions.

A. Photorefractive diffraction efficiency

Equations (66) and (67) can be inserted into Eqs. (38) and (55) to obtain the diffraction efficiency of photorefractive transmission and reflection gratings. We give here only the special cases valid for phase-matched Bragg diffraction in nonabsorbing materials. One gets

$$\eta=\sin^2\left[\frac{\pi}{2\lambda}\left(\frac{g_s g_p}{\cos\theta_s\cos\theta_p}\right)^{1/2}(n_s n_p)^{3/2}r_{\mathrm{eff}}E_{\mathrm{sc}}d\right] \quad (68)$$

for transmission grating in a medium of thickness d, and

$$\eta = \tanh^2\left[\frac{\pi}{2\lambda}\left(\frac{g_s g_p}{\cos\theta_s \cos\theta_p}\right)^{1/2}(n_s n_p)^{3/2} r_{\text{eff}} E_{\text{sc}} d\right] \tag{69}$$

for reflection gratings.

B. Photorefractive two-beam coupling

Two-beam coupling differs from the situations treated till now in the boundary conditions. For two-beam coupling the wave s is injected and has a nonzero amplitude at the entrance surface of the medium. Under such conditions one observes often an energy and/or phase transfer from one wave to the other, which depend on the phase relationship between the waves and the grating. It is easy to show that maximum energy coupling between the waves p and s is observed when the interference fringes produced by p and s are shifted in phase by $\pm\pi/2$ with respect to a phase grating, or by 0 or π with respect to an absorption grating [16]. One can distinguish two different beam coupling situations. In the first the two waves interact with a preexisting (fixed) grating that is not modified by the waves themselves. In the second, the waves themselves generate the grating at which they interact. The latter case occurs, for instance, for photorefractive two-beam coupling and for many other nonlinear-optical effects.

We first comment on the case of fixed gratings. The ingredients to solve this problem have all been given in Secs. III and IV. Let us suppose we want to know the amplitude of the signal wave s after coupling with the wave p at a fixed transmission grating. This amplitude is a coherent superposition of the transmitted amplitude when the wave p is not present and the amplitude diffracted from the wave p in direction of the signal wave s, when the latter has zero amplitude at the entrance face. Therefore one first calculates the transmitted amplitude $E_{s,t}$ using Eq. (30) letting s take the role of the pump wave in Sec. III. Second, the amplitude $E_{s,d}$ scattered from p into the general direction of s is calculated using Eq. (29) extracting the correct wave propagation direction $\vec{k}_{s,d}$ from Eq. (10). Finally the two waves are added and combined with the phase factors to obtain $E_{s,t}\exp(i\vec{k}_s\cdot\vec{r}) + E_{s,d}\exp(i\vec{k}_{s,d}\cdot\vec{r})$ as the electric-field amplitude of the wave s at the exit of the grating region. In case of perfect phase matching $\vec{k}_{s,d}\equiv\vec{k}_s$ and the transmitted and in-diffracted wave are not distinguishable. In general, geometries where the latter equality holds can be found also for small deviation from perfect phase matching of the two coherent waves to the preexistent grating. We notice that for reflection holograms the same procedure outlined above is followed. One uses Eqs. (49) and (47) instead of Eqs. (30) and (29) to obtain the transmitted and in-diffracted amplitudes.

The case where a dynamic grating is recorded by the two interacting waves themselves is more interesting. These waves automatically fulfill the real phase-matching condition $\hat{k}_{s,r} - \vec{k}_{p,r} \pm \vec{K} = \vec{0}$. We will treat here only the case where a phase grating is created as a result of the photorefractive effect. The theory of Kukhtarev-Vinetskii [23] describes the formation of a space-charge field $\vec{E}^{\text{sc}}(\vec{r}) = \hat{\vec{K}} E^{\text{sc}}(\vec{r})$ under the action of two interfering waves. The scalar space-charge electric field has the form

$$\begin{aligned} E^{\text{sc}}(\vec{r}) &= m E_{\text{sc}} \exp(i\vec{K}\cdot\vec{r}) + \text{c.c.} \\ &= m(E_{\text{sc},r} + iE_{\text{sc},i})\exp(i\vec{K}\cdot\vec{r}) + \text{c.c.}, \end{aligned} \tag{70}$$

where the real component $\cos(\vec{K}\cdot\vec{r})$ of the $\exp(i\vec{K}\cdot\vec{r})$ term is in phase with the light interference fringes generated by the waves s and p, and $E_{\text{sc},r}, E_{\text{sc},i}$ are the in-phase and 90° out-of-phase components of the space-charge field amplitude, respectively. The quantity m is a modulation index and Eq. (70) is valid only for $m \ll 1$. In this section we will be limited to the calculation of the two-beam coupling gain in this limit (undepleted pump approximation). The modulation index m needs some further consideration. In Ref. [23] and all later theories of the photorefractive effect, the modulation index was taken as the modulation of the light intensity inside the material. This driving quantity was used to calculate the photoexcitation rate of charge carriers with density N as $\partial N/\partial t \propto I_0(1 + m\cos\vec{K}\cdot\vec{r})$. This assumption is allowed in the case of isotropic materials and a transmission grating geometry. However, the rate of photoexcitation actually depends on the locally dissipated energy and not on the net energy flux through a material. This fact is particularly evident when one considers a reflection grating recorded by two counterpropagating waves of equal intensity. In this case there is no net energy flux (no light intensity) in any direction but there are still photoexcitations. It is therefore more correct to assume $\partial N/\partial t \propto U_0(1 + m\cos\vec{K}\cdot\vec{r})$ and thus to define m as the modulation of the dissipated optical energy density U in the material. The dissipated optical energy is related to the imaginary dielectric tensor $\overleftrightarrow{\epsilon}_i^{\,0}$ [17]

$$U(\vec{r},t) \propto \tfrac{1}{2}\epsilon_0 \vec{\mathcal{E}}(\vec{r},t)\cdot\overleftrightarrow{\epsilon}_i^{\,0}\cdot\vec{\mathcal{E}}^*(\vec{r},t), \tag{71}$$

where $\vec{\mathcal{E}}(\vec{r},t)$ is given by Eq. (1) and ϵ_0 is the permittivity of vacuum. Equation (71) can be used to calculate the number density of mobile photoexcited charge carriers if every absorbed photon produces such a carrier, i.e., when the quantum efficiency for photoexcitation is unity. However, in most photorefractive materials the quantum efficiency ϕ has often been observed to be considerably lower, in the extreme case of photorefractive polymers the usual quantum efficiencies are in the order of 10^{-4}–10^{-2} [10]. It is therefore necessary to replace the tensor $\overleftrightarrow{\epsilon}_i^{0}$ in Eq. (71) by a similar tensor $\overleftrightarrow{\kappa}$ that takes into account only those useful absorption processes that give rise to movable charge carriers. One can define the elements of $\overleftrightarrow{\kappa}$ as

$$\kappa_{kl} = \phi_{kl}\epsilon^0_{i,kl}, \tag{72}$$

where the quantities ϕ_{kl} describe the polarization dependence of the quantum efficiency. In Eq. (72) no summing over equal indices is performed. The characteristics of the tensor $\overleftrightarrow{\kappa}$ may be determined by photoconductivity measurements as a function of light polarization. If the quantum efficiency for photogeneration of charge carriers is unity, the tensors $\overleftrightarrow{\epsilon}_i^{0}$ and $\overleftrightarrow{\kappa}$ are identical. In view of the above argumentation we can now write the useful dissipated energy as

$$U(\vec{r},t)\propto\vec{\mathcal{E}}(\vec{r},t)\cdot\vec{\vec{\kappa}}\cdot\vec{\mathcal{E}}^*(\vec{r},t)=U_0\left(1+\frac{m}{2}e^{i\vec{K}\cdot\vec{r}}+\frac{m^*}{2}e^{-i\vec{K}\cdot\vec{r}}\right), \tag{73}$$

where for simplicity we have used a complex modulation index m that takes fully into account all phase shifts in the light fringes due to phase coupling between the waves. One can assume that m is real at the input surface of the medium. Using Eqs. (73) and (1) we find the general expression for the modulation of a transmission grating as

$$m(\vec{r})=\frac{2E_s(\vec{r})E_p^*(\vec{r})(\hat{\vec{e}}_s\cdot\vec{\vec{\kappa}}\cdot\hat{\vec{e}}_p)e^{-(\alpha_s+\alpha_p)\hat{\vec{\zeta}}\cdot\vec{r}}}{|E_p(\vec{r})|^2(\hat{\vec{e}}_p\cdot\vec{\vec{\kappa}}\cdot\hat{\vec{e}}_p)e^{-2\alpha_p\hat{\vec{\zeta}}\cdot\vec{r}}+|E_s(\vec{r})|^2(\hat{\vec{e}}_s\cdot\vec{\vec{\kappa}}\cdot\hat{\vec{e}}_s)e^{-2\alpha_s\hat{\vec{\zeta}}\cdot\vec{r}}}. \tag{74}$$

For small modulation one has

$$|E_p|^2(\hat{\vec{e}}_p\cdot\vec{\vec{\kappa}}\cdot\hat{\vec{e}}_p)\exp(-2\alpha_p\hat{\vec{\zeta}}\cdot\vec{r})\gg|E_s|^2(\hat{\vec{e}}_s\cdot\vec{\vec{\kappa}}\cdot\hat{\vec{e}}_s)\exp(-2\alpha_s\hat{\vec{\zeta}}\cdot\vec{r}) \tag{75}$$

and Eq. (74) simplifies to

$$m(\vec{r})=\frac{2E_s(\vec{r})}{E_p(\vec{r})}\frac{(\hat{\vec{e}}_s\cdot\vec{\vec{\kappa}}\cdot\hat{\vec{e}}_p)}{(\hat{\vec{e}}_p\cdot\vec{\vec{\kappa}}\cdot\hat{\vec{e}}_p)}e^{(\alpha_p-\alpha_s)\hat{\vec{\zeta}}\cdot\vec{r}}. \tag{76}$$

Note that the modulation index that would be obtained by using the light intensity as a driving term is similar to Eq. (76) but contains the ratio $(n_s/n_p)^{1/2}$ instead of $(\hat{\vec{e}}_s\cdot\vec{\vec{\kappa}}\cdot\hat{\vec{e}}_p)/(\hat{\vec{e}}_p\cdot\vec{\vec{\kappa}}\cdot\hat{\vec{e}}_p)$. Using Eqs. (66), (17), and (18) and neglecting any absorption modulation we obtain the coupled wave equations for transmission grating two-beam coupling in photorefractive materials

$$\frac{\partial E_s}{\partial\vec{r}}\cdot\hat{\vec{u}}_s=\frac{k_0}{4n_sg_s}[-iRmE_pE_{\rm sc}e^{(\alpha_s-\alpha_p)\hat{\vec{\zeta}}\cdot\vec{r}}], \tag{77}$$

$$\frac{\partial E_p}{\partial\vec{r}}\cdot\hat{\vec{u}}_p=\frac{k_0}{4n_pg_p}[-iRm^*E_sE_{\rm sc}^*e^{(\alpha_p-\alpha_s)\hat{\vec{\zeta}}\cdot\vec{r}}], \tag{78}$$

where $R=n_s^2n_p^2g_sg_pr_{\rm eff}$ and $r_{\rm eff}$ is given by Eq. (67). If the inequality (75) is fulfilled one can neglect the second differential equation (78) and by using Eq. (76) the equation for the evolution of the signal wave amplitude becomes

$$\frac{\partial E_s}{\partial\vec{r}}\cdot\hat{\vec{u}}_s=\frac{k_0R}{2n_sg_s}a[E_{{\rm sc},i}-iE_{{\rm sc},r}]E_s, \tag{79}$$

where we have introduced the quantity

$$a=\frac{(\hat{\vec{e}}_s\cdot\vec{\vec{\kappa}}\cdot\hat{\vec{e}}_p)}{(\hat{\vec{e}}_p\cdot\vec{\vec{\kappa}}\cdot\hat{\vec{e}}_p)}. \tag{80}$$

Equation (79) has a solution of the form

$$E_s(\hat{\vec{\zeta}}\cdot\vec{r}=d)=E_{s0}e^{(\Gamma/2)d}e^{i\delta d}, \tag{81}$$

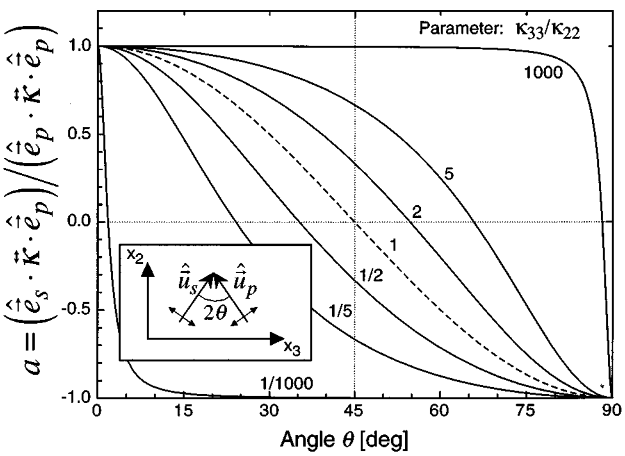

FIG. 7. Dependence of the correcting factor $a=(\hat{\vec{e}}_s\cdot\vec{\vec{\kappa}}\cdot\hat{\vec{e}}_p)/(\hat{\vec{e}}_p\cdot\vec{\vec{\kappa}}\cdot\hat{\vec{e}}_p)$ on the half angle between the s and p waves in a symmetric geometrical configuration. Propagation in the x_2,x_3 plane with p-polarized waves is assumed, as shown in the inset. The dashed curve for $\kappa_{33}/\kappa_{22}=1$ corresponds to the case of isotropic photoexcitation, for which the conventional projection factor $a=(\hat{\vec{e}}_s\cdot\hat{\vec{e}}_p)$ applies.

with Γ being the small-signal gain expressed as

$$\Gamma=\frac{2\pi}{\lambda}\frac{n_sn_p^2}{\cos\theta_s}g_par_{\rm eff}E_{{\rm sc},i}, \tag{82}$$

which is positive if the product $r_{\rm eff}E_{{\rm sc},i}$ is positive, and

$$\delta=-\frac{\pi}{\lambda}\frac{n_sn_p^2}{\cos\theta_s}g_par_{\rm eff}E_{{\rm sc},r} \tag{83}$$

describes the phase drift of the wave along the propagation direction. We recall that θ_s is the angle between the Poynting vector of the wave S and the normal to the surface $\hat{\vec{\zeta}}$. It should be noted that, while the sign of $E_{{\rm sc},r}$ does not depend on whether the space-charge field is created in the photorefractive crystal by means of electron charge transport or hole charge transport, the sign of $E_{{\rm sc},i}$ is opposite for the two cases. One has $E_{{\rm sc},i}<0$ for dominant electron transport, and $E_{{\rm sc},i}>0$ for dominant hole transport. Therefore the sign of the gain Γ is influenced accordingly. We also notice that the constants Γ and δ of Eqs. (82) and (83) are influenced by the dielectric anisotropy within the factors $\cos\theta_s$ and g_p and are influenced by the absorption anisotropy within the quantity a. Figure 7 shows an example of the dependence of the correcting factor a of Eq. (80) on the half-angle θ between the energy propagation directions of the waves s and p. We have assumed a material cut along the main axes x_1,x_2,x_3 of the absorption ellipsoid and symmetric propagation in the x_2,x_3 plane, as shown in the inset of the figure. The waves have polarization in the same plane so that only the elements κ_{22} and κ_{33} of the tensor $\vec{\vec{\kappa}}$ are active. The different curves in Fig. 7 are for different values of the anisotropy ratio κ_{33}/κ_{22}. If the absorption is isotropic (dashed curve) the factor a reduces to $(\hat{\vec{e}}_s\cdot\hat{\vec{e}}_p)$, corresponding to the term obtained for isotropic theories. It can be noticed that in case of absorption anisotropy the correction due to the factor a can be quite relevant if the directions of the optical electric field vectors $\hat{\vec{e}}_s$ and $\hat{\vec{e}}_p$ deviate from a main axis of the modified absorption tensor $\overleftrightarrow{\kappa}$. For $\kappa_{33}/\kappa_{22}=1/2$ the correction with respect to $(\hat{\vec{e}}_s\cdot\hat{\vec{e}}_p)$ is about 25% already if the deviation angles are of the order of 20°.

We finally express the evolution of the signal wave intensity I_s. It is obtained using Eq. (82) for the gain and the definition of the optical electric field and propagation vector (1) and (2), which gives

$$I_s(\hat{\vec{\zeta}}\cdot\vec{r}=d)=I_{s0}e^{(\Gamma-2\alpha_s)d}, \tag{84}$$

with $I_{s0}=I_s(\hat{\vec{\zeta}}\cdot\vec{r}=0)$. The same path can be followed also for two-beam coupling in a reflection grating configuration. For small m, one obtains in analogy to Eq. (84)

$$I_s(\hat{\vec{\zeta}}\cdot\vec{r}=0)=I_s(\hat{\vec{\zeta}}\cdot\vec{r}=d)e^{-(\Gamma+2\alpha_s)d}, \tag{85}$$

where Γ is still given by Eq. (82) and it should be remembered that $\cos\theta_s$ is negative for reflection gratings.

VI. CONCLUSIONS

We have extended the coupled wave theory of Kogelnik [1] to the case of anisotropic materials with grating vector and medium oriented in arbitrary directions. Spatial modulation of both refractive index and absorption constant with a common grating wave vector and an arbitrary phase shift between each other have been allowed for. Solutions for the wave amplitudes, diffraction efficiencies, and angular sensitivities have been given in transmission and reflection configurations for the case of moderately absorbing materials. The special case of phase holograms in optically inactive photorefractive crystals has been considered in detail revealing simple expressions permitting one to calculate diffraction efficiencies and small-signal two-beam coupling gains in any given geometrical situation. The insights provided by the present approach should be particularly important for the analysis of all anisotropic diffraction processes and also of isotropic interaction of light with volume holograms recorded in materials with strong birefringence, such as organic crystals, ordered polymers, and liquid crystalline cells.

ACKNOWLEDGMENTS

We are very grateful to P. Günter and R. Ryf for useful discussions.

APPENDIX: LINEAR PROPAGATION

We treat here the problem of linear propagation in a medium with a complex dielectric constant with the aim of extracting a relationship between the imaginary part of the dielectric tensor $\overleftrightarrow{\epsilon}_i^0$ and the effective amplitude absorption constants α used in this paper. We consider a wave of the form $\vec{\mathcal{E}}\exp[i(\vec{k}\cdot\vec{r}-\omega t)]$ with complex wave vector of the form $\vec{k}=\vec{k}_r+i\vec{k}_i$. If this wave is an eigenwave of the medium it has to fulfill the wave equation (3), which translates into

$$-E[(\hat{\vec{e}}\times(\vec{k}_r+i\vec{k}_i))\times(\vec{k}_r+i\vec{k}_i)]e^{i\vec{k}_r\cdot\vec{r}}e^{-\vec{k}_i\cdot\vec{r}} \\ =k_0^2[\overleftrightarrow{\epsilon}_r^{\,0}+i\overleftrightarrow{\epsilon}_i^{\,0}]E\hat{\vec{e}}e^{i\vec{k}_r\cdot\vec{r}}e^{-\vec{k}_i\cdot\vec{r}}, \tag{A1}$$

where we have used $\vec{\mathcal{E}}=E\hat{\vec{e}}$. Equation (A1) is the same as Eq. (9) of Sec. II. The real part of the space-independent terms in (A1) reads

$$E[(\hat{\vec{e}}\times\vec{k}_r)\times\vec{k}_r-(\hat{\vec{e}}\times\vec{k}_i)\times\vec{k}_i]=-Ek_0^2\overleftrightarrow{\epsilon}_r^{\,0}\cdot\hat{\vec{e}}, \tag{A2}$$

Under our general assumption of moderate absorption ($|\vec{k}_i|\ll|\vec{k}_r|$), we recognize Eq. (A2) as the equation describing linear propagation in lossless anisotropic media, as can be checked in many textbooks [14,15,24]. The remaining (imaginary) terms of Eq. (A1) are related to the effective scalar absorption constant we are looking for, they read

$$-iE[(\hat{\vec{e}}\times\vec{k}_r)\times\vec{k}_i+(\hat{\vec{e}}\times\vec{k}_i)\times\vec{k}_r]=iEk_0^2\overleftrightarrow{\epsilon}_i^{\,0}\cdot\hat{\vec{e}}. \tag{A3}$$

We are considering here a boundary value problem, the wave should have a constant intensity at the input surface of the medium defined by the condition $\hat{\vec{\zeta}}\cdot\vec{r}=0$ (Fig. 1). A constant intensity implies

$$|\vec{\mathcal{E}}e^{i\vec{k}\cdot\vec{r}}|(\hat{\vec{\zeta}}\cdot\vec{r}=0)=\text{const}, \tag{A4}$$

which can be fulfilled only if the vector $\vec{k}_i$ is parallel to the surface normal $\hat{\vec{\zeta}}$. One can write

$$\vec{k}_i=\alpha(\pm\hat{\vec{\zeta}}), \tag{A5}$$

where the effective absorption constant α is positive, and the $(+)$ sign in Eq. (A5) holds if the wave enters the medium at the plane $\hat{\vec{\zeta}}\cdot\vec{r}=0$, while the $(-)$ sign holds if it enters at the plane $\hat{\vec{\zeta}}\cdot\vec{r}=d$ and exits at $\hat{\vec{\zeta}}\cdot\vec{r}=0$. Therefore, the vectors $\vec{k}_r$ and $\vec{k}_i$ are in general, noncollinear and the wave is inhomogeneous even in the absence of a nonlinear grating. Inserting Eq. (A5) into Eq. (A3) and using the vector identity $(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$ one gets

$$\alpha k_0n[(\hat{\vec{e}}\cdot\hat{\vec{\zeta}})\hat{\vec{k}}_r+(\hat{\vec{e}}\cdot\hat{\vec{k}}_r)\hat{\vec{\zeta}}-2(\hat{\vec{k}}_r\cdot\hat{\vec{\zeta}})\hat{\vec{e}}]=-k_0^2\overleftrightarrow{\epsilon}_i^{\,0}\cdot\hat{\vec{e}}, \tag{A6}$$

where n is the refractive index seen by the wave and we have used $\vec{k}_r=k_0n\hat{\vec{k}}_r$. We are looking for an expression for α, which is obtained by multiplying Eq. (A6) on the left by $\hat{\vec{e}}$

$$2\alpha k_0n[(\hat{\vec{e}}\cdot\hat{\vec{\zeta}})(\hat{\vec{k}}_r\cdot\hat{\vec{e}})-(\hat{\vec{k}}_r\cdot\hat{\vec{\zeta}})]=-k_0^2(\hat{\vec{e}}\cdot\overleftrightarrow{\epsilon}_i^{\,0}\cdot\hat{\vec{e}}). \tag{A7}$$

The expression in the square brackets on the left-hand side is related again to the unit vector $\hat{\vec{u}}$ along the energy propagation vector (Poynting vector) of the wave and can be rewritten as

$$[(\hat{\vec{e}}\cdot\hat{\vec{\zeta}})(\hat{\vec{k}}_r\cdot\hat{\vec{e}})-(\hat{\vec{k}}_r\cdot\hat{\vec{\zeta}})]=\hat{\vec{\zeta}}[\hat{\vec{e}}(\hat{\vec{k}}_r\cdot\hat{\vec{e}})-\hat{\vec{k}}_r]=-g(\hat{\vec{\zeta}}\cdot\hat{\vec{u}}) = -g\cos\theta, \tag{A8}$$

where the second equality makes use of Eq. (16) and the angle θ is defined by the last equality. We recall that $g=\hat{\vec{e}}\cdot\hat{\vec{d}}$, with $\hat{\vec{d}}$ being the unit vector pointing in the direction of the dielectric displacement vector (polarization). Combining Eqs. (A7) and (A8) we obtain finally the value of α which fulfills Eq. (3), that is

$$\alpha=\frac{k_0(\hat{\vec{e}}\cdot\overleftrightarrow{\epsilon}_i^0\cdot\hat{\vec{e}})}{2ng|\cos\theta|}=\frac{\pi(\hat{\vec{e}}\cdot\overleftrightarrow{\epsilon}_i^0\cdot\hat{\vec{e}})}{\lambda ng|\cos\theta|}, \tag{A9}$$

where λ is the vacuum wavelength. Equation (A9) can also be used to derive the elements of the tensor $\overleftrightarrow{\epsilon}_i^0$ from measurements of the amplitude absorption constant α for selected wave propagation directions and polarizations.

[1] H. Kogelnik, Bell Syst. Tech. J. **48**, 2909 (1969).
[2] K. Kojima, Jpn. J. Appl. Phys. **21**, 1303 (1982).
[3] K. Rokushima and J. Yamakita, J. Opt. Soc. Am. **73**, 901 (1983).
[4] R. V. Johnson and J. A. R. Tanguay, Opt. Eng. **25**, 235 (1986).
[5] E. N. Glytsis and T. K. Gaylord, J. Opt. Soc. Am. A **4**, 2061 (1987).
[6] F. Vachss and L. Hesselink, J. Opt. Soc. Am. A **4**, 325 (1987).
[7] I. C. Khoo, IEEE J. Quantum Electron. **22**, 1268 (1986).
[8] W. E. Moerner and S. M. Silence, Chem. Rev. **94**, 127 (1994).
[9] K. Sutter, J. Hulliger, and P. Günter, Solid State. Commun. **74**, 867 (1990).
[10] G. Montemezzani and P. Günter, in *Notions and Perspectives in Nonlinear Optics*, edited by O. Keller (World Scientific, Singapore, 1996).
[11] S. J. Stepanov, M. P. Petrov, and A. A. Kamshilin, Sov. Tech. Phys. Lett. **3**, 345 (1977).
[12] T. K. Gaylord and M. G. Moharam, Appl. Opt. **20**, 3271 (1981).
[13] M. G. Moharam and T. K. Gaylord, J. Opt. Soc. Am. **71**, 811 (1981).
[14] V. M. Agranovich and V. L. Ginzburg, in *Crystal Optics with Spatial Dispersion and Excitons*, 2nd ed., edited by H. J. Queisser, Springer Series in Solid-State Sciences Vol. 42 (Springer-Verlag, Berlin, 1984).
[15] M. Born and E. Wolf, *Principles of Optics*, 6th ed. (Pergamon, Oxford, 1980).
[16] *Photorefractive Materials and their Applications I: Fundamental Phenomena*, edited by P. Günter and J. P. Huignard, Topics in Applied Physics, Vol. 1 (Springer Verlag, Berlin, 1988).
[17] G. N. Ramachandran and S. Ramaseshan, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1961), Vol. 25/1, pp. 85–96.
[18] E. Guibelalde, Opt. Quantum Electron. **16**, 173 (1984).
[19] S. R. Marder, J. W. Perry, and C. P. Yakymyshyn, Chem. Mater. **6**, 1137 (1994).
[20] F. Pan, G. Knöpfle, C. Bosshard, S. Follonier, R. Spreiter, M. S. Wong, and P. Günter, Appl. Phys. Lett. **69**, 13 (1996).
[21] P. Günter and M. Zgonik, Opt. Lett. **16**, 1826 (1991).
[22] D. F. Nelson and M. Lax, Phys. Rev. Lett. **24**, 1187 (1970).
[23] N. V. Kukhtarev, V. B. Markov, S. G. Odulov, M. S. Soskin, and V. L. Vinetskii, Ferroelectrics **22**, 949 (1979).
[24] A. Yariv and P. Yeh, *Optical Waves in Crystals*, edited by J. W. Goodman, Wiley Series in Pure and Applied Optics (Wiley, New York, 1984).